Momentum maps and integrability in noncommutative algebraic geometry

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Abstract

One aspect of non-commutative algebraic geometry consists in following Kontsevich-Rosenberg's philosophy, which states that the non-commutative version of a structure defined on a non-commutative algebra should yield the corresponding classical structure on the representation spaces of this algebra. The work of Van den Bergh [3] deals with the introduction of non-commutative Poisson and quasi-Poisson geometry under this principle, as well as defining the necessary objects to perform (quasi-)Hamiltonian reductions on the corresponding representation spaces.

Our aim is to explain part of this work and how to to search for integrable systems in this context. We also illustrate how Van den Bergh's theory applied to cyclic quivers is a good starting point to study systems in the Calogero-Moser family. To be precise, we explain the formalism needed to recover the non-spin and spin versions of the rational CM system [2], the non-spin trigonometric Ruijsenaars-Schneider system [1], but also the spin generalisation of the trigonometric RS model, which is the object of a forthcoming publication with O. Chalykh.

References

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