

MERGER INCENTIVES AND THE FAILING FIRM DEFENSE*

JAN BOUCKAERT[†]

PETER M. KORT[‡]

Abstract

The merger incentives between profitable firms differ fundamentally from the incentives of a profitable firm to merge with a failing firm. We investigate these incentives under different modes of price competition and Cournot behavior. Our main finding is that firms strictly prefer exit of the failing firm to acquisition. This result may imply that other than strategic reasons, like economies of scale, must be looked for to understand why firms make use of the failing firm defense. However, when products are sufficiently heterogenous, we find that (i) the failing firm defense can be welfare enhancing and (ii) a government bail-out increases total welfare when the number of firms is sufficiently low.

Keywords: failing firm defense, merger analysis.

JEL Classification: L13, L44

1 INTRODUCTION

COMPETITION AUTHORITIES TYPICALLY BLOCK A MERGER WHENEVER the danger exists that it would result in a “substantial lessening of competition” (SLC in the UK and US) or a “significant impediment to effective competition” (SIEC in the EU). Importantly, they can block a merger when these anti-competitive effects happen as a *direct* consequence of the merger. In some special cases, however, competition authorities may clear a merger on the basis of “the

*We would like to thank Jérôme Pouyet, Pierre Piccard, and especially Patrick Rey for pointing out how to model in a consistent way the exit of a firm. Also, we thank Thomas Peeters for his critical remarks. Finally, we thank Stefka Petrova who co-authored this work in an earlier stage of this project.

[†]University of Antwerp. *email: jan.bouckaert@ua.ac.be.*

[‡]Tilburg University and University of Antwerp. *email: kort@wt.nl.*

failing firm defense” argument, even though competition is likely to be substantially weakened or impeded. Typically, the following three criteria are used when judging these “rescue mergers”: (i) one of the firms involved in the merger must be failing; (ii) there is no alternative buyer who could provide for a less anti-competitive solution; (iii) the assets of the failing firm would otherwise exit the market and its market shares would be acquired by the acquiring firm. This last criterion tells that SLC or SIEC would also be the outcome without the merger. In other words, the merger itself is not responsible for the creation of SLC or SIEC. As a result, SLC or SIEC cannot be considered as an argument to block the merger. An extreme version of the failing firm defense considers a homogeneous goods duopoly where firm 1 would either acquire the failing firm 2 or not. Either choice by firm 1, however, results in the monopoly outcome. The failing firm defense may therefore be considered as a way to offer a legal answer to a lessening of competition that results from an increased market concentration with or without the merger.

The concept of the failing firm defense is recognized both in the US and in Europe. In the US, the failing firm defense is legally established in the 1992 Horizontal Merger Guidelines as implemented by the FTC and the DOJ. The failing firm defense is not part of the Merger Regulation in Europe. However, it is incorporated into the 2004 Merger Guidelines (paragraphs 89-91), and in the UK, the OFT’s substantive assessment guidance for mergers extensively discusses the failing firm defense (paragraphs 4.36-39). The failing firm defense, at the same time, has not been applied frequently in the clearing of mergers. Early examples from the US case-law include *International Shoe Co. v. FTC* (1930), *Granader v. Public Bank* (1969), among others.¹ In continental Europe, the European Commission applied the failing firm defense in *Kali and Salz/Mitteldeutsche Kali (Treuhand)* in 1994 and when *BASF* acquired *Eurodiol* and *Pantochim* in 2001. The OFT applied the failing firm defense only on a few occasions since the Enterprise Act 2002.² While there are some minor differences in how the failing firm defense is applied across the different jurisdictions, all share the common requirement that post-merger outcome in the relevant market should be no worse than the market outcome in the absence of the merger.³ Interestingly, the failing firm defense has received renewed attention during the recent financial crisis.⁴

While the incentives for profitable firms to merge and their economic and welfare consequences are a topic of intense research, the incentives for firms to merge with a failing firm are less well documented. This paper shows that firms’ incentives to merge with a failing firm differ fundamentally from mergers between profitable firms. To see this, consider a market, where we

denote by Π^N the profit of a firm when N symmetric firms sell substitutable products, Π^{N-1} as the profit of a firm when only $N - 1$ symmetric firms compete for customers, and Π^m as the profit of one product variety within a merger m . Typically, it holds that profits increase when the number of firms decreases, or $\Pi^{N-1} > \Pi^N$. There is a private incentive for profitable firms to merge whenever each firm's profits increase as a result of the merger, or

$$\Pi^m > \Pi^N.$$

Such a private incentive exists under Bertrand competition with differentiated products, as e.g. shown by Davidson and Deneckere (1985). This result, however, stands in sharp contrast with Cournot competition, where firms find it only profitable to merge when (i) their combined market share becomes sufficiently large, or (ii) their capacity to produce differs, or (iii) the efficiency gains are considerable, or (iv) goods are highly differentiated, as shown by Salant et al. (1983), Perry and Porter (1985), Farrell and Shapiro (1990), and Motta (2004).

The private incentives for a profitable firm to merge with a failing firm, however, amounts to comparing the profits of the two merging entities with the profit that a profitable firm will receive in the event the failing firm leaves the market. The private incentive for a profitable firm to acquire the failing firm should therefore satisfy

$$2\Pi^m > \Pi^{N-1},$$

which clearly differs from the condition when all firms are profitable.

This paper investigates whether firms competing in prices or quantities have an incentive to acquire a failing firm or not. We start out by studying the representative consumer model with a linear Shubik-Levitan demand. In addition, we consider Salop's circular location model, the logit model, and Cournot competition with differentiated products. We find that $2\Pi^m < \Pi^{N-1}$ across all forms of competition. In other words, all remaining profitable firms have no strategic incentives to acquire the failing firm; they prefer the failing firm to disappear from the market instead of rescuing it. This result may imply that other reasons, like e.g. important economies of scope in their fixed or variable operation costs, must be looked for to explain why firms make use of the failing firm defense.

From a social perspective, allowing a failing firm defense is optimal when

$$2\Pi^m + (N - 2)\Pi^o + CS^m > (N - 1)\Pi^{N-1} + CS^{N-1},$$

where CS stands for consumer surplus, and Π^o refers to the profits of the firms outside the merger. The advantage of CS^m over CS^{N-1} is that a merger maintains the degree of product

variety in the market. The social benefit from a rescue merger further depends on the resulting prices to see whether CS^m is really greater than CS^{N-1} . Since we always find that $2\Pi^m + (N - 2)\Pi^o < (N - 1)\Pi^{N-1}$, consumer surplus from product variety must play an important role should we favor the failing firm defense from a social perspective. We find that there is a social incentive to undertake the failing firm defense if goods are sufficiently heterogeneous. This finding can easily be explained since the rescue merger maintains product variety and lowers prices as compared to a market structure without the failing firm.

We also investigate the welfare effects when the government bails out the failing firm as a possible alternative to the failing firm exiting from the market. While a bail-out does not increase producer surplus, consumer welfare drastically goes up; this form of government intervention preserves product variety while at the same time maintains competition between all firms at the level prior to the failure or merger. We find that a public bail-out is welfare increasing whenever product heterogeneity is high enough, the shadow social cost of consumer taxation is low enough, and the number of firms is sufficiently low.

The number of papers that analyze the failing firm defense and its policy implications is rather limited. Persson (2005) uses a Cournot oligopoly model to evaluate the welfare consequences of the failing firm defense in the EU and US merger laws. In his model, failing firms are sold to potential buyers by means of an endogenous valuations auction model and the identity of the buyer affects the profits of all firms. He shows that applying the failing firm defense increases consumer welfare, but does not always ensure a socially optimal merger, as it favours small and possibly inefficient buyers. Mason and Weeds (2010) employ a dynamic model with stochastically changing demand to assess the effects of the failing firm argument on entry and entrepreneurial activity. In their model, firms are allowed to use the failing firm defense as a (preferred) alternative to exit if they are facing poor market conditions. The authors find that, when future profitability turns out to be too low, an option to make use of the failing firm defense stimulates entry and, therefore, competition. Based on that, they derive an optimal merger policy in the form of a threshold, beyond which it is welfare increasing to approve mergers.⁵ Vasconcelos (2012) shows that firms may strategically set up an efficiency improving merger so as to induce a failure of outsiders. When the merging parties can make use of the failing firm defense, the market may end up completely monopolized. The availability of the failing firm defense, therefore, may not always serve the consumers' interests. Our paper, in contrast, focuses on profitable firms' incentives to acquire the failing firm. We show that firms have no strategic incentives to make use of the failing firm defense, and strictly prefer the failing firm to

disappear from the market.

Section 2 presents our insights in a model with a representative consumer and symmetric price competition with linear demand. Section 3 shows that our main results also hold for two other standard models of price competition (Salop and the logit model) and Cournot competition. Section 4 concludes.

2 THE MODEL

The failing firm defense typically applies to industries with a high degree of market concentration. Therefore, in the main text we focus our analysis to a concentrated industry where the number of firms $N = 3$. By doing so, we analyze market structures where mergers between profitable firms would be blocked with a high probability by antitrust authorities for anti-competitive reasons.⁶ We further consider that each firm i , with $i \in \{1, 2, 3\}$, has identical constant marginal costs, which we normalize to zero for convenience. We assume that firm 1 fails for exogenous reasons because of differences in fixed costs F_i across firms. In particular, we assume that $F_1 > \{F_2, F_3\}$.⁷ Two obvious, possible interpretations result in firm 1 failing. First, when the size of the market declines sufficiently slowly, firm 1 will fail first when it has the highest fixed cost. Second, when the size of the market remains constant but fixed costs increase sufficiently slowly over time for all firms, firm 1 fails first.

Since our analysis assumes that firm 1 is failing, we consider the two available options: either firm 1 leaves the market or one of the solvent firms acquires the failing firm. We compare the incentives for the two solvent firms to adopt one or the other option. Since firms 2 and 3 only differ with respect to their fixed costs, they have identical incentives to acquire the failing firm or not. For convenience, we denote the acquiring firm as firm 2 (the insider) whereas firm 3 (the outsider) does not participate in the merger. Finally, we assume a fixed aggregate market demand.

In our main model, we consider symmetric price competition with linear demand. We make use of the Shubik-Levitan (1980) quasi-linear utility function

$$U^3 = \sum_{i=1}^3 q_i - \frac{3}{2(1+\gamma)} \left[\sum_{i=1}^3 q_i^2 + \frac{\gamma}{3} \left(\sum_{i=1}^3 q_i \right)^2 \right] + y, \quad (1)$$

where a representative consumer has income y and demand q_i for three differentiated goods i . The positive parameter $\gamma \in [0, \infty]$ captures the degree of substitutability between the goods. Goods become more homogeneous when γ increases. The representative consumer maximizes

her utility subject to her income constraint, and the interior first-order condition results in the inverse linear demand

$$p_i = 1 - \frac{1}{1 + \gamma} (3q_i + \gamma \sum_{j=1}^3 q_j)$$

for each good i . Rewriting the system of three equations gives the direct symmetric demand function

$$q_i = \frac{1}{3} \left(1 - (1 + \gamma) p_i + \frac{\gamma}{3} \sum_{j=1}^3 p_j \right). \quad (2)$$

When the three firms compete in prices for customers, the objective of firm i is

$$\max_{p_i} \Pi_i^3(p_i; p_{-i}) \equiv \max_{p_i} \frac{1}{3} \left(1 - (1 + \gamma) p_i + \frac{\gamma}{3} \sum_{j=1}^3 p_j \right) p_i - F_i, \quad (3)$$

where the superscript refers to the number of competing firms. From the necessary and sufficient symmetric first-order conditions

$$1 - 2 \left(1 + \frac{2}{3} \gamma \right) p_i + \frac{\gamma}{3} \left(\sum_{i \neq j} p_j \right) = 0,$$

each firm i 's best-response can be written as

$$p_i = \frac{3 + 2\gamma p_j}{2(3 + 2\gamma)}. \quad (4)$$

Symmetry implies that the equilibrium price equals

$$p_i^* = \frac{3}{2(\gamma + 3)},$$

resulting in equilibrium net profits of

$$\Pi_i^3(p_i^*) = \frac{2\gamma + 3}{4\gamma^2 + 24\gamma + 36} - F_i.$$

2.1 *failing firm defense*

When firm 2 acquires the failing firm 1, the merged entity m maximizes its net profit

$$\max_{p_1, p_2} \Pi^m(p_1, p_2) \equiv \max_{p_1, p_2} \sum_{i=1}^2 \Pi_i^3(p_i; p_{-i}).$$

This gives two symmetric first-order conditions

$$1 - 2 \left(1 + \frac{2}{3} \gamma \right) p_1 + \frac{\gamma}{3} (2p_2 + p_3) = 0, \text{ and } 1 - 2 \left(1 + \frac{2}{3} \gamma \right) p_2 + \frac{\gamma}{3} (2p_1 + p_3) = 0.$$

Subtracting the two equations gives $p_1 = p_2$, so that we can rewrite the first-order conditions into the common best-response for both product varieties

$$\frac{3 + \gamma p_3}{2(3 + \gamma)} = p_j \quad (5)$$

with $j = 1, 2$. The profit-maximizing problem of the outsider coincides, of course, with Eq. (3) and its corresponding best-response with Eq. (4). Solving for Eqs. (4) and (5) results in the equilibrium prices

$$p_1^m = p_2^m = \frac{5\gamma + 6}{2(6\gamma + \gamma^2 + 6)} \text{ and } p^o = \frac{2\gamma + 3}{6\gamma + \gamma^2 + 6} \quad (6)$$

for the merged varieties and the outsider, resp. As expected, the merged firm charges higher prices than the outsider because the merged entity internalizes the competitive externality its varieties impose on each other. The equilibrium net profit for the merged entity, by consequence, equals

$$\Pi^m(p_1^m, p_2^m) = (\gamma + 3) \frac{(5\gamma + 6)^2}{18(6\gamma + \gamma^2 + 6)^2} - F_1 - F_2,$$

and

$$\Pi^o(p^o) \equiv \frac{(2\gamma + 3)^3}{9(\gamma^2 + 6\gamma + 6)^2} - F_3$$

for the outsider.

2.2 *the failing firm exits the market*

When the failing firm exits the market, we have a duopoly with firms 2 and 3 only. Product variety has reduced from three goods to two with corresponding utility function

$$U^2 = \sum_{i=2}^3 q_i - \frac{3}{2(1 + \gamma)} \left[\sum_{i=2}^3 q_i^2 + \frac{\gamma}{3} \left(\sum_{i=2}^3 q_i \right)^2 \right] + y, \quad (7)$$

where we impose that $q_1 = 0$, since firm 1 has disappeared from the market. The corresponding inverse demand functions equal

$$p_i = 1 - \frac{1}{1 + \gamma} ((3 + \gamma) q_i + \gamma q_j),$$

with $i, j = 2, 3$ and $i \neq j$. Importantly, these inverse demand functions still reflect consumers' preferences for the three goods.⁸ Substitution results in

$$q_i = \frac{\gamma + 1}{6\gamma + 9} (3 - (3 + \gamma) p_i + \gamma p_j) \quad (8)$$

for firm i . Firm i maximizes

$$\Pi_i^2(p_i; p_j) = \max_{p_i} q_i p_i - F_i,$$

where the superscript refers to the remaining number of competing firms. The first-order condition results in the best-response

$$p_i = \frac{3 + \gamma p_j}{2(3 + \gamma)},$$

which, for a symmetric equilibrium where $p^* \equiv p_i = p_j$, yields

$$p^* = \frac{3}{\gamma + 6}.$$

Equilibrium profits can now be written as

$$\Pi_i^2(p^*, p^*) = \frac{9(\gamma + 1)(\gamma + 3)}{(6\gamma + 9)(\gamma + 6)^2} - F_2.$$

Since firm 1's profits when failing are

$$\Pi_1^3(p_i^*) = \frac{2\gamma + 3}{4\gamma^2 + 24\gamma + 36} - F_1 = 0, \quad (9)$$

we obtain after substitution that

$$\Pi_i^2(p^*, p^*) > \Pi^m(p_1^m, p_2^m),$$

wherefrom firms 2 and 3 have *no* private incentive to engage in acquiring the failing firm. Instead they prefer the failing firm to leave the market.

Proposition 1 *There is no private incentive for solvent firms to merge with a failing firm. Solvent firms strictly prefer that the failing firm leaves the market.*

2.3 social incentives for the failing firm defense

We denote total surplus by TS^x , where $x = m$ when the failing firm defense is applied, and $x = 2$ when the failing firm has exited from the market. Total surplus is defined as the sum of consumer surplus CS^x and producer surplus PS^x . We define producer surplus by the sum of all producers' profits. Consumer surplus is obtained by subtracting equilibrium consumer expenditures from the corresponding equilibrium utility levels, and depends positively on the number of products on offer; the larger is the variety of products, the higher is consumer surplus. In this respect, consumers prefer a merger to an exit of the failing firm. At the same time, consumer surplus is negatively affected by price increases. Both immediate exit and merger result in a price increase, whereby the equilibrium price p^* following exit exceeds the outsider's and the insider's price since

$$p_i^* < p^o < p^m < p^*.$$

This can be explained as follows. First, when the failing firm exits, product variety decreases, implying that demand goes up for the other products. This demand effect invites firms to charge higher prices. Second, since prices are lower and product variety is larger when the failing firm is acquired by one of the solvent firms, it follows that consumer surplus is lower when the failing firm exits the market. However, Proposition 1 implies that exit of the failing firm results in a larger producer surplus. Thus, it is a priori unclear which regime maximizes welfare. The following proposition shows that, when goods are sufficiently heterogenous, the merger results in a higher total welfare than immediate exit.

Proposition 2 *For sufficiently low values of γ it holds that the failing firm defense is socially preferred to immediate exit as it results in higher total welfare. Consumers always prefer a failing firm defense to an exit of the failing firm.*

Proof. Consider producer surplus first. It equals

$$PS^2 = \frac{6(\gamma + 1)(\gamma + 3)}{(3 + 2\gamma)(\gamma + 6)^2} - F_2 - F_3$$

when the failing firm has exited the market. In contrast, when the failing firm defense applies, producer surplus amounts to

$$PS^m = \frac{64\gamma^5 + 663\gamma^4 + 2682\gamma^3 + 5346\gamma^2 + 5184\gamma + 1944}{36(\gamma^3 + 9\gamma^2 + 24\gamma + 18)^2} - F_2 - F_3$$

since (9) holds.

Next, we derive the consumer surplus when the failing firm exits. Evaluation of Eq. (8) at p^* and substituting into (7) yields a consumer surplus of

$$\begin{aligned} CS^2 &= U^2 - 2p^*q^* \\ &= \frac{(\gamma + 1)(\gamma + 3)^2}{(\gamma + 6)^2(2\gamma + 3)}. \end{aligned}$$

Consumer surplus under a failing firm defense is obtained by inserting Eq. (6) into Eq. (2), resulting in equilibrium quantities

$$q^m = \frac{(\gamma + 3)(5\gamma + 6)}{18(\gamma^2 + 6\gamma + 6)}, \text{ and } q^o = \frac{(2\gamma + 3)^2}{9(\gamma^2 + 6\gamma + 6)},$$

for each product variety in the merged entity and for the outsider, resp. Substituting these quantities into Eq. (1), we obtain that

$$\begin{aligned} CS^m &= U^3 - 2p^mq^m - p^oq^o \\ &= \frac{(18\gamma^4 + 133\gamma^3 + 351\gamma^2 + 396\gamma + 162)}{36(\gamma^2 + 6\gamma + 6)^2}. \end{aligned}$$

Comparison shows that $CS^m > CS^2$, or, consumers prefer the failing firm defense to exit of the failing firm. Subtracting the welfare when only two firms serve the market from the welfare when one of the solvent firms acquires the failing firm gives

$$\begin{aligned} & TS^m - TS^2 & (10) \\ = & \frac{-20\gamma^8 - 345\gamma^7 - 2520\gamma^6 - 9531\gamma^5 - 14823\gamma^4 + 16524\gamma^3 + 94770\gamma^2 + 122472\gamma + 52488}{36(2\gamma + 3)(\gamma^4 + 15\gamma^3 + 78\gamma^2 + 162\gamma + 108)^2}, \end{aligned}$$

which is clearly positive for γ small enough. Therefore, we can conclude that when goods are sufficiently heterogenous, the failing firm defense results in a higher welfare level than immediate exit. ■

The result is intuitive since consumers value product variety more when goods are more differentiated. Since under the failing firm defense the number of different goods is one larger than when the failing firm exits, the difference in consumer surplus between the two outcomes is larger when goods are more heterogenous. This explains why, for γ small enough, applying the failing firm defense enhances welfare.

2.4 *bail-out vs. exit: a welfare comparison*

An important criterion of the failing firm defense requires that no alternative buyer could provide for a less anti-competitive solution. The second condition of the EC Merger Guidelines reads as “there is no less anti-competitive alternative purchase than the notified merger”. Similarly, the third condition of the 1992 US Horizontal Merger Guidelines reveals that “[the failing firm] has made unsuccessful good-faith efforts to elicit reasonable alternative offers of acquisition of the assets of the failing firm that would both keep its tangible and intangible assets in the relevant market and pose a less severe danger to competition than does the proposed merger.” Suppose, therefore, that the Merger Guidelines or Merger Regulation frameworks regarding the failing firm defense would not exclude that the government could provide for a less anti-competitive solution. In this section, we study a bail-out by the government while at the same time looking at the welfare consequences.

We modify the model by assuming that the market is declining over time.⁹ In particular, we impose that the market declines at a per-period rate μ , $\mu \in (0, 1)$. For this reason, we replace the demand function (2) by

$$q_i = \frac{1}{3}\mu^t \left(1 - (1 + \gamma)p_i + \frac{\gamma}{3} \sum_{j=1}^3 p_j \right),$$

where $t \in [0, \infty)$ is discrete time. Assume firm 1 fails at $t = 0$.¹⁰ This implies that its profits, $\Pi_1^3(t)$ are zero for $t = 0$, i.e.

$$\Pi_1^3(0) \equiv \frac{2\gamma + 3}{4\gamma^2 + 24\gamma + 36} - F_1 = 0. \quad (11)$$

As time goes on, firm 1's profits become increasingly negative. To prevent firm 1 from exiting, the government needs to subsidize its activities so as to make it break-even. Denoting this subsidy by $S(t)$, it follows that

$$S(t) \equiv -\Pi_1^3(t) = (1 - \mu^t) \frac{(2\gamma + 3)}{4\gamma^2 + 24\gamma + 36}. \quad (12)$$

From (11) and (12) we obtain that $S(0) = 0$, $S'(t) > 0$, and $S''(t) < 0$. If the aim of a bail-out policy is to increase welfare, it should be implemented whenever

$$PS^S(t) + CS^S(t) > PS^2(t) + CS^2(t), \quad (13)$$

where the superscript S refers to producer and consumer surplus under the bail-out regime. Since consumer and producer surplus are decreasing in time in a declining market, and since $S(t)$ increases with time, a bail-out will be carried out during a time interval $[0, T_1^S)$, where T_1^S satisfies the equality

$$PS^S(T_1^S) + CS^S(T_1^S) = PS^2(T_1^S) + CS^2(T_1^S),$$

if we establish that (13) holds at time zero. From the proof of Proposition 2 we derive that

$$PS^2(t) + CS^2(t) = \frac{6(\gamma + 1)(\gamma + 3)\mu^t}{(3 + 2\gamma)(\gamma + 6)^2} - F_2 - F_3 + \frac{(\gamma + 1)(\gamma + 3)^2\mu^t}{(\gamma + 6)^2(2\gamma + 3)}.$$

Total welfare when the government bails out firm 1 consists of

$$\begin{aligned} PS^S(t) &\equiv PS^3(t) + S(t) = \Pi_2^3(t) + \Pi_2^3(t) = 2 \frac{(2\gamma + 3)\mu^t}{4\gamma^2 + 24\gamma + 36} - F_2 - F_3, \\ CS^S(t) &\equiv CS^3(t) - (1 + \lambda)S(t) \end{aligned}$$

where we assume that the bail-out subsidy $S(t)$ is financed from a lump sum tax on consumers, and each unit of subsidy implies a deadweight cost of $\lambda > 0$ representing the shadow social cost to the tax payers.¹¹ To evaluate (13) at time zero, we need to assess the value of $CS^3(0)$. To do so, we derive the consumer surplus when all three firms are competing, or

$$CS^3 = U^3 - 3p_i^*q_i^* = \frac{(2\gamma + 3)^2}{8(\gamma + 3)^2}.$$

As a result,

$$CS^S(0) - CS^2(0) = \frac{28\gamma^4 + 246\gamma^3 + 675\gamma^2 + 756\gamma + 324}{8(2\gamma + 3)(\gamma^2 + 9\gamma + 18)^2} - (1 + \lambda)S(0) > 0,$$

when the shadow cost of taxation is low enough. Furthermore,

$$PS^S(0) - PS^2(0) = -\frac{\gamma(8\gamma^3 + 60\gamma^2 + 135\gamma + 108)}{2(2\gamma + 3)(\gamma^2 + 9\gamma + 18)^2} < 0,$$

and

$$PS^S(0) + CS^S(0) - PS^2(0) - CS^2(0) = \frac{-4\gamma^4 + 6\gamma^3 + 135\gamma^2 + 324\gamma + 324}{8(2\gamma + 3)(\gamma^2 + 9\gamma + 18)^2} - \lambda S(0).$$

We derived the following proposition.

Proposition 3 *Compared to exit, a government bail-out of the failing firm provides higher consumer surplus and lower producer surplus, while raising total welfare, provided the goods are sufficiently heterogenous and the deadweight cost of the bail-out is not too high.*

In the Appendix, we show that for N firms, and when goods are sufficiently heterogenous, the difference in total surplus between a bail-out and exit of the failing firm decreases with the number of firms for two reasons. First, when the number of firms augments, prices increase to a lesser extent when the failing firm exits from the market. Second, the amount of capital needed to finance the bail-out and make the least efficient firm break-even is higher, since oligopoly profits decrease in the number of firms. That is, $S(N)$ is increasing in N . As a result, when the shadow cost $\lambda S(N)$ to the tax payers is taken into account, a bail-out may become socially more costly than clearing the merger. As a result, there exists a N^* such that $TS^S(N^*) \leq TS^{N^*-1}(N^*)$ for all $N \geq N^*$.

3 ALTERNATIVE DEMAND SPECIFICATIONS

As in section 2, and for each mode of competition that we present now, our analysis starts where firm 1 is failing. We consider the two available options: either firm 1 leaves the market or one of the competing solvent firms acquires the failing firm. Likewise, all models assume a fixed aggregate market demand. In line with the previous model of price competition, we find no private incentive to undertake the failing firm defense. While we do not report our calculations, scenarios where there is a social incentive to merge can be detected in each mode of competition.

3.1 the Salop model

We consider first the Salop (1979) model with three symmetrically located firms. For convenience, suppose firm 1 is located at twelve o'clock, firm 2 at four o'clock, and firm 3 at eight o'clock. Consumers are uniformly located on a circle with perimeter one and have unit density. Transportation costs are linear, with unit costs τ . We consider consumers' willingness to pay to be high enough so that the market is covered and aggregate demand is constant. When all three firms are competing, we have that firm i maximizes its gross profit

$$\max_{p_i} S_i^3(p_i; p_{-i}) \equiv \max_{p_1} \left(p_1 \left(\frac{p_2 - p_1}{2\tau} + \frac{1}{6} \right) + p_1 \left(\frac{1}{6} - \frac{p_1 - p_3}{2\tau} \right) \right)$$

with respect to its price p_1 . The necessary and sufficient first-order condition gives

$$\frac{p_2 - 2p_1}{2\tau} + \frac{1}{3} + \frac{p_3 - 2p_1}{2\tau} = 0.$$

In a symmetric equilibrium we have $p_i^* \equiv p_i = \tau/3$ for all firms i , and gross profits for all firms amount to $S_i^*(p_i^*) = \tau/9$.

3.1.1 failing firm defense

Suppose firm 2 acquires the failing firm 1. This merged entity m maximizes its gross profit

$$\begin{aligned} \max_{p_1, p_2} S^m(p_1, p_2; p_3) &\equiv \max_{p_1, p_2} \sum_{i=1}^2 S_i^3(p_i; p_{-i}) \\ &= \max_{p_1, p_2} p_1 \left(\frac{1}{3} + \frac{p_2 - 2p_1 + p_3}{2\tau} \right) + p_2 \left(\frac{1}{3} + \frac{p_3 - 2p_2 + p_3}{2\tau} \right). \end{aligned}$$

This gives the first-order conditions

$$6p_2 - 12p_1 + 3p_3 + 2\tau = 0 \text{ and } 6p_1 - 12p_2 + 3p_3 + 2\tau = 0.$$

Firm 3, denoted as the outsider o , maximizes

$$S^o(p_3; p_1, p_2) \equiv \max_{p_3} \left(p_3 \left(\frac{1}{6} + \frac{p_2 - p_3}{2\tau} \right) + p_3 \left(\frac{1}{6} - \frac{p_3 - p_1}{2\tau} \right) \right),$$

which leads to

$$3p_1 + 3p_2 - 12p_3 + 2\tau = 0.$$

This results, respectively, in equilibrium prices and gross equilibrium profits

$$p_3 \equiv p^o = \frac{4}{9}\tau; \quad p_1 = p_2 \equiv p^m = \frac{5}{9}\tau \text{ and } S^m = \frac{25}{81}\tau; \quad S^o = \frac{16}{81}\tau.$$

Again we see that, for the same reason as in section 2, the merged firm charges higher prices. This leads to higher profits for the merged entity than what the outside firm earns.

3.1.2 the failing firm exits the market

When firm 1 leaves the market, firms 2 and 3 compete for customers while remaining positioned on their original locations. Firm 2 maximizes its gross profit

$$\max_{p_2} S_2(p_2; p_3) \equiv \max_{p_2} \left(p_2 \left(\frac{p_3 - p_2}{2\tau} + \frac{1}{3} \right) + p_2 \left(\frac{1}{6} - \frac{p_2 - p_3}{2\tau} \right) \right).$$

In a similar fashion, firm 3 maximizes its gross profits, wherefrom we obtain that their equilibrium prices and profits are

$$p^* = \frac{\tau}{2} \text{ and } S_i^2 = \frac{\tau}{4}.$$

We conclude that, in contrast to the model of the previous section, $p^* < p^m$. When the merged multi-product firm increases the price of one product, demand for its other product increases so much that a higher price is optimal as compared to the profit-maximizing price when the failing firm is no longer present.¹²

A simple comparison between the net profits for firm 2 from the failing firm defense and its net profits when firm 1 exits the market shows that there is no incentive to acquire the failing firm. Since the failing firm breaks even at the moment it leaves the market, we have that $F_1 = \tau/9$. Consequently, we find that

$$S_2 - F_2 = \frac{1}{4}\tau - F_2 > S^m - F_1 - F_2 = \frac{25}{81}\tau - \frac{\tau}{9} - F_2 = \frac{16}{81}\tau - F_2.$$

This leads us to the following proposition:

Proposition 4 *In the Salop circle model, there is no private incentive for solvent firms to merge with a failing firm. Solvent firms strictly prefer that the failing firm leaves the market.*

3.2 Logit demand

We follow Anderson and de Palma (1992) and assume that a consumer's utility for good i satisfies $U_i = V_i + \lambda\varepsilon_i$, where $V_i = a - p_i$ is identical across consumers. At the same time, ε_i is a consumer specific random variable with unit variance and zero mean, and λ is a positive parameter that increases with the heterogeneity in consumers' tastes for horizontally differentiated goods. We make the assumption that all consumers purchase and therefore that buying the outside good results in infinitely low utility. This assumption also buys us the result that aggregate market demand is constant. The demand D_i for good i is given by

$$D_i \equiv \frac{e^{-\frac{p_i}{\lambda}}}{\sum_{j=1}^3 e^{-\frac{p_j}{\lambda}}}.$$

Firm i maximizes its gross profits

$$\max_{p_i} \mathcal{L}_i^3(p_i; p_{-i}) \equiv \max_{p_i} p_i \frac{e^{-\frac{p_i}{\lambda}}}{\sum_{j=1}^3 e^{-\frac{p_j}{\lambda}}}.$$

The first-order condition is

$$\frac{1}{\left(\sum_{j=1}^3 e^{-\frac{p_j}{\lambda}}\right)^2} \left(\sum_{j=1}^3 e^{-\frac{p_j}{\lambda}} \left(e^{-\frac{p_i}{\lambda}} - \frac{p_i}{\lambda} e^{-\frac{p_i}{\lambda}} \right) + \frac{1}{\lambda} e^{-\frac{p_i}{\lambda}} p_i e^{-\frac{p_i}{\lambda}} \right) = 0,$$

wherefrom

$$p_i = \frac{\lambda}{1 - \frac{e^{-\frac{p_i}{\lambda}}}{\sum_{j=1}^3 e^{-\frac{p_j}{\lambda}}}}.$$

For a symmetric equilibrium we obtain

$$p_i^* = \frac{3\lambda}{2} \text{ and } \mathcal{L}^3(p_i^*) = \frac{\lambda}{2}.$$

3.2.1 the failing firm defense

Suppose that firms 1 and 2 merge. This merged firm maximizes its gross profit

$$\begin{aligned} \max_{p_1, p_2} \mathcal{L}^m(p_1, p_2; p_3) &\equiv \max_{p_1, p_2} \sum_{i=1}^2 \mathcal{L}_i^3(p_i; p_{-i}) \\ &= \max_{p_1, p_2} \left(p_1 \frac{e^{-\frac{p_1}{\lambda}}}{\sum_{j=1}^3 e^{-\frac{p_j}{\lambda}}} + p_2 \frac{e^{-\frac{p_2}{\lambda}}}{\sum_{j=1}^3 e^{-\frac{p_j}{\lambda}}} \right) \\ &= \max_{p_1, p_2} \frac{1}{\sum_{j=1}^3 e^{-\frac{p_j}{\lambda}}} \left(p_1 e^{-\frac{p_1}{\lambda}} + p_2 e^{-\frac{p_2}{\lambda}} \right). \end{aligned}$$

The first-order conditions are, after simplifying,

$$1 - \frac{p_1}{\lambda} + \frac{p_1}{\lambda} \frac{e^{-\frac{p_1}{\lambda}}}{\sum_{j=1}^3 e^{-\frac{p_j}{\lambda}}} + \frac{p_2}{\lambda} \frac{e^{-\frac{p_2}{\lambda}}}{\sum_{j=1}^3 e^{-\frac{p_j}{\lambda}}} = 0,$$

and

$$1 - \frac{p_2}{\lambda} + \frac{p_2}{\lambda} \frac{e^{-\frac{p_2}{\lambda}}}{\sum_{j=1}^3 e^{-\frac{p_j}{\lambda}}} + \frac{p_1}{\lambda} \frac{e^{-\frac{p_1}{\lambda}}}{\sum_{j=1}^3 e^{-\frac{p_j}{\lambda}}} = 0.$$

The first-order condition of Firm 3 equals

$$1 - \frac{p_3}{\lambda} + \frac{p_3}{\lambda} \frac{e^{-\frac{p_3}{\lambda}}}{\sum_{j=1}^3 e^{-\frac{p_j}{\lambda}}} = 0.$$

Defining $p^m \equiv p_1 = p_2$ and $p^o \equiv p_3$, this gives

$$p^m = \frac{\lambda}{1 - \frac{2e^{-\frac{p^m}{\lambda}}}{2e^{-\frac{p^m}{\lambda}} + e^{-\frac{p^o}{\lambda}}}} = \frac{\lambda \left(2e^{-\frac{p^m}{\lambda}} + e^{-\frac{p^o}{\lambda}} \right)}{e^{-\frac{p^o}{\lambda}}} = \lambda \left(2e^{\frac{p^o - p^m}{\lambda}} + 1 \right) \quad (14)$$

$$p^o = \frac{\lambda}{1 - \frac{e^{-\frac{p^o}{\lambda}}}{2e^{-\frac{p^m}{\lambda}} + e^{-\frac{p^o}{\lambda}}}} = \frac{\lambda \left(2e^{-\frac{p^m}{\lambda}} + e^{-\frac{p^o}{\lambda}} \right)}{2e^{-\frac{p^m}{\lambda}}} = \lambda \left(1 + \frac{1}{2} e^{\frac{p^m - p^o}{\lambda}} \right). \quad (15)$$

The following lemma shows that, like in the previous models, also here the merged firm charges a higher price for its products.

Lemma 5 $p^m > p^o$, which implies that

$$e^{\frac{(p^m - p^o)}{\lambda}} < 2. \quad (16)$$

Proof. If, on the contrary, $p^m \leq p^o$, we get from (14) and (15):

$$\begin{aligned} \frac{\lambda}{1 - \frac{2e^{-\frac{p^m}{\lambda}}}{2e^{-\frac{p^m}{\lambda}} + e^{-\frac{p^o}{\lambda}}}} &\leq \frac{\lambda}{1 - \frac{e^{-\frac{p^o}{\lambda}}}{2e^{-\frac{p^m}{\lambda}} + e^{-\frac{p^o}{\lambda}}}} \\ \frac{2e^{-\frac{p^m}{\lambda}}}{2e^{-\frac{p^m}{\lambda}} + e^{-\frac{p^o}{\lambda}}} &\leq \frac{e^{-\frac{p^o}{\lambda}}}{2e^{-\frac{p^m}{\lambda}} + e^{-\frac{p^o}{\lambda}}} \\ 2e^{-\frac{p^m}{\lambda}} &\leq e^{-\frac{p^o}{\lambda}}, \end{aligned}$$

which does not hold for $p^m \leq p^o$. Hence, the lemma must be true. Since we now know that $p^m > p^o$, we have

$$\frac{\lambda}{1 - \frac{2e^{-\frac{p^m}{\lambda}}}{2e^{-\frac{p^m}{\lambda}} + e^{-\frac{p^o}{\lambda}}}} > \frac{\lambda}{1 - \frac{e^{-\frac{p^o}{\lambda}}}{2e^{-\frac{p^m}{\lambda}} + e^{-\frac{p^o}{\lambda}}}},$$

so that

$$2e^{-\frac{p^m}{\lambda}} > e^{-\frac{p^o}{\lambda}} \Rightarrow e^{\frac{(p^m - p^o)}{\lambda}} < 2.$$

■

For the profits we get

$$\begin{aligned} \mathcal{L}^m &= \frac{2p^m e^{-\frac{p^m}{\lambda}}}{2e^{-\frac{p^m}{\lambda}} + e^{-\frac{p^o}{\lambda}}} = \frac{2 \frac{\lambda \left(2e^{-\frac{p^m}{\lambda}} + e^{-\frac{p^o}{\lambda}} \right)}{e^{-\frac{p^o}{\lambda}}} e^{-\frac{p^m}{\lambda}}}{2e^{-\frac{p^m}{\lambda}} + e^{-\frac{p^o}{\lambda}}} = 2\lambda e^{\frac{(p^o - p^m)}{\lambda}}, \\ \mathcal{L}^o &= \frac{p^o e^{-\frac{p^o}{\lambda}}}{2e^{-\frac{p^m}{\lambda}} + e^{-\frac{p^o}{\lambda}}} = \frac{\frac{\lambda \left(2e^{-\frac{p^m}{\lambda}} + e^{-\frac{p^o}{\lambda}} \right)}{2e^{-\frac{p^m}{\lambda}}} e^{-\frac{p^o}{\lambda}}}{2e^{-\frac{p^m}{\lambda}} + e^{-\frac{p^o}{\lambda}}} = \frac{1}{2} \lambda e^{\frac{(p^m - p^o)}{\lambda}}. \end{aligned}$$

Due to (16) we conclude that the merged entity's profits are higher than the profit of the outsider.

3.2.2 the failing firm exits the market

When firm 1 exits from the market, its demand $D_1 \equiv e^{-\frac{p_1}{\lambda}} / \sum_{j=1}^3 e^{-\frac{p_j}{\lambda}}$ equals zero if and only if p_1 is infinitely large. As a result, equilibrium prices and gross profits for firms 2 and 3 are

$$p^* = 2\lambda \text{ and } \mathcal{L}^2(p^*; p_1 = \infty) = \lambda,$$

respectively. From (14)-(16) it follows, as in the Salop model and for the same reasons as there, that $p^o < p^* < p^m$. The insider's profits can be written as

$$\begin{aligned} \mathcal{L}^m - F_1 - F_2 &= 2\lambda e^{\frac{(p^o - p^m)}{\lambda}} - F_1 - F_2 \\ &= 2\lambda e^{\frac{(p^o - p^m)}{\lambda}} - \frac{\lambda}{2} - F_2, \end{aligned}$$

since the failing firm's break-even condition satisfies $F_1 = 0.5\lambda$. If there is a private incentive to merge, it must hold that the insider's profits are greater than in the case of no merger. This should imply that

$$2\lambda e^{\frac{(p^o - p^m)}{\lambda}} - \frac{\lambda}{2} > \lambda.$$

This is equivalent with

$$e^{\frac{(p^m - p^o)}{\lambda}} < \frac{4}{3}, \quad (17)$$

or

$$\frac{(p^m - p^o)}{\lambda} < \ln \frac{4}{3} = 0.28768. \quad (18)$$

If we subtract (15) from (14), and divide by λ , we obtain that in equilibrium it must hold that

$$\frac{p^m - p^o}{\lambda} = \frac{2}{e^{\frac{p^m - p^o}{\lambda}}} - \frac{1}{2} e^{\frac{p^m - p^o}{\lambda}}. \quad (19)$$

From (18) we get that, if there is a private incentive to merge, the LHS of (19) has a value below 0.28768. On the other hand, after realizing that the RHS of (19) is decreasing in $e^{\frac{p^m - p^o}{\lambda}}$, we can conclude from (17) that, whenever there is a private incentive to merge, the RHS has a value above

$$\frac{2}{4/3} - \frac{1}{2} \frac{4}{3} = \frac{5}{6}.$$

We conclude that the equilibrium condition (19) can never be satisfied when there is a private incentive to merge, i.e. when conditions (17) and (18) hold. This implies that we have proved the following proposition.

Proposition 6 *In the price competition model with logit demand, there is no private incentive for solvent firms to merge with a failing firm. Solvent firms strictly prefer that the failing firm leaves the market.*

3.3 the Cournot model

We make use of the Levitan-Shubik utility function

$$U = \sum_{i=1}^3 q_i - \frac{3}{2(1+\gamma)} \left[\sum_{i=1}^3 q_i^2 + \frac{\gamma}{3} \left(\sum_{i=1}^3 q_i \right)^2 \right] + y$$

and refer to Section 2 for more details. The first-order conditions result in the inverse linear demand

$$p_1^C = 1 - \frac{3+\gamma}{1+\gamma} q_1 - \frac{\gamma}{1+\gamma} q_2 - \frac{\gamma}{1+\gamma} q_3$$

for every good i . The objective of firm 1 is therefore

$$\Pi_1^C(q_1; q_2, q_3) = \max_{q_1} \left(1 - \frac{3+\gamma}{1+\gamma} q_1 - \frac{\gamma}{1+\gamma} q_2 - \frac{\gamma}{1+\gamma} q_3 \right) q_1 - F_1,$$

which results in the necessary and sufficient first-order condition

$$1 - \frac{6+2\gamma}{1+\gamma} q_1 - \frac{\gamma}{1+\gamma} q_2 - \frac{\gamma}{1+\gamma} q_3 = 0.$$

Symmetry implies that

$$1 - \frac{6+2\gamma}{1+\gamma} q^C - \frac{\gamma}{1+\gamma} q^C - \frac{\gamma}{1+\gamma} q^C = 0 \text{ or } q_i^{C*} = \frac{1+\gamma}{6+4\gamma},$$

where q_i^C denotes the equilibrium Cournot quantity. Further, we obtain that

$$p_i^C = \frac{3+\gamma}{6+4\gamma}.$$

The equilibrium Cournot profit for every firm i now becomes

$$\Pi_i^C(q_i^{C*}) = \frac{(1+\gamma)(3+\gamma)}{4(3+2\gamma)^2} - F_i.$$

3.3.1 the failing firm defense

When firm 2 acquires the failing firm, the merged entity m maximizes

$$\begin{aligned} & \Pi_m^C(q_1, q_2; q_3) \\ &= \max_{q_1, q_2} \left(\left(1 - \frac{3+\gamma}{1+\gamma} q_1 - \frac{\gamma}{1+\gamma} q_2 - \frac{\gamma}{1+\gamma} q_3 \right) q_1 + \left(1 - \frac{3+\gamma}{1+\gamma} q_2 - \frac{\gamma}{1+\gamma} q_1 - \frac{\gamma}{1+\gamma} q_3 \right) q_2 \right). \end{aligned}$$

This gives the first-order conditions

$$\begin{aligned} 1 - \frac{6+2\gamma}{1+\gamma} q_1 - \frac{2\gamma}{1+\gamma} q_2 - \frac{\gamma}{1+\gamma} q_3 &= 0, \\ 1 - \frac{6+2\gamma}{1+\gamma} q_2 - \frac{2\gamma}{1+\gamma} q_1 - \frac{\gamma}{1+\gamma} q_3 &= 0. \end{aligned}$$

From the problem of the outsider we obtain the first-order condition

$$1 - \frac{6 + 2\gamma}{1 + \gamma} q_3 - \frac{\gamma}{1 + \gamma} q_2 - \frac{\gamma}{1 + \gamma} q_1 = 0.$$

This results in

$$\begin{aligned} q_1^* &= q_2^* = q^m = \frac{(6 + \gamma)(1 + \gamma)}{6(\gamma^2 + 6\gamma + 6)}, \\ q_3^* &= q^o = \frac{(3 + \gamma)(1 + \gamma)}{3(\gamma^2 + 6\gamma + 6)}, \end{aligned}$$

and a merger profit equal to

$$\Pi_m^C(q_1^*, q_2^*, q_3^*) = \frac{1}{18} (\gamma + 1) (\gamma + 6)^2 \frac{2\gamma + 3}{(6\gamma + \gamma^2 + 6)^2}.$$

3.3.2 the failing firm exits the market

After the failing firm exits the market we have a duopoly with firms 2 and 3. Firm 2 now maximizes

$$\Pi_2^{2C}(q_2; q_3) = \max_{q_2} \left(1 - \frac{1}{1 + \gamma} (3q_2 + \gamma \sum_{j=2}^3 q_j) \right) q_2 - F_2,$$

where the inverse demand contains no longer the quantities of good 1 that were offered by the failing firm, while still reflecting consumers' preference for the three goods. The first-order condition gives

$$1 - \frac{6 + 2\gamma}{1 + \gamma} q_2 - \frac{\gamma}{1 + \gamma} q_3 = 0,$$

which for a symmetric equilibrium implies that

$$q_2^{2C*} = q_3^{2C*} = \frac{1 + \gamma}{6 + 3\gamma} \text{ and } p^* = \frac{3 + \gamma}{6 + 3\gamma}.$$

After substitution, equilibrium profits are

$$\Pi_2^{2C*}(q_2^{2C*}, q_3^{2C*}) = \frac{(1 + \gamma)(3 + \gamma)}{9(2 + \gamma)^2}.$$

To see whether there is a private incentive to engage in the failing firm defense, we check

$$\Pi_m^C - \Pi_2^{2C*} - F_1 > 0.$$

Since the failing firm breaks even when it fails, we know from the Cournot equilibrium profits that $\Pi_1^C = \frac{(1 + \gamma)(3 + \gamma)}{4(3 + 2\gamma)^2} - F_1 = 0$. Substitution learns that

$$\frac{1}{18} (\gamma + 1) (\gamma + 6)^2 \frac{2\gamma + 3}{(6\gamma + \gamma^2 + 6)^2} - \frac{(1 + \gamma)(3 + \gamma)}{9(2 + \gamma)^2} - \frac{(1 + \gamma)(3 + \gamma)}{4(3 + 2\gamma)^2} < 0,$$

so that we arrive at the following proposition.

Proposition 7 *In the Cournot competition model there is no private incentive for solvent firms to merge with a failing firm. Solvent firms strictly prefer that the failing firm leaves the market.*

4 CONCLUSIONS

This paper has shown that profitable firms have no strategic incentives to make use of the failing firm defense and engage in a rescue merger. Instead, they strictly prefer to let the firm fail and exit from the market. Acquisition of a failing firm does not follow from a number of standard models of price competition, like the representative consumer model with a Levitan-Shubik linear demand, Salop's circular model, the Logit demand model, as well as from Cournot competition. Finally, we also show that a temporary public bail-out as an alternative to exit of the failing firm may be superior for welfare as long as the goods are sufficiently heterogenous, the deadweight cost of tax collection is not too stringent, and the number of firms is not too high.

We would like to make two further comments to our analysis. First, there may be other reasons why firms could have an interest in making use of the failing firm defense. For example, when the acquiring firm runs the merged entity, we have assumed that fixed costs per offered variety did not change. Clearly, when we would introduce economies of scope or scale at the level of fixed or variable costs, acquisition of the failing firm may become a profitable strategy.¹³ In the extreme case where the fixed costs of the failing firm disappear, all the models that we considered may offer room for a profitable acquisition. Second, the literature on predation has shown that the failing firm defense, under certain circumstances, could alter firms' incentives to engage in predatory pricing to decrease the asset values of a weak firm (Saloner, 1987). Our analysis, however, shows that firms' incentives to prey are not altered, because firms have no interest in acquiring the failing firm.

Competition authorities can, under strict conditions, rely on the failing firm defense to clear mergers when competition is likely to be substantially weakened. During the recent financial crisis, public authorities have paid renewed attention towards the appropriate failing firm defense policy. The OFT, for example, restated its existing strict guidelines regarding the failing firm defense, taking account of the prevailing economic and market conditions. It remains unclear, however, to what extent the failing firm defense will be accepted as an argument to clear certain mergers during economic downturn as compared to normal market conditions. It may, for example, be argued that, when the state of the economy is bad, the appropriate substitute counterfactual is not firm failure but government intervention. That is, it may well be the case that the failing firm defense has less bite when market conditions are bad. The takeover of the failing Scotland Halifax Bank Of Scotland (HBOS) by Lloyds TSB is illustrative. The OFT did not accept that the application of the failing firm defense was appropriate. The argument

was that “the OFT considers that the application of the failing firm defence in this case is not appropriate given that it is not realistic to consider that HBOS would have been allowed to fail [...] and therefore has ruled out failure/exit as a possible substitute counterfactual.” (OFT, 2008). The takeover has also been criticized because of its irreversible characteristic (Vickers, 2008). In times of high economic uncertainty or when markets decline only temporarily, clearing a merger when the acquired party is in financial distress, with or without reference to the failing firm defense may, in the long run, be a costly decision. Temporary government intervention, through e.g. a bail-out, could, therefore, improve long-run welfare implications. Of course, important challenges remain to be dealt with, like how to distinguish failing firms from flailing firms, or to what extent different types of uncertainties (e.g. firm specific, industry specific, or economy wide shocks) influence the effects of different forms of government intervention on long-run total welfare. These are interesting topics for evaluating existing or proposed industrial policies.

5 Appendix: linear demand analysis with N firms

This appendix shows that our results with three firms generalize to N firms. We consider our linear demand function where firms are competing in prices. While in the main text we restrict ourselves to three firms, here we let the number of firms be N . Consumer utility is

$$U^N = \sum_{i=1}^N q_i - \frac{N}{2(1+\gamma)} \left[\sum_{i=1}^N q_i^2 + \frac{\gamma}{N} \left(\sum_{i=1}^N q_i \right)^2 \right] + y.$$

Maximizing utility yields

$$q_i = \frac{1}{N} \left(1 - (1+\gamma)p_i + \frac{\gamma}{N} \sum_{j=1}^N p_j \right).$$

In a symmetric equilibrium we have

$$p^N = \frac{N}{2N + (N-1)\gamma} \text{ and } \Pi_i^N = \frac{N + (N-1)\gamma}{(2N + (N-1)\gamma)^2} - F_i.$$

5.1 failing firm defense

Suppose that firm 1 and firm i merge. This gives

$$\begin{aligned}
 p^o &= \frac{N + (N - 1)\gamma}{(N - 2)\gamma^2 + 3(N - 1)\gamma + 2N}, \\
 p^m &= \frac{2N + (2N - 1)\gamma}{2((N - 2)\gamma^2 + 3(N - 1)\gamma + 2N)}, \\
 \Pi^m &= \frac{(N - 2)(2N - 1)^2\gamma^3 + 3N(2N - 1)(2N - 3)\gamma^2 + 12(N - 1)N^2\gamma + 4N^3}{2N^2((N - 2)\gamma^2 + 3(N - 1)\gamma + 2N)^2} - F_1 - F_i, \\
 \Pi^o &= \frac{((N - 1)\gamma + N)^3}{N^2((N - 2)\gamma^2 + 3(N - 1)\gamma + 2N)^2}.
 \end{aligned}$$

5.2 The failing firm exits from the market

With only $N - 1$ firms left, we obtain that

$$\begin{aligned}
 p^{N-1} &= \frac{N}{2N + (N - 2)\gamma} \\
 \Pi^{N-1} &= (1 + \gamma) \frac{N(N + (N - 2)\gamma)}{(N + (N - 1)\gamma)(2N + (N - 2)\gamma)^2} - F_i.
 \end{aligned}$$

When failing, the profit of firm 1 equals zero, i.e.

$$F_1 = \frac{N + (N - 1)\gamma}{(2N + (N - 1)\gamma)^2}.$$

We obtain after substitution that

$$\Pi^{N-1} > \Pi^m.$$

This results in the following proposition.

Proposition 8 *The remaining $N - 1$ solvent firm have no incentive to acquire the insolvent firm. They strictly prefer that the failing firm leaves the market.*

5.3 Social incentives for the failing firm defense

Now we calculate the consumer surplus. From the utility functions

$$U^N = \sum_{i=1}^N q_i - \frac{N}{2(1 + \gamma)} \left[\sum_{i=1}^N q_i^2 + \frac{\gamma}{N} \left(\sum_{i=1}^N q_i \right)^2 \right] + y$$

and

$$U^{N-1} = \sum_{i=2}^N q_i - \frac{N}{2(1 + \gamma)} \left[\sum_{i=2}^N q_i^2 + \frac{\gamma}{N} \left(\sum_{i=2}^N q_i \right)^2 \right] + y,$$

we obtain

$$\begin{aligned} CS^{N-1} &= U^{N-1} - (N-1)p^{N-1}q^{N-1} \\ &= \frac{1}{2} \frac{(\gamma+1)(N-1)}{(N+(N-1)\gamma)} \frac{(N-2\gamma+N\gamma)^2}{(2N-2\gamma+N\gamma)^2}, \end{aligned}$$

and

$$\begin{aligned} CS^m &= U^N - (N-2)p^o q^o - 2p^m q^m \\ &= \frac{2N^2(N-2)^2\gamma^4 + (N-2)(8N^3 - 8N^2 - 4N + 1)\gamma^3}{4N^2((N-2)\gamma^2 + 3(N-1)\gamma + 2N)^2} \\ &\quad + \frac{3N(4N^3 - 8N^2 + 3)\gamma^2 + 4N^2(2N^2 - 2N - 1)\gamma + 2N^4}{4N^2((N-2)\gamma^2 + 3(N-1)\gamma + 2N)^2}. \end{aligned}$$

Taking the difference we find that the failing firm defense raises consumer surplus:

$$CS^m - CS^{N-1} = \frac{D}{4N^2(N+(N-1)\gamma)(2N-2\gamma+N\gamma)^2((N-2)\gamma^2 + 3(N-1)\gamma + 2N)^2} > 0,$$

where

$$\begin{aligned} D &\equiv (5N - 12N^2 + 6N^3 - 1)(N-2)^3\gamma^6 \\ &\quad + 5N(4N-3)(-4N+2N^2+1)(N-2)^2\gamma^5 \\ &\quad + N^2(N-2)(387N - 378N^2 + 108N^3 - 110)\gamma^4 \\ &\quad + 2N^3(N-2)(-169N + 76N^2 + 83)\gamma^3 \\ &\quad + 2N^4(-171N + 59N^2 + 115)\gamma^2 \\ &\quad + 24N^5(2N-3)\gamma \\ &\quad + 8N^6. \end{aligned}$$

Hence, consumers prefer the failing firm defense to an exit of the failing firm. From a welfare perspective, allowing a failing firm defense is optimal when

$$\Pi^m + (N-2)\pi^o + CS^m > (N-1)\pi^{N-1} + CS^{N-1}.$$

To check whether this is the case here, we calculate

$$\Pi^m + (N-2)\pi^o + CS^m - (N-1)\pi^{N-1} - CS^{N-1},$$

which equals

$$\frac{A}{4N^2(N+(N-1)\gamma)(2N+3(N-1)\gamma+(N-2)\gamma^2)^2(2N+(N-2)\gamma)^2(2N+(N-1)\gamma)^2},$$

where

$$\begin{aligned}
A \equiv & (-N + 4N^2 - 2N^3 + 1) (N - 1)^2 (N - 2)^3 \gamma^8 \\
& + (-24N + 42N^2 - 16N^3 + 11) N (N - 1)^2 (N - 2)^2 \gamma^7 \\
& + (-235N + 406N^2 - 411N^3 + 218N^4 - 44N^5 + 62) N^2 (N - 2) \gamma^6 \\
& + 2 (N + 47N^2 - 5N^3 - 12N^4 - 25) N^3 (N - 2) \gamma^5 \\
& + 2 (-743N + 964N^2 - 447N^3 + 63N^4 + 153) N^4 \gamma^4 \\
& + 8 (257N - 188N^2 + 39N^3 - 98) N^5 \gamma^3 \\
& + 8 (-127N + 40N^2 + 86) N^6 \gamma^2 + 32N^7 (5N - 8) \gamma + 32N^8.
\end{aligned}$$

It is straightforward to verify that the terms in front of γ^i , with $i \in \{0, 1, 2, 3\}$ are positive for $N \geq 3$, the term associated with γ^4 is negative for $N = 3$ but positive for $N > 3$, whereas the terms in front of γ^i , with $i \in \{5, 6, 7, 8\}$ are negative for $N \geq 3$. It follows that results are exactly the same as in the case with three firms, i.e. the following proposition holds.

Proposition 9 *Take the number of firms be equal to $N \geq 3$. For sufficiently low values of γ it holds that from a social perspective the failing firm defense is preferred to immediate exit as it results in higher total welfare. Consumers always prefer a failing firm defense to exit of the failing firm.*

5.4 *bail-out vs. failing firm defense with N firms: a welfare comparison*

The demand function for N firms is given by

$$q_i = \frac{1}{N} \mu^t \left(1 - (1 + \gamma) p_i + \frac{\gamma}{N} \sum_{j=1}^N p_j \right).$$

Assume firm 1 fails at time zero. As a result, its profits $\Pi_i^N(t)$ are zero for $t = 0$, or

$$\Pi_i^N(0) = \frac{N + (N - 1) \gamma}{(2N + (N - 1) \gamma)^2} - F_1 = 0.$$

As time passes, firm 1's profit becomes negative. To prevent firm 1 from exiting, the government needs to compensate firm 1 so as to keep its profits equal to zero. Denoting the compensation payment at time t by $S^N(t)$, it follows that

$$S^N(t) = -\Pi_i^N(t) = (1 - \mu^t) \frac{(N + (N - 1) \gamma)}{(2N + (N - 1) \gamma)^2},$$

where the superscript N refers to the number of firms, and where $S^N(0) = 0$, $S^{N'}(t) > 0$, and $S^{N''}(t) < 0$. In addition, since profits decrease when the industry is less concentrated, the amount needed to subsidy the failing firm increases with the number of firms, or $\partial S^N / \partial N > 0$. If the aim of a bail-out policy is to increase welfare, it should be carried out only as long as

$$PS^S(t) + CS^S(t) > PS^{N-1}(t) + CS^{N-1}(t). \quad (20)$$

Or, the bail-out policy is socially optimal only during a time interval $[0, T_1^S)$, where T_1^S satisfies the equality

$$PS^S(T_1^S) + CS^S(T_1^S) = PS^{N-1}(T_1^S) + CS^{N-1}(T_1^S).$$

To verify that a bail-out policy is indeed welfare improving, we have to show that there is a time interval $[0, T_1^S)$ during which (20) holds. From our analysis of N firms, we obtain that

$$\begin{aligned} PS^{N-1}(t) + CS^{N-1}(t) &= \sum_{i=2}^N \Pi_i^{N-1} + CS^{N-1} \\ &= (1 + \gamma) \frac{N(N-1)(N + (N-2)\gamma)\mu^t}{(N + (N-1)\gamma)(2N + (N-2)\gamma)^2} \\ &\quad - \sum_{i=2}^N F_i + \frac{1}{2} \frac{(\gamma+1)(N-1)(N-2\gamma+N\gamma)^2}{(N + (N-1)\gamma)(2N - 2\gamma + N\gamma)^2}. \end{aligned}$$

From the bail-out analysis we have that

$$\begin{aligned} PS^S(t) &= PS^N(t) + S(t) = \sum_{i=2}^N \Pi_i^N = (N-1) \frac{(N + (N-1)\gamma)\mu^t}{(2N + (N-1)\gamma)^2} - \sum_{i=2}^N F_i, \\ CS^S(t) &= CS^N(t) - (1 + \lambda)S^N(t). \end{aligned}$$

Due to the declining market assumption, the producer and consumer surpluses all continuously decrease over time. So, if we establish that (20) holds at time zero, we know that this inequality will hold during some time interval starting from time zero on.

What is still needed for evaluating (20) at time zero is to derive $CS^N(0)$. So, we must derive the consumer surplus in case of N competing firms. With utility function

$$U^N = \sum_{i=1}^N q_i - \frac{N}{2(1+\gamma)} \left[\sum_{i=1}^N q_i^2 + \frac{\gamma}{N} \left(\sum_{i=1}^N q_i \right)^2 \right] + y$$

we obtain the consumer surplus

$$CS^N = U^N - Np^N q^N = \frac{1}{2} \frac{(N - \gamma + N\gamma)^2}{(2N - \gamma + N\gamma)^2}.$$

We find that the sign of

$$\begin{aligned}
& CS^S(0) - CS^{N-1}(0) \\
= & \frac{1}{2(N - \gamma + N\gamma)(N^2\gamma^2 + 4N^2\gamma + 4N^2 - 3N\gamma^2 - 6N\gamma + 2\gamma^2)^2} \\
& ((3N^4 - 14N^3 + 23N^2 - 16N + 4)\gamma^4 + (14N^4 - 49N^3 + 55N^2 - 20N)\gamma^3 \\
& + (23N^4 - 55N^3 + 33N^2)\gamma^2 + (16N^4 - 20N^3)\gamma + 4N^4) - (1 + \lambda)S^N,
\end{aligned}$$

is positive when the social cost of funding is sufficiently low. Moreover,

$$\begin{aligned}
& PS^S(0) - PS^{N-1}(0) \\
= & (N - 1) \frac{(N + (N - 1)\gamma)}{(2N + (N - 1)\gamma)^2} - (1 + \gamma) \frac{N(N - 1)(N + (N - 2)\gamma)}{(N + (N - 1)\gamma)(2N + (N - 2)\gamma)^2} \\
= & \frac{-\gamma}{N - 1} \frac{(N - \gamma + N\gamma)(N^2\gamma^2 + 4N^2\gamma + 4N^2 - 3N\gamma^2 - 6N\gamma + 2\gamma^2)^2}{((2N^3 - 8N^2 + 10N - 4)\gamma^3 + (8N^3 - 22N^2 + 14N)\gamma^2 + (10N^3 - 15N^2)\gamma + 4N^3)} \\
< & 0,
\end{aligned}$$

is always negative. Defining $W^G(0) \equiv PS^S(0) + CS^S(0) - PS^{N-1}(0) - CS^{N-1}(0)$, where W^G stands for the welfare gain obtained by the bail-out policy, we find that

$$W^G(0) = \frac{Z}{2(N - \gamma + N\gamma)(N^2\gamma^2 + 4N^2\gamma + 4N^2 - 3N\gamma^2 - 6N\gamma + 2\gamma^2)^2} - (1 + \lambda)S^N.$$

where $Z \equiv ((-N^4 + 6N^3 - 13N^2 + 12N - 4)\gamma^4 + (-2N^4 + 11N^3 - 17N^2 + 8N)\gamma^3 + (3N^4 - 5N^3 + 3N^2)\gamma^2 + (8N^4 - 12N^3)\gamma + 4N^4)$. The term in Z associated with γ^4 is negative for $N \geq 3$, the term in front of γ^3 is negative for $N > 3$, while the other terms are all positive for $N \geq 3$. Furthermore,

$$\begin{aligned}
\frac{\partial W^G(0)}{\partial N} = & \frac{1}{4(N - \gamma + N\gamma)^2(N^2\gamma^2 + 4N^2\gamma + 4N^2 - 3N\gamma^2 - 6N\gamma + 2\gamma^2)^4} \\
& ((2N^8 - 24N^7 + 124N^6 - 360N^5 + 642N^4 - 720N^3 + 496N^2 - 192N + 32)\gamma^9 \\
& + (22N^8 - 260N^7 + 1280N^6 - 3452N^5 + 5602N^4 - 5616N^3 + 3400N^2 - 1136N + 160)\gamma^8 \\
& + (94N^8 - 1144N^7 + 5368N^6 - 13120N^5 + 18434N^4 - 15088N^3 + 6720N^2 - 1264N)\gamma^7 \\
& + (170N^8 - 2524N^7 + 11536N^6 - 24952N^5 + 28578N^4 - 16880N^3 + 4072N^2)\gamma^6 \\
& + (-24N^8 - 2544N^7 + 12896N^6 - 24400N^5 + 20768N^4 - 6720N^3)\gamma^5 \\
& + (-696N^8 + 160N^7 + 6080N^6 - 10976N^5 + 5608N^4)\gamma^4 \\
& + (-1376N^8 + 2944N^7 - 512N^6 - 1472N^5)\gamma^3 + (-1312N^8 + 2624N^7 - 1024N^6)\gamma^2 \\
& + (-640N^8 + 768N^7)\gamma - 128N^8) - \lambda \partial S^N / \partial N.
\end{aligned}$$

The term associated with γ^9 is positive for $N \geq 3$, the term associated with γ^8 is positive for $N \geq 4$ (and negative for $N = 3$), the term associated with γ^7 is positive for $N \geq 5$ (and negative for $N = \{3, 4\}$), the term associated with γ^6 is positive for $N \geq 9$ (and negative for $N = \{3, \dots, 8\}$), whereas all other terms are always negative.

The above results are collected in the following proposition.

Proposition 10 *A government bail-out of the failing firm provides a higher consumer surplus and a lower producer surplus. Total welfare increases, provided the goods are sufficiently heterogeneous and the social cost for funding the bail-out is not too costly. Total welfare gains from a bail-out policy decrease with the number of firms.*

Proposition 10 implies that the loss in consumer surplus, caused by taxing consumers to finance the bail-out, is smaller than the loss consumers suffer from the price increase when the failing firm exits the market. The difference, however, decreases with the number of firms for two reasons. First, when the number of firms augments, prices increase to a lesser extent when the failing firm defense applies. Second, the amount of capital needed to finance the bail-out and make the least efficient firm break-even, increases as competition between firms is more intense. Since oligopoly profits decrease in the number of firms, $S(N)$ is increasing in N . As a result, when the shadow cost λS to the tax payers is taken into account, a bail-out may become socially more costly than exit of the failing firm. That is, there exists a N^* such that $TS^N(N^*) - \lambda S^N(N^*) \leq TS^{N^*-1}(N^*)$ for all $N \geq N^*$.

Notes

¹See Persson (2005) for a more extensive list.

²Scherer and Ross (1990), however, observe that failing targets account for almost 6% in a large merger sample (see Persson, 2005).

³For the purposes of this paper, we will consider the treatment of the failing firm defense as equivalent. For an in-depth discussion and comparison of the European and US rules, see Persson (2005).

⁴See OECD (2009) Report for a recent overview.

⁵Earlier work by Campbell (1984) and Friedman (1986) show that the failing firm defense prevents liquidation of productive assets and may therefore be economically sound.

⁶We show in the Appendix that our analysis for a symmetric price competition with linear demand also applies to any number N of firms.

⁷Differences in marginal costs between firms would only make calculations more tedious without adding insights. More importantly, the assumption that firms only differ with respect to fixed costs is more restrictive and therefore strengthens our results.

⁸See Höffler and Schmidt (2008) and Höffler (2008).

⁹The other alternative where F_i increases over time yields similar insights.

¹⁰For simplicity, we assume that the rate at which the market declines is sufficiently slow so that only the least efficient firm fails.

¹¹The government could also collect its tax revenues from the solvent firms. This approach, however, could change the exit structure of the industry. To avoid this from happening and stay within our framework, we prefer to collect the tax revenues from the consumers. The subsidy is *de facto* a transfer from consumers to the failing firm. It pays for a 3-firm symmetric market to continue longer, and can be thought of as the price of preserving competition and product variety. Observe that when firm 1 fails and exits from the market, a transfer from consumers to producers takes place as well: producer profits, and thus producer surplus, increase whereas consumer expenditures increase as the prices of the two remaining products increase. Both events result in a monetary flow from consumers to producers.

¹²An alternative way to model exit of the failing firm is when firm 1 charges a prohibitive price $\bar{p}_1 = p_2 + t/3 = p_3 + t/3$, resulting in no demand for firm 1. Then, firm 2, say, optimally charges

$$t = \bar{p}_2^* \equiv \arg \max_{p_2} p_2 \left[\frac{p_2 + t/3 - p_2}{2t} + \frac{1}{6} + \frac{1}{6} - \frac{p_2 - p_3}{2t} \right].$$

Remark that competition with only two firms restores the result that competition among fewer firms results in an equilibrium price higher than what a merged entity would charge in equilibrium, since $p^m < \bar{p}_2^*$.

¹³Persson (2005) e.g. makes use of an increasing returns to scale setting.

6 references

Anderson S. and A. de Palma, 1992, The logit as a model of product differentiation, *Oxford Economic Papers*, 44, 1, 51-67.

Campbell, T., 1984, The efficiency of the failing company defense, *Texas Law Review*, 63, 251-283.

Deneckere, R. and C. Davidson, 1985, Incentives to form coalitions with Bertrand competition, *RAND Journal of Economics*, 16 (4), pp. 473-486.

Farrell, J. and Shapiro, C., 1990, "Horizontal mergers: an equilibrium analysis", *American Economic Review*, 80(1), 107-126.

Friedman, R., 1986, Untangling the failing company doctrine, *Texas Law Review*, 64, 1375-1426.

Höffler, F., 2008, On the consistent use of linear demand systems if not all varieties are available, WHU – *Working Paper Series in Economics*, WP 08/1.

Höffler, F. and K. Schmidt, 2008, Two tales on resale, *International Journal of Industrial Organization*, 26, 6, 1448-1460.

Mason, R. and H. Weeds, 2010. *Merger policy, entry, and entrepreneurship*, mimeo, University of Essex, Department of Economics.

- Motta, M., 2004, Competition policy: theory and practice, *Cambridge University Press*.
- OECD, 2009, The Failing Firm Defence, Policy Roundtables, 38, Directorate for Financial and Enterprise Affairs, Competition Committee.
- OFT, 2008, *Restatement of OFT's position regarding acquisitions of 'failing firms'*, December, OFT1047.
- Perry, M. and Porter, R. H., 1985, "Oligopoly and the Incentive for Horizontal Merger", *American Economic Review*, 75(1), 219-227.
- Persson, L., 2005, The failing firm defense, *Journal of Industrial Economics*, 53, 2, 175-201.
- Salant, S.W., Switzer, S. and R.J. Reynolds, 1983, Losses from horizontal merger: the effects of an exogenous change in industry structure on Cournot-Nash equilibrium, *Quarterly Journal of Economics*, 1983, 98, 185-99.
- Saloner, G., 1987, Predation, mergers, and incomplete information", *RAND Journal of Economics*, 18, 2, 165-186.
- Salop, S., 1979, Monopolistic competition with outside goods, *Bell Journal of Economics*, 10, Spring, 141-156.
- Shubik, M. and R. Levitan, 1980, *Market Structure and Behavior*. Harvard University Press.
- Vasconcelos, H, 2012, Can the failing firm defense rule be counterproductive?, forthcoming *Oxford Economic Papers*.