Integrable Hamiltonian Systems: Problems 8

Prof. dr. Sonja Hohloch (lectures) & dr. Marine Fontaine (problems)
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Problem 8.1. Let the two-dimensional torus $\mathbb{T}^2 = \mathbb{S}^1 \times \mathbb{S}^1$ with standard coordinates (α, β) and symplectic form $\omega = -d\alpha \wedge d\beta$. Show that the $\mathbb{S}^1$-action $\vartheta : \mathbb{S}^1 \times \mathbb{T}^2 \to \mathbb{T}^2$ defined by $\vartheta(t, (\alpha, \beta)) = (\alpha + t, \beta)$ is symplectic but not hamiltonian.

Problem 8.2. Show that an effective hamiltonian action of a torus $\mathbb{T}^n$ on a connected symplectic manifold (M^{2n}, ω) gives rise to a completely integrable system.

Problem 8.3. Let the two-dimensional sphere $\mathbb{S}^2 \subset \mathbb{R}^3$ with cylindrical coordinates (θ, h) and symplectic form $\omega = -d\theta \wedge dh$. Show that the smooth maps $\varphi, \psi : \mathbb{S}^1 \times \mathbb{S}^2 \to \mathbb{S}^2$ given by $\varphi(t, (\theta, h)) = \varphi_t(\theta, h) = (\theta + t, h)$ and $\psi(s, (\theta, h)) = \psi_s(\theta, h) = (hs + \theta, h)$ give rise to a symplectic $\mathbb{T}^2$-action on $\mathbb{S}^2$. Determine whether this action is effective or not.