The elastic trefoil is the twice covered circle

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To describe the behavior of knotted loops of springy wire with a mathematical model, we minimize the classic bending energy together with a small multiple of ropelength (i.e., the quotient length over thickness) in order to penalize self-intersection. Our main objective is to characterize limit configurations of energy minimizers of the total energy as the regularization parameter tends to zero. The latter will be referred to as elastic knots.

The elastic unknot turns out to be the singly covered circle with bending energy $(2\pi)^2$. For all (non-trivial) knot classes for which the natural lower bound $(4\pi)^2$ for the bending energy is sharp, the respective elastic knot is the doubly covered circle.

The knot classes for which $(4\pi)^2$ is sharp are precisely the $(2, b)$-torus knots for odd b with $|b| \geq 3$ (containing the trefoil knot). In particular, the elastic trefoil knot is the doubly covered circle.

This is a joint work with Heiko von der Mosel and Henryk Gerlach.