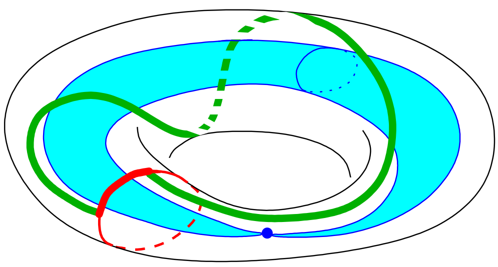
Conference

Geometric aspects of momentum maps and integrability

Congressi Stefano Franscini (CSF)
Ascona/Switzerland
April 8-13, 2018



This conference is sponsored by

Congressi Stefano Franscini

FNSNF
Swiss National Science Foundation

SwissMAP
The Mathematics of Physics
National Centre of Competence in Research

ETH zürich

Geometric aspects of momentum maps and integrability

(CSF, Ascona/Switzerland, April 8-13, 2018)

Organizers

Anton Alekseev (University of Geneva),
Sonja Hohloch (University of Antwerp),
Tudor Ratiu (Shanghai Jiao Tong University & Geneva).

Speakers

Bolsinov, Alexey (Loughborough),
Bytsko, Andrey (Geneva),
Dullin, Holger (University of Sydney),
Efstathiou, Konstantinos (Groningen),
Gukov, Sergei (CalTech),
Hanßmann, Heinz (Utrecht),
Kappeler, Thomas (Zurich),
Izosimov, Anton (University of Arizona),
Lane, Jeremy (Geneva),
Matveev, Vladimir (Jena),
Palmer, Joseph (Rutgers),
Pelayo, Álvaro (UC San Diego),
Romão, Nuno (IHES & Augsburg),
Roubtsov, Vladimir (Angers & ITEP Moscow),
Sabatini, Silvia (Cologne),
Sepe, Daniele (Universidade Federal Fluminense),
Shatashvili, Samson (IHES & Dublin),
Suris, Yuri (TU Berlin),
Szenes, András (Geneva & Budapest),
Tarama, Daisuke (Ritsumeikan University),
Tolman, Susan (Urbana-Champaign),
Volkov, Alexander (Steklov Institute),
Wacheux, Christophe (IBS Center for Geometry and Physics).

Invited Participants

Jaume Alonso (Antwerpen), *poster presentation*,

Damien Bouloc (Toulouse),

Alexander Caviedes Castro (Cologne),

Isabelle Charton (Cologne),

Sean Dawson (Sydney), *poster presentation*,

Roisin Dempsey Braddell (Barcelona),

Maxime Fairon (Leeds), *poster presentation*,

Marine Fontaine (Augsburg), *poster presentation*,

Tamás F. Görbe (Szeged),

Yannick Gullentops (Antwerpen),

David Hoffman (Cornell), *poster presentation*,

Yohann Le Floch (Strasbourg),

Yanpeng Li (Geneva), *poster presentation*,

Bohuan Lin (Groningen),

Nikolay Martynchuk (Groningen), *poster presentation*,

Anastasia Matveeva (Angers),

Riccardo Montalto (Zurich),

Diana Nguyen (Sydney), *poster presentation*,

Nikita Nikolaev (Geneva), *poster presentation*,

Cédric Oms (Barcelona),

Milena Pabiniak (Cologne),

Gleb Smirnov (Trieste),

Alexander Spies (Erlangen-Nürnberg),

Xiudi Tang (UC San Diego), *poster presentation*.

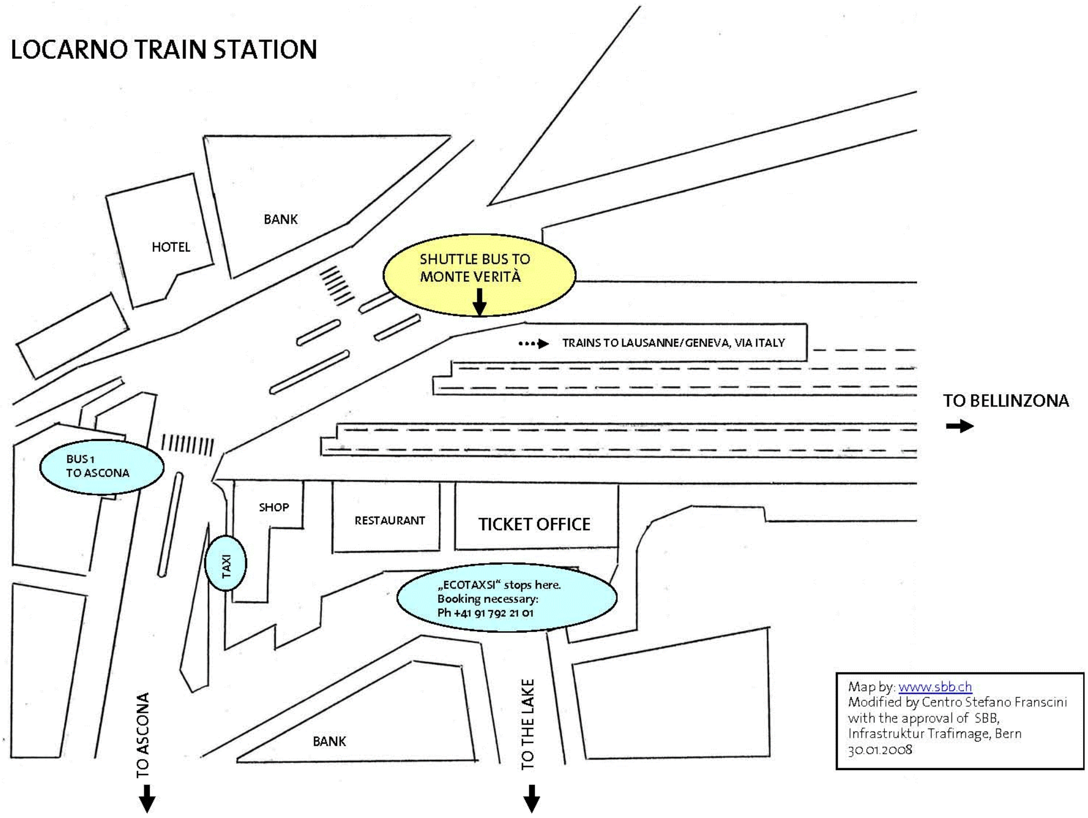
Schedule

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday
	Breakfast **(served between 7:00 – 10:00 am)**				
	9:00 – 9:15 **CSF Welcome** 9:15 – 10:05 **Bolsinov**	9:00 – 9:30 **Wacheux** 9:40 – 10:10 **Palmer**	9:00 – 9:30 **Sepe** 9:40 – 10:10 **Tarama**	9:10 – 10:00 **Matveev**	9:00 – 9:10 **– CSF Award –** 9:15 – 10:05 **Tolman**
	10:05 – 10:30 **Coffee break**	10:10 – 10:30 **Coffee break**	10:10 – 10:30 **Coffee break**	10:00 – 10:30 **Coffee break**	10:05 – 10:30 **Coffee break**
	10:30 – 11:20 **Izosimov** 11:30 – 12:00 **Lane**	10:30 – 12:20 **Poster session** **(in Balint room)**	10:30 – 11:20 **Shatashvili** 11:30 – 12:20 **Efstathiou**	10:30 – 11:20 **Suris** 11:30 – 12:20 **Roubtsov**	10:30 – 11:20 **Szenes**
	Lunch **(starts at 12:15)**	**Lunch** **(starts at 12:30)**			**Lunch** **(starts at 12:15)**
	14:00 – 14:10 **Video Monte Verita** 14:10 – 15:00 **Gukov**				
	15:00 – 15:30 **Coffee break**	15:00 – 15:30 **Coffee break**		15:00 – 15:30 **Coffee break**	
	15:30 – 16:20 **Kappeler** 16:30 – 17:20 **Bytsko**	15:30 – 16:20 **Pelayo** 16:30 – 17:20 **Hanßmann** 17:30 – 18:20 **Dullin**		15:30 – 16:20 **Sabatini** 16:30 – 17:20 **Volkov** 17:30 – 18:20 **Romão**	
Dinner **“buffet style”** **19:30** **to** **21:00**	**Dinner** **(starts at 19:00)**				

Free Shuttle Bus

The minibus can accommodate up to 9 people per ride and will run 6 times between Locarno Train station and the CSF / Monte Verità Hotel on “arrival” Sunday afternoon/evening and “departure” Friday noon/afternoon:

1) Locarno train station to **CSF/ Monte Verità** on **SUNDAY, April 8, 2018**:



Shuttle departures from Locarno train station on SUNDAY, April 8, 2018:

16:50h
17:30h
18:10h
18:50h
19:30h
20:10h

If you missed the last shuttle on Sunday or if you arrive on another day, please use a taxi or public transport, see conference web page for more information [The walk from the public bus stop in Ascona to the CSF Ascona/ Monte Verita seems to take about 20min and goes UPHILL].

2) CSF/ Monte Verità to **Locarno train station** on **FRIDAY, April 13, 2018**:

During the conference, we will collect everybody's departure time and schedule the 6 shuttle rides (spread over 3-4h) on Friday during the "peak departure times" to accommodate a maximum number of people, i.e., whoever is leaving *outside* this time window *has to take a taxi*.

Talks

(alphabetically ordered)

Alexey Bolsinov (Loughborough University and Moscow State University)

Symplectic invariants of integrable Hamiltonian systems: the case of degenerate singularities

Abstract.

The nature of symplectic invariants for non-degenerate singularities of integrable Hamiltonian systems has been studied and clarified (both in the local and semi-local setting) in fundamentally important papers by Vey, Eliasson, Dufour, Toulet, Miranda, Zung and San Vu Ngoc. The talk is devoted to some new ideas and techniques that can be used for studying symplectic invariants of degenerate singularities.

As an example, I would like to discuss normal forms and symplectic invariants of parabolic orbits and cuspidal tori in integrable Hamiltonian systems with two degrees of freedom. Such singularities appear in many integrable systems in geometry and mathematical physics and can be considered as the simplest example of degenerate singularities.

Tetrahedron equation, totally positive matrices, and (quantum) dilogarithm identities

Andrei Bytsko

(University of Geneva)

The tetrahedron equation,

$$R_{123}\, R_{124}\, R_{134}\, R_{234} = R_{234}\, R_{134}\, R_{124}\, R_{123},$$

is a 3–simplex counterpart of the Yang-Baxter equation. Starting with the RTT presentation of an upper triangular quantum group, one can construct a family of solutions to the tetrahedron equation [2]. In this construction, the key building block is the q–exponential function,

$$\langle x \rangle_q = \sum_{n \geq 0} \frac{x^n}{(q)_n}, \qquad (q)_n \equiv (q-1)\dots(q^n-1).$$

In turn, the obtained solution to the tetrahedron equation can be used to derive a family of quantum dilogarithm identities involving products of q–exponential functions whose non-commutative arguments are monomials in the generators of a quantum torus algebra associated to a certain quiver [3]. Remarkably, these identities are invariant under the action of the symmetric group S_3.

The quasi-classical limit [5] of a quantum dilogarithm identity yields an identity involving the Rodgers dilogarithm, $L(x) = Li_2(x) + \frac{1}{2}\log x\, \log(1-x)$. A family of identities involving Rodgers dilogarithms whose arguments are minors of totally positive matrices can be derived by using two involutions (close analogues of the BFZ twist [1]) on the variety of upper triangular totally positive matrices [4]. These identities admit a form manifestly invariant under the action of the symmetric group S_3 and correspond conjecturally to the quasi-classical limit of the quantum dilogarithm identities related to the tetrahedron equation.

Based on joint work with Alexander Volkov.

References

[1] A. Berenstein, S. Fomin, A. Zelevinsky, *Parametrizations of canonical bases and totally positive matrices,* Adv. Math. **122** (1996), 49–149.

[2] A. Bytsko, A. Volkov, *Tetrahedron equation, Weyl group, and quantum dilogarithm,* Lett. Math. Phys. **105** (2015), 45–61.

[3] A. Bytsko, A. Volkov, *Tetrahedron equation and cyclic quantum dilogarithm identities,* Int. Math. Res. Not. **2015** (2015), 1075–1100.

[4] A. Bytsko, A. Volkov *Totally positive matrices and dilogarithm identities*, arXiv:1708.08445.

[5] L.D. Faddeev, R.M. Kashaev, *Quantum dilogarithm,* Mod. Phys. Lett. **A9** (1994), 427–434.

Monodromy in the Kepler Problem

Holger R. Dullin, University of Sydney, joint work with
Holger Waalkens, University of Groningen
Phys. Rev. Lett. 120, 020507 (2018)

What could possibly be said about the Kepler Problem that is new? It is well known that this superintegrable system can be separated in different coordinate systems, and each such separation defines a distinct Liouville integrable system. We show that for separation in prolate spheroidal coordinates the resulting integrable system has Hamiltonian monodromy. This is a semi-toric system with two degrees of freedom on $S^2 \times S^2$ that is obtained by symplectic reduction of the S^1 action generated by the Kepler Hamiltonian. Analogous results are obtained for the corresponding quantum integrable system, where the eigenfunctions are spheroidal harmonics. Similar analysis can be done for many prominent superintegrable systems, for example the harmonic oscillator, the free particle or the geodesic flow on the sphere. In all these cases the resulting reduced systems correspond to well known special functions, but the quantum monodromy in their joint spectrum is reported here for the first time.

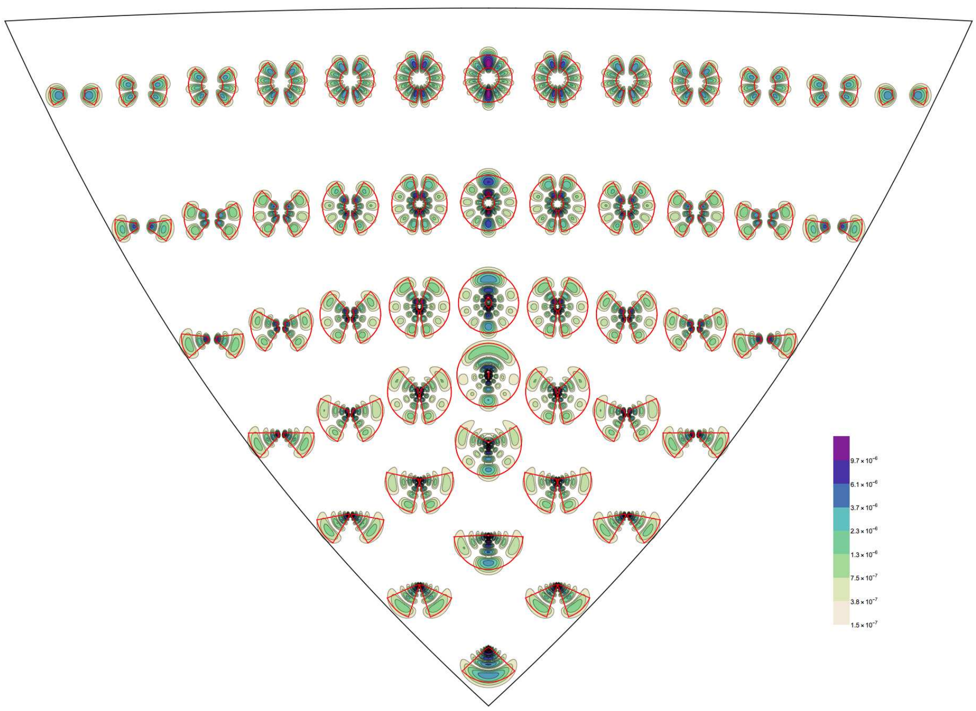


Fig. 1: 49 spheroidal eigenfunctions of the Kepler problem for principal quantum number $n = 7$ plotted at the location of the corresponding joint eigenvalues of angular momentum (horizontal axis, the global S^1 action) and the separation constant G (vertical axis). The eigenvalue of the operator $\hat{G}$ is the eigenvalues of the corresponding spheroidal wave function. For each wave function the corresponding classical caustic is shown in red.

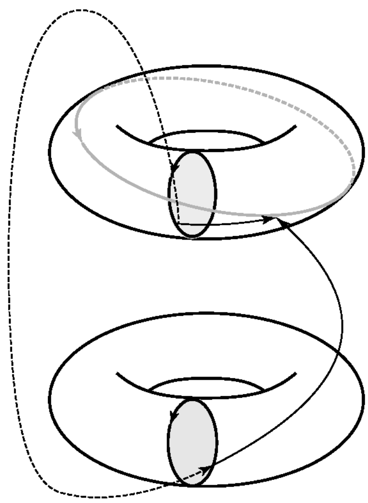
Rotation Forms and Non-compact Monodromy

Konstantinos Efstathiou

Johann Bernoulli Institute for Mathematics and Computer Science
University of Groningen

Abstract

The monodromy of torus bundles associated with completely integrable systems can be computed using geometric techniques (constructing homology cycles) or analytic arguments (computing discontinuities of abelian integrals). In this talk, based on [1], I give a general approach to the computation of monodromy that resembles the analytical one. This involves the definition of *rotation 1-forms* and the computation of their residues. Applying this approach to the case of non-degenerate focus-focus singularities, one re-obtains the classical results. An advantage is that the residue-like formula can be shown to be local in a neighborhood of a singularity, hence allowing the definition of monodromy also in the case of non-compact fibers. This idea has been introduced in the literature under the name of scattering monodromy [2, 3]. It turns out that the different definitions of monodromy for systems with non-compact fibers having a circle action coincide. Finally, I discuss recent results on scattering monodromy in Euler's two-center problem [4].



References

[1] K. Efstathiou, A. Giacobbe, P. Mardešić, and D. Sugny. "Rotation forms and local Hamiltonian monodromy". In: *Journal of Mathematical Physics* 58.2 (2017), p. 022902. DOI: 10.1063/1.4975215.

[2] L. M. Bates and R. H. Cushman. "Scattering monodromy and the A_1 singularity". In: *Central European Journal of Mathematics* 5.3 (2007), pp. 429–451. DOI: 10.2478/s11533-007-0022-4.

[3] H. R. Dullin and H. Waalkens. "Nonuniqueness of the Phase Shift in Central Scattering due to Monodromy". In: *Physical Review Letters* 101.7 (2008), p. 070405. DOI: 10.1103/PhysRevLett.101.070405.

[4] N. Martynchuk, H. R. Dullin, K. Efstathiou, and H. Waalkens. *Scattering invariants in Euler's two-center problem.* 2018. arXiv: 1801.09613 [math-ph].

Sergei Gukov
(Caltech)

Title: Sigma-model on a disk and trisections

Abstract: The star of the show will be a topological sigma-model (A-model) on a disk with various symplectic target manifolds that originate from topology, *e.g.* moduli spaces of flat G-connections on a genus-g Riemann surface F_g.

Surprisingly, this relatively simple sigma-model setup (illustrated in Figure 2 below) allows to compute various invariants of smooth 4-manifolds: Seiberg-Witten invariants, Donaldson invariants, Vafa-Witten invariants, and various generalizations associated with topological twists of 4d non-Lagrangian theories [1]. The key element of this construction is a projection that generalizes the moment map of a toric surface with respect to the torus action.

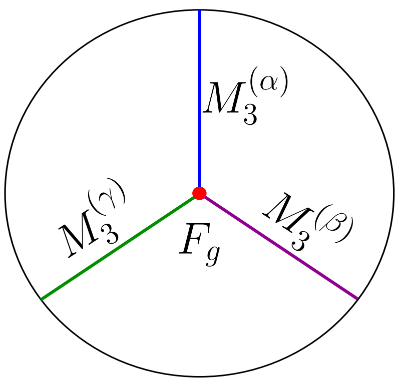


Figure 1: Cutting a 4-manifold is like cutting a cake: with only three skillful cuts, any 4-manifold can be trisected into three basic pieces.

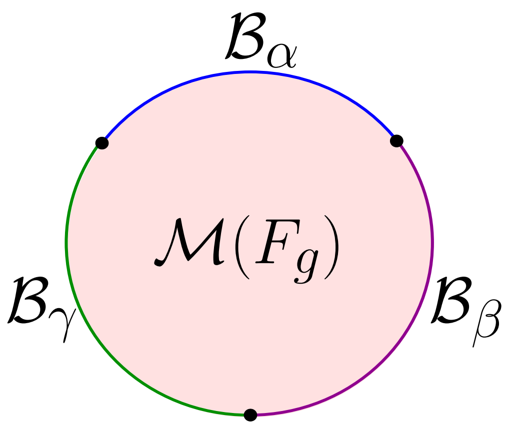


Figure 2: Disk amplitude in the A-model of $\mathcal{M}(F_g)$, with three Heegaard boundary conditions, dual to the trisection in Figure 1.

References

[1] S. Gukov, "Trisecting non-Lagrangian theories," JHEP **1711**, 178 (2017) arXiv:1707.01515 [hep-th].

Bifurcations and Monodromy of the Axially Symmetric 1:1:−2 Resonance

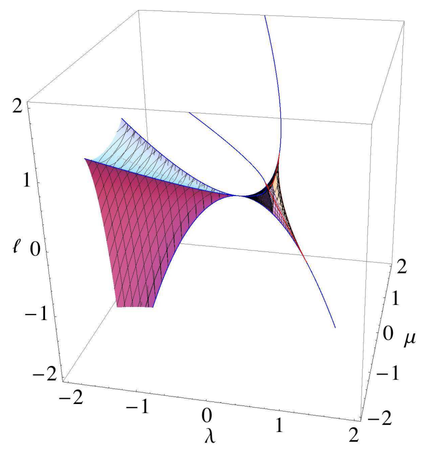
Heinz Hanßmann (Universiteit Utrecht)

Abstract

We consider integrable Hamiltonian systems in three degrees of freedom near an elliptic equilibrium in 1:1:−2 resonance. The integrability originates from averaging along the periodic motion of the quadratic part and an imposed rotational symmetry about the vertical axis. Introducing a detuning parameter we find a rich bifurcation diagram, containing three parabolas of Hamiltonian Hopf bifurcations that join at the origin. We describe the monodromy of the resulting ramified 3–torus bundle as variation of the detuning parameter lets the system pass through 1:1:−2 resonance.

Joint work with Konstantinos Efstathiou (Groningen) and Antonella Marchesiello (Prague).

As there is room for a pictue, here is the bifurcation diagram:



Pentagram maps and refactorization in Poisson-Lie groups

Anton Izosimov
University of Arizona

The pentagram map was introduced by Richard Schwartz in 1992, and is now one of the most renowned discrete integrable systems which has deep connections with many different subjects such as integrable PDEs, cluster algebras, dimer models etc. The definition of the pentagram map is illustrated in Figure 1: the image of the polygon P under the pentagram map is the polygon P' whose vertices are the intersection points of consecutive "short" diagonals of P (i.e., diagonals connecting second-nearest vertices).

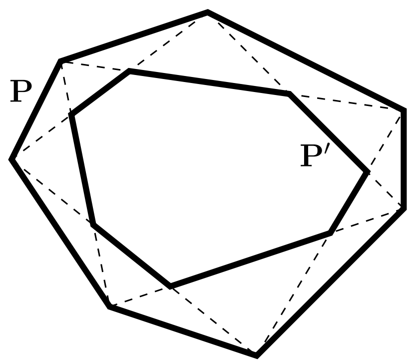


Figure 1: The pentagram map.

In this talk I will present a geometric construction which identifies the pentagram map, as well as its various multidimensional generalizations, with refactorization-type mappings in Poisson-Lie groups. This provides invariant Poisson structures and commuting first integrals for all these maps.

On a version of the Arnold-Liouville theorem in infinite dimension: a case study

T. Kappeler (University of Zurich), P. Topalov (Northeastern University)

March 10, 2018

Abstract: It is well known that the focusing nonlinear Schroedinger (fNLS) equation is an integrable PDE. When considered on the circle, the periodic eigenvalues of the Zakharov-Shabat (ZS) operator, appearing in the Lax pair formulation of the fNLS equation, form an infinite set of integrals of motion. In contrast to other integrable PDEs such as the defocusing nonlinear Schroedinger equation, the fNLS equation exhibits features of hyperbolic dynamics, in particular homoclinic orbits. In this talk I present a version of the Arnold-Liouville theorem for the fNLS equation: any connected level set of the above mentioned integrals of maximal and hence necessarily infinite dimension is a torus. On an invariant open neighborhood of such a torus we construct normal coordinates by developing the method of analytic continuation in the framework of normal form theory.

The $U(n)$ Gelfand-Zeitlin system as a limit of Ginzburg-Weinstein diffeomorphisms

Jeremy Lane
Université de Genève

It was recently shown by [1] that in the $t \to \infty$ limit of certain t-dependent coordinates (cluster coordinates given by generalized minors), the Poisson bracket on the dual Poisson Lie group K^*, of a compact Lie group K, converges to global action-angle coordinates (over a certain convex cone). In the special case $K = U(n)$, the action-angle coordinates defined in the limit are isomorphic to the Gelfand-Zeitlin integrable system over the interior of the Gelfand-Zeitlin cone [3].

For any compact connected Lie group K, there exist Poisson diffeomorphisms, called Ginzburg-Weinstein diffeomorphisms, from the Lie algebra dual $\mathfrak{k}^*$ to the dual Poisson Lie group K^*. As remarked in [3, 1], one might hope that integrable systems with global action-angle coordinates can be constructed on $\mathfrak{k}^*$ as a $t \to \infty$ limit of Ginzburg-Weinstein diffeomorphisms (composed with the coordinates studied in [1]).

In this work, we study the $t \to \infty$ limit of Ginzburg-Weinstein diffeomorphisms defined on $\mathfrak{u}(n)^*$ by [2] and show that the limit recovers the Gelfand-Zeitlin functions.

This is joint work with Anton Alekseev and Yanpeng Li (work on the case of general K is also joint with Benjamin Hoffman). The speaker is supported by NCCR SwissMAP of the Swiss National Science Foundation.

References

[1] Alekseev A, Berenstein A, Hoffman B, Li Y. 2017. Poisson structures and potentials. arXiv:1709.09281v2.

[2] Alekseev A, Meinrenken E. 2007. Ginzburg-Weinstein via Gelfand-Zeitlin. *J. Differential Geom.* **76**.

[3] Alekseev A, Davydenkova I. 2014. Inequalities from Poisson brackets. *Indag. Math. (N.S.)* **25**.

Symplectic geometry of Finsler metrics of constant curvature.

Vladimir Matveev

(Friedrich-Schiller-Universität Jena)

I will discuss Finsler metrics of positive constant flag curvature (definition, previous results and geometry will be recalled) on closed 2-dimensional surfaces. The main result is that the geodesic flow of such a metric is symplectically conjugate to that of a Katok metric. Recall that Katok metrics are easy and well-understood examples of two-dimensional Finsler metrics of positive constant flag curvature; I explain what they are describe their geodesic flows. As a corollary, either all geodesics are closed, and at most two of them have length less than the generic one, or all geodesics but two are not closed; in the latter case there exists a Killing vector field.

The proof goes through finite dimensional integrable systems, the main technical step is that the geodesic flow of such a Finsler metric is integrable (in fact, in all dimensions). I then use results on sympectic geometry of integrable systems (e.g. action-angle variables) and of Zoll metrics.

The results are almost contained in the papers arXiv:1710.03736 , arXiv:1710.01281 coauthored in different constellations with R. Bryant, P. Foulon, S. Ivanov and W. Ziller.

Some results on semitoric integrable systems

Joseph Palmer
Rutgers University

A *semitoric integrable system* is a 4-dimensional integrable system

$$(M, \omega, F = (J, H))$$

where J is a proper with periodic Hamiltonian flow, and the singularities of F are assumed to be relatively nice[1]. These generalize 4-dimensional toric integrable systems (for which H would also have periodic flow), and one of the main differences is that semitoric systems admit singularities of so-called *focus-focus* type, which do not appear in toric systems.

Semitoric integrable systems were classified by Pelayo-Vũ Ngọc [PVuN09, PVuN11] in terms of five invariants, one of which is a family of polygons generalizing the Delzant polygons which classify 4-dimensional toric integrable systems. The classification includes a description of how to construct a semitoric system from a given set of admissible invariants via a gluing procedure.

This talk is concerned with some work that has been done, by the speaker and others, to gain a better understanding of these systems, and the five invariants, in the time since the classification. There will be special emphasis on constructing explicit examples with (J, H) globally defined and deformations of semitoric systems which change the number of focus-focus points.

Portions of the work which will be presented are joint with Sonja Hohloch, Daniel M. Kane, Yohann Le Floch, and Álvaro Pelayo.

References

[PVuN09] Á. Pelayo and S. Vũ Ngọc. Semitoric integrable systems on symplectic 4-manifolds. *Invent. Math.*, 177(3):571–597, 2009.

[PVuN11] Á. Pelayo and S. Vũ Ngọc. Constructing integrable systems of semitoric type. *Acta Math.*, 206(1):93–125, 2011.

[1] namely, that they be non-degenerate with no hyperbolic components (in the Eliasson-Miranda-Zung normal form)

Geometric aspects of momentum maps and integrability, Ascona, April 8-13, 2018

Classical and semiclassical integrable systems with symmetries

Álvaro Pelayo
University of California, San Diego, USA

I will discuss some classical and recent works on finite dimensional integrable systems with toric features. These are integrable systems $f_1, \ldots, f_n$ for which a collection $f_i, i \in I \subset \{1, \ldots, n\}$, generate a periodic flow, so their joint map is the momentum map of a Hamiltonian action of a torus of dimension $|I|$. In the extreme case that $|I| = n$, the theory is essentially understood under suitable conditions on the underlying manifold and the components $f_1, \ldots, f_n$. If $|I| = n - 1$ and $n = 2$, these systems have been extensively studied in the past ten years and a lot is known about them. The talk will emphasize the interplay between classical and semiclassical aspects of integrable systems.

Nuno M. Romão[*]

L^2 geometry of symplectic vortices

Abstract: Symplectic vortices (on a Riemann surface Σ) generalise pseudo-holomorphic curves to an equivariant setting: the target manifold X is equipped with a Hamiltonian group action, and one needs to take gauge symmetry into account. The relevant moduli spaces support interesting Kähler metrics g_{L^2}, which encode information about underlying classical and quantum field theories — e.g. gauged sigma-models.

For a compact target X, the moduli spaces are typically noncompact, even if Σ is also compact. My talk will cover recent results on the asymptotic behaviour of the metrics g_{L^2} at the boundaries, in some concrete examples. Time permitting, I will also illustrate a few implications of such results for the field theories. (Based on joint work with Ákos Nagy and Martin Speight.)

[*]University of Augsburg, Germany

Non-commutative Painlevé equations and Calogero-Moser systems

Vladimir Roubtsov

LAREMA, UMR 6093 du CNRS, Département de Mathématiques,
Université d'Angers,

Theory Division, ITEP, Moscow.

All Painlevé equations can be written as a time-dependent Hamiltonian system, and as such they admit a natural generalization to the case of several particles with an interaction of Calogero type (rational, trigonometric or elliptic). Recently, these systems of interacting particles have been proved to be relevant the study of β-models.

An almost two decade old open question by Takasaki asks whether these multi-particle systems can be understood as isomonodromic equations, thus extending the Painlevé correspondence. I shall give an (affirmative) answer by displaying explicitly suitable isomonodromic Lax pair formulations. As an application of the isomonodromic representation we provide a construction based on discrete Schlesinger transforms, to produce solutions for these systems for special values of the coupling constants, starting from uncoupled ones; the method is illustrated for the case of the second Painlevé equation.

This is a joint work with Marco Bertola (SISSA-CRM, Montreal) and Mattia Cafasso (LAREMA, Angers).

References

M. Bertola, M. Cafasso, V. Roubtsov, Non-commutative Painlevé equations and systems of Calogero type, arXiv:1710.00736, 25 pp.

K. Takasaki. Painlevé-Calogero correspondence revisited. J. Math. Phys., 42(3):1443–1473, 2001.

HAMILTONIAN S^1-SPACES WITH LARGE MINIMAL CHERN NUMBER

SILVIA SABATINI

Consider a compact symplectic manifold of dimension $2n$ which is acted on by a circle in a Hamiltonian way with isolated fixed points; we refer to it as a Hamiltonian S^1-space. In [3] it is proved that the minimal Chern number N is bounded above by $n+1$, bound which is expected for all positive monotone compact symplectic manifolds. Assuming that the Hamiltonian S^1-space is monotone (i.e. the first Chern class is a multiple of the class of the symplectic form) in [2] several bounds on the Betti numbers are proved, these bounds depending on N. I will first discuss the ideas behind the proofs of the aforementioned facts, and then concentrate on $N = n+1$. In this case my student Isabelle Charton [1] proved that the manifold must be homotopically equivalent to a complex projective space of dimension $2n$.

References

[1] Charton, "Hamiltonian manifolds with high index". *Master thesis.* University of Cologne, 2017.

[2] Godinho, von Heymann, Sabatini "12, 24 and beyond", *Advances in Mathematics*, **319** (2017), 472 – 521.

[3] Sabatini "On the Chern numbers and the Hilbert polynomial of an almost complex manifold with a circle action", *Communications in Contemporary Mathematics*, **19**, No. 04 (2017).

Mathematisches Institut, Universität zu Köln, Germany
E-mail address: sabatini@math.uni-koeln.de

On the rigidity of Lagrangian products

Daniele Sepe*

The problem of finding obstructions to symplectic embeddings is one of the driving questions in symplectic topology. Recently, some symplectic submanifolds of the cotangent bundle to Euclidean space, known as Lagrangian products, have come to the fore primarily because of their connection to billiards. For instance, Ramos has calculated the optimal symplectic embeddings of the 4-dimensional Lagrangian bidisc into a ball and an ellipsoid. The aim of this talk is to show that for a large class of Lagrangian products of any dimension, the corresponding symplectic embedding problem is rigid, *i.e.* the natural inclusion is the best possible embedding. The proof of the result is inspired by Ramos' techniques and combines ideas from the theory of integrable systems with two symplectic capacities, namely the Gromov width and the cube capacity recently introduced by Gutt and Hutchings. This is joint work with Vinicius G. B. Ramos.

*Universidade Federal Fluminense, Instituto de Matemática, Campus do Valonguinho CEP 24020-140, Niterói (Brazil), email: `danielesepe@id.uff.br`.

(TBA)

Samson Shatashvili

(IHES & Trinity College Dublin)

TBA.

COMMUTATIVITY IN LAGRANGIAN AND HAMILTONIAN MECHANICS

YURI B. SURIS (TU BERLIN)

Let M be a finite dimensional manifold, and $L_1, L_2 \in C^\infty(TM)$ two non-degenerate Lagrange functions (with locally invertible Legendre transformations). Let S_1, S_2 be their principal action functions. Let $H_1, H_2 \in C^\infty(T^*M)$ be the corresponding Hamilton functions on the phase space T^*M with its canonical Poisson bracket.

Theorem 1. *If the principal action functions satisfy*

$$\min_{q_1 \in M} \Big(S_1(q_0, q_1, t_1) + S_2(q_1, q_{12}, t_2) \Big) = \min_{q_2 \in M} \Big(S_2(q_0, q_2, t_2) + S_1(q_2, q_{12}, t_1) \Big) \tag{0.1}$$

for all (q_0, q_{12}) from some neighborhood of the diagonal in $M \times M$ and for sufficiently small $t_1, t_2 > 0$, then the Hamilton functions Poisson commute, $\{H_1, H_2\} = 0$.

A discrete time counterpart of Theorem 1 is very instructive and enlightening, concerning both the statement and the proof. Let $\Lambda_1, \Lambda_2 \in C^\infty(M \times M)$ be two discrete time Lagrange functions, generating two symplectomorphisms F_1, F_2 of T^*M.

Theorem 2. *If the discrete time Lagrange functions satisfy*

$$\min_{q_1 \in M} \Big(\Lambda_1(q_0, q_1) + \Lambda_2(q_1, q_{12}) \Big) = \min_{q_2 \in M} \Big(\Lambda_2(q_0, q_2) + \Lambda_1(q_2, q_{12}) \Big) \tag{0.2}$$

for all (q_0, q_{12}) from some neighborhood of the diagonal in $M \times M$, then the symplectic maps F_1, F_2 commute.

The converse statements to Theorems 1 and 2 are also easily shown. This relates those theorems to the pluri-Lagrangian theory of commuting Hamiltonian flows and commuting symplectic maps developed in:

Yu.B. Suris. *Variational formulation of commuting Hamiltonian flows: multi-time Lagrangian 1-forms.* J. Geometric Mechanics (2013) **5**, No. 3, 365–379.

This is a joint work with A. Sridhar.

Research supported by the DFG Collaborative Research Center TRR 109 “Discretization in Geometry and Dynamics”.

(TBA)

András Szenes

(Geneva & Budapest)

TBA.

Stability analysis for generalized free rigid body dynamics on a real semi-simple Lie algebra with respect to an arbitrary Cartan subalgebra

Daisuke Tarama*

This talk deals with the stability analysis for equilibria of free rigid body dynamics on real semi-simple Lie algebras. Around 1980, Mishchenko and Fomenko have introduced a family of generalized Euler equations associated to an arbitrary Cartan subalgebra $\mathfrak{h}$ in the real semi-simple Lie algebra $\mathfrak{g}$:

$$\frac{\mathsf{d}}{\mathsf{d}t}X = [X, \varphi_{a,b,D}(X)], \qquad X \in \mathfrak{g}, \tag{1}$$

where $\varphi_{a,b,D} : \mathfrak{g} \to \mathfrak{g}$ is an operator associated to the elements $a, b \in \mathfrak{h}$ (a is assumed to be regular) and a symmetric operator $D : \mathfrak{h} \to \mathfrak{h}$. Mishchenko and Fomenko have proved that the restriction of Euler equation to a generic adjoint orbit is a completely integrable Hamiltonian system in the sense of Liouville with respect to the orbit symplectic form. (See e.g. [4].)

It is rather recent that the stability of equilibria has been analyzed for Euler equations (1) on real semi-simple Lie algebras of type A, on compact real Lie algebras, and on split real form of complex semi-simple Lie algebras. (See [3, 5] and the references therein.)

In this talk, based on the methods for the bi-Hamiltonian systems developed in [1, 2], the stability analysis is carried out for isolated equilibria on a generic adjoint orbit for above Euler equation (1) on any real semi-simple Lie algebra associated with an arbitrary Cartan subalgebra. The stability property of the equilibria is characterized by the types of the roots corresponding to the complexification of the Cartan subalgebra. The talk in based on a joint project with Tudor S. Ratiu (Shanghai Jiao Tong University).

References

[1] A. Bolsinov and A. Izosimov, Singularities of bi-Hamiltonian systems, Commun. Math. Phys., 331, 2014, 507-543.

[2] A. V. Bolsinov and A.A. Oshemkov, Bi-Hamiltonian structures and singularities of integrable systems, Regul. Chaotic Dyn., 14(4-5), 2009, 431-454.

[3] A. Izosimov, Singularities of integrable systems and algebraic curves, Int. Math. Res. Not., 00, 2016, 1–50.

[4] A. S. Mishchenko and A. T. Fomenko, Euler equations on finite-dimensional Lie groups, Izvest. Akad. Nauk SSSR, Ser. Matem., 42, 1978, 396-415.

[5] T. S. Ratiu and D. Tarama, The $U(n)$ free rigid body: integrability and stability analysis of the equilibria, *I. Differential Equations*, 259, 2015, 7284–7331.

*Department of Mathematical Sciences, Ritsumeikan University. Partially supported by Grant-in-Aid for Young Scientists (B), JSPS KAKENHI Grant Number 26870289. E-mail: dtarama@fc.ritsumei.ac.jp

Susan Tolman

University of Illinois at Urbana-Champaign

Title: On toric degenenerations and cohomological ridigity for symplectic toric manifolds

Abstract

Given an integral polytope $\Delta \subset \mathbb{R}^n$, there is a naturally associated toric variety X_Δ, that is, an n-dimensional algebraic variety with an n-dimensional torus action. We say that X_Δ is a *toric manifold* if the variety is smooth. In this case, X_Δ inherits a symplectic form $\omega_\Delta \in \Omega^2(X_\Delta)$. A conjecture, called "cohomological rigidity", posits that any toric manifolds with isomorphic cohomology rings are diffeomorophic. This has been proved in various special cases, but is still open. There's a natural symplectic analog to this conjecture: If there is an isomorphism of cohomology rings which preserves the symplectic cohomology class, then the manifolds are symplectomorphic. Unfortunately, this has been difficult to prove, because it's hard to construct symplectomorphisms. Adapting ideas from Harada and Kaveh, we use toric degenerations to find symplectomorphism between certain toric manifolds. This generalizes the well-known isomorphisms between Hirzebruch surfaces. We use this to prove the symplectic analog of cohomological rigidity under certain assumptions. For example, it holds when the cohomology ring is isomororphic to the cohomology ring of the product of two-spheres. This talk is based on a work-in-progress, which is joint with Milena Pabiniak

QUANTUM DILOGARITHM IDENTITIES

ALEXANDER YU. VOLKOV
STEKLOV MATHEMATICAL INSTITUTE, ST. PETERSBURG

The quantum dilogarithm function (previously known as the q-exponential) is perhaps the most ubiquitous tool in anything related to quantum integrability. Just like its classical counterpart, the quantum dilogarithm satisfies a plethora of interesting functional relations (identities), which this time involve non-commuting variables. I will address this topic from the perspective of so-called Y-systems.

Pleading for a functorial approach to the Delzant correspondence

Christophe Wacheux, *

March 20, 2018

Abstract

In 1982 Atiyah - Guillemin & Sternberg theorem showed that the image of the moment map of a Hamiltonian torus action was a rational convex polytope. In 1987, Delzant proved that in the integrable case (and for an effective action), the polytope classifies all symplectic toric manifolds.

Category theory is a unifying language that has proven powerful in many areas of mathematics. One of the "mantras" in this theory is that more than its objects, the morphisms between them gives the shape of a category.

In this talk, we will discuss what could be good candidates for morphisms of polytopes and toric systems, and the expected relations they should satisfy. We also want to give motivations for this functorial approach.

This is a work in progress with Damien Lejay.

*IBS-Center for Geometry and Physics, cwacheux@ibs.re.kr

Posters

(alphabetically ordered)

The Taylor series invariant of semi-toric systems

Jaume Alonso

Department of Mathematics and Computer Science
University of Antwerp
Middelheimlaan 1, 2020 Antwerp, Belgium

Abstract

Semi-toric systems are a specific class of completely integrable systems in four dimensions where one of the first integrals is proper and induces a global $\mathbb{S}^1$-action, together with some other assumptions. Á. Pelayo and S. Vũ Ngọc presented some years ago a classification of semi-toric systems in terms of five symplectic invariants. One of these invariants, the so-called *Taylor series invariant*, consists of the association of a Taylor series to each of the singularities of the semi-toric system that are of focus-focus type. We present the recent explicit calculation of this invariant for two systems, the coupled spin-oscillator and the coupled angular momenta. The latter depends on some parameters, which can make the singularity of focus-focus type turn into a singularity of elliptic-elliptic type. We also show how the Taylor series invariant reflects this dependence.

This work is joint with H. Dullin (University of Sydney) and S. Hohloch (University of Antwerp).

Quantum Monodromy and Symplectic Invariants of the Spheroidal Harmonics System

Sean Dawson, Holger Dullin and Diana Nguyen (University of Sydney)

March 19, 2018

In 2018, Dullin and Waalkens showed that the Hydrogen atom in prolate spheroidal coordinates has quantum monodromy [1]. This means that a global assignment of quantum numbers is impossible. By semi-classical Bohr Sommerfeld quantisation, the eigenvalues of a quantum integrable system can be approximated by action integrals. Semi-global symplectic invariants, introduced by Vu Ngoc in 2003, describe the behaviour of the actions near the Focus-Focus point [2].

In this poster, we show that the quantum integrable system obtained by separating the Laplacian in prolate spheroidal coordinates possesses quantum monodromy around its Focus-Focus point. We call this the spheroidal harmonics system. We prove the existence of monodromy by showing that the pre-image of a critical Focus-Focus value of the energy momentum map is a doubly pinched torus in the phase space. After discrete symmetry reduction, this becomes a singly pinched torus. By computing the associated actions, we obtain a semi-classical approximation of the eigenvalues of the spheroidal wave equation.

We compute the Taylor series invariants for this integrable system. We show that the reduced spheroidal harmonics system and the spherical pendulum are not equivalent integrable systems.

References

[1] Holger Dullin and Holger Waalkens. Defect in the joint spectrum of hydrogen due to monodromy. *Phys. Rev. Lett.*, 120:020507, 2018.

[2] VN San. On semi-global invariants for focus-focus singularities. 42:365–380, 2003.

Momentum maps and integrability in noncommutative algebraic geometry

Maxime Fairon

School of Mathematics, University of Leeds
Leeds, LS2 9JT, UK
email: mmmfai @ leeds.ac.uk

Abstract

One aspect of non-commutative algebraic geometry consists in following Kontsevich-Rosenberg's philosophy, which states that the non-commutative version of a structure defined on a non-commutative algebra should yield the corresponding classical structure on the representation spaces of this algebra. The work of Van den Bergh [3] deals with the introduction of non-commutative Poisson and quasi-Poisson geometry under this principle, as well as defining the necessary objects to perform (quasi-)Hamiltonian reductions on the corresponding representation spaces.

Our aim is to explain part of this work and how to to search for integrable systems in this context. We also illustrate how Van den Bergh's theory applied to cyclic quivers is a good starting point to study systems in the Calogero-Moser family. To be precise, we explain the formalism needed to recover the non-spin and spin versions of the rational CM system [2], the non-spin trigonometric Ruijsenaars-Schneider system [1], but also the spin generalisation of the trigonometric RS model, which is the object of a forthcoming publication with O. Chalykh.

References

[1] Chalykh, O., Fairon, M.: *Multiplicative quiver varieties and generalised Ruijsenaars-Schneider models.* J. Geom. Phys. 121, 413–437 (2017); e-print : arXiv:1704.05814.

[2] Chalykh, O., Silantyev, A.: *KP hierarchy for the cyclic quiver.* J. Math. Phys. 58, 071702 (2017); e-print : arXiv:1512.08551.

[3] Van den Bergh, M.: *Double Poisson algebras.* Trans. Amer. Math. Soc., 360 no. 11, 5711–5769 (2008); e-print : arXiv:math/0410528.

Lyusternik-Schnirelmann category and Localization Formula
Marine Fontaine (Augsburg University)

The LS-category of a topological space is a homotopical invariant, introduced originally in a course on the global calculus of variations by Lyusternik and Schnirelmann, to estimate the number of critical points of a smooth function. When the topological space is a smooth manifold equipped with a proper action of a Lie group, we give a localization formula to calculate the equivariant analogue of this category in terms of the minimal orbit-type strata. The formula holds provided that the manifold admits a specific cover. We show that such a cover exists on every symplectic toric manifold. The known result stating that the LS-category of a symplectic toric manifold is equal to the number of fixed points of the torus action follows from our localization formula. This work is in collaboration with James Montaldi.

Hamiltonian stacks and their moment polytopes

Benjamin Hoffman
Cornell University

We develop the theory of Hamiltonian stacks, which are the stacky analogue of Hamiltonian symplectic manifolds. Hamiltonian stacks are equipped with the action of a Lie group stack and can have irrational moment polytopes. We give necessary and sufficient conditions for the existence of a stacky symplectic reduction, and prove an analogue of the Duistermaat-Heckman theorem.

Langlands dual and Poisson Lie dual via cluster theory and tropicalization

Yanpeng Li

Université de Genéve

Abstract

We define the dual of a cluster variety, and build a correspondence between the cluster variables of these two cluster varieties. As an example, the cluster algebras on the double Bruhat cells of G and $G^\vee$ are dual to each other, where G is a complex semisimple Lie group, and $G^\vee$ is the Langlands dual of G. The correspondence gives us an injective crystal map of Berenstein-Kazhdan lattice cones $\tau\colon \mathcal{C}(G) \to \mathcal{C}(G^\vee)$. Using this map, we show that tropicalization of the Poisson Lie dual G^* preserves the symplectic leaves and their volume.

This is a joint work with A. Alekseev, A. Berenstein, and B. Hoffman.

SCATTERING INVARIANTS OF INTEGRABLE SYSTEMS

NIKOLAY MARTYNCHUK

University of Groningen

Abstract

We demonstrate how topological scattering can be studied in Hamiltonian systems with the following two structures:

- The structure of a scattering system and
- Liouville integrability.

In particular, we generalize the notions of scattering [1, 2] and non-compact monodromy [3] to such systems and make a connection to Knauf's index [4]. Our motivation comes from [4] and [2, 7].

This is a report on my joint works with H.R. Dullin, K. Efstathiou and H. Waalkens [5, 6].

References

1. L. Bates and R. Cushman, *Scattering monodromy and the A1 singularity*, Central European Journal of Mathematics **5** (2007), no. 3, 429–451.
2. H. Dullin and H. Waalkens, *Nonuniqueness of the phase shift in central scattering due to monodromy*, Phys. Rev. Lett. **101** (2008).
3. K. Efstathiou, A. Giacobbe, P. Mardešić, and D. Sugny, *Rotation forms and local Hamiltonian monodromy*, Submitted (2016).
4. A. Knauf, *Qualitative aspects of classical potential scattering*, Regul. Chaotic Dyn. **4** (1999), no. 1, 3–22.
5. N. Martynchuk, H.R. Dullin, K. Efstathiou, and H. Waalkens, *Scattering invariants in Euler's two-center problem*, Submitted, arXiv:1801.09613.
6. N. Martynchuk and H. Waalkens, *Knauf's degree and monodromy in planar potential scattering*, Regular and Chaotic Dynamics **21** (2016), no. 6, 697–706.
7. H. Waalkens, H.R. Dullin, and P.H. Richter, *The problem of two fixed centers: bifurcations, actions, monodromy*, Physica D: Nonlinear Phenomena **196** (2004), no. 3-4, 265–310.

Address: Johann Bernoulli Institute for Mathematics and Computer Science, University of Groningen, P.O. Box 407, 9700 AK Groningen, The Netherlands.
E-mail: N.Martynchuk@rug.nl

Integrable systems arising from separation of variables on S^3

Diana M.H Nguyen *with Holger Dullin*

School of Mathematics and Statistics

The University of Sydney

Inspired by Schöbel's work on the classification of separable coordinates on S^3 as well as Vu Ngoc's classification of semitoric systems, we study the separation of variables of the geodesic flow on S^3 and the resulting integrable systems. This poster presents a variety of interesting preliminary results about these systems.

The geodesic flow on the 3-sphere S^3 is a superintegrable system with a global Hamiltonian S^1-action. The quotient of T^*S^3 by this action is isomorphic to $S^2 \times S^2$. We obtain an explicit parametrisation of this symplectic manifold in terms of the angular momenta L_{ij}. The Hamiltonian-Jacobi equation of the geodesic flow on S^3 separates in the general spherical-elliptical coordinates as well as the 5 degenerate coordinates: prolate, oblate, Lamé-subgroup reduction, spherical and cylindrical coordinates. These six distinct Stäckel's systems give rise to six integrable systems on S^3 and consequently six integrable systems on $S^2 \times S^2$. We produce the image of the integral map (for a fixed energy) for all six systems, with the general elliptical case consisting of four lines and a quadratic curve. Resounding similarities are found between the momentum maps of our systems and those from the geodesic flow on an ellipsoid and the Neumann problem.

We also show that image of the corresponding action map for the general elliptical coordinates is the equilateral triangle given by the intesection of the plane $I_1 + I_2 + I_3 = 1$ with the positive quadrant. With an appropriate projection of this triangle onto $\mathbb{R}^2$, we show that the image of the action map has the boundary being a Delzant triangle with critical curves on the interior of this polygon.

Abelianisation of $\mathsf{SL}_2(\mathbb{C})$-Connections and Darboux Coordinates on their Moduli Spaces

Nikita Nikolaev

Section de Mathématiques
Université de Genève

We describe a new approach to studying flat connections on holomorphic vector bundles over Riemann surfaces. Such connections are put in correspondence with much simpler objects: flat connections on line bundles. This approach is called *abelianisation*, because the nonabelian monodromy data of a connection on a vector bundle is put in equivalence with abelian monodromy data of a connection on a line bundle (a.k.a., abelian connection). An elementary analogy is the approach in linear algebra to describe a linear transformation equivalently in terms of its eigenvalues and eigenspaces.

Take a holomorphic $\mathsf{SL}_2(\mathbb{C})$-vector bundle with a flat connection $(\mathcal{E}, \nabla)$ with simple singularities over a Riemann surface X. By studying the behaviour of flat sections of ∇ at the singularities, we can extract certain asymptotic data characterising the connection. We then encode this asymptotic data in a holomorphic line bundle with a flat connection $(\mathcal{E}^{\mathrm{ab}}, \nabla^{\mathrm{ab}})$ with simple singularities over another Riemann surface Σ which is a double cover $\pi : \Sigma \to \mathsf{X}$. The procedure $(\mathcal{E}, \nabla) \mapsto (\mathcal{E}^{\mathrm{ab}}, \nabla^{\mathrm{ab}})$ is a functor, called an *abelianisation functor* π^{ab}. Moreover, the abelianisation functor is an equivalence of categories from flat $\mathsf{SL}_2(\mathbb{C})$-connections to connections on line bundles.

One significance of this equivalence is that the corresponding moduli spaces (i.e., sets of equivalence classes) of flat connections are holomorphic symplectic manifolds, and the abelianisation functor π^{ab} descends to a symplectomorphism. Furthermore, the moduli space of flat connections on line bundles is a very simple space: it is just an algebraic torus, and so the abelianisation functor can be interpreted as a Darboux coordinate chart on the moduli space of $\mathsf{SL}_2(\mathbb{C})$-connections. Such coordinates were first considered by Fock-Goncharov in 2006 and later in 2013 by Gaiotto-Moore-Neitzke.

Semiglobal symplectic invariants of focus-focus singular fibers with multiple pinched points

Xiudi Tang

(UC San Diego)

We classify, up to symplectomorphisms, a neighborhood of a singular fiber of an integrable system (which is proper and has connected fibers) containing $k > 1$ non-degenerate focus-focus critical points. Our result shows that there is a one-to-one correspondence between such neighborhoods and k formal power series, up to a $(\mathbb{Z}_2 \times D_k)$-action, where D_k is the k-th dihedral group. This proves a conjecture of San Vu Ngoc from 2002.