Tetrahedron equation, totally positive matrices, and (quantum) dilogarithm identities

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The tetrahedron equation,

$$R_{123}\, R_{124}\, R_{134}\, R_{234} = R_{234}\, R_{134}\, R_{124}\, R_{123},$$

is a 3–simplex counterpart of the Yang-Baxter equation. Starting with the RTT presentation of an upper triangular quantum group, one can construct a family of solutions to the tetrahedron equation [2]. In this construction, the key building block is the q–exponential function,

$$\langle x \rangle_q = \sum_{n \geq 0} \frac{x^n}{(q)_n}, \qquad (q)_n \equiv (q-1)\dots(q^n-1).$$

In turn, the obtained solution to the tetrahedron equation can be used to derive a family of quantum dilogarithm identities involving products of q–exponential functions whose non-commutative arguments are monomials in the generators of a quantum torus algebra associated to a certain quiver [3]. Remarkably, these identities are invariant under the action of the symmetric group S_3.

The quasi-classical limit [5] of a quantum dilogarithm identity yields an identity involving the Rodgers dilogarithm, $L(x) = Li_2(x) + \frac{1}{2}\log x\, \log(1-x)$. A family of identities involving Rodgers dilogarithms whose arguments are minors of totally positive matrices can be derived by using two involutions (close analogues of the BFZ twist [1]) on the variety of upper triangular totally positive matrices [4]. These identities admit a form manifestly invariant under the action of the symmetric group S_3 and correspond conjecturally to the quasi-classical limit of the quantum dilogarithm identities related to the tetrahedron equation.

Based on joint work with Alexander Volkov.

References

[1] A. Berenstein, S. Fomin, A. Zelevinsky, *Parametrizations of canonical bases and totally positive matrices,* Adv. Math. **122** (1996), 49–149.

[2] A. Bytsko, A. Volkov, *Tetrahedron equation, Weyl group, and quantum dilogarithm,* Lett. Math. Phys. **105** (2015), 45–61.

[3] A. Bytsko, A. Volkov, *Tetrahedron equation and cyclic quantum dilogarithm identities,* Int. Math. Res. Not. **2015** (2015), 1075–1100.

[4] A. Bytsko, A. Volkov *Totally positive matrices and dilogarithm identities*, arXiv:1708.08445.

[5] L.D. Faddeev, R.M. Kashaev, *Quantum dilogarithm,* Mod. Phys. Lett. **A9** (1994), 427–434.