A Short Research Overview

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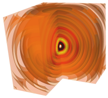
Helmholtz equation and preconditioning

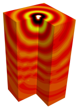
$$-\left[\triangle + k(\mathbf{x})^2\right] u(\mathbf{x}) = f(\mathbf{x}), \qquad \mathbf{x} \in \Omega \subseteq \mathbb{R}^d. \qquad (1)$$

[H. von Helmholtz, 19th century]

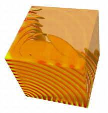
- Applications: scattering models, inverse problems (forward problem)
- Numerical discretization using FDM, FVM, FEM, BEM, ...
- Iterative solution: MG-preconditioned Krylov methods (CG, GMRES)
- Complex shifted Laplacian (CSL) preconditioner [Erlangga et al. 2004]



Homogeneous (radar/sonar)



Wedge problem (seismics)



Coulomb problem (quantum break-up)

k_0 $n_x \times n_y$	1 128^2	2 256^2	4 512^2
MG-FGMRES	51 (5.40)	92 (46.8)	174 (813)
MG-FGMRES(10)	66 (5.52)	140 (46.7)	250 (421)
LVL-MG	44 (3.42)	83 (25.3)	149 (215)

2D ionization problem - iterations (CPU time in s.)

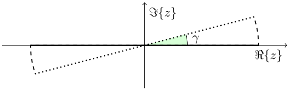
[Cools et al. 2013 (NLAA)]

Far field map for Helmholtz (HH) & Schrödinger (SCH)

$$F(\alpha) = \int_{\Omega} G(\mathbf{x}, \alpha) \left(k^2(\mathbf{x}) - k_0^2\right) \left(u_{sc}(\mathbf{x}) + u_{in}(\mathbf{x})\right) d\mathbf{x}. \qquad (2)$$

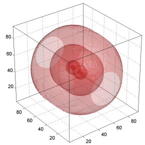
[Colton et al. 1998]

- Oscillatory integral [Huybrechts et al. 2006]
- Computational bottleneck: solve HH or SCH system for $u_{sc}(\mathbf{x})$
- Reformulation to complex integral

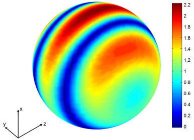


Schematic representation: 1D complex contour
[Cools et al. 2014 (SISC)]

- Advantage: fast iterative computation of HH or SCH system (CSL)



3D object of interest



Far field map

3D Helmholtz scattering problem (256^3-pt. FDM)

→ Previously: hard to compute
supercomputing infrastructure required

→ Now: efficient iterative computation
solvable in $\pm$ 8 min. on laptop

Wavelet-MG preconditioner for ATR

ATR = algebraic tomographic reconstruction - regularized linear system

$$\min_{\mathbf{x}}\{\|W\mathbf{x}-b\|_2+\lambda\|L\mathbf{x}\|_2\}. \quad (3)$$

- Classic reconstruction algorithms (ART, SIRT, etc.) converge poorly

  Slow resolvance of low-frequency modes! (~~sharp edges~~)

- Krylov methods require suitable preconditioner: WMG-preconditioner
  - (1) **wavelet-projection** on low-, semi- & high-oscillatory coarse subspaces
    $\rightarrow$ resolve **all** modes
  - (2) **multigrid-type error correction** scheme
    $\rightarrow$ coarse grid representations **correct** initial guess

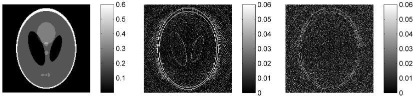


Image   Error SIRT   Error WMG-Krylov

method	iter	CPU	error L_2
SIRT	1000	81.1 s.	0.1385
BiCGStab	100	9.35 s.	0.1074
WMG-BiCGStab	14	5.44 s.	0.1083

[Cools et al. 2014 (under review)]