A Note on Paul Samuelson's Early Methodology

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Abstract: This paper bears on Paul Samuelson's early methodology as presented in his *Foundations of Economic Analysis* (1947). The paper sheds new light on Samuelson's approach towards mathematical economics by reflecting on the similarity between his mathematical economics and Edwin B. Wilson's mathematics. Wilson was Samuelson's professor of advanced mathematical and statistical economics; he was also a protégé of Josiah Willard Gibbs. Wilson defined mathematics as a vernacular language that consisted of three interconnected aspects: postulational, axiomatic and operational. In his *Foundations*, in a Wilsonian style, Samuelson wrote in the opening page, "Mathematics is a Language" and claimed that he offered "operationally meaningful theorems." In this paper, it is argued that these maxims embodied Wilson's approach, which framed Samuelson's mathematical and statistical thinking around 1940 and which led him to present his work as being mathematically, theoretically and empirically well founded.

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Introduction

In the opening page of his *Foundations of Economic Analysis* (1983 [1947]), Paul Samuelson (1915-2009) wrote "Mathematics is a Language", and attributed this motto to Josiah Willard Gibbs. In *Foundations*, Samuelson also argued that he offered "operationally meaningful theorems," which he presented as being of observational and empirical significance because they could be tested in idealized conditions. Historians, philosophers and methodologists of economics have often studied Samuelson's methodology by using *ex-post* frameworks of rational reconstructions. In these accounts, Samuelson's ideas about operational and meaningful knowledge are analyzed in reference to comprehensive philosophies such as operationalism, logical positivism, empiricism, falsificationism, descriptivism or behaviorism, or in reference to the methodological positions of other economists who themselves explicitly adopted some of these philosophical positions.[1] In these accounts, Samuelson's ideas about operational, meaningful, observational and testable knowledge appear philosophically naïve and unintelligible because they do not properly fit with these rationalizing reference points.

Often, if not systematically, the contributions used to reconstruct Samuelson's methodological ideas are comments from Samuelson during the 1960s.[2] These comments were related to a controversy over the realism of hypotheses in economics.[3] In them, Samuelson invites us to look back to his *Foundations,* which was an extension of his 1941 doctoral thesis (1941), and to interpret them as consistently related to some of the above-mentioned comprehensive philosophies. His comments are surely enlightening for our understanding of Samuelson's (naïve) philosophical pretentions in the 1960s, and

[1] See for example (Hands 2001; Mongin 2000; Cohen 1995; Hausman 1992; Boland 1989; Caldwell 1982; Blaug 1980; Wong 1978, 1973).
[2] See for example Archibald, Simon, and Samuelson 1963; Samuelson 1964; Samuelson 1965.
[3] See Friedman 1953; Nagel 1961; 1963; Machlup 1964; Garb 1965; Massey 1965; Lerner 1965.

probably for contextualizing in methodological terms his work in economics in the 1960s and thereafter. They are, however, misleading in regards to his earlier work: there is no well-documented evidence that around 1940 Samuelson seriously engaged with any of these comprehensive philosophies.

Around 1940, instead, Samuelson was committed to Edwin Bidwell Wilson's (1879-1964) ideas about mathematics and science. Wilson was an American mathematician, trained around 1900 at Harvard University, Yale University and the *Ecole Normale Supérieure* in Paris. Over the years, Wilson marginalized himself from the American mathematical community, as he engaged professionally with mathematical physics at the Massachusetts Institute of Technology (1907-1922), and in statistics, social science and economics at Harvard University while directing the chair of biostatistics at the Harvard School of Public Health (1922-1945). During the 1900s, Wilson was one of the most active young mathematicians in American mathematics; during the 1910s, he made relatively important contributions within the American scene in mathematical physics, especially in relativity; during the 1920s, he made significant contributions in statistics, which led to his election as president of the American Statistical Association in 1929; during the 1920s and the 1930s, Wilson was also key for the rise of mathematical economics in America: he played a crucial role in the establishment of the Econometric Society in America and offered the first program in advanced mathematical economics and mathematical statistics at Harvard during the 1930s. Over the years, Wilson developed strong ideas about the foundations of mathematics and science; he defined mathematics as a language and presented himself as the best representative of Josiah Willard Gibbs's American style of (applied) mathematics.[4]

By reflecting on Wilson's definition of mathematics as a language, this note discusses Samuelson's methodology as presented in *Foundations*. It will be argued that Samuelson's *operationally meaningful theorems* strongly reflected Wilson's definition of mathematics as a language, indeed a vernacular, which consisted of three interconnected aspects: a postulaional, which imposed structure; an axiomatic, which conveyed meaning;

[4] On Wilson and his influence on American sciences, economics and Samuelson see respectively (Carvajalino 2017, Forthcoming, 2016).

and operational, which connected the latter two. In its operational aspect, meaning was preeminent over mathematical structure.

E. B. Wilson: Mathematics as a Language[5]

Wilson thought that mathematics was applicable to (natural and social) science because there must be something that could be taken as invariant (Wilson 1903a, 85). He suggested two sources of invariance: the human capacity for learning any kind of language (Wilson 1940), and scientific conventions of specific subject matters. Wilson defined mathematics as a language, indeed as a vernacular, because he believed that everybody had the cognitive capacity to learn and eventually use its (algebraic and statistical) techniques in their professional and practical lives. This was the case, because

> "there [seemed] to be no present conclusive evidence that learning a particular technique [was] impossible to any person […] and, therefore, each could presumably learn any technique and use it in much the same sense as he could learn any language and write in it" (Wilson 1940, 664).

By defining mathematics as a language, he also meant that mathematics and science should be regarded as interconnected and interdependent. Mathematics for Wilson consisted of a permanent back-and-forth between certain elements found in mathematics and certain elements found in subject matters, to which he respectively referred as postulates and axioms, and which represented two distinct but interrelated categories of scientific hypothesis.

Wilson defined each term as well as the desired relationship between them. For this purpose, he used some aspects of David Hilbert's, Bertrand Russell's and Henri Poincaré's ideas about mathematics, while suggesting that his mathematics best

[5] This section is largely inspired by (Carvajalino 2017).

represented Gibbs's American mathematical style.[6] Eventually he suggested that mathematics consisted of three interconnected and interdependent aspects: one postulational, one axiomatic and one operational.[7]

Postulates, Wilson believed, were elements that belonged to pure mathematics and logic. They represented self-evidences or *a priori* truths for professional mathematicians. They did not have an empirical existence. They only existed in the minds of mathematicians who had learned in a classroom the criteria that validated their existence. Hilbert and Russell, Wilson believed, offered distinct but complementary criteria determining the validity of postulates. Wilson stressed that Hilbert had "created an epoch in the technique of mathematics" (Wilson 1904, 77). Russell, Wilson thought, based mathematics on logical inference and offered the best philosophical foundations for pure mathematics as he "presupposes a mind capable of rational, that is, non-selfcontradictory ratiocinative processes" (Wilson 1904, 79). In this way, "[p]ure mathematics [became] the class of all proposition of the form 'p implies q'" (Wilson 1904, 76). For Wilson, Hilbert and Russell had improved the technical and logical criteria determining the validity of postulates. Postulates represented hence a sort of valid grammar with which to structure (pure) mathematical thinking.

Postulates, however, were only necessary, not sufficient, to found mathematics. Hilbert and Russell, Wilson argued, were only concerned with pure technique and pure logic. While Hilbert detached mathematics from intuition and judgment—two central notions for Wilson's mathematics—Russell reduced mathematics to formalism and symbolism, which Wilson rejected. As grammar interacted with semantics to convey meaning, postulates needed to be interconnected with judgment and intuition related to axioms found in subject matters in order to produce meaning. Alone, postulates were only meaningless—and therefore useless—techniques and pure philosophical devices.[8]

Axioms, for Wilson, represented convenient conventional working hypotheses found in subject matters, which studied specific phenomena. By this, following Poincaré,

[6] See in particular (Wilson 1903a, 1903c, 1904, 1906) and
[7] E. Wilson, unpublished and undated paper (Papers of Edwin Bidwell Wilson (PEBW), Harvard University Archives (HUA), HUG4878.214, Folder: Miscellaneous Papers, Chapter I. General Introduction).
[8] See in particular (Wilson 1903c, 1904).

Wilson believed that knowledge in science was never to be regarded as structural or universally true; it resulted from a variety of working hypotheses.[9] It could only be held as partial, probable and approximate.[10] As the criterion of prevalence of a working hypothesis over another was not necessarily evident, Wilson adopted a conventionalist epistemology. In this vein, axioms embodied the social and consensual nature of science; they represented agreement among scientists for the "best" way they could mediate between theoretical and empirical emphases in their work, namely between different sets of theory and data.[11] In the pragmatic sense of convenience, axioms corresponded to Wilson's second source of invariance in science, as they represented things that "change so slowly that we may regard them for practical purposes as non-changing or at any rate can assign limits to their change in amount and not [in] time."[12] As conventional convenient invariances, Wilson pointed out, axioms often became self-evidences, as they became things that students learned in a classroom and that subsequently framed their intuition and judgment about the "actual" facts that they studied.[13] During their professional lives, scientists gained experience in dealing with theory and data. If they understood that axioms only corresponded to a working knowledge, not a structural or universal one, Wilson believed that with better experience, scientists would acquire better intuition and better judgment to mediate between theory and data.[14] Axioms, however, should never be taken as endorsed principles, as Wilson thought Poincaré often did.[15] Pragmatism did not imply that scientists could forget about the thing being studied. If they did, axioms became pure theoretical entities detached from phenomena, hence useless devices.

Mathematics as a language, for Wilson, thus implied that mathematics consisted of imposing certain mathematical structures, as grammatical rules, to subject matters. This meant using postulates, namely sophisticated (algebraic and statistical) techniques and

[9] For Wilson, for example, the conservation of energy and continuity were only working hypotheses, namely temporary convenient conventions (Wilson 1914).
[10] See in particular (Wilson 1920).
[11] On Wilson's ideas about statistics and statistical inference see (Wilson 1923a, 1923b, 1926a, 1926b, 1927b, 1927a, 1930).
[12] E. Wilson to C. Snyder, 2 June 1934 (PEBW, HUA, HUG4878.203, Box 24).
[13] See (Wilson 1904).
[14] See references of footnote 12.
[15] See (Wilson 1906). See also (Gray 2013).

advanced logic when mediating between theory and data of subject matters. It also implied that subject matters in return provided intuition and judgment, indeed meaning as semantics, to the mathematics. In other words, for Wilson, mathematics as a language implied that while mathematics was indispensable for science, science provided immediate intelligibility and applicability to the mathematics. By interconnecting mathematics and science, mathematicians/scientists could better mediate between theory and data and could eventually produce *structured* and *meaningful* scientific knowledge.

The establishment of the interconnections, indeed correspondences, between postulates and axioms represented for Wilson a process of translation that he referred to as the operational aspect of mathematics. This operational aspect was different from the postulational (intellectual) and the axiomatic (practical) aspects of mathematics, as it was not in itself "of practical or intellectual interest." [16] This "operational […] side," implied using and sometimes creating new algebraic/statistical "series of rules of operation often both dull and unintelligible,"[17] but which were not more abstract than the well-known and intuitive arithmetic operations of division and multiplication. During this operational process, Wilson insisted that the emphasis had to be put on creating (structured) meaning; this implied that in the study of phenomena, intuition of subject matters must prevail over pure mathematical consistency: axioms were preeminent over postulates. Axioms attached theories of subject matters to immediate concerns of scientists, namely to the data of the "real" world. This emphasis on intuition and meaning implied an impressionist approach towards mathematics for Wilson, who favored mathematically incomplete scientific contributions in which the mathematical gaps were filled with intuition of the subject matter, than contributions that were merely concerned with mathematical consistency. In his words:

> "whether the [work] is mathematically complete or not does not interest me; this is unimportant. Science advances not so much by the completeness or

[16] *Idem*, (p.2).
[17] E. Wilson, unpublished and undated paper (PEBW, HUA, HUG4878.214, Folder: Miscellaneous Papers, Chapter I. General Introduction, p.1).

elegance of its mathematics as by the significance of its facts." (Wilson 1928a, 244).

All in all, by defining mathematics as a language, Wilson suggested that scientists needed to have sufficient knowledge of certain mathematical structures (postulates), deep understanding of the conventional working hypotheses (axioms) of their field, and be confident enough with their operational knowledge, in particular in (vector) algebra, advanced calculus and statistics, in order to establish correspondences between postulates and axioms. In this operational aspect of mathematics, Wilson suggested that scientists should create *much science with little mathematics* by focusing mainly on meaning and intuition and not only on mathematical consistency, and this even if they used highly sophisticated mathematical tools.[18]

Wilson's ideas about mathematics were rooted in his belief that modern (pure) mathematics and many scientific fields lacked sound scientific foundations. As a result, mathematicians and scientists tended to adopt wrong methodological approaches in their professional practices. This yielded confusion in science. Wilson, a pedagogue, believed that if students were instructed in the right kind of mathematics (namely his), they would eventually adopt a sound mathematical and scientific approach in their professional practices, within (and outside) academia. Eventually they would develop their fields based on sound mathematical and scientific foundations. During his whole career, Wilson argued for reforms of mathematical education in American colleges.[19] In the 1930s, he established the first program of advanced mathematical and statistical economics at the Harvard Department of Economics.[20]

[18] See also Wilson 1928b.
[19] Wilson pleaded for an intermediate kind of mathematics, namely not only concerned with pure mathematical consistency and acting as intermediary between mathematics and subject matters. This intermediate matheamtics was more accessible for students, he thought. For Wilson's pleas, see (Wilson 1903b, 1911b, 1913, 1915); for the kind of mathematics that he regarded as intermediate, see (Wilson 1911a).
[20] See (Carvajalino Forthcoming).

Reflections on Samuelson's Early Methodology[21]

Having graduated in economics from the University of Chicago, Samuelson entered the graduate program of economics in 1935 at Harvard thanks to a highly selective pre-doctoral scholarship awarded by the Social Science Research Council. He was then 20 years old and particularly well-trained in college mathematics. During the springs of 1936 and of 1937, he took Wilson's courses on Mathematical Statistics and Mathematical Economics respectively. He succeeded in both and impressed Wilson. Eventually Samuelson committed to Wilson's mathematics, which framed his early mathematical and statistical thinking and which led him to believe, as he attested in the opening page of his 1941 thesis and his 1947 *Foundations*, that "Mathematics was a Language." Rightly or wrongly, Samuelson attributed this motto to Gibbs. With it, Samuelson invited the reader to relate his work to Gibbs's American mathematics, of which Wilson had always claimed to be the best representative.

In *Foundations*, Samuelson argued that he offered *operationally meaningful theorems*. They embodied his commitment to Wilson's mathematics, as they represented Samuelson's effort of interconnecting certain mathematical structures (postulates) with standard conventions in economics (axioms) using his operational knowledge of mathematics. To support the argument of a commitment, let us interpret how Samuelson's *operationally meaningful theorems* reflected Wilson's postulational, axiomatic and operational aspects of mathematics.

First of all, Samuelson's *operationally meaningful theorems* have a clear postulational pretention, as he effectively interconnected his work in economics with certain mathematical structures. More particularly, he imposed the "structural characteristics of the equilibrium set" (Samuelson 1941, 15) to the study of the individual and the aggregate levels of the economy, which at the same time could be regarded as composing a singular comprehensive system. Samuelson went even further and claimed that in various fields of economics he had found that certain discrete inequalities provided the necessary and sufficient conditions for attaining a stable equilibrium

[21] This section is largely inspired by (Carvajalino 2016).

position. These inequalities, he claimed, acted as a structural pillar that unified the different subfields of economics. Eventually, he argued, they unified the different chapters of *Foundations*. This claim is misleading, however. It would imply that *Foundations* was unified by a formal analogy, which was not necessarily the case.

Like Wilson did for all sciences, Samuelson diagnosed that standard economics lacked sound scientific foundations, which led economists to adopt wrong methodological approaches. For this reason, there was not yet agreement about the applied and theoretical concerns in economics; economics therefore still lacked unity as a scientific discipline. Emphasizing that unity was necessary, Samuelson built on two working hypotheses, which he presented fully conventional. First, he supposed that individuals—consumers and firms—could be regarded as optimizing separated and isolated systems that were in stable equilibrium. This implied that at the individual level of the economy, the notions of stability and optimality were simultaneously defined and determined by the demanded and/or supplied quantities that corresponded to the individual's equilibrium position. Simple summations of these individual's quantities formed the respective aggregate quantity at given moments in time in the economy. Second, Samuelson assumed that the evolution over time of the aggregate system of the economy, namely the interaction through time of aggregate variables, could be regarded as a being in dynamical stable equilibrium. This only implied an emphasis on stability of equilibrium at the aggregate level of the economy. With these two working hypotheses, structured by the mathematical conditions of equilibrium, Samuelson presented the study of the economy as a system in which the individual and the aggregate levels could be regarded as interconnected or separated, depending on the emphasis of the study.[22]

Reflecting on Wilson, Samuelson thought that to convey (postulational) meaning, his mathematical economics had to help economists develop better ways of mediating between their theories and data. There were two problems to overcome. First, Samuelson stated, there was not yet enough economic data (Samuelson 1941, 54). In this vein, by reflecting on Wilson's skepticism of classical statistics and econometrics, Samuelson did

[22] See chapters 1-3 of *Foundations*.

not use any standard statistical tests or econometric regressions.[23] Second, standard mathematical economics remained empirically empty, Samuelson claimed. The two working hypotheses in the literature of mathematical economics on which he built were developed on the basis of marginal and differential calculus, which for Wilson and Samuelson represented an overly abstract kind of mathematics for economics because it was based on continuous formulas, while economic data always came in a discrete form.[24]

The operational solution that Samuelson offered to overcome these two problems consisted of establishing correspondences between continuous formulas (defined at the margin) and formulas defined in the discrete. In his graduate courses to economists at Harvard, after talking about thermodynamical equilibrium, Wilson had precisely defined the equilibrium position in consumer theory as being determined by discrete inequalities. In opposition to the continuous, the discrete was more general and more cogent with economic data.[25] In this spirit, Samuelson claimed that his *operationally meaningful theorems* could be tested in idealized conditions, and that they were of observational significance. Samuelson, however, did not use any statistical test and did not present any observed data upfront. As he connected the continuous with the discrete, Samuelson believed that his operationally meaningful work significantly contributed to improving the way economists mediated between theory and data, where data meant observable discrete magnitudes, not observed quantitative information.

Besides mathematical appendixes, three main sections composed Samuelson's thesis and *Foundations*. The first section was introductory. The second and third focused respectively on the individual and aggregate levels of the economy and their proper definition of stable equilibrium.

In the second section, building on the basis of his first working hypothesis, Samuelson presented his work on consumer theory and on production and cost theory.

[23] Even if Wilson was a founding member of the Econometric Society, and even if he had played a central role in the American establishment of the Society, Wilson rapidly distanced himself from the econometric movement. Wilson was highly critical of Karl Pearson and Ronald Fisher's mathematical statistics, playing significant influence in the econometric movement. Wilson was also critical of Ragnar Frisch's econometrics.
[24] See (Samuelson 1983 [1947], 46).
[25] see (Carvajalino Forthcoming).

In the chapters on consumer theory, Samuelson explained his work on the idealized individual's consumer behavior, which he had developed in separate papers published during his doctoral years.[26] With it Samuelson was able to connect utility analysis with observable data. More specifically, based on observable information contained in demand functions, preference fields and in expenditure, Samuelson was able to develop certain inequalities composed by formulas of discrete magnitudes, which he subsequently showed to correspond to the well-known standard restrictions of maximization procedures of demand functions. Because he thought that he had *translated* the continuous into the discrete, Samuelson claimed to have offered the empirical foundations of utility and Marshallian demand abstract theories.

In the chapter on production and cost theory, Samuelson used the Le Chatelier Principle, which Wilson had briefly covered in one of his courses. Wilson had explained that it corresponded to a principle of stability of equilibrium in the case of a marginal change of a parameter.[27] For Samuelson, it implied that the greater the number of constraints a system had, the more stable its equilibrium position remained when it was affected by the marginal change of one parameter. In the problem of the firm, for Samuelson, this principle corresponded well to the economic intuition that no matter the number of constraints a firm had to confront, there was no possible way of improving its profits if it already was in equilibrium. Samuelson then generalized the Le Chatelier Principle and claimed that it also held for finite changes, if the system remained within the limits of equilibrium. With this generalization into the discrete, Samuelson was able to study the optimization problem of the firm in cases in which the production function was discontinuous or in which there were boundary issues reflecting the possibility that certain inputs were not used.

In the third section of *Foundations*, on the basis of his second working hypothesis, Samuelson studied the aggregate level of the economy and the dynamic stability of its equilibrium. He defined stability—as perfect stability of the first kind—as the specific case in which "from any initial conditions all the variables approach their equilibrium values in the limit as time becomes infinite" when the system was affected by the change

[26] See (1938a, 1938b, 1938c).
[27] See (Carvajalino Forthcoming).

of a parameter (Samuelson 1983 [1947], 261). For Samuelson, his second working hypothesis implied something that he called the Correspondence Principle, which for him established a correspondence between comparative statics and dynamics. With it, he (intuitively) connected the static (and therefore discrete) problem of individuals' optimizing behavior and the evolution through time (and therefore continuous) of the interactions between aggregates of the system. On the basis of this principle, in his dynamics, Samuelson focused on (discrete) comparative statics rather than on dynamical, namely (continuous) moving, equilibrium. In this vein, Samuelson studied the stability of aggregate systems as found in business cycles and in supply-and-demand dynamical systems. In all these cases, he established certain correspondences between systems of differential equations and systems of difference equations. Such correspondences, like the Correspondence Principle, reflected Wilson's ideal that the continuous and the discrete could be interconnected by one-to-one correspondences. Samuelson argued indeed that dynamical systems helped clarify comparative static problems, and vice versa.

At the individual and aggregate levels of the system, Samuelson explained that the necessary and sufficient conditions of achieving a stable equilibrium position corresponded to certain inequality relationships. Nevertheless, by reflecting on Wilson's impressionist mathematics, and even sometimes directly following Wilson's advice, Samuelson did not necessarily and systematically emphasize mathematical consistency of his work. To some extent, he filled the mathematical gaps with economic intuition.

Samuelson's postulate of consistency in consumer behavior, with which he claimed to have offered the new empirical foundations of consumer theory, for example, did not display the necessary and sufficient conditions for effectively connecting his work with actual sets of data. Such work would appear only thirty years later.[28] Samuelson had *translated* standard (mathematical) consumer theory in terms of discrete magnitudes. He did not break with standard conventions of the subfield. He had "only" put it in a form, which was more cogent with data and which could eventually reinforce the consensus

[28] See (Afriat 1967). For a historical perspective, see (Hands 2014)

(among mathematical economists) that consumers could be studied as optimizing isolated systems that were in stable equilibrium.[29]

In a similar vein, in his cost and production theory, Samuelson was not at all persuaded that the generalization of the Le Chatelier Principle into the discrete world of finite differences held up as a matter of mathematical formalization. Despite this, Samuelson introduced it in his work, "By making use of Professor E. B. Wilson's suggestion that [the Le Chatelier Principle] is essentially a mathematical theorem applicable to economics" (Samuelson 1983 [1947], 81) With the Le Chatelier Principle, Samuelson used and reinforced the convention (among mathematical economists) that for the firm there was no movement outside equilibrium that would improve its profits: firms could be studied as optimizing isolated systems that were in stable equilibrium.

Similarly, in his dynamics of aggregate systems, by using his Correspondence Principle, Samuelson treated comparative statics as a special case of dynamics, while focusing on the former because it was more general and fundamental; this provided a great sense of unity to his whole *Foundations* in a highly intuitive way. Indeed, Samuelson suggested that his Correspondence Principle implied that the static and therefore discrete individual's optimizing behavior was a special case of the continuous evolution over time of the aggregate system at large. Individuals were therefore necessarily optimizing and in stable equilibrium at every discrete moment in time during the dynamical movements of the system. As the aggregates of the dynamical system resulted from individuals' optimizing behavior at discrete moments in time, their behavior "affords a unified approach" in economics (Samuelson 1983 [1947], 23). Eventually it can be argued that for Samuelson, following Wilson's impressionist mathematics, comparative statics and dynamics, as the individual and the aggregate levels of the economy, were interconnected with each other as the discrete and the continuous were interrelated. Like Wilson's claims about the relation between the continuous and the discrete, Samuelson's Correspondence Principle was based on highly intuitive ideas. This limited his thinking and did not compel him to study the possibility of a moving dynamic equilibrium point. Such study would only be conducted a few years

[29] See also (Moscati 2007).

later.[30] Samuelson, however, reinforced the already common idea (among mathematical economists) that the evolution over time of the economic system could be regarded as a being in dynamic stable equilibrium.

Samuelson's postulate of consistency, his Le Chatlier Principle, as well as his Principle of Correspondence, indeed his whole *Foundations*, would be of tremendous significant relevance for the subsequent development of economics. Samuelson presented his work on consumer theory, cost and production theory as well as dynamics as *operationally meaningful theorems*. However, in Wilsonian style, the mathematics of *Foundations* was not necessarily complete; Samuelson did not necessarily offer fully mathematically consistent *theorems.* Instead he filled the mathematical gaps with economic intuition, when necessary. It can thus be argued that he offered *much economics with little mathematics*, while still using highly sophisticated mathematical tools for the time in economics. Eventually, with his *operationally meaningful theorems*, he presented his work as being mathematically, theoretically and empirically founded. As a consequence, the notion of stable equilibrium at the individual and aggregate levels of the economy could be and would henceforth be regarded as mathematically structured, theoretically based and empirically evident.

Concluding Remarks

When Samuelson defended his thesis, he was only twenty-five years old. *Foundations* was only an expansion of the thesis. Despite his young age, in 1941 and 1947, Samuelson argued that he was providing economics with new foundations of a mathematical kind: "Mathematics is a Language," he wrote in the opening page of *Foundations*. He attributed this motto to Gibbs, whose mathematics Wilson claimed to best represent. Samuelson could have attributed that motto to Wilson. Indeed Wilson, who was Samuelson's professor of mathematical and statistical economics, defined

[30] See (Weintraub 1991)

mathematics as vernacular language. For Wilson, this implied that mathematics and science were indispensable for each other. It also meant that mathematics consisted of three interconnected aspects: a postulational, an axiomatic and an operational. It also implied that mathematics could remain formally incomplete, as long as the mathematical gaps were meaningfully filled with intuition of the subject matter. With his "Mathematics is a Language," since the first page of *Foundations*, Samuelson announced his adoption of Wilson's impressionist attitude towards mathematics. In the same vein, his *operationally meaningful theorems* embodied Wilson's three aspects of mathematics and their interrelation. Through them, Samuelson revealed the (mathematical and scientific) attitude that he had adopted. These mottos reflected his approach: he wanted to impose certain mathematical structures of equilibrium upon conventional working hypothesis in economics, when studying the individual and the aggregate levels as being in stable equilibrium; he also aimed at improving the way economists mediated between theory and data.

For this purpose he translated well-established formulas defined in the continuous into formulas defined in the discrete. The continuous was abstract; the discrete was more general and more cogent with data. In his operational work of "translation," Samuelson favored economic meaning over mathematical structure; his work was not necessarily and systematically consistent mathematically. However, he convincingly presented it as been mathematically structured, theoretically sound, and empirically grounded. Eventually, with Wilson, it can be argued that Samuelson offered *much economics with little mathematics*.

References

Afriat, S. N. 1967. "The Construction of Utility Functions from Expenditure Data." *International Economic Review* 8 (1): 67–77.

Archibald, G. C., Herbert A. Simon, and Paul Samuelson. 1963. "Discussion." *The American Economic Review* 53 (2): 227–36.

Blaug, Mark. 1980. *The Methodology of Economics: Or, How Economists Explain*. Cambridge: Cambridge University Press.

Boland, Lawrence. 1989. *The Methodology of Economic Model Building : Methodology after Samuelson*. London ; New York: Routledge,.

Caldwell, Bruce. 1982. *Beyond Positivism: Economic Methodology in the Twentieth Century*. London: George Allen and Unwin.

Carvajalino, Juan. Forthcoming. "Edwin B. Wilson and the Rise of Mathematical Economics in America, 1920-1940." *History of Political Economy*.

———. 2016. "Edwin B. Wilson, More than a Catalytic Influence for Paul Samuelson's Foundations of Economic Analysis." *R&R*.

———. 2017. "Edwin Bidwell Wilson and Mathematics as a Language." *Submitted*.

Cohen, Joshua. 1995. "Samuelson's Operationalist-Descriptivist Thesis." *Journal of Economic Methodology* 2 (1): 53–78.

Friedman, Milton. 1953. *Essays in Positive Economics*. Chicago and London: University of Chicago Press.

Garb, Gerald. 1965. "Professor Samuelson on Theory and Realism: Comment." *The American Economic Review* 55 (5): 1151–53.

Gray, Jeremy. 2013. *Henri Poincaré: A Scientific Biography*. Princeton, N.J.: Princeton University Press.

Hands, Wade. 2001. *Reflection Without Rules: Economic Methodology and Contemporary Science Theory*. Cambridge [etc.]: Cambridge University Press.

———. 2014. "Paul Samuelson and Revealed Preference Theory." *History of Political Economy* 46 (1): 85–116.

Hausman, Daniel M. 1992. *The Inexact and Separate Science of Economics*. Cambridge: Cambridge University Press.

Lerner, Abba P. 1965. "Professor Samuelson on Theory and Realism: Comment." *The American Economic Review* 55 (5): 1153–55.

Machlup, Fritz. 1964. “Professor Samuelson on Theory and Realism.” *The American Economic Review* 54 (5): 733–35.

Massey, Gerald J. 1965. “Professor Samuelson on Theory and Realism: Comment.” *The American Economic Review* 55 (5): 1155–64.

Mongin, Philippe. 2000. “La Méthodologie Économique Au Xxème Siècle. Les Controverses En Théorie de L’entreprise et La Théorie Des Préférences Révélées.” In *Nouvelle Histoire de La Pensée Économique*, edited by Béraud and Gillbert Faccarello, 3:340–78. Paris: Éditions La Découverte.

Moscati, Ivan. 2007. “History of Consumer Demand Theory 1871 – 1971: A Neo-Kantian Rational Reconstruction.” *The European Journal of the History of Economic Thought* 14 (1): 119–56.

Nagel, Ernest. 1961. *The Structure of Science: Problems in the Logic of Scientific Explanation*. New York: Harcourt, Brace & World.

———. 1963. “Assumptions in Economic Theory.” *The American Economic Review* 53 (2): 211–19.

Samuelson, Paul. 1938a. “A Note on the Pure Theory of Consumer’s Behaviour.” *Economica* 5 (17): 61–71.

———. 1938b. “A Note on the Pure Theory of Consumer’s Behaviour: An Addendum.” *Economica* 5 (19): 353–54.

———. 1938c. “The Empirical Implications of Utility Analysis.” *Econometrica* 6 (4): 344–56.

———. 1941. “Foundations of Analytical Economics: The Observational Significance of Economic Theory.” Cambridge Mass.: Harvard University.

———. 1964. “Theory and Realism: A Reply.” *The American Economic Review* 54 (5): 736–39.

———. 1965. “Professor Samuelson on Theory and Realism: Reply.” *The American Economic Review* 55 (5): 1164–72.

———. 1983. *Foundations of Economic Analysis*. Enlarged edition. Cambridge, Mass., and London, England: Harvard University Press.

Weintraub, Roy. 1991. *Stabilizing Dynamics Constructing Economic Knowledge*. Cambridge, England: Cambridge University Press.

Wilson, Edwin. 1903a. "Mach's Mechanic's." *Bulletin of the American Mathematical Society* 10 (2): 80–86.

———. 1903b. "Review: Höhere Analysis Für Ingenieure, by John Perry." *Bulletin of the American Mathematical Society* 9 (9): 504–6.

———. 1903c. "The So-Called Foundations of Geometry." *Archiv Der Mathematik Und Physik* 6: 104–22.

———. 1904. "The Foundations of Mathematics." *Bulletin of the American Mathematical Society* 11 (2): 74–93.

———. 1906. "The Foundations of Science." *Bulletin of the American Mathematical Society* 12 (4): 187–93.

———. 1911a. *Advanced Calculus*. Boston, New York: Ginn and company.

———. 1911b. "Mathematical Physics for Engineers." *Bulletin of the American Mathematical Society* 17 (7): 350–61.

———. 1913. "Let Us Have Calculus Early." *Bulletin of the American Mathematical Society* 20: 30–36.

———. 1914. "Review of The Theory of Relativity by Robert D. Carmichael." *Science* 39 (998): 251–52.

———. 1915. "Some Books on Calculus." *Bulletin of the American Mathematical Society* 21 (9): 471–76.

———. 1920. "Space, Time, and Gravitation." *The Scientific Monthly* 10 (3): 217–35.

———. 1923a. "Keynes on Probability." *Bulletin of the American Mathematical Society* 29 (7): 319–22.

———. 1923b. “The Statistical Significance of Experimental Data.” *Science* 58 (1493): 93–100.

———. 1926a. “Empiricism and Rationalism.” *Science* 64 (1646): 47–57.

———. 1926b. “Statistical Inference.” *Science* 63 (1629): 289–96.

———. 1927a. “Probable Inference, the Law of Succession, and Statistical Inference.” *Journal of the American Statistical Association* 22 (158): 209–12.

———. 1927b. “What Is Statistics?” *Science* 65 (1694): 581–87.

———. 1928a. Review of *Review of The Abilities of Man, Their Nature and Measurement*, by C. Spearman. *Science* 67 (1731): 244–48.

———. 1928b. “Too Little Mathematics--and Too Much.” *Science* 67 (1725): 52–59.

———. 1930. “Mathematics and Statistics.” *Journal of the American Statistical Association* 25 (169): 1–8.

———. 1940. “Methodology in the Natural and the Social Sciences.” *American Journal of Sociology* 45 (5): 655–68.

Wong, Stanley. 1973. “The ‘F-Twist’ and the Methodology of Paul Samuelson.” *The American Economic Review* 63 (3): 312–25.

———. 1978. *Foundations of Paul Samuelson’s Revealed Preference Theory: A Study by the Method of Rational Reconstruction*. London: Routledge & Kegan Paul.