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Price competition between subsidized organizations

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Abstract

Many firms and organizations compete for customers while at the same time receiving substantial funding from outside sources, such as government subsidies. In this paper, we study the effects of two commonly observed subsidy systems on the strategic behavior of competing firms. We compare a per unit subsidy to a subsidy allocated according to the firms' market shares. We show that, holding the total subsidy budget constant, the per unit subsidy results in lower prices, higher output, lower profits and higher overall welfare as compared to the market-share based alternative. However, we also find that a market-share based subsidy makes collusive behavior between firms much harder. Our results suggest a potential trade-off between short-run and long-run objectives: subsidy systems designed to widen participation may favor collusive behavior. The welfare implications of this trade-off are discussed. Our findings have important policy implications for the design of subsidy systems.

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1. Introduction

The revenues of many firms and organizations do not only stem from the prices they charge to their customers. In many instances, other sources of income represent a considerable fraction of their direct revenues. For example, theatres, operas, museums, and other artistic organizations compete for audience while receiving significant subsidies from diverse governmental agencies and sponsoring firms.[1] In most countries, universities and other institutes of higher education directly compete for students, but at the same time they benefit from considerable government funding.[2] In some industrial sectors, firms fight for customers while receiving government subsidies for making use of environmentally-friendly production techniques. Of course, outside sources are not limited to government subsidies. For example, it is not unusual for sports leagues to allocate funds from a common revenue source, like broadcasting rights, among their league members. For the competing league members, this often constitutes a considerable part of their revenues.

A government may have a variety of reasons for granting subsidies: (i) It may subsidize artistically related activities to prevent the local cultural heritage from disappearing; (ii) Institutes of higher education may be subsidized to support economic growth or, for distributional reasons and to limit social exclusion, to further expand participation rates; (iii) Governments may provide subsidies when there is a social concern about the output level of a monopolistic firm[3] ; (iv) Subsidies may be granted to cope with negative externalities. For example, governments may subsidize clean-producing firms in order to promote the use of renewable energy sources. Finally, private organizations may also have specific incentives to provide funding. The sports leagues referred to above redistribute common broadcasting revenues among their league members mainly to preserve the degree of competitive balance between the different competing teams. They subsidize the weaker league members at the cost of the stronger teams. The

[1] For example, the revenues in 2009 of the Royal Opera House in London amounted to 90 million pounds, of which 40 million pounds came from ticket sales and 27 million pounds from government funds. The remaining part came from individual funds and other commercial activities (*Forbes.com*, September 19, 2009). See also Van der Ploeg (2006) for an overview of cultural expenditures in European countries.

[2] Government subsidies to public institutes of higher education are considerable. Heckman (2000) estimates for the US that, on average, students attending public institutes of higher education pay less than 20% of the total cost of education. See, among others, Winston (1999), Barr (2004) and Santiago et al. (2008)) for surveys on the economics of higher education funding.

[3] See Segal (1998).

allocation of subsidies to firms or organizations in a particular sector can take many forms.[4] In this paper, we study the competitive and welfare effects of different allocation rules for subsidizing, or distributing funds to, competing organizations. We develop a stylized model with two price-competing firms offering a differentiated product, and we consider the effects of two subsidy systems. In the first one the government provides a fixed per unit subsidy. The second system assumes that a fixed subsidy budget is available for the sector as a whole; allocation to individual firms is then based on their market shares in industry output.[5] Within this setting, we have two specific but important objectives. First, we analyze the effects of both subsidy systems for prices, output, profits, and welfare. Second, we study the implications of the two different types of subsidies for the long-run incentives to collude.

We obtain the following results. First, we show that the per-unit system is more efficient in the sense of Anderson et al. (2001a, b): holding the total subsidy budget constant, a per unit subsidy results in more fierce price competition than a market-share based subsidy. As a result, the per unit subsidy generates a larger market output and is, therefore, more effective at stimulating participation than a market-share based subsidy. Firms enjoy larger profits under the market-share based system. Total welfare is higher for the per unit subsidy. Second, we find that variations in a per unit subsidy have no effect on the incentives to collude. However, increasing the budget available for a market-share based subsidy decreases the incentives to collude. We show that this implies there is less potential for collusion in a market-share based system. Third, interestingly, our results imply that a trade-off between short-run and long-run objectives may exist: a market-share based system performs worse if the objective is to stimulate participation or short-run welfare, but the long-run benefit may be that there are lower incentives to collude.

[4] They can be granted as a lump sum, or they can be linked to particular criteria such as input use (e.g. size of the orchestra, maintenance cost of existing infrastructure, …), outputs (e.g. consumption units, attendance at a theatre performance, number of hospital patients or passing students…), market share, etc. Moreover, the system can be closed-ended (an a priori fixed budget is allocated), open-ended or a mixture of the two (for example, an open-ended system with some form of cap to prevent excessive subsidy payments). Note that there are also numerous implicit ways of granting subsidies through favorable tax regimes, such as tax breaks for revenues, R&D investments, or private donations.

[5] Note that our analysis does not look at the competitive and welfare implications of state aid, where e.g. subsidies favor one firm (or group of firms) to the disadvantage of competing firms that do not receive subsidies (see e.g. Friederiszick et al. (forthcoming)). Our analysis therefore differs from Garcia and Neven (2005) who assess the competitive distortions of state aid granted to a specific subgroup of firms. It also differs from Collie (2000) who studies competition between firms of different member countries of the European Union, when each country provides a per-unit optimal production subsidy (optimal in the sense of maximizing domestic welfare) to the local firm. He shows that each country indeed has an incentive to provide such subsidies, and that from a European Union viewpoint such subsidies are undesirable. Our model is of a different nature. We investigate the welfare implications of subsidy allocation systems that treat all firms in a sector equally.

Finally, when the conditions for collusion hold under both subsidy systems, we find that the per-unit system overall performs better since its collusive price is always lower than the collusive price under the market-share based system.

While real-world subsidies typically involve complex allocation rules, examples of subsidies that are broadly consistent with one of the systems just described are frequently observed in practice. Examples of per-unit production subsidies are found in international trade (see e.g. Collie (2000)), in the cultural sector (Van der Ploeg (2006)), and in education (Jacobs and Van der Ploeg (2005)). A prominent education example is Denmark's "taximeter" model, in which universities get funding per passing student (see Kalpazidou et al. (2007)). Examples of market-share based subsidy systems are also observed in education (for example, the funding of universities in Belgium), in environmental economics (output-based refunding of environmental taxes in the US and Sweden (Fischer (2003), Gersbach and Requate (2004)), and in international trade (Krishna et al. (2001)).

To the best of our knowledge, a formal comparison of the effects of per-unit and market-share based subsidy systems is not available in the literature. First, a number of papers have explicitly focused on per-unit subsidies. For example, in an early paper Hansmann (1981) studied the implications of subsidies for the behavior of artistic organizations (museums, theatres, etc.) under various different objective functions. He considers different types of subsidies (lump-sum and various other subsidies like a per-unit price subsidy, or a matching subsidy per dollar of revenues raised through donations) and analyzes the effects on ticket prices, output, and welfare. More recently, Fethke (2005, 2006) offers a theoretical framework of competitive behavior in the subsidized market for public higher education; his focus is on a per-unit enrollment subsidy. Second, several papers have focused on market-share based subsidy schemes. Krishna, Roy and Thursby (2001) study market access requirements (MAR), whereby an importing country voluntarily agrees a minimum share of its home market for a good from a foreign country. They look at a subsidy scheme where each targeted firm gets a subsidy proportional to its individual share of the market, provided the market access requirement is met at the aggregate level.[6] Output-based refunding of environmental taxes in imperfectly

[6] The model assumes that, in the first stage of the game, the government announces that each targeted firm will receive part of a total subsidy equal to its individual market share, provided the aggregate US share meets the minimum level specified by the MAR. Referring to Sen (1966), the authors argue that share-based subsidies are

competitive markets is another example of this type of subsidies (see, e.g., Fischer (2003) and Gersbach and Requate (2004)). Under this system, producers are taxed according to their emissions, and total revenues are refunded based on the firm's market share in total output. Third, at least one study has formally compared a closed-ended with an open-ended subsidy. Fuest and Tillessen (2005) focus on subsidies to support entrepreneurial investment. However, the closed system they consider is of a totally different nature than the market-share based system studied in this paper.[7] Moreover, although our per unit subsidy can in principle be interpreted as an open-ended system, our comparison studies market outcomes and welfare for a given budget.

Finally, the most detailed study of the implications of taxes and subsidies in a differentiated oligopoly is probably Anderson et al. (2001a, b). They specifically study the relative efficiency of ad valorem and per-unit taxes, defining the relative efficiency of the two tax systems according to whether a system yields higher tax revenues (lower subsidy costs) for given outputs or, alternatively, higher output for given tax revenue (subsidy costs). They show, among others, that ad valorem taxes are (both for Cournot and Bertrand competition) welfare superior to unit taxes if production costs are identical across firms; cost asymmetries make the case for ad valorem stronger under Cournot, but under Bertrand the opposite may hold. Although the current paper also deals with alternative forms of subsidies, it differs from Anderson et al. (2001a, b) on several accounts. We compare a per unit subsidy with a subsidy system where the available budget is allocated according to firms' market shares; they study per-unit versus ad valorem subsidies. Moreover, we study both the welfare effects and the implications of different subsidy systems for the incentive to collude.

A formal comparison of the effects of per-unit and market-share based subsidy systems may be important for several reasons. One is that subsidy systems widely differ across countries, even within the same sector. As noted above, the Danish system of education finance comes closest to a per unit subsidy, whereas other countries have opted for what is better described as a market-share based system, or a complex mixture of the two. Moreover, recent policy changes in some countries could be interpreted as a shift away from one system in the direction of the other.

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more high-powered than specific subsidies, because they imply an externality: when one firm gets more, the other necessarily gets less.

[7] They study a subsidy that is limited to a maximum amount and compare it with an open system that affects investment decisions at the margin. They show that, contrary to expectations, the closed system is welfare superior when capital markets are subject to asymmetric information.

It is then useful to understand the effects of such policy changes.[8] Another justification for the analysis of the current paper is that very little is known about the long-run effects of different financing arrangements on the structure of the industry. To the extent that different subsidy systems have different implications for the incentives to collude, ways of funding that hamper long-run competition may be less desirable, independent of their effects on pricing and output in the short-run.

Our model highlights that different subsidy schemes may significantly alter the strategic behavior between organizations. However, the analysis is highly stylized and simplifies many of the complexities of existing subsidy schemes. For example, the basic version of the model focuses on price competition between profit- or revenue-maximizing organizations. This assumption may not be suited for all subsidized organizations we observe. Hansmann (1981) has argued that many artistic companies (theatres, etc.) probably do not care too much about profit; instead, they may care about quality, attendance, revenues, etc. Similarly, the appropriate objective function for institutes of higher education is likely to be highly multi-dimensional (see, e.g., Winston (1999)). We do think that profit or revenue maximizing behavior by the relevant firms or organizations may serve as a useful benchmark. Price competition in the US higher education market has increased significantly (see e.g. Bok (2004) and Winston and Zimmerman (2000)); moreover, the further globalization of education is likely to result into more tuition competition between European universities. Similarly, although large sports teams no doubt are also interested in winning games and drawing large audiences, their entry on the stock market in the UK and France, among others, suggests that profit maximization will become the rule rather than the exception, be it within the limits set by the league (Szymanski (2003)). We therefore believe that our model does capture important ingredients of a broad range of subsidy systems. However, to allow for possible other organizational objectives apart from profit, we also briefly discuss an extension of the model in which the firm also cares for the consumer surplus of its customers (see Section 4.3 below),

The structure of the paper is as follows. In Section 2, we present the structure of the model and explain the properties of the per unit and market-share based subsidy systems. Section

[8] Belgium recently moved from a system with substantial open-ended funding to a more closed-ended system in financing its universities. They now receive part of the closed-ended funding according to their market share in the number of (entering or passing) students, publications, and citations. Also, see Barr (2004) for the recent reform in the UK.

3 derives the market outcomes under Bertrand competition for each of the two subsidy systems. Section 4 reports on a detailed welfare analysis of the two subsidy systems. In Section 5 we consider the implications of the two types of subsidies for the incentive to collude on the output market. We summarize our conclusions in Section 6.

2. The model

A representative consumer's utility is defined by[9]

$$u(q_1, q_2, q_0) = q_1 + q_2 - 0.5\,[q_1^2 + 2dq_1q_2 + q_2^2] + q_0 .$$

In this specification, q_0 denotes the quantity of the composite numéraire good, and q_1 and q_2 are the quantities of goods 1 and 2, respectively. Preferences are assumed to be quasi-linear (so that all the income effects are captured by the numéraire good) and quadratic in the two other goods. The consumer's budget constraint can be written as $y = q_0 + p_1q_1 + p_2q_2$, where y is the available budget and p_1 and p_2 are the unit prices of goods 1 and 2, respectively. We further assume that $0 \le d < 1$, so that a consumer considers goods 1 and 2 as imperfect substitutes in consumption: the marginal utility of one good declines with more consumption of the other. The goods are independent when $d = 0$, and become more substitutable when d augments.

The first-order conditions directly result in the consumer's inverse demand functions

$$\begin{aligned} p_1(q_1, q_2) &= 1 - q_1 - dq_2 \\ p_2(q_1, q_2) &= 1 - q_2 - dq_1. \end{aligned} \tag{1}$$

By inversion, the demand system is readily obtained as

$$\begin{aligned} q_1(p_1, p_2) &= \frac{1}{1-d^2}[(1-d) - p_1 + dp_2] \\ q_2(p_1, p_2) &= \frac{1}{1-d^2}[(1-d) - p_2 + dp_1] \end{aligned} \tag{2}$$

[9] See Vives (1985) and Singh and Vives (1984) for a more general specification. For the purposes of our analysis, this simpler specification has the advantage of keeping the analysis tractable while at the same time preserving the main qualitative insights.

as long as quantities demanded for both commodities remain positive.[10] Note that our assumption on the parameter d implies positive and symmetric cross-price effects between both goods. One easily shows, using (2), that higher values of d yields larger cross-price effects.

We consider a duopolistic industry where one firm offers good 1 and the other firm sells good 2. Let each firm i maximize its profits, with $i = 1, 2$. Assume zero production costs to simplify the analysis. Firms receive revenues from *two* sources. First, they charge a price p_i (admission fee, ticket price, etc.) for their goods or services. Second, they receive funding from an outside source which, for purposes of concreteness, we will assume to be a government subsidy. As a result, profit of firm i is given by

$$\pi_i = p_i q_i + S_i,$$

where S_i denotes the subsidy firm i receives.

We study two subsidy systems[11]. In the first one, the sponsoring organization provides a pre-determined fixed subsidy γ per unit of output. The total subsidy received by each firm *i* then equals

$$S_i = \gamma q_i. \tag{3}$$

Under the second system we assume that the sponsoring organization has a given fixed amount β available for funding the industry. The available budget β has been determined ex ante by, for example, an exogenous political or budgetary process. In our set-up, the firms in the sector receive a fraction of β according to their respective market shares. Accordingly, firm *i* receives a subsidy of

$$S_i = \beta \frac{q_i}{q_1 + q_2}. \tag{4}$$

[10] When firm *j* charges a price that is too high, firm *i*'s demand shows a kink at some critical price of firm *j* for which *i* has a monopoly position (see Deneckere (1983) and Singh and Vives (1984) for a complete characterization). In this paper, we do not consider firm *i*'s incentives to exclude firm *j* and become a monopolist when subsidies are available.

[11] For purposes of concreteness, we will frame the model in terms of different types of government subsidies. However, the analysis is more generally applicable to other settings where funds are allocated to competing organizations according to particular allocation rules.

3. Bertrand competition between subsidized firms: prices, output, and profits

In this section, we study the impact of different subsidy systems on market outcomes. We assume throughout this section that both firms maximize profit and compete in prices. As noted in the Introduction, it could be argued that some subsidized organizations may have different objectives than pure profit maximization. We return to this issue in Section 4.3 below.

3.1. Bertrand competition with a per unit subsidy

When the government provides a producer subsidy γ per unit, each firm maximizes its profit by

$$\max_{p_i} \pi_i = \max_{p_i} (p_i + \gamma) q_i .$$

Using (2), the resulting necessary and sufficient first-order condition is given by

$$1 - d - p_i + dp_j = (p_i + \gamma), \tag{5}$$

where $j = 1, 2$ and $i \neq j$. Firm *i*'s reaction function can be written as

$$\begin{aligned} p_i^o &= 0.5(1 - d - \gamma + dp_j) \quad \text{if } 0 \leq p_j < \overline{p}_j^o \\ p_i^o &= 0.5(1 - \gamma) \quad \text{if } p_j > \overline{p}_j^o \end{aligned} \tag{6}$$

where the superscript '*o*' refers to the per unit subsidy system and $\overline{p}_j^o$ is the cutoff price of firm *j* that results in firm *i* becoming a monopoly. Since we focus in what follows only on the case where both firms serve a positive share of the market, we assume throughout that $0 \leq p_j < \overline{p}_j^o$.

Reaction functions are upward sloping so that prices are strategic complements. Moreover, a higher degree of substitutability (a larger *d*) increases the slope and decreases the intercept of each firm's reaction curve. The total effect of increased substitutability is that firms optimally reduce their price for any given price set by their competitor. The slope of the reaction functions

$$\frac{\partial p_i^o}{\partial p_j} = \frac{d}{2} \tag{7}$$

lies between zero and one half, so that a stable Nash equilibrium is guaranteed. Solving the two reaction functions for prices, the (symmetric) Nash equilibrium looks like

$$p_i^{o*} = p_j^{o*} = \frac{1 - \gamma - d}{2 - d}. \tag{8}$$

In what follows, we focus only on values for d and γ such that (8) remains non-negative. [12]

When goods are independent ($d=0$) and there are no subsidies ($\gamma=0$), each firm charges the monopoly price of 0.5. Expression (8) further shows that equilibrium prices decrease when goods become better substitutes (higher d) or a higher per unit subsidy γ is provided. Finally, note that

$$\frac{\partial^2 p_i^{o*}}{\partial\gamma\partial d} = -\frac{1}{(2-d)^2} < 0.$$

Since the per unit subsidy reduces prices, it follows that the impact of the subsidy rises when goods become better substitutes.

Using (2) and (8), each firm's equilibrium quantity with a per unit subsidy equals

$$q_i^{o*} = q_j^{o*} = \frac{1+\gamma}{(1+d)(2-d)}. \tag{9}$$

Firms sell more when the per unit subsidy augments. The impact of higher substitution between goods (a higher d) is ambiguous in general. Increasing d reduces output for relatively modest substitutes ($d<0.5$), it raises output for good substitutes ($d>0.5$).[13]

Finally, using (8) and (9) we find that profit amounts to

$$\pi_i^{o*} = \pi_j^{o*} = \frac{(1-d)(1+\gamma)^2}{(1+d)(2-d)^2}. \tag{10}$$

Obviously, it increases with the size of the per unit subsidy.

3.2. Bertrand competition with market-share based funding

In the market-share based subsidy system that we consider, the sponsoring organization (e.g., the government) fixes its total subsidy budget β beforehand. Each firm competes for the

[12] That is, we assume throughout the analysis that $1-d \geq \gamma$. One could, of course, argue that in extreme cases subsidies may be so large that prices become negative. However, we believe that negative prices are not realistic for the industry examples that we have discussed in the Introduction.

[13] There are two effects when d rises. On the one hand, it follows from (8) that increasing d reduces equilibrium prices, but less so at high values of d; on the other hand, demand system (2) implies that it also makes demand more elastic. Combination of these two effects implies that the overall impact on demand is ambiguous a priori.

scarce resources and receives a fraction of the budget according to its market share.[14] The problem for firm *i* is then to

$$\max_{p_i} \pi_i = p_i q_i + \beta \left[\frac{q_i}{q_i + q_j} \right].$$

The first-order condition can be written as, using (2):

$$1 - d - 2p_i + dp_j - \beta \frac{q_j + dq_i}{(q_i + q_j)^2} = 0. \tag{11}$$

Substituting the demand functions (2) in (11), this expression implicitly defines the reaction function $p_i^c(p_j; \beta, d)$ for any price $p_j \in [0, \overline{p}_j^c]$ set by firm j, where the superscript c refers to the market-share based funding system. As before, we focus on equilibrium outcomes where both firms serve a positive market share, since any price equal or larger than $\overline{p}_j^c$ is the price of j that makes i a monopoly. The second-order condition for a maximum is satisfied when

$$-\beta(1-d)(q_j + dq_i) < (q_i + q_j)^3.$$

This inequality holds whenever $d < 1$, as we assumed.

From the first-order condition (11), we derive the slope of the reaction function as:

$$\frac{\partial p_i^c}{\partial p_j} = \frac{d(q_i + q_j)^3}{2(q_i + q_j)^3 + 2\beta(1-d)(q_j + dq_i)} > 0. \tag{12}$$

Therefore, as in the case of the per unit subsidy system, the reaction functions are upward sloping. Again, as $d < 1$, the slope is less than one, which guarantees a stable Nash equilibrium.

Interestingly, comparing (7) and (12), we see that a market-share based subsidy system implies less responsive reactions of firms to price changes by the competitor than a per unit subsidy system. Since $d < 1$, the slope of the reaction function in (12) is necessarily less than $0.5d$, the slope under the per-unit subsidy derived in (7). The intuition for this difference in price-responsiveness is that, under the market-share based system, a price increase by one firm imposes a positive externality on the competitor. To see this, we easily obtain, using (2), that

[14] As noted in the introduction, European soccer leagues, e.g., share part of their pre-determined broadcast revenues according to league position or TV appearances. Universities in Belgium receive an important part of their funding according to their market share in the number of (entering or passing) students, publications, and citations. Of course, other allocation rules could be used.

$$\frac{\partial\left(\frac{\beta}{(q_i+q_j)}\right)}{\partial p_i}=\frac{\beta}{(1+d)(q_i+q_j)^2}>0.$$

This positive effect of a price increase on the resulting subsidy expressed per unit, also raises each firm's incentive to increase its own price. By doing so, it sells less but at the same time raises the subsidy it receives per unit of demand. More importantly, however, a price increase by one firm raises the subsidy per unit for the competitor, *ceteris paribus*.[15] This windfall revenue gain induces a less pronounced reaction of firm *j* to the price increase by firm *i*. Note, by contrast, that a price increase under the per-unit subsidy system does not affect the level of the unit subsidy γ.[16]

Not surprisingly, using (11) we further find that raising β, the total budget available for subsidies, shifts the reaction functions downwards, or

$$\frac{\partial p_i^c}{\partial \beta}=-\frac{(q_j+dq_i)}{2(q_i+q_j)^2}<0. \qquad (13)$$

For any price charged by firm j, firm i optimally charges a lower price to benefit from the increased subsidy budget.

Note that the demand structure of our model implies a symmetric equilibrium with $q_i=q_j$. In any symmetric Nash equilibrium, $p_i^{c*}=p_j^{c*}=p^{c*}$, so that, after substitution in (1), we find that

$$\begin{aligned}&(1+d)q=(1-p^c)\\&2q=2(1-p^c)/(1+d).\end{aligned}$$

Substituting these results in the first-order condition as expressed in (11), the latter can be written as

[15] In fact, starting from symmetric market shares, one can show that a firm's unilateral price increase actually raises the total subsidy received by the competitor.

[16] Of course, in either funding system a price increase by firm i reduces both its market share and total demand. Formally, we have that $\frac{\partial(q_i+q_j)}{\partial p_i}<0$ and $\frac{\partial\left(\frac{q_i}{(q_i+q_j)}\right)}{\partial p_i}<0.$

$$1-d-(2-d)p-\beta\frac{(1+d)^2}{4(1-p)}=0.$$

Solving for p and choosing the only economically sensible root (the other root yields negative demand) yields the symmetric Nash equilibrium prices

$$p_i^{c*}=p_j^{c*}=\frac{(3-2d)-\sqrt{1+\beta(1+d)Z}}{2(2-d)} \tag{14}$$

where $Z\equiv(2-d)(1+d)$. Profit-maximizing quantities per firm are given by

$$q_i^{c*}=\frac{1+\sqrt{1+\beta(1+d)Z}}{2Z}. \tag{15}$$

Each firm's equilibrium profit equals

$$\pi_i^{c*}=p_i^{c*}q_i^{c*}+0.5\beta.$$

From equations (14) and (15), the profits for the market-share based subsidy system can be expressed as

$$\pi_i^{c*}=\pi_j^{c*}=0.5\beta+\frac{2(1-d)\left[1+\sqrt{1+\beta(1+d)Z}\right]-\beta(1+d)Z}{4(2-d)Z}. \tag{16}$$

Simple differentiation shows that an increase in the overall subsidy β raises firms' profits.

4. Unit subsidies versus market-share based funding: comparing efficiency and welfare

We noted above that, compared to a per unit subsidy, a market-share based funding system implies (i) lower price responsiveness to a price change by the competitor, and (ii) an augmented incentive for each player to increase its price. In this section, we provide a more detailed comparison of the two systems in terms of prices, output, profits, and welfare. We first compare the subsidies' relative efficiency in stimulating output with a given budget; next we proceed to a comparison of profits and welfare. In a final subsection, we discuss some extensions of the basic model.

4.1. Relative efficiency of the two subsidy systems

Anderson et al. (2001) propose two ways to evaluate the efficiency of different taxes or subsidies. First, a subsidy instrument is more efficient than another if the former yields a higher output for a given subsidy budget. Second, a funding system is more efficient than another when it reaches the same output with a lower budget.[17] To fix ideas, we focus on the first definition. We comment on the second one at the end of this sub-section.

We compare the funding subsidy system based on market shares with a per unit subsidy by assuming that the government decides to move from the former to the latter, while holding the total subsidy cost constant. Let the total cost of the subsidy under the initial market-share based system be β. Let us denote the per unit subsidy that accomplishes an unchanged total subsidy cost as $\hat{\gamma}$. In other words, this per unit subsidy must satisfy $\beta = 2\hat{\gamma}\hat{q}_i^o$, where $\hat{q}_i^o$ is the equilibrium quantity per firm that results from the per unit subsidy $\hat{\gamma}$; by symmetry, we have $\hat{q}_i^o = \hat{q}_j^o$.

Using the equilibrium quantities under a per unit subsidy system (see (9)) we have that:

$$\beta = 2\hat{\gamma}\left[\frac{1+\hat{\gamma}}{(1+d)(2-d)}\right].$$

Solving the resulting quadratic equation for $\hat{\gamma}$, we find that

$$\hat{\gamma} = \frac{-1+\sqrt{1+2\beta Z}}{2} \qquad (17)$$

where, as before, $Z \equiv (2-d)(1+d)$.[18] From (8), this subsidy per unit implies a Nash equilibrium price of

$$\hat{p}_i^{o*} = \frac{1-\hat{\gamma}-d}{2-d}.$$

Substituting in (17) and working out yields

$$\hat{p}_i^{o*} = \frac{3-2d-\sqrt{1+2\beta Z}}{2(2-d)}. \qquad (18)$$

The equilibrium quantity demanded per firm is, substituting (17) in (9), equal to

$$\hat{q}_i^{o*} = \frac{1+\sqrt{1+2\beta Z}}{2Z}. \qquad (19)$$

Finally, simple algebra shows that profit per firm is, using (17), (18) and (19), equal to

[17] Anderson et al. (2001) show the two definitions are equivalent under mild conditions.

[18] The other root involves a per-unit tax yielding a tax revenue of β.

$$\hat{\pi}_i^{o*} = (\hat{p}_i^{o*} + \hat{\gamma})\hat{q}_i^{o*} = \frac{(1-d)[1+\sqrt{1+2\beta Z}\,]^2}{4Z(2-d)}. \tag{20}$$

Using the above results, we are now in a position to directly compare the two subsidy systems under the maintained assumption that the total subsidy cost to the government is equal. A comparison of (18) with the price under the initial system as in (14), shows that

$$\hat{p}_i^{o*} - p_i^{c*} = \frac{\sqrt{1+\beta(1+d)Z} - \sqrt{1+2\beta Z}}{2(2-d)} < 0 \tag{21}$$

since $0 < d < 1$. Of course, comparing demands (see (19) and (15)) we find after simple algebra that $q_i^{c*} < \hat{q}_i^o$.

We have shown the following proposition.

Proposition 1: *The efficiency of a given budget β available for subsidies is higher for a per unit subsidy than for a market-share based subsidy; a per unit subsidy results in fiercer price competition and a larger output than a market-share based subsidy.*

When a given subsidy budget β is available, there exists an appropriately chosen subsidy $\hat{\gamma}$ per unit that fully exhausts the available budget. This unit subsidy $\hat{\gamma}$ results in lower consumer prices and a higher output as compared to the market-share subsidy system. The intuition for this result is as follows. A price increase under the per unit subsidy system reduces a firm's demand and total demand, but it has no effect on the subsidy per unit. In a market-share based subsidy system, when firm *i* changes its price by the same amount, its demand changes in a similar fashion. However, it also raises the subsidy per unit for both players, since fewer units are sold. This change in the subsidy per unit explains why firms are less inclined to set a lower price in a market-share based subsidy system as compared to the per unit funding system. This results in a higher equilibrium price and, consequently, a lower total output.

Finally, following Anderson et al. (2001), in Appendix 1 we show a corollary to the result of this section, viz. that a smaller total subsidy is needed to generate the same output effect under a per unit subsidy than under a subsidy based on market shares.

4.2. Welfare comparison of per-unit and market-share based subsidies

In this subsection, we compare the welfare effects of the two subsidy systems, taking account of consumers' surplus and firms' profits. The cost of the subsidy to the government can be ignored in the comparison as it is, by assumption, equal in both cases.

As the per unit subsidies imply lower prices, consumers will be better off. This is confirmed by calculating consumers' surplus. This is easily shown to equal

$$C_s = (1+d)q^2$$

for both subsidy systems, where q is the output level per firm. Using (19) and (15), the difference in total consumer surplus on the market can, after simple algebra, be written as

$$\hat{C}_S^{o*} - C_S^{c*} = \left[\frac{1+d}{4Z^2}\right]\left\{2\left[\sqrt{1+2\beta Z} - \sqrt{1+\beta(1+d)Z}\right] + \beta Z(1-d)\right\}. \tag{22}$$

This is necessarily positive, since $d < 1$. Straightforward algebra shows that it increases in the size of the subsidy budget β.

Next, we compare profits under the two systems, holding subsidies constant. The profit difference per firm can be written as

$$\hat{\pi}_i^{o*} - \pi_i^{c*} = \hat{p}_i^{o*}\hat{q}_i^{o*} - \hat{p}_i^{c*}\hat{q}_i^{c*}.$$

Noting that $\beta = 2\hat{\gamma}\hat{q}_i^o$, using previous results and working out, we find that

$$\hat{\pi}_i^{o*} - \pi_i^{c*} = \frac{(1-d)\left\{2\left[\sqrt{1+2\beta Z} - \sqrt{1+\beta(1+d)Z}\right] - \beta Z\right\}}{4(2-d)Z} \quad . \tag{23}$$

The profit difference tends to zero when goods become very close substitutes (d approaching 1) or when the subsidy is zero (β=0). Moreover, in Appendix 2 we show that, for positive subsidies and $0 \le d < 1$, the profit difference (23) is necessarily negative. This result implies that the market-share based subsidy yields higher profits for firms at all positive subsidy levels and imperfect substitution between goods.

Finally, to compare total welfare (denoted W), let us define welfare as the sum of consumer surplus and sector profits (minus the cost of the subsidy to the government, which is equal by assumption). Taking into account that profits were calculated per firm, we find that

$$\hat{W}^o - W^c = \frac{(1+d)}{4Z^2}\left\{\left[\sqrt{1+2\beta Z} - \sqrt{1+\beta(1+d)Z}\right]\left[2(3-2d)\right] - \beta Z(1-d)\right\}. \tag{24}$$

A simple numerical example will be instructive to understand the sensitivity of the results to parameter values. In Table 1 we report, for various combinations of (d, β), equilibrium outcomes under the per unit subsidy and the market share based system, respectively. Specifically, we provide information on prices and quantities, and on the difference in total consumer surplus, total industry profit and overall welfare between the two systems.

The results suggest that the price difference between the two systems rises strongly in the total size of the subsidy. Although prices decline substantially when goods become better substitutes (larger d), the price difference between subsidy systems appears to be rather insensitive to variations in d. Consumers are always better off under the per unit system. The difference in consumer surplus rises in the subsidy but declines in d. For all parameter combinations considered, the allocation based on market shares yields higher profit for firms; the profit advantage is more pronounced at higher subsidy levels and when goods are better substitutes. Finally, assuming positive prices, overall welfare is always higher under the per unit subsidy. The difference is most important for large subsidies and when goods are not too close substitutes.

Summarizing our findings, we have shown that, moving from a system based on allocation according to market shares to a per unit subsidy, where the total budgetary cost is kept constant at the initial level, leads to a lower price, more demand, and higher net consumer surplus. The unit subsidy is, therefore, more efficient in the sense of Anderson et al. (2001a,b): the same total subsidy yields more output. The unit subsidy system is preferred by consumers, whereas producers prefer the system based on market shares. Overall welfare is higher for the per unit subsidy; the difference rises in the size of the subsidy and declines when goods are better substitutes. We have the following proposition.

Proposition 2: *For a given available budget for subsidies, consumers prefer the per unit price subsidy to the market-share based subsidy system. Firms have higher profits under the allocation system according to market share. Overall welfare is higher for per unit price subsidies.*

Our results suggest that if there is a fixed government budget, and the social concern for consumer participation or output is the most important objective, the per unit subsidy is to be favored. An important example may include the public promotion of educational participation programs.[19] If, however, the main concern is to convince firms to participate, the market-share based system seems to be most effective, as this generates the highest increase in producer surplus. For example, suppose the government wants to introduce stricter and more costly production standards that favor product safety or reduce the impact on the environment. Producers will be more willing to invest in costly effort under a market-share based subsidy system since their marginal profits increase more.[20]

[19] Clearly, if participation is of so much importance, a government may find it optimal to impose *mandatory* participation, like it very often does for primary and secondary education. In that event, our two alternative subsidy systems coincide. Any compulsory participation would, by construction, result in identical participation and have no effect on the market size. For example, in a simple Hotelling model with two firms at opposing ends of the unit interval, either firm would attract half of the fixed number of consumers under both subsidy systems. With identical budget availability, this results in identical consumer prices. Our set-up, therefore, is only relevant when no mandatory participation is imposed, as in the case of higher education.

[20] Suppose the increased environmental-friendly production standard requires a fixed expenditure of F_e. Firms are only willing to introduce this new standard under the market–share based system whenever $\hat{\pi}_i^{o*}(\hat{\gamma}) - \hat{\pi}_i^{o*}(0) < F_e < \pi_i^{c*}(\beta) - \pi_i^{c*}(0)$.

d	β	$\hat{p}_i^{o*}$	$\hat{q}_i^{o}$	p_i^{c*}	q_i^{c*}	$\hat{C}_S^{o*} - C_S^{c*}$	$\hat{\pi}_i^{o*} - \pi_i^{c*}$	$\hat{W}^o - W^c$
0	**0**	0.500	0.500	0.500	0.500	0	0	0
0	**0.1**	0.454	0.546	0.476	0.524	0.023	-0.003	0.020
	0.3	0.379	0.621	0.433	0.566	0.065	-0.02	0.044
	0.5	0.317	0.683	0.396	0.604	0.102	-0.046	0.057
0.1	**0.1**	0.423	0.524	0.445	0.505	0.022	-0.005	0.017
	0.3	0.342	0.589	0.395	0.55	0.061	-0.025	0.036
	0.5	0.274	0.660	0.375	0.590	0.095	-0.052	0.043
0.5	**0.1**	0.265	0.489	0.281	0.480	0.015	-0.009	0.006
	0.3	0.156	0.563	0.194	0.537	0.042	-0.033	0.009
	0.5	0.066	0.623	0.120	0.587	0.068	-0.059	0.009
0.9	**0.1**	0.004	0.524	0.008	0.522	0.006	-0.004	0.002
	0.3	<0		<0				
	0.5	<0		<0				

Table 1: numerical welfare comparison between the per unit and market-share based subsidy system.

4.3. Extensions of the basic model

We conclude the analysis of this section with a brief discussion of two extensions of the basic model.

First, we assumed throughout Sections 3 and 4 that both organizations maximized profit. However, as noted in the introduction, several authors have argued that many subsidized organizations, such as schools and theatres, are unlikely to be pure profit-maximizers. Moreover, the recent literature on corporate social responsibility (see, e.g., Bénabou and Tirole (2010)) casts similar doubts as to the validity of this assumption. We therefore extended the model assuming that the organizations not only care about profit but also attach some importance to the surplus generated for their customers. Not surprisingly, this substantially complicates the comparison of the two subsidy systems.

In Appendix 3, we derive the equivalent of expression (21), the price difference that was the driving force behind many of our results. We show that the sign of

$$\hat{p}_i^{o*} - p_i^{c*}$$

is no longer unambiguous. It is increasing in *d* and declining in β. Most importantly, it increases in the weight (denoted by λ) the firm gives to consumer surplus. Simple numerical simulations (similar to those reported in Table 1) confirmed that, for given substitutability and a given subsidy budget, the price difference will become positive at a sufficiently high value for λ. As a consequence, we find that the market-share based system will perform better than the per unit subsidy if the weight of consumer surplus is sufficiently large.

Why this is the case can be understood as follows. Intuitively, a higher weight associated with consumer surplus implies that, conditional on a given per unit subsidy, the price goes down and the quantity consumed increases (see expression (A3.2) in Appendix 3 for a formal proof). Since we are comparing both subsidy systems for a given subsidy budget β, this implies that the per unit subsidy $\hat{\gamma}$ that yields the same budgetary cost will necessarily decline. However, a lower per unit subsidy drives up the price $\hat{p}_i^{o*}$ (see (8)), worsening the relative performance of the per unit subsidy system. In other words, the conclusion that the per unit subsidy system outperforms the market-share based system has to be qualified when subsidized organizations care substantially about consumer surplus.

Second, we extended the model to see whether anything can be said about the welfare optimal subsidies under each of the two systems. In Appendix 4 we briefly study this issue, assuming that the government maximizes a particular welfare function that takes into account the firms' profits, the surplus of consumers, and the cost of the subsidy to the government; this cost is weighted by a 'cost of funds'-parameter that reflects the shadow price of collecting government funds through distortionary taxation. As expected, we find that both the optimal unit subsidy γ and the optimal global subsidy β to be allocated according to market share are declining functions of the cost of funds to the government; more substitutability (a higher *d*) also reduces the optimal subsidies. In the case of the per unit subsidy, we reproduce the highly intuitive result that, if the cost of funds equals one, the optimal subsidy pushes down Nash equilibrium prices to zero. This is indeed the first-best outcome: as our model assumed zero marginal production cost for the firm and there are no externalities, the optimal price is zero[21].

5. Subsidy systems and incentives to collude.

The previous section focused on the welfare effects of different types of subsidies to competing firms. In this section, we study a different question that relates to long-run differences in industry behavior. Specifically, we compare firms' incentives to collude under the two subsidy systems. This is relevant, because short-run advantages of a particular system may have to be traded-off against long-run and potentially undesirable implications on industry conduct.

To find out under which subsidy system collusion is more likely, we return to profit maximizing behavior, and we assume that firms make use of grim trigger strategies. Each firm compares the discounted stream of its profits under collusion with its profits from deviating (in the sense of undercutting its rival, who is assumed to stick to the collusive price), plus all future profits if the rival retaliates. The game reverts to the static Nash equilibrium outcome for all future periods after one of the players deviated from the collusive outcome. Assuming a common discount factor $0 < \delta < 1$, collusion is beneficial to an arbitrary firm whenever[22]

$$\frac{\pi^{coll}}{1-\delta} \geq \pi^{dev} + \frac{\delta}{1-\delta}\pi^{NE}$$

where $\pi^{coll}, \pi^{dev}, \pi^{NE}$ are, respectively (i) each firm's profits when all firms respect the collusive arrangement (i.e. π^{coll} is half the industry monopoly profit), (ii) the profits when the firm deviates and undercuts its rival, and (iii) the profits in the Nash equilibrium. Rearranging, we have that the condition

$$\delta \geq \frac{\pi^{dev} - \pi^{coll}}{\pi^{dev} - \pi^{NE}}$$

is sufficient for collusion to arise. We now study how subsidies affect the incentives to collude.

5.1. The per unit subsidy system

When firms collude under the per unit system, the profit maximization problem reduces to

[21] Unfortunately, direct comparison of the optimal welfare levels evaluated at the optimal subsidies under the two subsidy systems is not straightforward. For more details, see Appendix 4.

[22] In this section, we drop firm-specific subscripts to simplify notation.

$$\text{Max}_{p^o} \quad 2(p^o + \gamma)\frac{1}{1-d^2}[(1-d) - p^o + dp^o].$$

The collusive price and quantity per firm are easily shown to be

$$p^o_{coll} = \frac{1-\gamma}{2} \text{ and } q^o_{coll} = \frac{1+\gamma}{2(1+d)} \ . \tag{25}$$

Substituting and working out shows that the total collusive profit per firm equals

$$\pi^o_{coll} = \frac{(1+\gamma)^2}{4(1+d)}. \tag{26}$$

When a firm deviates from the collusive agreement and undercuts its rival, it solves

$$\text{Max}_{p^o_{dev}} \quad \pi = (p^o_{dev} + \gamma)\frac{1}{1-d^2}[(1-d) - p^o_{dev} + dp^o_{coll}]$$

where the rival firm charges the collusively agreed price p^o_{coll}. The optimal price is given by

$$p^o_{dev} = \frac{2(1-\gamma) - (1+\gamma)d}{4}$$

wherefrom profits can be written as

$$\pi^o_{dev} = \frac{[(1+\gamma)(2-d)]^2}{16(1-d^2)}. \tag{27}$$

Finally, Nash equilibrium profit was previously derived in (10); it reads

$$\pi^o_{NE} = \frac{(1-d)(1+\gamma)^2}{(1+d)(2-d)^2}. \tag{28}$$

We want to find out how an increase in the subsidy γ affects the condition

$$\delta \geq \frac{\pi^o_{dev} - \pi^o_{coll}}{\pi^o_{dev} - \pi^o_{NE}} \equiv \delta^o$$

for collusion. Substitution of (26), (27) and (28) in the inequality for the discount rate immediately shows that the critical discount factor δ^o only hinges on the degree of product differentiation d , and is independent of γ. With hindsight, this is no surprise, as the subsidy acts like a profit-increasing reduction in the constant marginal cost for both firms.[23] A per unit subsidy system has, therefore, no effect on the likelihood of collusive arrangements among firms.

5.2. The market-share based subsidy system

The total government budget is fixed and denoted by β. Each firm gets a fraction of the budget according to its market share. When the firms collude, the profit maximization problem for an arbitrary firm reduces to

$$\text{Max}_p \quad 2pq(p) = \frac{2p}{1-d^2}[(1-d) - p + dp] + \beta .$$

The optimal price and profit (for each firm) are denoted as

$$p^c_{coll} = \frac{1}{2} \text{ and } \pi^c_{coll} = \frac{1}{4} + \frac{\beta}{2}, \tag{29}$$

respectively. Clearly, as with monopoly, the price is now independent of the subsidy, which has a lump sum character when the firms collude.

Denote by π^c_{NE} the Nash equilibrium profit, and let π^c_{dev} be the profit a firm receives when it optimally deviates from the collusive price. We want to find out how the subsidy β affects the condition

$$\delta \geq \frac{\pi^c_{dev} - \pi^c_{coll}}{\pi^c_{dev} - \pi^c_{NE}} \equiv \delta^c .$$

Differentiating δ^c with respect to β, we have

$$\frac{\partial \delta^c}{\partial \beta} = \left[\frac{1}{\pi^c_{dev} - \pi^c_{NE}}\right]^2 \left\{ \left(\pi^c_{dev} - \pi^c_{NE}\right)\left(\frac{\partial \pi^c_{dev}}{\partial \beta} - \frac{\partial \pi^c_{coll}}{\partial \beta}\right) - \left(\pi^c_{dev} - \pi^c_{coll}\right)\left(\frac{\partial \pi^c_{dev}}{\partial \beta} - \frac{\partial \pi^c_{NE}}{\partial \beta}\right)\right\}. \tag{30}$$

To analyze the sign of $\frac{\partial \delta^c}{\partial \beta}$, we first show that

$$\frac{\partial \pi^c_{coll}}{\partial \beta} = 0.5; \qquad \frac{\partial \pi^c_{dev}}{\partial \beta} = \frac{q^c_{dev}}{q^c_{dev} + q_{coll}} > 0.5; \qquad \frac{\partial \pi^c_{NE}}{\partial \beta} = 0.5 . \tag{31}$$

The first expression follows from differentiating (29). The second and the third result are obtained by noting that

[23] To see why incentives to collude are independent of γ, note that our model implies that $\pi^o_{dev} \equiv \rho \pi^o_{coll}$ and $\pi^o_{NE} \equiv \sigma \pi^o_{coll}$, where ρ and σ are coefficients that are independent of the unit subsidy, and it is easily shown that $0 \leq \sigma \leq 1 \leq \rho$. Since marginal costs are constant, the critical δ is independent of the unit subsidy.

$$\frac{\partial \pi_k^c}{\partial \beta} = \frac{\partial \pi_k^c}{\partial \beta}\bigg|_{p_k^c} + \frac{\partial \pi_k^c}{\partial p_k^c}\frac{\partial p_k^c}{\partial \beta} \qquad \text{with } k = dev, NE.$$

The first term on the right hand side equals the firm's market share, and the second term on the right hand side equals zero by the first-order condition for profit maximizing behavior. Using these insights produces the two final results in (31).

Substituting the derivatives (31) into (30), it then easily follows that

$$\frac{\partial \delta^c}{\partial \beta} = \left[\frac{1}{\pi_{dec}^c - \pi_{NE}^c}\right]^2 \left\{\left(\frac{q_{dev}^c}{q_{dev}^c + q_{coll}^c} - \frac{1}{2}\right)\left(\pi_{coll}^c - \pi_{NE}^c\right)\right\}.$$

Since the deviating firm has a market share exceeding 50% and collusion yields higher profit than the Nash equilibrium, it follows that

$$\frac{\partial \delta^c}{\partial \beta} > 0$$

for all $\beta > 0$. Put differently, the cut-off discount rate that makes collusion profitable is increasing in the amount available for subsidizing the sector. The incentives to collude therefore decline when the available subsidy budget rises.

Of course, in the special case without subsidies ($\beta = 0$), all results for the per unit and market-share based subsidy systems coincide. Importantly, the conditions for collusion do not depend on the size of the subsidy for the per unit system. However, collusion under the market-share based system becomes more difficult when the available subsidy to the industry augments. Therefore, we can conclude that the per unit system offers more possibilities to successfully collude for all positive subsidy budgets. We have the following proposition.

Proposition 3: *When the total subsidy budget increases, the incentives to deviate augment under a market-share based funding system; hence, the incentives to collude are a declining function of the subsidy available for the sector. On the contrary, the level of the per unit subsidy does not affect firms' incentives to collude. Therefore, the market-share based subsidy system makes collusion harder as compared to the per unit system.*

The intuition for this result is easy to grasp. Observe that the collusive price is independent of the total subsidy β. As a result, when firms charge the collusive price, an increase in β does not result in a positive total market effect so that each firm's demand remains constant as well. Consequently, more subsidies necessarily lead to an increase in profit by $0.5\,\Delta\beta$ for both firms. However, when a firm deviates, the deviating firm reduces its price so that its market share rises above one half. An increase in the total subsidy therefore makes deviating behavior relatively more attractive as compared to collusion. Hence, larger subsidies reduce the incentives for collusion.

Our result suggests that raising the subsidy in a market-share based subsidy system reduces the potential for collusive behavior. If we take the zero subsidy case as starting point (or consider a move from a per unit to a market-share based system whereby the subsidy rises) then this suggests that a market-share based system hampers the potential for collusion. Alternatively, we showed before that, holding market demand constant, a market-share based system raises the total subsidy cost compared to a per unit system. Hence, there is less potential for collusion in a market-share based system.

Proposition 3 tells us that a market-share based system makes collusion less probable, since $\delta^c \geq \delta^o$ for any given subsidy budget $\beta \geq 0$. However, this does not necessarily imply that it leads to lower welfare than the per unit system; this depends on the collusive price levels resulting from the two subsidy schemes.[24] To analyze this further, two cases can be distinguished. First, whenever $\delta \geq \delta^c \geq \delta^o$, collusion will be sustained under both systems. In that case, the per unit subsidy system outperforms the market-share based system, since $p^c_{coll} = \frac{1}{2} > p^o_{coll} = \frac{1-\gamma}{2}$ for any positive subsidy. Second, when $\delta^c \geq \delta \geq \delta^o$ collusion will occur only under the per unit system. However, despite collusive behavior, this system will be socially preferred to the market-share based system provided that $p^c_{NE} > p^o_{coll}(\hat{\gamma})$. The equilibrium price under the market-share based system p^c_{NE} is given by (14), and the collusive price under the per unit subsidy is $p^o_{coll}(\hat{\gamma}) = \frac{1-\hat{\gamma}}{2}$, with $\hat{\gamma}$ given by (17). Comparison of the two prices reveals that the per unit subsidy system yields higher welfare than the market-share based system whenever

[24] We are grateful to the referees for this important remark.

$$2\left[\sqrt{1+2\beta Z}-\sqrt{1+\beta(1+d)Z}\right]>d\left[1+\sqrt{1+2\beta Z}\right].$$

If the two goods are independent (*d=0*) this inequality necessarily holds; for perfect substitutes (*d=1*), the condition never holds. This suggests that the per unit subsidy system is welfare-superior at low substitution levels, despite collusive behavior. Intuitively, if the goods are very poor substitutes, the difference between competitive and collusive prices under the per unit subsidy system are very small; in fact, for (*d=0*), they coincide (compare (8) and (25)). Welfare losses due to collusive behavior are therefore very small. When the services offered by the two firms are very good substitutes, the welfare effects of collusion are much larger; in that case the market-share based system may be welfare-superior.

5.3. Policy implications

Combining our findings of the previous sections leads to clear policy implications In Sections 3 and 4 we showed that, in the short run, the consumer pays higher prices under a market-share based system, and overall welfare is lower than with a per unit subsidy (a short-term welfare loss). However, in Section 5 we found that the allocation of subsidies based on firms' market shares implies lower incentives to collude. This implies that a potential trade-off may arise between short-run output and welfare effects on the one hand, and the long-run welfare implications of industry conduct on the other hand.

We found that the per unit subsidy system may be preferable from a short-run perspective of yielding lower prices and higher welfare, while at the same time the market-share based system may lead to higher long-run welfare once the implications on collusive behavior are taken into account. This will be the case when conditions are such that collusion occurs in the per unit system only, and the collusive price under the per unit subsidy system is higher than the competitive price under the market-share based subsidy. We showed that this is likely to be the case if the goods are very good substitutes. Then a per unit subsidy system performs better at increasing participation in the short-run (higher output), but it goes at the long-run expense of higher potential collusion between firms in the sector.

The potential conflict does not always arise, however. Whenever conditions are such that collusion is sustainable under both subsidy systems, the per unit system certainly performs better, since its competitive and collusive price are both lower than in the market-share based system. Moreover, the per unit subsidy system also remains welfare-superior -- both in the short-run and taking account of the incentives to collude -- whenever collusion occurs in the per unit system only, but the collusive price under the per unit subsidy system is lower than the competitive price under the market-share based system. We showed that this will be the case if the goods are very poor substitutes. In that case the per unit system is to be preferred, despite the higher incentives to collude.

6. Conclusions

Governments often offer subsidies to competing firms or organizations. Prominent examples can be found in, among others, the arts and the education industry. This paper has studied the implications of different, commonly observed subsidy systems on the strategic behavior of firms in the industry. We analyzed a stylized model with two price-competing, subsidized firms offering a differentiated product. Two existing subsidy systems were considered. In the per unit system, the government provides a per unit subsidy that is known by firms ex ante. In the alternative market-share based system, a fixed subsidy is available for the sector as a whole, and the allocation rule is based on firms' market shares in industry output.

We have shown that, holding the total subsidy budget constant, a per unit subsidy results in fiercer price competition than a market-share based subsidy. As a result, it generates a larger market output and, therefore, it is more effective at stimulating wider participation. Firms' profits are higher under the market-share based system. Overall welfare, however, is necessarily higher under the per unit subsidy. Finally, a market-share based subsidy makes collusive behavior between firms more difficult than a per unit subsidy. Our results, therefore, may imply a potential trade-off between short-run and long-run objectives of governments. For example, subsidy systems that are better suited to stimulate participation may also provide more incentives for firms to collude. We showed that this trade-off is likely to occur if the firms offer highly substitutable goods. However, policy makers should certainly prefer the per unit system when both its competitive and collusive price are lower. This happens when (i) collusion is likely to take place under either subsidy system, or (ii) collusion only occurs under the per unit system but at a price level lower than the competitive price of the market-share based system; this is likely to be the case when the goods considered are poor substitutes.

Our model was deliberately simple, and a number of extensions can be thought of. First, the specification of consumer utility was quite restrictive. Although it highlighted the role of substitutability between goods in a highly transparent way, more realistic specifications might provide additional information on, for example, the role of explicit differences in preferences for one of the goods (for example, consumers may have intrinsic preferences in favor of a particular theatre or university). Second, the model could be extended to an oligopolistic market structure with more than two firms; however, this exercise is unlikely to affect the main insights.[25] Third, it has been argued above that organizations may compete in quality as well as price. Incorporating quality competition seems a useful extension, although initial efforts along these lines suggest that transparent analytical results may be hard to derive. Finally, our model implies that, when firms strategically interact, the design of subsidy schemes matters for both consumers and producers in a conflicting way. A useful extension would be to incorporate subsidy schemes in a more elaborate political economy model in which all interest groups lobby for one or the other subsidy system. This might explain the variety of different systems observed in practice.

[25] Another extension is to consider Cournot competition. In the working paper version of this paper, we show that our result that a market-share based subsidy system gives lower participation and, therefore, lower total welfare is maintained when firms compete in a Cournot fashion with homogeneous goods. Likewise, the potential for collusion decreases in the market-share based subsidy system when the available subsidy budget increases, while the incentives to collude are invariant towards budget size when subsidies are granted on a per-unit basis.

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Appendix 1

In this appendix we consider, for completeness sake, a move from a market-share based to a per unit subsidy system in which the user price (and hence demand) is kept constant. As we focus on symmetric solutions, we delete firm-specific subscripts. Denote the subsidy, the price and demand after the move to the per unit system as $\bar{\gamma}, \bar{p}, \bar{Q}$, respectively. We then require:

$$\bar{p} = p^* \quad -> \quad \frac{1-\bar{\gamma}}{2-d} = \frac{(3-2d)-\sqrt{1+(1+d)Z}}{2(2-d)(1-d)} \quad . \qquad \text{(A1)}$$

Solving leads to

$$\bar{\gamma} = \frac{-1+\sqrt{1+(1+d)Z}}{2(1-d)}. \qquad \text{(A2)}$$

This unit subsidy implies the same price and the same demand ($\bar{p} = p^*, \bar{Q} = Q^*$, respectively) that we had under the market-share based system. However, it implies a lower budgetary cost to the government. To see this, note that the total cost is $2\bar{\gamma}\bar{Q}$. Using (A2) and (9) we find:

$$2\bar{\gamma}\bar{Q} = \frac{(1+d)Z}{2(2-d)(1-d)}.$$

Using the definition of *Z*, we have

$$2\bar{\gamma}\bar{Q} = \frac{\beta(1+d)}{2}.$$

Since $d < 1$ we have that $2\bar{\gamma}\bar{Q} < \beta$.

Hence, the unit subsidy is more efficient in the sense of Anderson et al (2001). This is just a collorary of the result we showed above (higher output and hence consumer surplus for given subsidy cost). The intuition is clear. The unit subsidy is more efficient in stimulating demand, so that a smaller subsidy is needed to generate the same demand effect as the subsidy according to market share. Note that, using (17) and (A2), we also see that $\hat{\gamma} > \bar{\gamma}$.

Appendix 2

In this appendix, we show that the market-share based system necessarily yields higher profits for firms for all positive subsidies and $0 \le d < 1$. To see this, differentiate (23) with respect to β to find:

$$\frac{\partial(\hat{\pi}_i^{o*}-\pi_i^{c*})}{\partial\beta}=\left[\frac{1-d}{4(2-d)}\right]\left[\frac{2}{\sqrt{1+2\beta Z}}-\frac{(1+d)}{\sqrt{1+\beta(1+d)Z}}-1\right].$$

The sign of the final term between brackets on the right-hand side is the same as the sign of

$$\left[2\sqrt{1+\beta(1+d)Z}-(1+d)\sqrt{1+2\beta Z}-(\sqrt{1+\beta(1+d)Z})(\sqrt{1+2\beta Z})\right].$$

Reformulation gives

$$\left[\sqrt{1+\beta(1+d)Z}-\sqrt{1+2\beta Z}\right]+\left[\sqrt{1+\beta(1+d)Z}\left(1-\sqrt{1+2\beta Z}\right)\right]-\left[d\sqrt{1+2\beta Z}\right].$$

This is necessarily negative, given $d<1$. Therefore, raising the subsidy necessarily reduces the right-hand side of (23). As the profit difference is zero at zero subsidies this implies that the market-share based subsidy system yields higher profits for firms for all positive subsidies and less than perfect substitution between goods.

Appendix 3

In the main body of the paper, we assumed that the organization maximized profit. In this appendix, we extend the firms' objective functions. Specifically, we assume that firms not only care about profit, but they also attaches some importance to the surplus generated for consumers.

First reconsider the case of a per unit subsidy. Given a subsidy γ, we write the objective function as

$$(p_i+\gamma)q_i+\lambda\left[\int_0^{q_i}p_i(q_i,q_j)dq_i-p_iq_i\right].$$

The first term is profit, the second term is net consumer surplus weighted by a factor λ. The larger λ, the more important is net consumer surplus relative to profit.

Using demand specification (2), it easily follows that the optimization problem facing firm *i* can be rewritten as

$$\max_{p_i}\quad (p_i+\gamma)q_i+0.5\lambda(q_i)^2.$$

Following the same steps as in Section 3.1, and assuming both firms serve a positive share of the market, the reaction function for firm *i* can be shown to read

$$p_i^o=\frac{(1-d^2)\left(1-d-\gamma+dp_j\right)-\lambda\left(1-d+dp_j\right)}{2(1-d^2)-\lambda}.$$

Solving the two reaction functions and assuming symmetry yields the following Nash equilibrium prices

$$p_i^{o*} = p_j^{o*} = \frac{(1+d)(1-d-\gamma)-\lambda}{(2-d)(1+d)-\lambda}. \qquad \text{(A3.1)}$$

For $\lambda = 0$, this result just reproduces (8). Moreover, simple differentiation shows that Nash equilibrium prices are declining in λ. For future reference we note that, using (2) in the main body of the paper, Nash equilibrium quantities are given by

$$q_i^{o*} = q_j^{o*} = \frac{1+\gamma}{Z-\lambda} \qquad \text{(A3.2)}$$

where, as always, $Z = (2-d)(1+d)$.

Next take the case of market-share based funding. Given a subsidy budget β, the objective function for the firm can be written as

$$\max_{p_i} \quad p_i q_i + \beta \frac{q_i}{q_i + q_j} + 0.5\lambda (q_i)^2.$$

Following the same steps as in Section 3.2, and assuming a symmetric Nash equilibrium, straightforward algebra produces the extended version of (14); the Nash equilibrium prices are given by

$$p_i^{c*} = \frac{(3-2d)-\lambda-\sqrt{\lambda(2+\lambda)+1+\beta(1+d)Z}}{2(2-d)}. \qquad \text{(A3.3)}$$

This reduces to (14) when the weight of consumer surplus is zero; moreover, Nash equilibrium prices are again declining in λ.

Reconsider, then, the comparison of the two subsidy systems. Suppose the total available subsidy budget is constant and given by β. The Nash equilibrium prices are then given by (A3). How do these prices compare to those under a system of per unit subsidies, holding the total cost of the subsidies constant at β? To find out, we first derive the Nash equilibrium prices that will be observed under a per unit subsidy system with a total cost of β. Denote the per unit subsidy that generates the same total subsidy cost β in the per unit system by $\hat{\gamma}$; in other words, $\beta = 2\hat{\gamma}\hat{q}_i^o$, where $\hat{q}_i^o$ is the quantity demanded in the per unit system if the subsidy is $\hat{\gamma}$. Using (A3.2), it immediately follows that

$$\beta = 2\hat{\gamma}\left(\frac{1+\hat{\gamma}}{Z-\lambda}\right).$$

Solving this expression for $\hat{\gamma}$ leads to

$$\hat{\gamma} = \frac{-1+\sqrt{1+2\beta(Z-\lambda)}}{2}.$$

Observe that the per unit subsidy is a declining function of the weight λ associated with net consumer surplus. Finally, using (A3.1) we find that this subsidy $\hat{\gamma}$ gives Nash equilibrium prices

$$\hat{p}_i^{o*} = \frac{(1+d)\left[3-2d-\sqrt{1+2\beta(Z-\lambda)}\right]-2\lambda}{2(Z-\lambda)}.$$

As should be the case, this boils down to (18) when $\lambda = 0$.

We want to compare the prices $\hat{p}_i^{o*}$ and p_i^{c*}. We have

$$\hat{p}_i^{o*} - p_i^{c*} = \left[\frac{(1+d)\left[3-2d-\sqrt{1+2\beta(Z-\lambda)}\right]-2\lambda}{2(Z-\lambda)}\right] - \left[\frac{(3-2d)-\lambda-\sqrt{\lambda(2+\lambda)+1+\beta(1+d)Z}}{2(2-d)}\right].$$

This can be reformulated after simple algebra as

$$\hat{p}_i^{o*} - p_i^{c*} = \left[\frac{1}{2(Z-\lambda)(2-d)}\right]* \left\{(Z-\lambda)\left[\lambda+\sqrt{\lambda(2+\lambda)+1+\beta(1+d)Z}\right]-\left[\lambda+Z\sqrt{1+2\beta(Z-\lambda)}\right]\right\}.$$

Of course, this expression boils down to (21) if the weight given to consumer surplus is zero, in which case it was shown to be negative. However, numerical simulations suggested that, depending on the parameter values, it can be negative or positive. It is easy to show that the price difference is increasing in λ and in *d*, and it is declining in β. In other words, for given substitutability and a given subsidy budget, the price difference will become positive at sufficiently high values of the weight the firm attaches to consumer surplus. As a consequence, we found the welfare difference between the per unit and the market-share based systems to be increasing in β, and decreasing in both *d* and in the weight of consumer surplus in the firm's objective function. This implies that the market-share based system will perform better than the per unit subsidy if the weight of consumer surplus is sufficiently large.

Appendix 4

Can anything be said about the optimal second-best subsidy for each of the two subsidy systems? To study this question, we return to the case of profit-maximizing firms for simplicity. In determining the optimal subsidy, we assume the government maximizes a standard welfare function, to be described below.

First, consider a per unit subsidy system. Write the government's objective function as

$$(p+\gamma)q+\left[\int_0^q p(q)dq-pq\right]-(1+\mu)\gamma q.$$

The first term is profit of the firm, the second term is surplus for the consumer, the third term is the cost of the subsidy to the government. A weight (1+ μ) larger than one ($\mu \geq 0$) is associated with the cost of the subsidy, reflecting the absence of lump sum taxation. It can be interpreted as the cost of funds. Assuming symmetry, we have deleted firm subscripts; everything is expressed on a per firm basis.

The government is interested in maximizing welfare with respect to the subsidy per unit γ . Using the linear demand functions (2), straightforward algebra shows that the optimal subsidy satisfies

$$\gamma=\frac{1-d-\mu(2-d)}{1+2\mu(2-d)}.$$

As expected, it follows by differentiation of this expression that an increase in the cost of funds μ reduces the optimal subsidy. Similarly, more substitutability (a higher *d*) reduces the optimal subsidy. Note that, if the cost of funds equals one ($\mu=0$) the optimal subsidy $\gamma=1-d$ will push down Nash equilibrium prices to zero, see (8). This is the first-best outcome: as our model assumed zero marginal production cost for the firm and there are no externalities, the optimal price is zero.

Optimal prices and quantities at the optimal subsidy are given by, respectively

$$p = \frac{\mu(3-2d)}{1+2\mu(2-d)}$$

$$q = \frac{1+\mu}{(1+d)\left[1+2\mu(2-d)\right]}.$$

Using these results, the total subsidy cost is

$$\gamma q = \left(\frac{1+\mu}{1+d}\right)\frac{\left[1-d-\mu(2-d)\right]}{\left[1+2\mu(2-d)\right]^2}.$$

Second, we turn to the optimal market-share based subsidy. The government maximizes

$$(pq+\beta)+\left[\int_0^q p(q)dq - pq\right]-(1+\mu)\beta.$$

with respect to β. Manipulating the first-order condition yields, after simple but substantial algebra, the optimal subsidy as

$$\beta = \left[\frac{\left[(1+d)^2(3-2d)\right]^2-\left[8Z\mu+(1+d)^2\right]^2}{\left[8Z\mu+(1+d)^2\right]^2(1+d)Z}\right].$$

Again, differentiation shows that the optimal subsidy is a downward sloping function of the cost of funds.

In principle, prices, quantities and the various welfare components (profit, consumer surplus, etc.) can be determined for both subsidy systems. This would allow a welfare comparison of the subsidy systems, assuming the government sets the subsidy levels at their second-best optimal values. However, although calculating welfare is not difficult for the per unit subsidy, doing the same for the market-share based system turned out to be highly complicated.[26] Moreover, both the optimal subsidies and optimal welfare are highly nonlinear functions of the shadow cost of funds and other model parameters. A formal comparison of the relative welfare performance of the two systems at their respective second-best optimum is therefore outside the scope of this paper.

[26] We were unable to obtain an explicit expression for optimal welfare in the case of a market-share based subsidy system.