

Integrable Hamiltonian Systems: Problems 9

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*The problems marked with * are due at the beginning of the class on Tuesday 4 December.*

Problem 9.1. Prove the Marsden-Weinstein reduction theorem for torus actions.

Problem 9.2*. (10 points) Show that the 2-dimensional trapezoids with vertices $(0, 0)$, $(0, 1)$, $(\ell, 1)$ and $(\ell + n, 0)$ with $n \in \mathbb{Z}_+$ and $\ell > 0$ are Delzant.

Reminder: A *Delzant polytope* Δ in \mathbb{R}^n is a polytope with the following properties:

- It is *simple*: there are n edges meeting at each vertex p .
- It is *rational*: the edges meeting at a vertex p are rational in the sense that each edge is of the form $p + tu_i$ with $t \geq 0$ and $u_i \in \mathbb{Z}^n$.
- It is *smooth*: for each vertex p the corresponding u_1, \dots, u_n can be chosen to be a \mathbb{Z} -basis of \mathbb{Z}^n .

Problem 9.3*. (20 points) Repeat the Delzant's construction for the polytope $\Delta \subset \mathbb{R}^2$ with vertices $(0, 0)$, $(0, 1)$ and $(1, 0)$ and whose normal vectors to the i th facet $i = 1, 2, 3$ are respectively given by $\nu_1 = (0, -1)$, $\nu_2 = (-1, 0)$ and $\nu_3 = (1, 1)$.