Stability analysis for generalized free rigid body dynamics on a real semi-simple Lie algebra with respect to an arbitrary Cartan subalgebra

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This talk deals with the stability analysis for equilibria of free rigid body dynamics on real semi-simple Lie algebras. Around 1980, Mishchenko and Fomenko have introduced a family of generalized Euler equations associated to an arbitrary Cartan subalgebra $\mathfrak{h}$ in the real semi-simple Lie algebra $\mathfrak{g}$:

$$\frac{\mathsf{d}}{\mathsf{d}t}X = [X, \varphi_{a,b,D}(X)], \qquad X \in \mathfrak{g}, \tag{1}$$

where $\varphi_{a,b,D} : \mathfrak{g} \to \mathfrak{g}$ is an operator associated to the elements $a, b \in \mathfrak{h}$ (a is assumed to be regular) and a symmetric operator $D : \mathfrak{h} \to \mathfrak{h}$. Mishchenko and Fomenko have proved that the restriction of Euler equation to a generic adjoint orbit is a completely integrable Hamiltonian system in the sense of Liouville with respect to the orbit symplectic form. (See e.g. [4].)

It is rather recent that the stability of equilibria has been analyzed for Euler equations (1) on real semi-simple Lie algebras of type A, on compact real Lie algebras, and on split real form of complex semi-simple Lie algebras. (See [3, 5] and the references therein.)

In this talk, based on the methods for the bi-Hamiltonian systems developed in [1, 2], the stability analysis is carried out for isolated equilibria on a generic adjoint orbit for above Euler equation (1) on any real semi-simple Lie algebra associated with an arbitrary Cartan subalgebra. The stability property of the equilibria is characterized by the types of the roots corresponding to the complexification of the Cartan subalgebra. The talk in based on a joint project with Tudor S. Ratiu (Shanghai Jiao Tong University).

References

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