Miniworkshop
“Semitoric systems and beyond”
Antwerp/Belgium, September 10-11, 2019

On the symplectic invariants of semitoric systems

Jaume Alonso

(Antwerpen)

Completely integrable systems are mathematical models that describe systems with many conserved quantities. Each possible state of the system corresponds to a point in the phase space. The time evolution of the system draws a curve in this space that must be confined in the common level sets of the conserved quantities. This allows us to use geometric tools to obtain dynamical results.
In the underlying work we focus on semitoric systems, a specific class of completely integrable systems in four dimensions, where one of the conserved quantities is a proper map that induces a circular action. We require further that all singularities are non-degenerate and have no hyperbolic components. These systems appear in different areas of science, such as quantum chemistry and quantum optics, and can exhibit interesting phenomena like monodromy, an obstruction to the existence of global action-angle coordinates related to the presence of focus-focus points.

Semitoric systems were classified a few years ago using five symplectic invariants. However, it was unclear how to compute these invariants in practise. In the underlying work we have addressed this situation by studying three families of semitoric systems: the coupled spin-oscillator, the coupled angular momenta and a special family of systems that can have two focus-focus points at the same time. By using mathematical software and exploiting the properties of elliptic integrals, we have developed different methods to compute their symplectic invariants and completed their symplectic classification. We have also discovered some extra properties of these families, such as superintegrability in a specific energy level set with vanishing twist, diverging coefficients in the Taylor series invariant and certain symmetries between the different components of the invariants.