Analysis & Geometry Seminar:

Introduction to regularization methods for inverse problems

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In many scientific fields and industrial applications, it is possible to predict the output of some process based on the input parameters. This is called the forward problem. However, often the "invisible" parameters that govern the system are of far greater interest than the observed output. Determining these parameters is called the inverse problem.

If the mathematical model is given by $F(x) = b$, for $x \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$, one of the main problems when solving inverse problems is that a perturbed version $\widetilde{b} = b + e$ of the output is measured. This error or noise $e \in \mathbb{R}^m$ is caused by small measurement errors and even though these can be very small, they can cause big numerical errors when naively trying to solve the inverse problem, i.e.

$$\min_{x \in \mathbb{R}^n} \left\| F(x) - \widetilde{b} \right\|_2 . \tag{1}$$

This has given rise to a whole field of regularization methods, which solve slightly modified versions of (1) in order to suppress the effect of the errors on the solution. One of the most widely used methods is Tikhonov regularization, which solves

$$\min_{x \in \mathbb{R}^n} \left\| F(x) - \widetilde{b} \right\|_2 + \alpha \left\| L(x) \right\|_2 \tag{2}$$

for some regularization operator L and regularization parameter $\alpha > 0$. However, choosing an "optimal" value for this parameter is a non trivial problem.

The aim of this talk is twofold. First, to give an introduction to inverse problems, to demonstrate the need for regularization and to show why it works. Secondly, to describe how the a good regularization parameter for the Tikhonov problem can be determined for both linear and non-linear inverse problems.

References

[1] Per Christian Hansen, *Discrete inverse problems: insight and algorithms.* Society for Industrial and Applied Mathematics, 2010.

[2] Silvia Gazzola and Paolo Novati, *Automatic parameter setting for Arnoldi-Tikhonov methods.* Journal of Computational and Applied Mathematics 256 (2014): 180-195.

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