# Resonant enhancement of the FFLO state by a 1D optical potential

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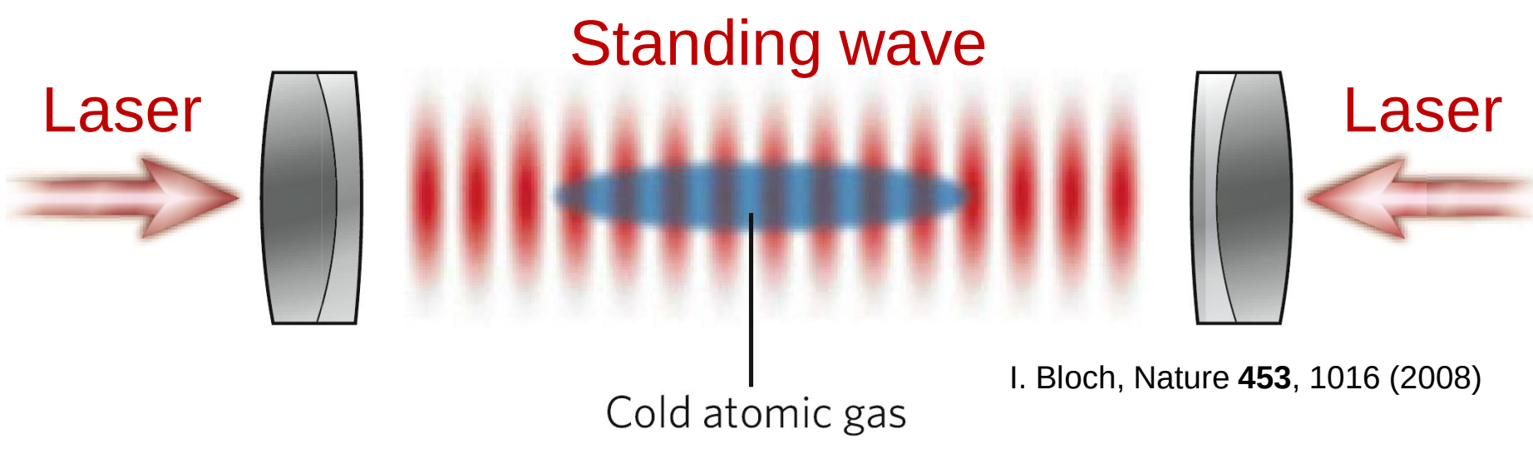
Theory of Quantum and Complex systems

## Introduction

We study a new mechanism for superfluidity in ultracold Fermi gases with spin imbalance: the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state.

### Ultracold Fermi gases

- Dilute gas of atoms cooled down to a few nanokelvin
- Fermions in two different states: spin-up & spin-down → form Cooper pairs → *superfluid state*
- Great control over system parameters: density, temperature, interaction strength, spin imbalance…
- Optical lattice: artificial crystals of light

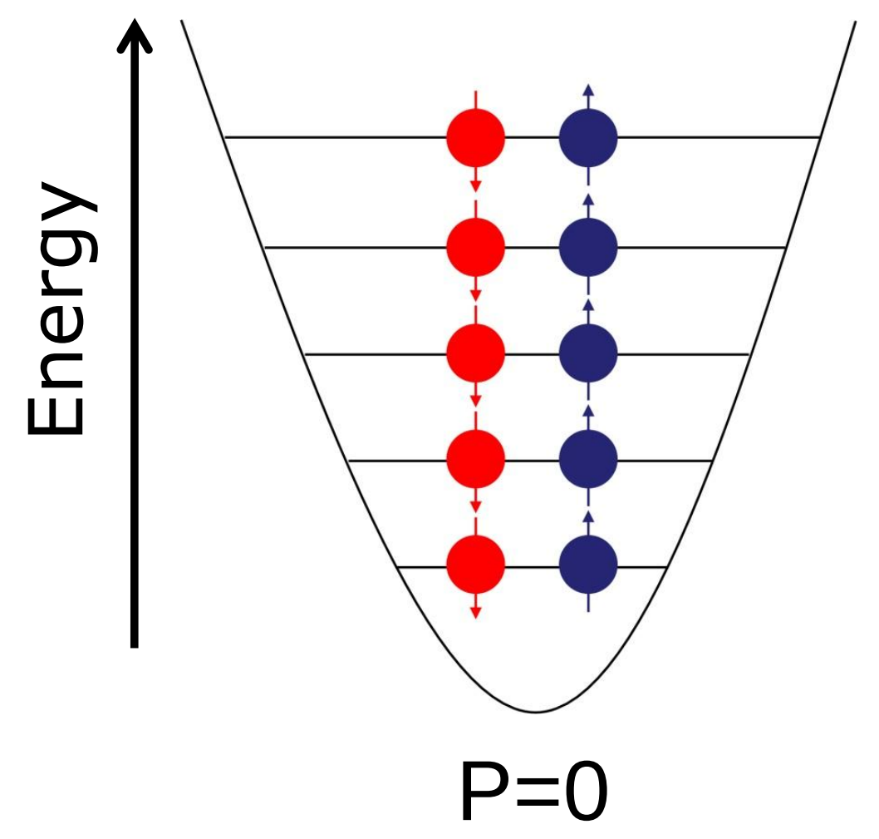


I. Bloch, Nature **453**, 1016 (2008)

=> Quantum simulators

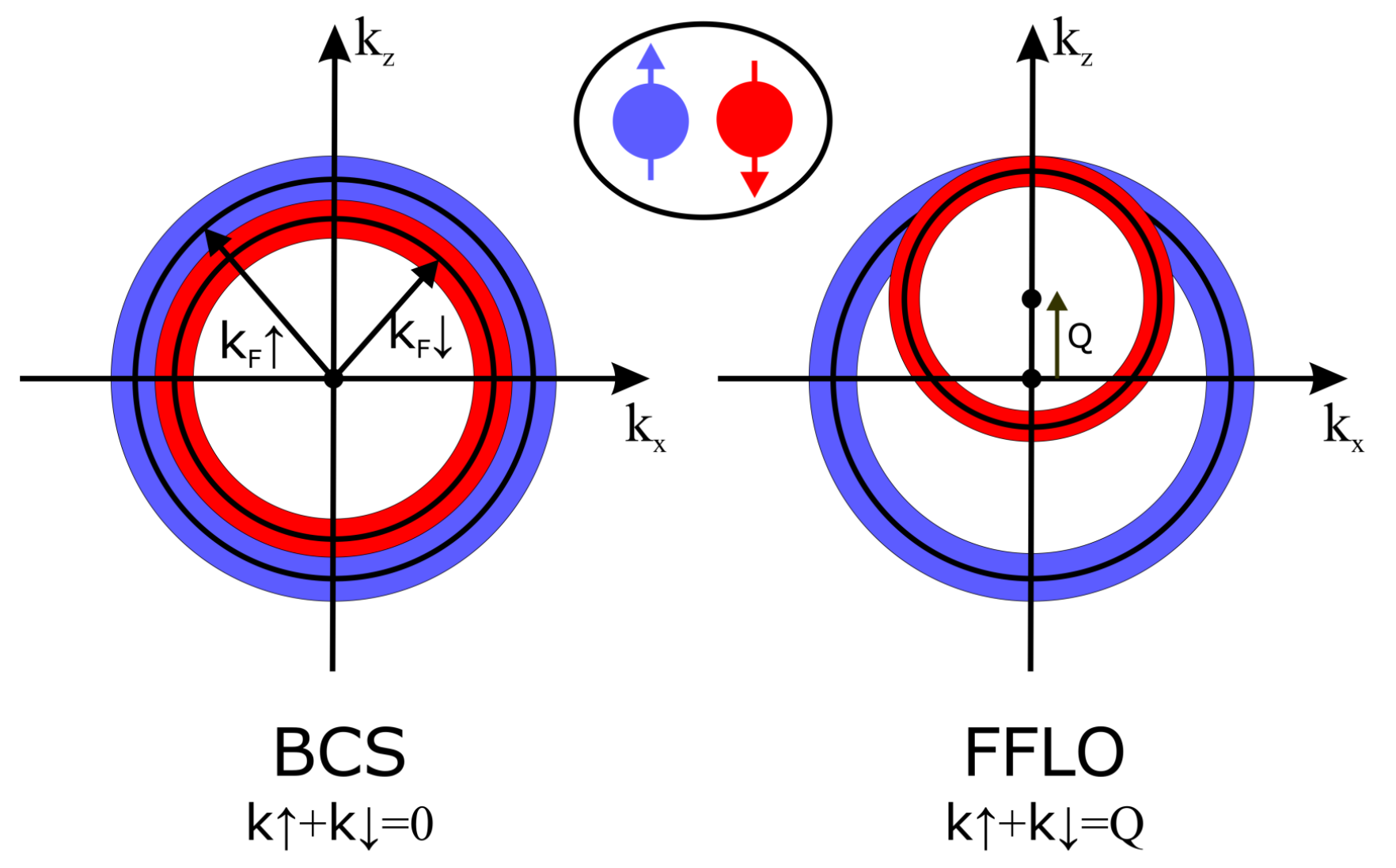
### Spin Imbalance

- Suppresses formation of Cooper pairs
- At critical polarization: superfluid → normal transition



### The FFLO state

- BCS: pairs have zero momentum
- FFLO: pairs have nonzero momentum Q → translates Fermi surface of minority spins by Q



## Method

Starting from the partition sum in path integral formulation we derive the free energy, which can be used to find the ground state of the system.

1) Partition sum of a 3D Fermi gas with spin imbalance

$$\mathcal{Z}=\int\mathcal{D}\bar{\psi}\mathcal{D}\psi\ \exp\left(-\sum_{\mathbf{k},\sigma}\bar{\psi}_{\mathbf{k},\sigma}\left(-i\omega_n+\xi_{\mathbf{k},\sigma}\right)\psi_{\mathbf{k},\sigma}-\frac{g}{\beta V}\sum_{\mathbf{k},\mathbf{q}}\bar{\psi}_{\frac{\mathbf{q}}{2}+\mathbf{k},\uparrow}\bar{\psi}_{\frac{\mathbf{q}}{2}-\mathbf{k},\downarrow}\psi_{\frac{\mathbf{q}}{2}-\mathbf{k},\downarrow}\psi_{\frac{\mathbf{q}}{2}+\mathbf{k},\uparrow}\right)$$

- without 1D potential $\longrightarrow$ $\xi_{\mathbf{k},\sigma}=k_x^2+k_y^2+k_z^2-\mu_\sigma$
- with 1D potential $\longrightarrow$ $\xi_{\mathbf{k},\sigma}=k_x^2+k_y^2+\delta\left[1-\cos\left(\pi k_z/Q_L\right)\right]-\mu_\sigma$

2) Hubbard-Stratonovich transformation: 4th order → 2nd order interaction term

$$\exp\left(-\sum_{\mathbf{q}}\bar{\psi}_{\mathbf{q}\uparrow}\bar{\psi}_{-\mathbf{q}\downarrow}\psi_{-\mathbf{q}\downarrow}\psi_{\mathbf{q}\uparrow}\right)=\int\mathcal{D}\bar{\Delta}\mathcal{D}\Delta\ \exp\left[\sum_{\mathbf{q}}\left(\frac{\bar{\Delta}_{\mathbf{q}}\Delta_{\mathbf{q}}}{g}+\bar{\Delta}_{\mathbf{q}}\psi_{-\mathbf{q}\downarrow}\psi_{\mathbf{q}\uparrow}+\Delta_{\mathbf{q}}\bar{\psi}_{\mathbf{q}\uparrow}\bar{\psi}_{-\mathbf{q}\downarrow}\right)\right]$$

3) Saddle point approximation

$$\Delta_{\mathbf{q}}=\sqrt{\beta V}\,\delta\left(\mathbf{q}-\mathbf{Q}\right)\Delta \quad\longrightarrow$$ FFLO state is allowed: Q≠0

4) Remaining fermionic path integral is quadratic and can be solved exactly

$$\int\mathcal{D}\bar{\psi}_{\mathbf{k}}\mathcal{D}\psi_{\mathbf{k}}\ \exp\left(-\sum_{\mathbf{k},\mathbf{k}'}\bar{\psi}_{\mathbf{k}}\mathbb{A}_{\mathbf{k},\mathbf{k}'}\psi_{\mathbf{k}'}\right)=\det\left(\mathbb{A}_{\mathbf{k},\mathbf{k}'}\right)$$

5) Free energy $\Omega_{sp}\left[\mu,\zeta;\Delta,Q\right]=-\frac{1}{\beta V}\ln\mathcal{Z}$

### Constructing the phase diagrams

Gap equation

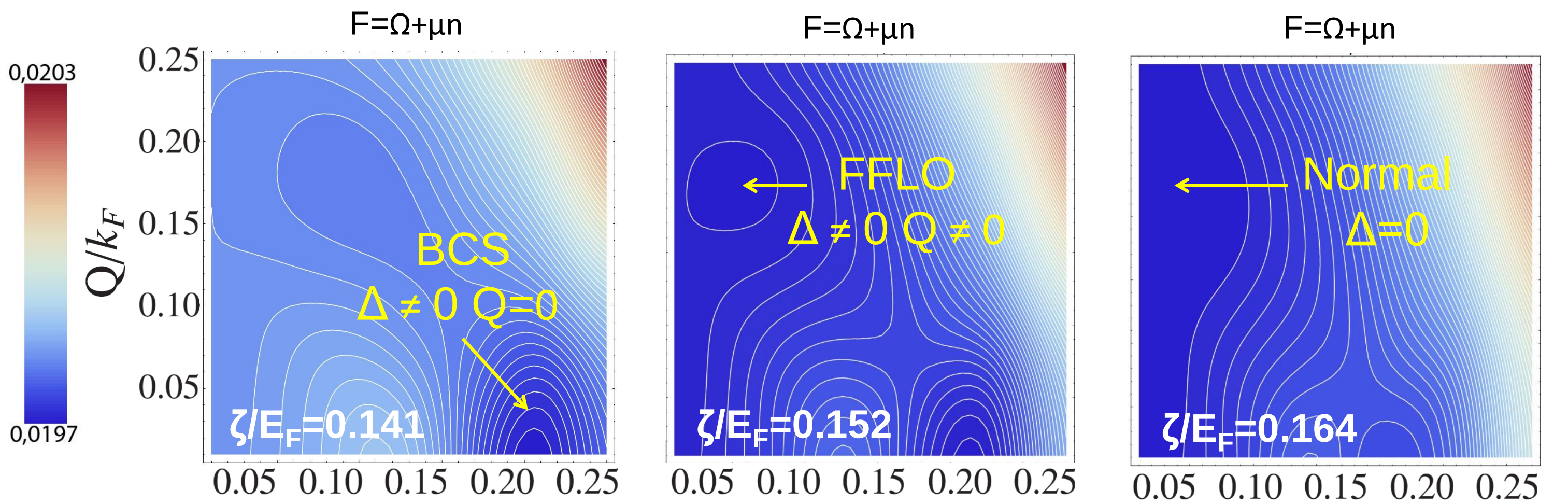
Finding de ground state

$$\frac{\partial\Omega_{sp}}{\partial\Delta}=0\ ,\ \frac{\partial\Omega_{sp}}{\partial Q}=0$$

Number equations

Transforming between chemical potentials & densities

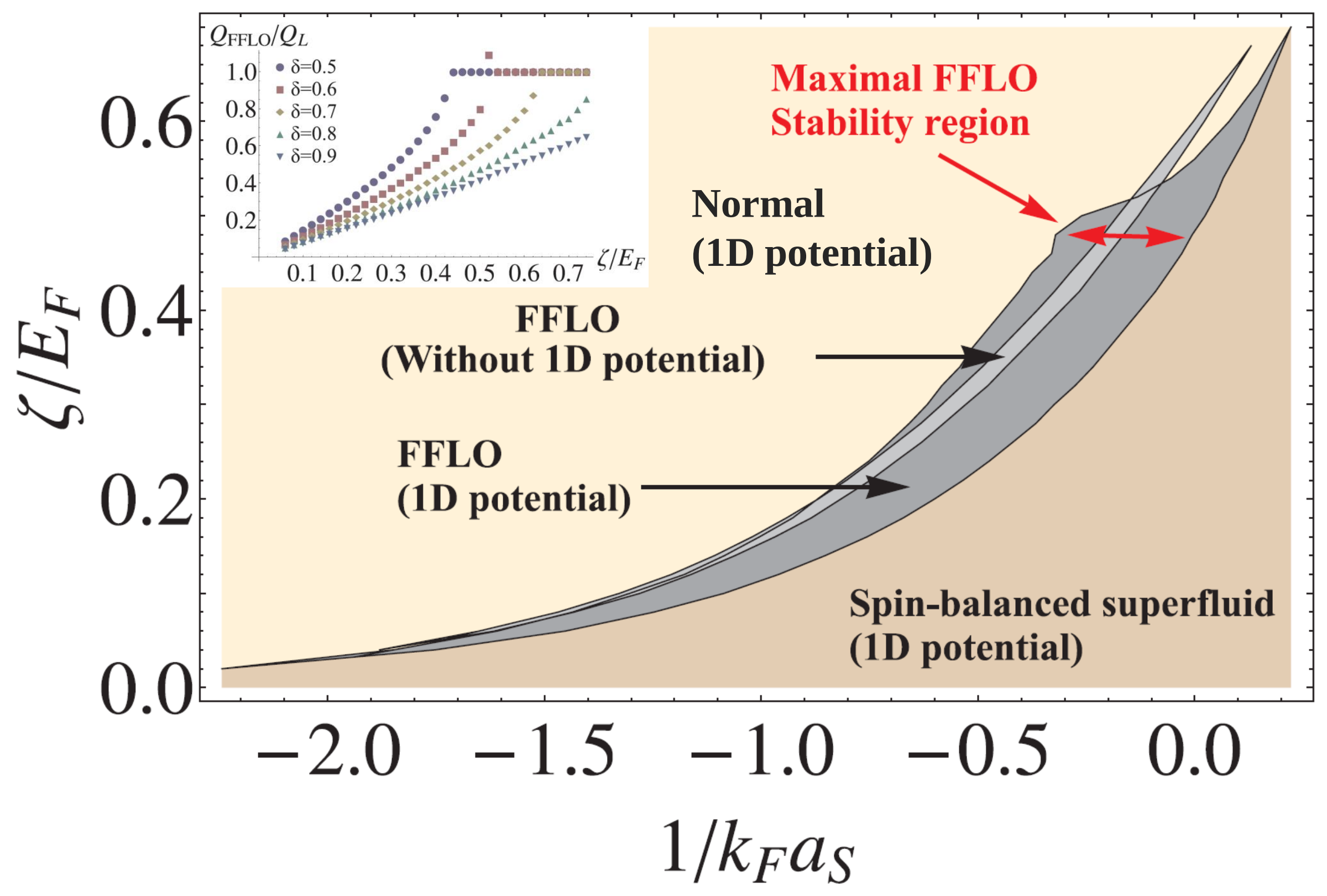
$$-\frac{\partial\Omega_{sp}}{\partial\mu}=n\ ,\ -\frac{\partial\Omega_{sp}}{\partial\zeta}=\delta n$$



## Results

We construct the phase diagrams of a spin-imbalanced 3D Fermi gas with and without a 1D optical potential and study the effects of the potential on the FFLO state.

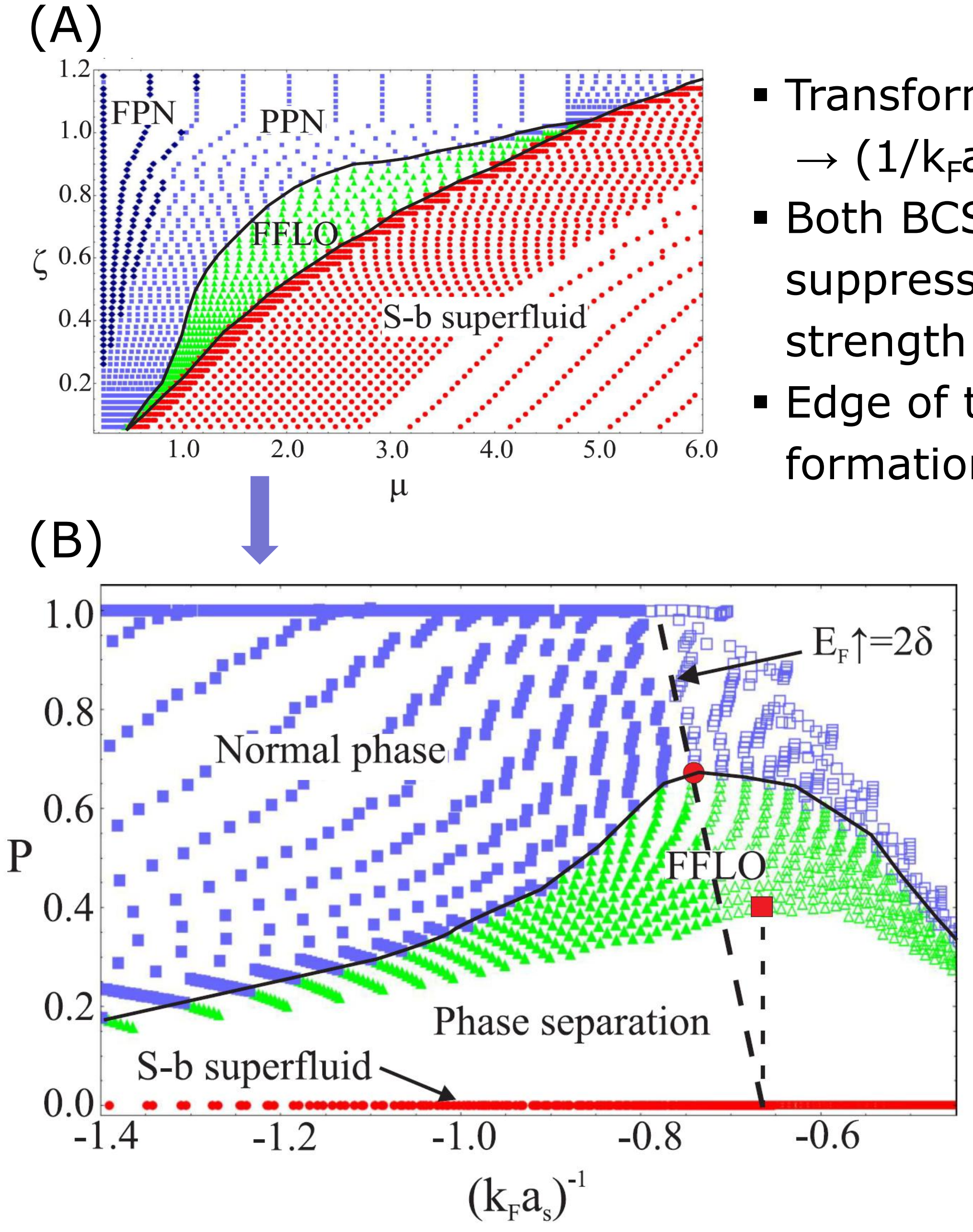
### I. Phase diagram at fixed density



- No potential: FFLO state occupies only a small area (light gray)
  With potential: FFLO area is significantly enlarged (dark gray)

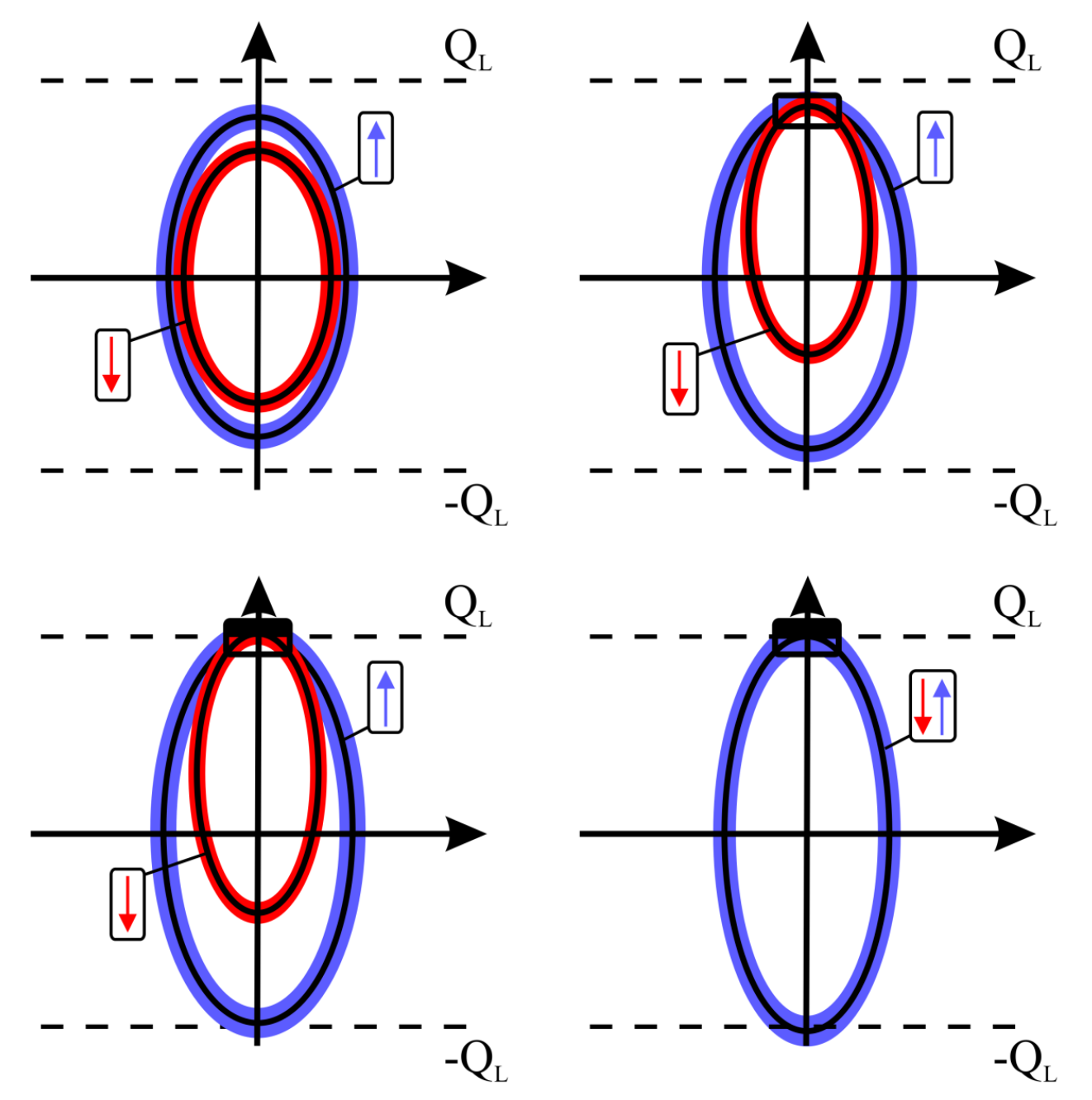
- The wave vector of FFLO grows when imbalance increases, but cannot become larger than the laser wave vector $Q_L$ (see inset)
  FFLO region is maximal when wave vector FFLO = wave vector laser

### II. Phase diagram at fixed scattering length



- Transformation:(μ,ζ)-phase diagram (A) → ($1/k_Fa_s$ ,P)- phase diagram (B)
- Both BCS- and FFLO pairing become suppressed above a certain interaction strength (red circle and red square in (B))
- Edge of the Brillouin zone hinders the formation of Cooper pairs

Pairing mechanism



## Conclusion

By adding a 1D optical potential, the presence of the FFLO state in the phase diagrams of a spin-imbalanced 3D Fermi gas is substantially enlarged. This enhancement is optimal when the wave vector of the FFLO state becomes equal to the wave vector of the laser.

References:
[1]Jeroen P. A. Devreese, Sergei N. Klimin, and Jacques Tempere, Phys. Rev. A **83**, 013606 (2011).
[2]Jeroen P. A. Devreese, Michiel Wouters, and Jacques Tempere, Journal of Physics B (in press). arXiv:1101.4868v1 [cond-mat.quant-gas].

Acknowledgements: Jeroen Devreese gratefully acknowledges a Ph.D. fellowship from “Fonds voor wetenschappelijk onderzoek – Vlaanderen” (FWO-Vlaanderen)

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