Fractional quantum Hall states of photons in an array of dissipative coupled cavities

R. O. Umucalılar and **I. Carusotto**

INO-CNR BEC Center and Department of Physics, University of Trento, Italy

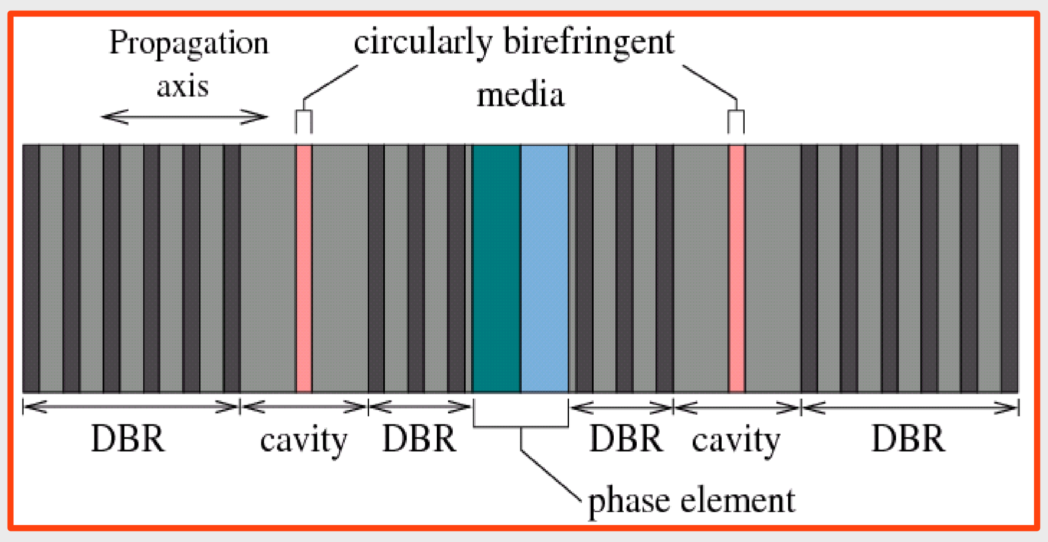
We report a theoretical study of the collective optical response of a two-dimensional array of nonlinear cavities in the impenetrable photon regime under a strong artificial magnetic field. Taking advantage of the non-equilibrium nature of the photon gas, we propose an experimentally viable all-optical scheme to generate and detect strongly correlated photon states which are optical analogs of the Laughlin states of fractional quantum Hall physics.

Artificial magnetic field for coupled cavities

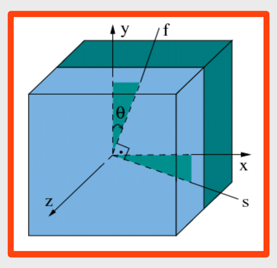
To impose a complex phase for evanescent wave tunneling between cavities, described by the Hamiltonian,

$$H_c = -Je^{i\phi}\,\hat{b}_R^\dagger\,\hat{b}_L + \text{h.c.}$$

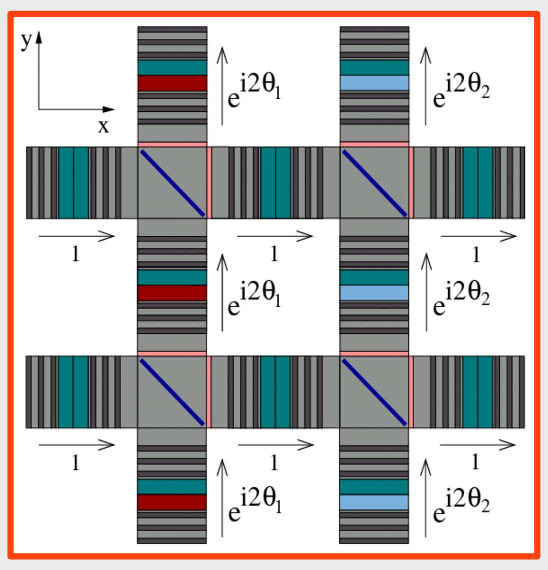
we propose to use a phase element, which consists of either a pair of half-wave slabs with a relative rotation angle between their optical axes, or a slab of optically active material. The ensuing phase is of geometric nature.



$$\sigma_+ \to \sigma_- \to e^{-i2\theta}\sigma_+$$
$$e^{i2\theta}\sigma_+ \leftarrow e^{i2\theta}\sigma_- \leftarrow \sigma_+$$



Generalization to 2D



Hopping phase between neighboring cavities can be written in terms of an artificial gauge potential $\mathbf{A}$:

$$\phi_{ij} = \frac{e}{\hbar}\int_{\mathbf{r}_j}^{\mathbf{r}_i} \mathbf{A}\cdot d\mathbf{l}$$

An artificial magnetic field B appears whenever the sum of tunneling phases around a unit cell is non-zero (modulo 2π):

$$\alpha = (2\pi)^{-1}\sum_{\Box}\phi_{ij}$$

Flux quanta per unit cell

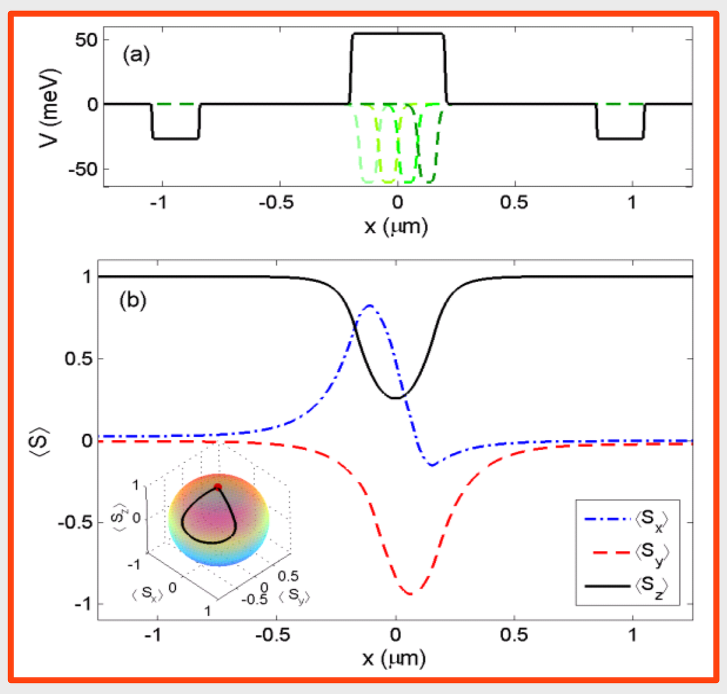
Landau gauge: $\mathbf{A} = (0, Bx, 0)$.
x and y modes couple due to a partially reflecting mirror within each cavity.

Laterally patterned planar microcavity scheme

In addition to a scalar potential that confines polaritons in two wells, we assume a position-dependent vector field is present that is coupled to the effective spin-1/2 system describing the photon polarization state:

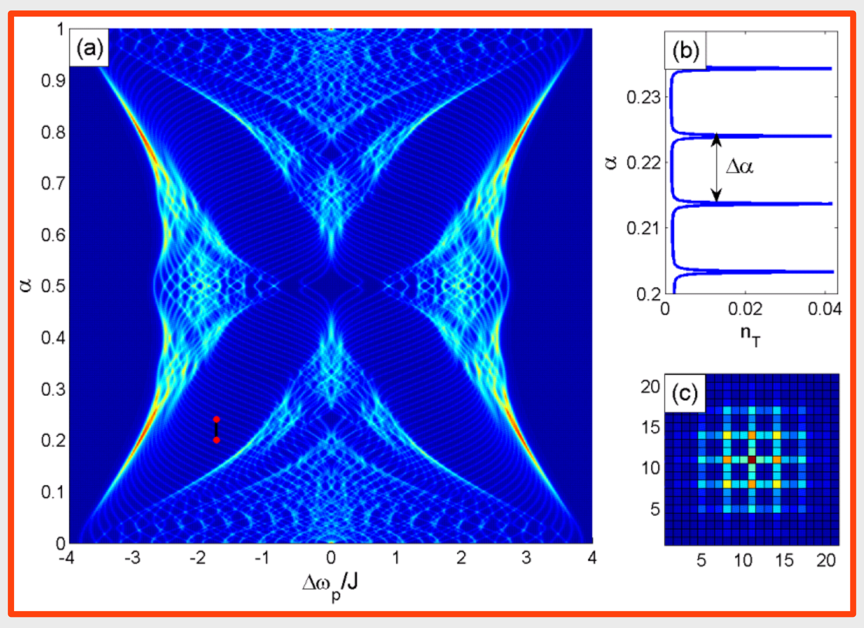
$$H = \frac{p_x^2}{2m} + V_{sc}(x) + V_z(x)\,\hat{\sigma}_z + \sum_j V_s(x-x_j)R_{\zeta_j}^{-1}\hat{\sigma}_x R_{\zeta_j}$$

Polarization adiabatically follows the local ground state determined by the direction of the local field and traces a closed loop on the Poincaré sphere, giving rise to a geometric tunneling phase.



Single particle spectrum

Transmission spectrum in the flux quanta per plaquette vs. pump frequency plane yields the Hofstadter butterfly (a), where one can also observe the edge states of quantum Hall physics (b). The artificial field manifests itself in the spatial periodicity of the photon occupation number (c).



Fractional quantum Hall (FQH) states of interacting polaritons

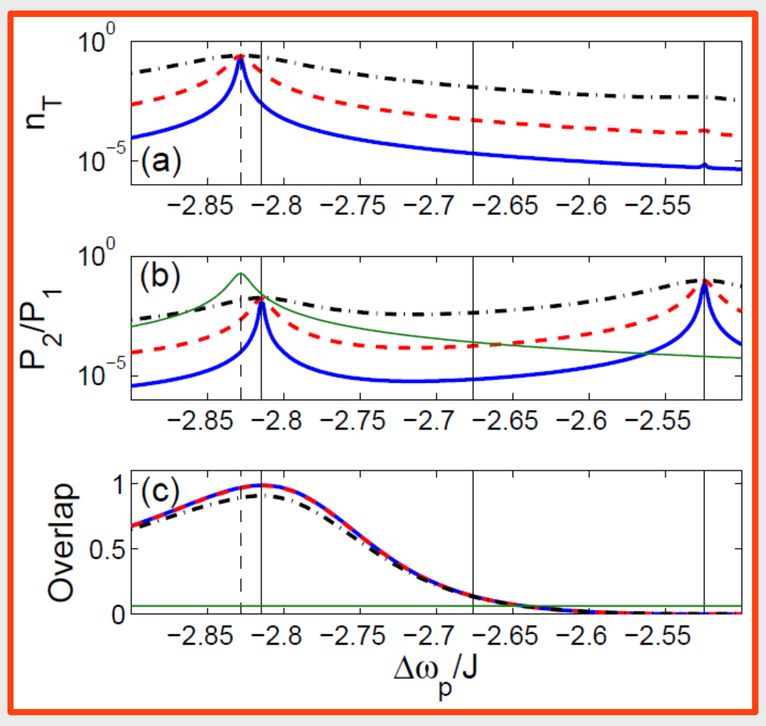
We consider the polaritonic Bose-Hubbard model for a square lattice of coupled cavities under a uniform effective magnetic field:

$$H_0 = \sum_i \hbar\omega_\circ \hat{b}_i^\dagger\hat{b}_i - \hbar J\sum_{\langle i,j\rangle}\hat{b}_i^\dagger\hat{b}_j e^{i\phi_{ij}} + \hbar\frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1)$$

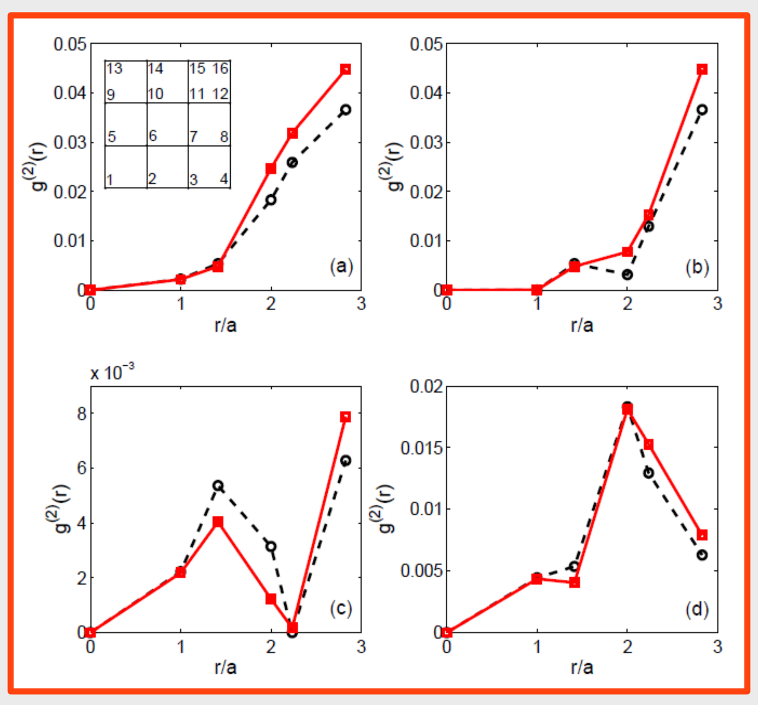
We look for the steady state properties of this system in a non-equilibrium setting with a driving pump and losses by solving for the stationary point of the master equation:

$$\partial_t\rho = \frac{i}{\hbar}[\rho, H_0 + H_{\text{drive}}] + \mathcal{L}(\rho)$$

We observed that it is possible to create a two-particle state at the expected pump frequency (b) having a large overlap with an FQH state (c), which in turn has a large overlap with the ground state of the isolated system (same observation goes for a three-particle state).



Second order correlation function measurements may be used to probe FQH states as they have peculiar signatures like the peaks and dips seen below.

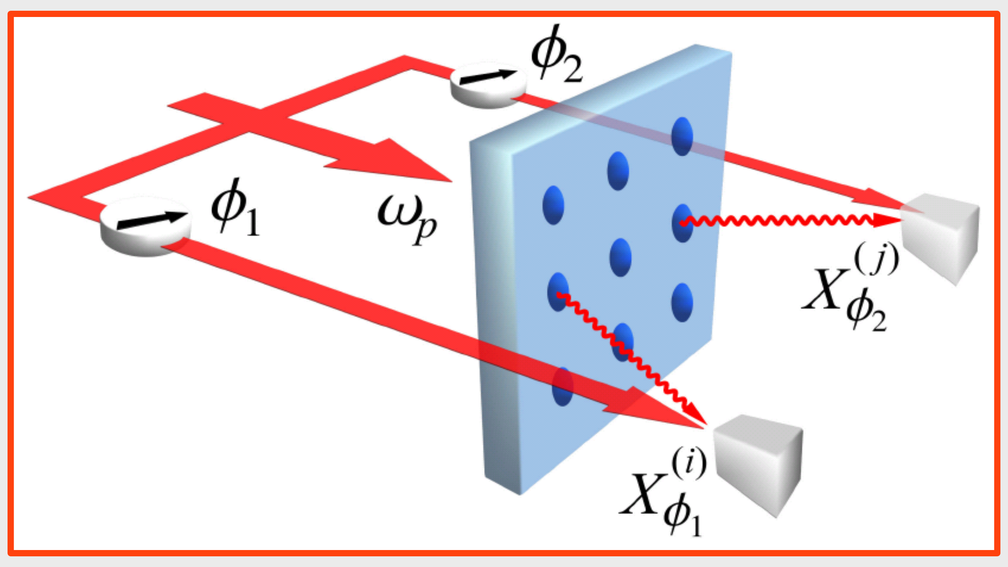


Extracting the many-photon amplitude

Microscopic structure of the many-body wave function can be experimentally extracted from the field quadratures of the secondary emission from the device:

$$\langle\hat{b}_i\hat{b}_j\rangle = \langle X_0^{(i)}X_0^{(j)}\rangle - \langle X_{\pi/2}^{(i)}X_{\pi/2}^{(j)}\rangle + i\langle X_0^{(i)}X_{\pi/2}^{(j)}\rangle + i\langle X_{\pi/2}^{(i)}X_0^{(j)}\rangle$$

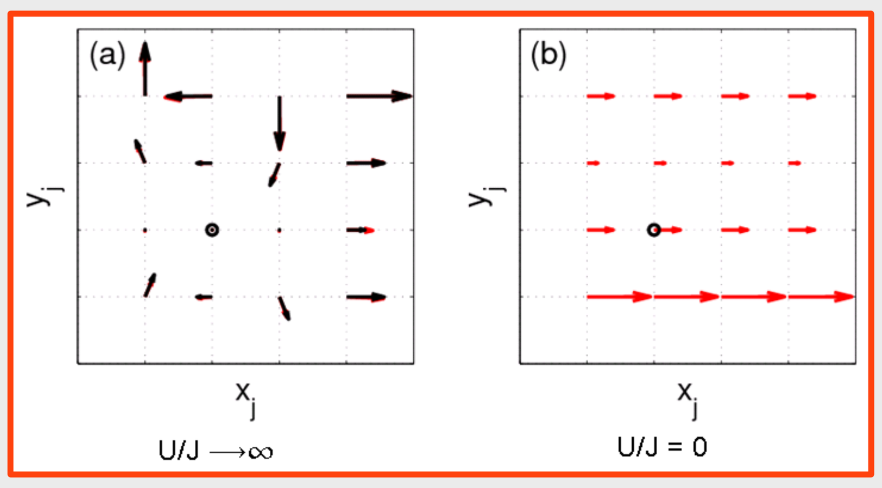
To construct an N-particle wave function, 2^N separate measurements are required. In the visible regime, field quadratures can be accessed through homodyne detection.



Quantum Hall nature of the many-photon amplitude can be assessed by calculating its overlap with a Laughlin wave function. The Laughlin wave function with filling $1/m$ (m is even for bosons) in the symmetric gauge for an infinite system is

$$\Psi(z_1,\ldots,z_N) \propto \prod_{i<j}(z_i - z_j)^m e^{-\sum_i |z_i|^2/2}$$

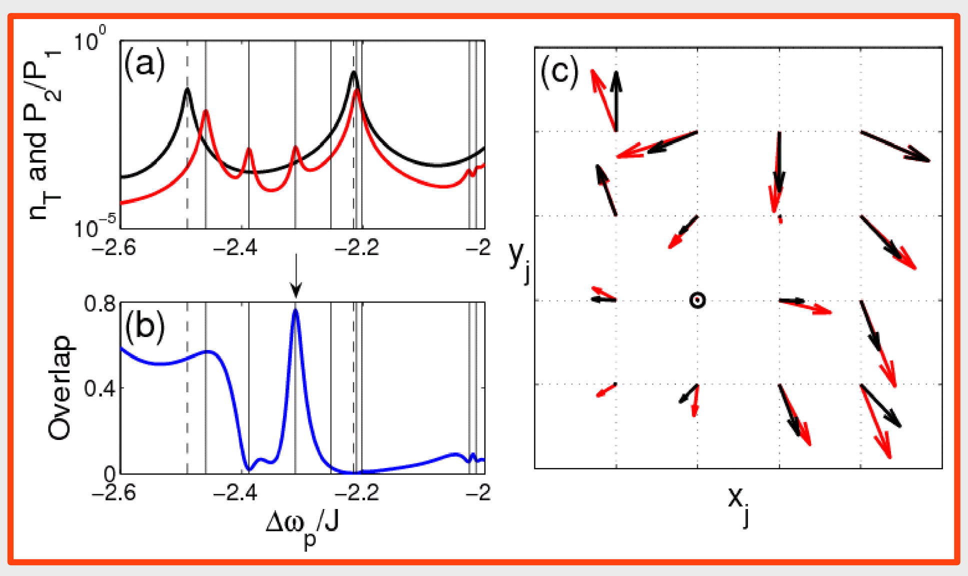
where z is the coordinate of a particle in the complex plane. We used a different form of this wave function generalized for a finite system with periodic boundary conditions.



The two-photon amplitude for interacting particles (a) has a non-trivial spatial structure, in contrast to the one for non-interacting particles (b).

Experimental imperfections

It looks that it is possible to drive a state that closely resembles an FQH state, even in a hard-wall geometry with moderate structural defects and a finite interaction strength.



Relevant work

[1] R. O. Umucalılar and I. Carusotto, Phys. Rev. A **84**, 043804 (2011).
[2] R. O. Umucalılar and I. Carusotto, arXiv:1110.6524 (to be published in Phys. Rev. Lett.).

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