FINANCIAL CRISES, CRISIS SPILLOVERS AND THE BUSINESS CYCLE

Stefan Straetmans*

School of Business and Economics, Maastricht University,
Tongersestraat53, 6211 LM Maastricht, The Netherlands

ABSTRACT

Financial crises typically occur both during economic recessions and expansions. The objective of this paper is to quantify the likelihood of financial crises and crisis spill-overs across the business cycle in order to assess whether and to what extent economic recession episodes are more inclined towards financial crises and crisis co-movements than expansion periods. Statistical extreme value analysis (EVT) is put at work to calculate these marginal and joint tail likelihoods for recessions and expansion subsamples. We find that tail risk is procyclical for different types of financial assets. Also, systemic risk indicators based on extreme co-movements between bank stocks are found to be procyclical which confirms earlier research on market-based systemic risk measures. Moreover, cross-asset crisis spillovers like flight-to-quality effects between stocks, bonds or gold become much more pronounced during recessions. Finally, we show that diversifying portfolio tail risk becomes more difficult during recessions. To our knowledge, applying EVT techniques to business cycle regimes (or other economically meaningful sample partitions) is novel to the literature on financial extremes and extreme value analysis. EVT measures can also be made dependent on multiple regimes and regime determination can be made endogenous.

Keywords: financial crises; business cycle; procyclicality; tail risk; systemic risk; flight-to-quality.

JEL classification: G21, G28, G29, G12, C49

* Corresponding author, email: s.straetmans@maastrichtuniversity.nl

1. Introduction

Crisis episodes like the dotcom bubble burst, the 2007-2009 banking crisis or the euro area sovereign debt crisis are a reminder to academics, institutional investors and policy makers that “co-crash” linkages in financial markets or, alternatively, asset substitution phenomena like “flight-to-quality” or “flight-to-liquidity” exist and can be severe especially during periods of market stress. The severity of financial crises and crisis spillovers also potentially determines how severe the impact may be on real economic activity. Alternatively, flight-to-quality or flight-to-liquidity spillovers may also enhance overall financial stability or increase the potential for diversifying the portfolio tail risk of large institutional investors like pension funds.

The vast majority of empirical studies on financial crises and crisis spillovers implemented some form of correlation analysis methodology see e.g. King and Wadwhani (1990); Lin et al. (1994); Susmel and Engle (1994), Bae et al. (2003), Manganelli et al. (2004), Cappiello et al. (2005) or more recently Bekaert et al. (2009, 2010, 2012) and White et al. (2013). Methodologies include, inter alia, multivariate versions of factor models, DCC-GARCH specifications, limited dependent variables models (multinomial probit and logit), multivariate quantile regressions etc. These articles typically study whether financial markets are more strongly co-moving during periods of market turbulence compared to periods of low market volatility and also question the direction of international spillovers. An increasingly important subset of this “market linkages” literature focuses on whether financial crises are "contagious" (see Forbes and Rigobon (2002); Bae et al. (2003); Chan-Lau et al. (2004); Bekaert et al. (2005)). Hartmann et al. (2006) argue that the financial contagion concept is far from unambiguously defined and discuss the most frequent interpretations that co-exist nowadays. Brownlees and Engle (2012), Acharya et al. (2010) and Adrian and Brunnermeier (2011) measure bank linkages between the top US financial institutions by introducing alternative indicators of systemic risk whose estimation is based on some form of covariance analysis.

The main objection against the (bulk of the) market linkages literature is twofold in that it (i) is very correlation-oriented and (ii) often does not capture true crisis episodes in the "systemic" sense of the word (i.e. very low frequency events) when focusing on high volatility regimes. Multiple correlation pitfalls imply that correlations can be very misleading indicators of dependence during crisis episodes. For example, Boyer et al. (1999) and Ang and Chen (2002) show that for the bivariate normal distribution the correlation varies considerably when conditioned on subsets of the overall distributional support. The conditional correlation eventually goes to zero when truncated on the tail area. This implies that when correlation analysis is employed in conjunction with the normality assumption to assess, say, indicators of systemic risk, this will almost certainly result in severe underestimation; moreover, truncated correlations differ across different classes of multivariate distributions; also, correlations can only capture linear dependence whereas one might suspect crisis spillovers to be fundamentally nonlinear phenomena.[1] For a more in-depth treatment of the pitfalls of correlation analysis, see e.g. Embrechts et al. (1999). Another problem with existing comovement approaches is that it is questionable whether they truly capture the low-incidence character of crises and crisis comovements. For example, co-quantile regressions typically focus on the 5% or 1% tail area which reflects crisis events that are expected to happen once in 20 days and once in 100 days, respectively. This can hardly be considered as extreme in the sense of large adverse shocks triggering companies into overnight insolvency. The latter types of extreme low-incidence events are the ones we are interested in.

Mainly because of the above concerns regarding the applicability of covariance analysis during periods of high market volatility, a growing body of literature applies extreme value analysis (EVT).[2] Loosely speaking, EVT enables one to estimate marginal and joint probabilities of infrequent tail events like crises without the need to resort to a

[1] The latter pitfall can nevertheless be partly remedied by integrating elements of non-linear time series analysis (e.g. jump diffusions) in multivariate comovement models based on GARCH or stochastic volatility frameworks.

[2] In univariate and bivariate settings EVT has been previously implemented to assess the severity of extreme market (co-) movements. For example, Koedijk et al. (1990, 1992) and Hols and de Vries (1991) study the (heavy) tails of foreign exchange rate returns. Jansen and de Vries (1991) and Longin (1996) analyze stock market booms and busts whereas de Haan et al. (1994) consider extreme up- and downturns in bond markets. Bivariate EVT has been employed to measure extreme stock market spillovers in either a parametric fashion (Longin and Solnik (2001)) or a semi-parametric way (see Straetmans, 2000; Poon et al., 2004). Hartmann et al. (2003, 2004) address various forms of currency and stock-bond spillovers. Finally, Hartmann et al. (2006), de Jonghe (2010), Zhou (2010) and Straetmans and Chaudhry (2013) apply EVT techniques to assess tail risk and systemic risk of financial institutions.

parametric probability law for the returns. As will be discussed in the estimation section of this paper, some mild conditions on the tail behavior of the returns suffice for the purpose of estimation and statistical inference. Also, extreme value analysis studies the tail behavior of the *unconditional* distribution of financial returns which constitutes a second methodological difference with the bulk of the comovement literature traditionally focusing on the *conditional* distribution of returns (all kinds of conditional GARCH, stochastic volatility, co-quantile regressions etc.).[3]

In this paper we take an intermediate position between these two long-standing traditions of either modeling the multivariate distribution in a conditional fashion (time dependent and typically with a short-term focus) or in an unconditional way (time independent and typically with a long-term focus). We develop the idea that the unconditional tail of financial return distributions (despite being stationary in an unconditional sense) may exhibit time dependent "regimes" of some kind. More specifically, we assume that the parameters governing the univariate (Pareto-type tail decline) and multivariate (tail dependence structure or tail copula)) tail behavior can change through time due to e.g. shifts in monetary or fiscal policy regimes, financial liberalization (e.g. liberalization of capital controls, prudential FX measures, exchange rate regimes etc.) or changes in certain macro-economic variables to name only a few possible causes.

In the aftermath of Hamilton's (1990) regime-switching paper, an "academic industry" developed on regime switching behavior in financial markets, see e.g. Engel and Hamilton (1990), Hamilton and Susmel (1994), Bekaert and Harvey (1995), Gray

[3] Conditional models enable one to identify dynamic (time varying) risk measures that are widely used by risk managers and investors. The latter agents typically exhibit short time horizons for sake of short-term volatility forecasting and portfolio rebalancing. However, for long-term investors, financial regulators and supervisors that care about e.g. assessing the likelihood of financial instability, the more relevant exercise to undertake seems the assessment of extreme risks and worst cases scenarios based on the *unconditional* tail of portfolio returns. The unconditional return distribution will typically render more conservative outcomes than dynamic risk measures based on the conditional return distribution. Although conservativeness in risk assessment may indeed be desirable from a regulatory or supervisory point of view[3], it may be less desirable for banks or portfolio investors themselves; financial institutions do not want to see their profitability squeezed too much and risk-averse portfolio investors may well care about tail risk events but typically also care about realizing some decent return.

(1996), Ang and Bekaert (2002), Baele (2005), Guidolin and Timermann (2008) or Ang and Timmermann (2011). In contrast, only a small number of papers tested for structural breaks in the unconditional tail behaviour of financial returns. The presence of structural breaks in univariate tail behavior (i.e. the tail index) has been investigated for different asset classes: exchange rates (Koedijk et al. (1990, 1992)), Bund Futures returns (Werner and Upper (2002)) and stock markets (Jansen and de Vries (1991), Pagan and Schwert (1990), Quintos et al. (2001), Galbraith and Zernov (2004) and Straetmans et al. (2008)). Straetmans and Candelon (2013) investigate the tail index constancy hypothesis for a variety of financial assets and summarize the univariate breakpoint literature. The general picture that emerges from this univariate EVT literature is that the tail index is remarkably stable: only emerging currency returns seem to exhibit jumps in the tail index, most probably due to changes in exchange rate regimes. As for temporal changes in the multivariate tail dependence structure, the number of studies is even more limited. Straetmans (1998) found only weak evidence for structural breaks in extreme linkages between international stock markets; on the other hand, the same author established that extremal spill-overs between European currencies expressed against the Dmark numeraire were seriously dampened due to the introduction of the European Monetary System (EMS). More recently, Straetmans et al. (2008) established a statistically significant "9/11 effect" in conditional co-crash probabilities between US sectoral indices and a market index. Finally, using alternative market-based (extreme value) indicators of systemic risk, Hartmann et al. (2006) and Straetmans and Chaudhry (2013) show that systemic risk has increased at both sides of the Atlantic.

The main contribution of this paper is to build further on this breakpoint literature by questioning the existence of regimes in the tail behavior of financial asset returns. More specifically, we investigate whether the tail behavior of asset returns (either single asset tail fatness or cross-asset strength of tail dependence) changes with the business cycle. Traditional applications of EVT do not take economic cycles into account and estimate tail features using a much return data as possible, i.e. the "full" sample. However, it seems natural to assume that the propensity towards financial crises or crisis spillovers is nonconstant over time and e.g. depends on the phase of the business cycle. If that is the case, full sample measures for e.g. tail-VaR or crisis spillover indicators will be biased estimates of the true state (recession-based or expansion-based estimates). If tail behavior varies with the business cycle, unbiased estimation of the tail characteristics requires splitting up the full sample into a "recession" and an "expansion" sample.[4] First, we consider regime-dependent proxies of tail risk for different financial asset classes (stock indices, bond indices, bank stocks, exchange rates and commodities). Next, we calculate market-based systemic risk measures conditioned on recessions and expansions using the stock prices of US financial institutions. Third, we compare the strength of co-crashes vs. flight-to-quality between stock and sovereign bond markets across recessions and expansions. Finally, we investigate to what extent business cycle phases matter for the potential of portfolio risk diversification. To that aim we estimate and minimize both portfolio variances and tail risks across the business cycle for simple two-stock portfolios selected from the Dow Jones index.

Anticipating our results, we find that univariate tail risk is strongly procyclical. Extreme downside risk is significantly more severe for the recession sample as compared to the expansion sample. Second, procyclicality is also present in tail risk and systemic risk indicators of selected US financial institutions. Third, previous EVT papers like e.g. Hartmann et al. (2004) established that stock-bond co-crashes and flight-to-quality phenomena are approximately equally likely over the sample period 1987-1999. In contrast, our results suggest that stock-bond co-crashes are less likely than flight-to-quality during times of market stress and that this asymmetry is largest during recessions. More specifically, flight-to-quality from stocks into bonds happens nearly twice as often as compared to stock-bond co-crashes during recessions; but the two likelihoods are much closer to each other for the expansion sample and the full sample. Finally, we establish that portfolio risk (both central risk measures like variance as well as tail risk) is much harder to diversify during recessions as compared to expansions, i.e., a diversification meltdown. The minimum tail risk portfolios are also substantially different across recessions and expansions which further confirms the rather different nature of the tail dependence structure during different phases of the business cycle.

---

[4] Notice, however, this also decrease estimation accuracy because recession and expansion samples are smaller than the full sample, i.e. the so-called bias-variance trade off in EVT estimation when varying the sample size.

The paper proceeds as follows. Section 2 summarizes theory on extreme value analysis and introduces regimes in the unconditional tails. Regimes are introduced by considering mixture models of univariate and multivariate distributional tails. Section 3 presents semi-parametric estimation procedures that are common in statistical EVT and that can be applied on recession-based and expansion-based subsamples. Section 4 contains empirical results. Conclusions are summarized in the final section 5.

2. Extreme linkages: probability theory and regime dependence

We first introduce our EVT-based measures of tail risk and tail co-movement (the so-called tail-β) based on the unconditional distribution, i.e., without considering regime dependence. The former is identified using a Pareto-type tail decline whereas the latter is expressed in terms of the so-called tail copula or stable tail dependence function (2.1). Next, we show how regime dependence of our tail risk and tail comovement measures can be easily introduced by means of distributional mixtures of univariate and bivariate tail models (2.2.).

2.1. Probability theory

Since Mandelbrot (1963), the stylized fact of fat tailed financial returns seems generally accepted within the financial economics profession. We employ it as an identification scheme for calculating tail likelihoods. Loosely speaking, it implies that the exceedance probability for a given exceedance level (or quantile) approximately evolves as a power law of the quantile (i.e. polynomial tail decay). Assume that we are interested in the distributional tail of financial return losses. Return losses will be denoted furtheron by a positive random variable X which implies we focus on the right tail.[5] The characterizing property for distributions with a power-type tail decay is the so-called regular variation at infinity property:

$$\lim_{s\to\infty}\frac{P\{X>sx\}}{P\{X>s\}}=x^{-\alpha},\quad x>0,\ \alpha>0. \tag{1}$$

[5] This corresponds with the negative of the (log) return series calculated on the original price series of the financial asset.

From this condition it follows that distributions, like e.g. the Student-t, the class of symmetric stable distributions or the GARCH class have bounded moments up to α, where α is known as the “tail index”. In contrast, distributions with exponentially decaying tails or with finite endpoints have all moments finite (bounded). Another way of characterizing the class of regularly varying functions is by factorizing them into a power law part $x^{-\alpha}$ and a slowly varying function part *L(x)*:

$$P\{X > x\} = L(x)x^{-\alpha}, \qquad (2)$$

with *x* large and where $L(x)$ is a slowly varying function, i.e., $\lim_{x\to\infty}\frac{L(sx)}{L(x)} = 1, \quad x > 0$. Estimation of the exceedance probability (2) for given quantile *x* (or, alternatively, of *x* for given exceedance probability *p*) will be discussed in the next section but basically amounts to estimating α and the linear (first order) part of *L(x)*.

Sofar the assumed power-type tail identification for sharp losses in financial returns. We now turn to the characterization of a tail dependence measure for pairs of extreme returns. Suppose one would like to know the likelihood that the market value of a bank’s equity or, more generally, a financial asset or market index, sharply drops given that the same happens for another bank, asset or market. Alternatively, the interest may lie in identifying the joint likelihood of a crashing asset and a booming asset as reflecting substitution effects during times of market stress (flight-to-quality from risky stocks into safe heavens wit lower perceived riskiness like e.g. bonds or gold). Let *x* and *y* be the quantiles (or “crisis barriers”) above which we speak of a financial crisis or crash (in case of a large loss) or, alternatively, a boom (in case of an exceptional gain).[6] From elementary probability theory (starting from the standard definition of conditional probability) we know that

[6] To study financial crisis comovements with EVT we use the convention to take the negative of a return so that all formulae are expressed in terms of the upper-upper quadrant; but in principle comovements in all four quadrants can be studied, i.e. co-crashes but also flight-to-quality from stocks into bonds, flight-to-liquidity or safe heaven substitution effects across sovereign debt markets etc.

$$CP_{XY} \equiv P\{X > x | Y > y\} = \frac{P\{X > x, Y > y\}}{P\{Y > y\}}$$

$$= \frac{P\{X > x\} + P\{Y > y\} - P\{X > x \text{ or } Y > y\}}{P\{Y > y\}} \tag{3}$$

with $P\{X > x \text{ or } Y > y\} \equiv 1 - P\{X < x, Y < y\}$.

The question arises how the conditional tail probability in (3) should be estimated. Upon assuming e.g. multivariate normality, estimating the first and second moments (means, variances and covariances) is sufficient in order to calculate the conditional probability (3). However, multivariate normality and the associated correlation structure are unsuited to assess extreme linkages between asset markets, mainly because the conditional probability CP_{XY} vanishes to zero regardless the covariance levels in the distributional centre. In other words, if extremal spillover potential is present in the data, it cannot be captured by the multivariate normal distribution. Imposing parametric models with tail dependent marginal distributions like e.g. the logistic dependence model (Ledford and Tawn (1996); Longin and Solnik (2001); Poon et al. (2004)) constitutes an alternative for multivariate normality because the logistic tail dependence allows for nonzero values of CP_{XY} beyond large crisis barriers x and y. However, within a parametric framework, one never knows what is the "true" underlying model and we therefore decided not to impose a parametric model for the tail dependence structure. Instead, we propose a semi-parametric estimation procedure for the tail dependence structure.

We aim to estimate CP_{XY} when the conditioning quantiles x and y become very large. This amounts to inverting the marginal distribution functions for X and Y in order to work out the asymptotic equivalent of our linkage measure (3) in terms of the (small) probabilities of having very extreme returns. In order to do this, we first introduce the upper quantile functions for the return losses X and Y respectively as

$$\begin{aligned} Q_1(tu) &\equiv (1 - F_1)^{-1}(tu) \\ Q_2(tv) &\equiv (1 - F_2)^{-1}(tv) \end{aligned},$$

for some small but positive values u and v and a scaling parameter t. Without loss of generality, we choose u,v and t such that tu and tv are smaller than one and thus interpretable as excess probabilities. Moreover set $Q_1(tu) = x$ and $Q_2(tv) = y$ corresponding with the original crash levels in (3).

Upon substituting the above quantile functions into (3), one obtains the following asymptotic equivalent for (3):

$$\lim_{t \to 0} CP_{XY} =$$

$$\frac{\lim_{t \to 0} t^{-1}\left[P\{X > Q_1(tu)\} + P\{Y > Q_2(tv)\} - P\{X > Q_1(tu) \text{ or } Y > Q_2(tv)\}\right]}{\lim_{t \to 0} t^{-1} P\{Y > Q_2(tv)\}} = \frac{u + v - l(u,v)}{v} \tag{4}$$

The consequence of letting t converge to 0 is that the excess probabilities <u>tu</u> and tv also tend to zero, and hence the corresponding quantiles Q_1 and Q_2 grow large. As for the limit function $l(u,v)$, it is defined as

$$l(u,v) = \lim_{t \to 0} t^{-1} P\{X > Q_1(tu) \text{ or } Y > Q_2(tv)\} = \lim_{t \to 0} t^{-1}\left[1 - P\{X \le Q_1(tu), Y \le Q_2(tv)\}\right]. \tag{5}$$

This is the so-called Stable Tail Dependence Function (STDF) and was introduced by Huang (1992). This limit function can also be interpreted as a tail version of the statistical copula between X and Y. Notice that the copula function that corresponds with a given joint distribution $F(x,y)$ boils down to:

$$D(u,v) \equiv F\left(F_1^{-1}(u), F_2^{-1}(v)\right), \quad 0 \le u,v \le 1,$$

and where $F_i^{-1}(\cdot)$ represent the general inverse functions of the marginal c.d.f. of X, Y (i=1,2). In other words, the copula is the joint distribution function with uniformized marginals which implies that the function solely reflects cross sectional dependence information about the random pair (X,Y); marginal information is filtered away by the marginal transforms.

The STDF can now alternatively be defined as a tail version of the copula function, i.e., a "tail" copula:

$$l(u,v) = \lim_{t\to 0} t^{-1}\left[1 - D(1-tu, 1-tv)\right],$$

see Huang (1992) for further discussion on this relation.

Multivariate extreme value theory deals with existence conditions, properties and estimators for this function, see Huang (1992) or De Haan and De Ronde (1998). More specifically, one can show that the STDF is one-to-one with the bivariate extreme value distribution of the joint extremes of *X* and *Y*. In contrast to univariate extreme value theory, however, there is no generally applicable parametric limit law for the joint extremes. This provides an additional argument to opt for a semi-parametric estimation approach for tail dependence structures.

The curvature of $l(u,v)$ completely determines the tail dependence structure between *X* and *Y*. Basic properties of $l(u,v)$ are its linear homogeneity, i.e., $l(\lambda u, \lambda v) = \lambda l(u,v)$ and the inequality

$$\max(u,v) \le l(u,v) \le u + v\,.$$

Equality holds on the left hand side if *X* and *Y* are completely dependent in the tail area, while equality on the right hand side obtains if *X* and *Y* are independent in the tail area. Note that independence means that for all Q_1 and Q_2

$$P\{X < Q_1, Y < Q_2\} = P\{X < Q_1\}P\{Y < Q_2\}\ ,$$

while tail independence only requires this factorization to hold asymptotically (i.e. for Q_1 and Q_2 growing large). Thus it may well be that non-extreme return pairs are dependent although their extremes are asymptotically independent, e.g. the earlier example of the bivariate normal distribution with tail independent marginals even in the presence of nonzero central correlation.

The STDF enables one to express joint exceedance probabilities as a function of marginal exceedance probabilities. In order to show this, let us first introduce some further notation for the marginal and joint tail probabilities that appear in (3): $p_1 = P\{X > x\}$, $p_2 = P\{Y > y\}$ and $p_{12} = 1 - P\{X \le x, Y \le y\}$. Exploiting the homogeneity property of $l(\cdot,\cdot)$ one can now easily show that the bivariate excess probability p_{12} and the marginal probabilities p_1 and p_2 are related via the STDF. For sufficiently small $t > 0$, it approximately holds that

$$l(u,v) \approx t^{-1} P\{X > Q_1(tu) \text{ or } Y > Q_2(tv)\}. \tag{6}$$

We can choose $tu = p_1$ and $tv = p_2$ such that $l(u,v) = l(t^{-1}p_1, t^{-1}p_2)$. STDF's linear homogeneity now implies that $tl(t^{-1}p_1, t^{-1}p_2) = l(p_1, p_2)$. Hence, for small values of p_1 and p_2 it approximately holds that

$$l(p_1, p_2) \approx p_{12}.$$

In other words, the joint probability p_{12} only depends on the marginal probabilities p_1 and p_2 and the tail dependence structure reflected by the curvature of $l(\cdot,\cdot)$. The linkage measure can thus be expressed as

$$CP_{XY} = \frac{p_1 + p_2 - l(p_1, p_2)}{p_2}. \tag{7}$$

Referring to expressions (3) and (4) as a "tail dependence" measure is somewhat misleading because it both reflects information about the tail dependence structure as well as the marginal inequality, i.e., $p_1 \neq p_2$. In financial risk management, however, it is common to calculate downside risk for common p-values of, say, 5% or 1%. Conditioning on a common p-value has the advantage that it makes downside risk calculations - as reflected by the quantile or VaR level - comparable across risky positions which is not the case with unrestricted p_1 and p_2. Moreover, by setting $p_1 = p_2 = p$, CP_{XY} solely reflects information on the tail dependence structure (STDF) because one controls for marginal inequalities; thus it becomes a "pure" tail dependence measure:

$$CP_{XY} = \frac{2p - l(p,p)}{p} \approx 2 - l(1,1). \tag{8}$$

The marginal-free expression in (8) holds due to the approximate homogeneity of $l(\cdot,\cdot)$ which implies that the marginal probability can be skipped from the numerator and denominator of (8). If both returns are completely dependent in the tails, i.e. $l(1,1) = \max(1,1) = 1$, then $CP_{XY} = 1$ and the pairs of assets, banks etc. co-crash with certainty. But without extreme co-movements in the two markets, $CP_{XY} \approx 0$ since $l(1,1) \approx 2$ (the case of full tail independence). If the second (conditioning) asset Y represents a nondiversifiable risk factor like e.g. a stock market index, we rename conditional probabilities like (3) or (8) as "tail-βs", see e.g. Straetmans (2008).

2.2. Mixtures of tail models

In line with the traditional assumptions of statistical extreme value analysis, the unconditional measures of tail risk and tail comovement introduced in the previous subsection are based on stationary unconditional return distributions. Loosely speaking, stationarity in this context implies that the parameters governing the univariate and multivariate tail behavior are constant in the long-term. However, this does not exclude short-term tail parameter variation by means of e.g. regime dependence. Arguably the simplest way to introduce regime dependence in our tail risk and tail comovement measures is by means of distributional mixture models. The unconditional distributions of these mixture models still exhibit stationarity and thus constancy of the tail parameters; but the short-term parameters – and thus risk indicators – are allowed to change between regimes for a distributional mixture.

Starting with univariate tail mixtures, assume that the (long-term unconditional) univariate marginal distribution of the random variable X is governed by n regimes, i.e.,

$$P\{X \le x\} = F(x) = \sum_{i=1}^{n} \lambda_i F_i(x),$$

and where λ_i is the probability that regime *i* occurs with corresponding c.d.f. $F_i(x)$. Obviously, $\sum_{i=1}^{n}\lambda_i = 1$. It automatically follows that the mixture also holds for the corresponding survivor functions, i.e.,

$$P\{X > x\} = 1 - F(x) = \sum_{i=1}^{n}\lambda_i(1 - F_i(x)) \tag{9}$$

Upon assuming that the regime dependent tail probabilities $1 - F_i(x)$ in (9) exhibit a power-type tail decay as described in (2), eq. (9) specializes to a mixture of regularly varying functions:

$$P\{X > x\} = \sum_{i=1}^{n}\lambda_i L_i(x)x^{-\alpha_i}. \tag{10}$$

From (10) it becomes clear that we allow for regime dependence in both the tail index and the slowly varying part *L(.)* determining the tail probabilities. It is straightforward to show that the regular variation property (1) also applies to the mixture of regularly varying functions in (10). In order to illustrate this, we assume (without loss of generality) *n*=2 regimes and $\alpha_1 > \alpha_2$. The mixture tail (10) can be factorized as:

$$P\{X > x\} = L(x)x^{-\alpha_2},$$

with $L(x) = \lambda L_1(x)x^{\alpha_2 - \alpha_1} + (1-\lambda)L_2(x)$. Showing that *L(x)* is also slowly varying is straightforward, i.e., $\lim_{x\to\infty}\frac{L(sx)}{L(x)} = 1, \quad x > 0$. Thus, the fattest of the mixture tail models dominates (i.e., the tail index of the mixture equals $\alpha_1 < \alpha_2$) and thus determines the tail decay of the unconditional (long-term) distribution.[7]

Turning to the multivariate case, it is equally straightforward to introduce mixtures into the tail dependence structure of asset return pairs. More specifically, analogous to the univariate case in (9), the joint survivor function can be written as a mixture of survivor

---

[7] Although this result is reminiscent of Feller's theorem on convolutions of heavy tailed random variables (preservation of the tail index for sums of fat tailed random variables), it should nevertheless not be mixed up with the former result, see Feller (1971).

functions across different regimes. Consequently, the mixture automatically also holds for the corresponding stable tail dependence functions, i.e.

$$l(\mathbf{u}) = \sum_{i=1}^{n} \lambda_i l_i(\mathbf{u}) \quad ,$$

with $\sum_{i=1}^{n} \lambda_i = 1$, $\mathbf{u}$ reflects a vector of uniform variables and where $l(\cdot), l_i(\cdot)$ stand for multivariate generalizations of the STDF in (5).

For sake of convenience, we limit ourselves to studying the regime dependence in bivariate tail dependence measures like the conditional co-crash probability in (3). Moreover, as in the univariate case we only assume the existence of n=2 regimes but all concepts are readily generalizable to multiple regimes and multiple assets. We earlier argued that the conditional probability (3) can be readily expressed in terms of the stable tail dependence function, see (4). Suppose now that the bivariate stable tail dependence structure exhibits two regimes, i.e.

$$l(1,1) = \lambda l_1(1,1) + (1-\lambda) l_2(1,1).$$

This mixture automatically implies a mixture in the conditional probability (8) via the following chain of equalities:

$$\begin{aligned} CP_{XY} &= 2 - l(1,1) \\ &= 2 - (\lambda l_1(1,1) + (1-\lambda) l_2(1,1)) \\ &= \lambda(2 - l_1(1,1)) + (1-\lambda)(2 - l_2(1,1)) \\ &= \lambda CP_{XY}^1 + (1-\lambda) CP_{XY}^2 \end{aligned} \tag{11}$$

3. Estimation and hypothesis testing

Univariate tail index and extreme quantile estimation (reflecting downside risk) as well as an estimation procedure for the STDF are presented in Subsection 3.1. In Subsection 3.2 we introduce equality tests in order to test the equality of tail indices, tail quantiles and conditional probabilities across regimes. Rejecting the null of equal values across regimes provides further support for our approach to split up the full sample into economically meaningful subsamples. The proposed estimation and testing approaches are semi-parametric in nature and mostly follow Hartmann et al. (2004) or Straetmans et al. (2008), the only difference being that we condition EVT-based measures on regime-dependent subsamples instead of the full sample.

3.1. Semi-parametric estimation procedures

We earlier assumed that the tails of financial returns are fat tailed, i.e. conditions (1)-(2), implying that tail likelihoods exhibit a power-type tail decay. The semi-parametric tail probability estimator from de Haan et al. (1994) is basically a first-order linear approximation of (2):

$$P\{X > x\} = \hat{p}_x = \frac{m}{n}\left(X_{n-m,n}\right)^{\alpha} x^{-\alpha} . \tag{12}$$

where the "tail cut-off point" $X_{n-m,n}$ is the *(n-m)*-th ascending order statistic (or loosely speaking the *m*-th smallest return) from a sample of size *n*. The constant $\frac{m}{n}\left(X_{n-m,n}\right)^{\alpha}$ can be seen as the linear part of the slowly varying function in (2). Notice also that for $x = X_{n-m,n}$ the corresponding tail likelihood equals the empirical frequency *m/n*. As for values $x > X_{n-m,n}$, the marginal exceedance estimator basically extends the empirical distribution further into the tail and even outside the domain of the historical sample ("out-of-sample"), i.e., $x > X_{n,n}$. The tail quantile or crisis barrier *x* is usually referred to as the "Value-at-Risk" (VaR). The VaR is chosen and the corresponding exceedance probability estimated in (12). However, in the empirical application, we will evaluate the regime dependence of the quantile or VaR levels for given p-values because it is the VaR fro a given p-value that is so widely assessed as downside risk measure in modern-day risk management, not the exceedance probability for a given VaR. The quantile estimator is simply obtained by inverting the tail likelihood estimator (12):

$$\hat{x}_p = X_{n-m,n}\left(\frac{m}{np}\right)^{\frac{1}{\alpha}} \tag{13}$$

Estimators (12)-(13) still require an estimator for the tail index α. We estimate the tail index by means of the popular Hill (1975) estimator:

$$\hat{\alpha} = \frac{1}{m}\sum_{j=0}^{m-1} \ln\left(\frac{X_{n-j,n}}{X_{n-m,n}}\right), \tag{14}$$

where *m* has the same value and interpretation as in (12). Further details are provided in Jansen and De Vries (1991) and the recent monograph by Embrechts et al. (1997).

The estimation of the bivariate excess probability p_{12} either requires adopting a specific functional form for the STDF, like e.g. in Poon et al. (2004), or proceeding semi-parametrically. Since there does not exist a unique parametrization for the STDF, we like to pursue a semi-parametric estimation method based on the highest order statistics, see Huang (1992). An intuitive derivation of this estimator proceeds as follows. We start by setting $t = k/n > 0$ in (6) where *n* equals the sample size and $1 \le k \le n$ represents a large real number such that $k(n) \to \infty$ and $k(n)/n \to 0$. The choice of *k* is discussed furtheron. Since the marginal probability estimates are available from the univariate step, we can also replace (u,v) in (6) by $(\hat{p}_1, \hat{p}_2)$:

$$\hat{l}(\hat{p}_1, \hat{p}_2) = \lim_{n\to\infty} \frac{n}{k} P\left\{ X \ge Q_1\left(\frac{k\hat{p}_1}{n}\right) \text{ or } Y \ge Q_2\left(\frac{k\hat{p}_2}{n}\right)\right\}. \tag{15}$$

In order to turn this expression into an estimator for $l(\cdot,\cdot)$, we replace *P*, Q_1 and Q_2 by their empirical counterparts such that

$$\hat{l}(\hat{p}_1, \hat{p}_2) = \frac{n}{k}\frac{1}{n}\sum_{i=1}^{n} 1\left\{X > X_{n-[k\hat{p}_1],n} \text{ or } Y > Y_{n-[k\hat{p}_2],n}\right\}, \tag{16}$$

where $1\{\cdot\}$ denotes the indicator function and where $[x]$ is the integer satisfying $x \le [x] < x+1$. The quantiles Q_1 and Q_2 have been replaced by order statistics. So, loosely speaking the estimator of $l(\cdot,\cdot)$ boils down to counting the instances at which one or both of the markets experience an extreme return within a given sample period.

However, the empirical probability measure (16) is still not operational at this stage because the marginal probability arguments of the STDF are typically smaller than the inverse of the sample size *n* which implies there are no exceedances to estimate the empirical measure.[8] However, one can exploit the linear homogeneity property of the STDF, i.e., $\hat{l}(\hat{p}_1, \hat{p}_2) \approx \lambda^{-1}\hat{l}(\lambda\hat{p}_1, \lambda\hat{p}_2)$ and choose $\lambda > 1$ such as to scale up the marginal probabilities in (15) and to obtain excess observations. In the remainder of the article, we assume that $p_1 = p_2$ and choose the scaling factor $\lambda = p^{-1}$. Exploiting the linear homogeneity of the STDF and assuming the above scaling factor, the estimator $\hat{l}(1,1)$ that is required in (8) boils down to:

$$\begin{aligned}\hat{l}(1,1) = \frac{\hat{l}(p,p)}{p} &= \frac{1}{pk}\sum_{i=1}^{n} 1\{X > X_{n-[kp],n} \text{ or } Y > Y_{n-[kp],n}\} \\ &\approx \frac{1}{k}\sum_{i=1}^{n} 1\{X > X_{n-k,n} \text{ or } Y > Y_{n-k,n}\}\end{aligned} \tag{17}$$

This estimator evaluates $l(\cdot,\cdot)$ inside the tail of the joint empirical distribution whereas in the original expression (16), there were no exceedances to employ nonparametric counting. Notice that this counting procedure of co-exceedances is also easily applicable in higher dimensions. A more extensive discussion on this derivation is provided in Huang (1992) or Straetmans (1998).

An estimator for the conditional probability of simultaneous crashes CP_{XY} now easily follows by combining (8) and (17):

$$\hat{P}_{XY} = 2 - \hat{l}(1,1) = 2 - \frac{1}{k}\sum_{i=1}^{n} 1\{X > X_{n-k,n} \text{ or } Y > Y_{n-k,n}\} \tag{18}$$

Conditional on the proper choice of the nuisance parameters *m* and *k*, the estimators of the Hill statistic and the stable tail dependence function (STDF) are asymptotically

---

[8] The order statistics that are supposed to estimate the quantiles Q_1 and Q_2 correspond with the historical sample boundaries $X_{n,n}$ and $Y_{n,n}$, respectively.

normally distributed. Goldie and Smith (1987) and Huang (1992) show that one can select m and k such as to minimize the respective asymptotic mean-squared errors (AMSE). Consequently, minimizing the sample MSE is the appropriate selection criterion. In small samples best practice is to plot the estimators as a function of the threshold, i.e. $\hat{\alpha} = \hat{\alpha}(m)$ and $\hat{P}_{XY} = \hat{P}_{XY}(k)$, and to select m and k in the region over which the estimators tend to be constant. More advanced algorithms for selecting m based on minimizing a sample equivalent of the AMSE also exist, see e.g. Beirlant et al. (1999). The value m=250 in Table 1 for the Hill statistic and the quantile stimator is made by both using Hill plots as well as employing the Beirlant et al. algorithm.

3.2. Hypothesis testing

We want to perform equality tests for estimates of the tail index, the tail quantile, the co-crash probability (or, alternatively, the STDF) and the Marginal Expected Shortfall (MES) either for single asset tails across business cycle regimes (unequal sample sizes) or across asset tails within a given regime (equal sample sizes). Asymptotic normality of the Hill statistic, the tail quantile estimator and the STDF has been established by Haeusler and Teugels (1985), de Haan et al. (1994) and Huang (1992), respectively.[9] Let *est* stand for an estimate of either of these three magnitudes. A simple T-test of the equality of *est* across regimes of asset tail within a given regime boils down to:

$$T = \frac{est_1 - est_2}{\sqrt{\sigma^2(est_1) + \sigma^2(est_2)}}, \tag{19}$$

which is asymptotically normal in sufficiently large samples.[10] The denominator still requires the calculation of asymptotic variances. Asymptotic variances for averages (in case of the MES) are straightforward to obtain whereas estimable expressions for the asymptotic variance of the Hill statistic, the quantile estimator and the STDF directly follow from the asymptotic normality derivations for these estimators.

[9] As for MES estimates, the Central Limit Theorem guarantees asymptotic normality because MES is a sample average.

[10] For sake of convenience, we assume that the covariance term between est1 and est2 equals zero which is an obvious simplification.

4. Empirical results

We first provide a description of our data sources in Section 4.1. Estimates of extreme downside risk (tail risk) for a variety of financial asset classes are discussed in Section 4.2. Section 4.3 reports estimates of systemic risk for a few representative US banks. Section 4.4 compares extreme linkages between stocks and bonds within G-5 countries. More specifically, we compare stock-bond co-crash likelihoods with flight-to-quality likelihoods from stocks into bonds. Finally, in section 4.5, we minimize the portfolio tail risk for a few representative stock pairs selected from the Dow Jones Index. Every EVT application considered distinguishes between full sample, recession-based and expansion-based estimates in order to assess whether the differences in tail behaviour across business cycle regimes are statistically and economically significant.

4.1. Data description

All considered financial data are extracted from Thomson Datastream. The financial time series consist of 7,282 daily prices for stocks, bonds, US$ nominal bilateral exchange rates, gold, silver and oil. The sample runs from 1 February 1985 until 31 December 2012. We consider 16 (dividend-adjusted) stock price series of representative US banks. Next to individual stock prices, we also downloaded G-5 (US, UK, France, Germany and Japan) stock indices and corresponding G-5 long-term (10 year maturity) government bond indices. In order to estimate systemic risk for our set of representative US banks, we condition tail-β's on the Datastream US bank index. We express stock and bond prices in local currency. Prices of oil (Brent Crude), gold and silver are in US$ per barrel or per troy ounce, respectively. Financial returns are expressed as log price differences between daily closes.[11]

The subsample partitioning of our financial time series into recession samples and expansion samples is based on different data sources for business cycle dummies. For

[11] We refer to G-5 stock and bond indices in the tables and the text using the following abbreviations: France (FR), Germany (GE), United Kingdom (UK), United States (US), Japan (JP). We consider total return indices (=dividend-adjusted and coupon-adjusted) for G-5 stock indices and 10-year benchmark government bonds.

the business cycle phases we make use of two different data sources. To date US business cycles, we use the National Bureau of Economic Research (NBER) peaks and troughs in economic activity to construct a binary variable that either reflects a recession phase or an expansion phase. The turning point dates published by the NBER represent a consensus chronology of the U.S. business cycle. As for France, Germany, Japan and the UK, business cycle data were downloaded from the Economic Cycle Research Institute (ECRI).[12] Comparable with what the NBER publishes on US business cycles, the ECRI reports binary (0-1) business cycle information without specifying either the raw data used or the dating algorithm for determining business cycle peaks and troughs. Both NBER and ECRI business cycle dummies are at the monthly frequency.

4.2. Extreme downside risk for different asset classes and the business cycle

Straetmans and Candelon (2013) recently studied the temporal stability in the tail index and accompanying quantile or "tail-VaR" for a wide variety of financial asset classes and found that structural breaks in univariate tail risk measures are relatively rare (except for emerging currency returns). This section's univariate analysis can be interpreted as an extension of this previous study because we consider a more general recession-based and expansion-based subsample partitioning instead of subsamples based on a single temporal breakpoint.

Table 1 contains full sample, recession-based and expansion-based estimation and testing results for the tail index α and the accompanying quantile or downside (tail) risk (conditioned on a p-value of 0.1%) and for a wide variety of asset classes. The "testing results" panel reports the equality test of the null hypothesis of equal recession-based and expansion-based tail indices and tail quantiles, respectively. Given that Straetmans and Candelon (2013) already showed that unconditional (long-term) asset tail risk may considerably differ across asset classes, we also consider a relatively large cross section of different asset types. More specifically, we distinguish between 5 different asset classes: US bank stocks (Panel A), G-5 stock indices (Panel B), G-5 bond indices (Panel

[12] The NBER and ECRI business cycle data are available at http://www.nber.org and http://www.businesscycle.com

C), US$ exchange rates (Panel D), and commodities (Panel E). The downside risk for banks reflects the downside risk in the market value of bank equity capital. For sake of completeness, we also report full sample and subsample average returns and volatilities.

[Insert Table 1 here]

The table reveals that recession-based estimates of the standard deviation, the tail index and the corresponding tail quantile nearly all exceed their expansion-based counterparts for basically all considered assets. It may not come as a surprise that the asymmetry in point estimates is most pronounced for bank stocks (panel A). For some banks the differences in tail properties across the business cycle is staggering. Half of the considered banks exhibits a tail index below 2 during recessions whereas none exhibits this property in the expansion phase. Notice this moment violation ($\alpha < 2$) implies that in recession phases, financial return variances are no longer properly defined (the distributional second moment is not finite and thus does not exist). The huge discrepancy in the tail index translates into huge differences in downside risk across the business cycle. For example, Bank of America's 0.1% Value-at-Risk equals a skyrocketing 76% during recessions but drops down to 12% during expansions. Most other banks with comparable discrepancies in the tail index also show huge discrepancies in their equity Value-at-Risk across business cycle regimes.

Business cycle asymmetries are also present in US$ foreign exchange risk despite the fact that we solely partition using the US business cycle whereas the very nature of exchange rates implies that two business cycles exhibit potential influence. Along the lines of Danielsson and de Vries (1997) or Straetmans and Candelon (2013), the regime dependent outcomes for foreign exchange tail-VaR can be used to determine regime dependent upper limits on open positions to foreign currency dealers by the treasurers of the forex dealing room of an international bank.[13] Our innovation here lies in the fact that we can make the trading limit regime dependent (higher limits during recessions and lower limits during expansions) whereas the previous papers propose trading limits that did not distinguish between recessions and expansion phases.

[13] See Danielsson and de Vries (1997) for a more elaborate discussion and for other applications of extreme quantile estimation for e.g. institutional investors.

Suppose a trading limit depends on the probability p on a single large negative currency return that can bring the bank's solvency in jeopardy. In this example, the level p is interpretable as the insolvency risk the management considers "acceptable". Suppose the management chooses a critical loss level $s < 0$ which stands for the maximum loss that can be incurred without running into solvency problems. A simple way to determine the maximum allowable investment I is to set $I = s/\hat{x}_p$ with $\hat{x}_p$ the extreme quantile estimator as defined in (13). Clearly, a full sample trading limit would be too conservative relative to the expansion regime and not sufficiently conservative relative to the recession regime, so a regime dependent trading limit seems desirable. Straetmans and Candelon (2013) illustrate that trading limits can hugely differ across subsamples when considering temporal breaks in the tail index; but the economic magnitudes of trading limit differences remains comparable for more complex subsample conditioning like the ones performed here. Apart from bank stocks and exchange rates, Table 1 clearly shows that other asset classes also exhibit tail asymmetries across business cycle regimes albeit to a lesser extent (except bond index returns that hardly show any regime dependence).

Turning to the statistical significance of the asymmetries (right panel "Testing results"), the tests reveal that the null hypothesis of equality between tail indices and tail quantiles across the business cycle is rejected in a majority of cases at the 1% significance level. Rejections are strongest for banks followed by commodities and exchange rates. Also, the equality of tail quantiles across regimes is slightly more often rejected than tail index equality. This is probably due to the fact that the quantile estimates also take account of the assets' scaling constants, i.e, even if the tail indices are invariant across regimes, tail quantiles can still be regime dependent if the scaling constants are.

Finally, notice that despite the fact that financial risk (either measured by the standard deviation or the tail quantile) is nearly always higher during recessions, this is mostly not reflected into a higher average return during recessions. On the contrary, the average recession return falls below the average expansion return in all cases. One interpretation of this outcome may be that the idiosyncratic (diversifiable) part of volatility and tail risk becomes a relatively more important component of total risk during recessions.

4.3. Systemic instability and the business cycle

In this section we discuss recession-based and expansion-based estimation and testing outcomes for three separate systemic risk measures: the linear correlation between a bank stock return and a banking market index, the tail-β (see e.g. Hartmann et al. (2006) or Straetmans and Chaudhry (2013)) and the marginal expected shortfall (MES), see e.g. Brownlees and Engle (2012). The latter two systemic risk measures are based on the same pairs of losses on individual bank stocks and a banking market index as the correlation is. The MES is defined as the expected loss on a (banking or general) market index conditional on a large adverse shock in the market price of an individual bank's equity capital. The tail-β was defined in (8) and estimated according to (16) using the stable tail dependence function whereas the MES is estimated non-parametrically by conditioning the banking market index loss on the 5% and 1% quantile exceedances of the empirical distribution of individual bank stock returns.

The three considered measures differ in the extent to which they explicitly focu on the tail behavior. First, the correlation is calculated for the full sample of data pairs (including the tails). The MES indicator is conditioned on the tail of the individual bank stocks returns but its nonparametric estimation approach does not allow to evaluate the indicator very deep in the bivariate tail. Finally, the tail-β and corresponding stable tail dependence function are evaluated "as if" one looks infinitely far into the tail: expression (18) does not depend on a cut off point or threshold such that it truly reflects an asymptotic value of the co-crash probability. In other words, the EVT-based tail-β looks much further in the tail than the MES but we nevertheless also included the MES for sake of comparison. Finally, in contrast to linear correlations, the tail-β and MES indicators can also detect non-linear comovements if present in the data.

The estimation results and testing results are summarized in Table 2. The table is organized in the same way as Table 1: the left and right part of the table correspond with estimation and testing results, respectively. We further distinguish full sample, recession-based and expansion-based estimates. The testing results panel reflects outcomes for testing the null hypothesis of equal recession-based and expansion-based MES and tail-β.

Turning to the results, we clearly see that our market-based measures are procyclical in the sense that their recession values always exceed their expansion values although there is still a large cross sectional heterogeneity. This seems to be in line with that part of the fundamentals-based banking crisis literature that claims that banking crises and systemic instability become more likely during recessions, see e.g. Gorton (1988). The economic interpretation of the numbers in the table is straightforward. For example, the 0.64 full sample value for Keycorp implies that a crash of the banking sector as a whole (i.e. a banking market index) coincides with a crash in the market value of Keycorp equity in 64% of cases; whereas thus number rises to 69% during recessions and drops to 54% during expansions. In general, however, the tail-β values are astonishingly high, even during periods of economic expansion, which is somewhat unexpected. The economic interpretation of the MES outcomes is equally straightforward. Whereas the tail-β is a probability, the MES reflects the expected severity of the aggregate loss resulting from an adverse shock in individual bank stock. For example, consider again Keycorp that exhibits a full sample MES value of 4%. This implies that a crash in Keycorp beyond the 5% historical tail quantile is expected to erode the aggregate banking market index with 4%. During recessions and expansions, the expected aggregate loss fluctuates between 13% and 3% . Some of the MES values - especially during expansions - seem quite low which may also be due to the fact that the nonparametric MES estimation approach is not looking very far into the bivariate tail which keeps the potential severity of the aggregate impact limited. 5 or 1% cut off values for the conditioning of MES hardly reflect extreme comovements between individual bank stocks and the market as a whole. If one would be able to evaluate MES much further in the tail, it is to be expected that the severity of the reported losses in Table 2 would also be much higher.

As concerns the magnitude of the tail-β or MES differences across the business cycle, they are sometimes quite spectacular (e.g. JP Morgan Chase) whereas for others they are quite small. In other words, banks seem to differ quite a lot in terms of their responsiveness to business cycle fluctuations. One possible explanation for this may be that banks that are engaged in more traditional banking activities generating interest-related revenues are more recession-prone than banks with a relatively important investment and trading division (non-interest related revenue sources). However, we leave the study of the determinants of this cross-regime variation for future research. It is also interesting to note that correlations between bank stock returns and the market as a whole are generally higher during recessions. We will also report "recession and expansion" correlations in the other tables still to be discussed in order to illustrate this alternative dimension of diversification meltdown.[14]

Turning to the testing outcomes in the right panel, it is obvious that recession-based and expansion-based systemic risk indicators are statistically significantly different from each other (typically at the 1% level). Upon looking at the gaps between the regime-dependent point estimates in the left panel, it is fair to say these differences are also strongly economically significant.

Notice that the considered systemic risk indicators can be turned into anticyclical indicators by simply applying the recession-based indicator values to expansions and vice versa. In this way, systemic risk is highest during expansions reflecting the build up of risks on and off the balance sheets when market volatility and correlations are low. On the contrary, systemic risk declines when the risk "bubble" bursts and the systemic crisis has struck.

Finally, it is interesting to observe in Table 2 that the correlation-based systemic risk rankings differ from the tail-β ranking. This may be due to the fact that tail-β′s capture non-linear spillovers during crisis periods whereas the linear correlations do not.[15]

[Insert Table 2 here]

[14]Traditionally, the term "diversification meltdown" characterizes a situation of rising correlations in highly volatile regimes, see e.g. Ang and Chen (2002). The question arises whether this correlation jumps are genuine changes in interdependence or induced by the rising volatilities themselves. Forbes and Rigobon (2002) constitutes the classic reference on disentangling financial contagion vs. interdependence in a context of diversification meltdown.

[15] This also holds when comparing rankings based on the CAPM-β and the tail-β, see Straetmans and Chaudhry (2013).

4.4. Co-crash versus Flight-to-quality effects and the business cycle

During crises, it is typically assumed that some assets that are perceived by investors as "safer" and/or more "liquid" may act as "flight to quality", "flight to liquidity" or "safe heaven" assets.[16] This implies that the investors sell off the asset whom they perceive as riskier and buy the supposedly safer asset. As a result, one expects opposite return movements and negative dependence between pairs of these asset returns. The literature on these types of substitution effects is suprisingly scant. Using bivariate statistical extreme value analysis Hartmann et al. (2004) estimate the potential of co-crashes between G-5 stock indices and government bonds, both domestically and cross-border. They find that flight-to-quality/liquidity effects into sovereign bonds has happened as frequently as co-crashes between stock and bond markets and hence their paper remains inconclusive as to which of both phenomena (co-crashes or flight-to-quality) dominates. Other more recent studies on whether government bonds or gold may act as safe heavens in case of stock crashes include, inter alia, Connolly et al. (2005); Baur and Lucy (2009); Brière et al. (2012); Baele et al (2013); but the evidence remains mixed and inconclusive.

We compare the likelihood of co-crashes (abbreviates as "CO") vs. flight-to-quality (abbreviated as "FTQ") for three assets (stocks, bonds and gold) by measuring the probability of stock-bond co-crashes and flight-to-quality using the conditional probability estimator in (18). Although also based on multivariate extreme value analysis, our comovement indicator differs from the one in Hartmann et al. (2004). Moreover, the latter paper did not condition on recession and expansion subsamples.

Although initially defined for pairs of return losses, the bivariate comovement measure (18) can be calculated for all four data quadrants. Stock-bond (SB), stock-gold (SG) or bond-gold (BG) co-crash probability measures boil down to:

$$
\begin{aligned}
P_{CO}^{SB} &= P\{B < Q_B(1-p) | S < Q_S(1-p)\} \\
P_{CO}^{SG} &= P\{G < Q_G(1-p) | S < Q_S(1-p)\} \qquad (20) \\
P_{CO}^{BG} &= P\{G < Q_G(1-p) | S < Q_B(1-p)\}
\end{aligned}
$$

[16] All three concepts are often used interchangeably in the literature.

Whereas flight-to-quality spillover probability expressions for the same asset pairs read:

$$
\begin{aligned}
P_{FTQ}^{SB} &= P\{B > Q_B(p) | S < Q_S(1-p)\} \\
P_{FTQ}^{SG} &= P\{G > Q_G(p) | S < Q_S(1-p)\} \qquad (21) \\
P_{FTQ}^{BG} &= P\{G > Q_G(p) | S < Q_B(1-p)\}
\end{aligned}
$$

In the above expressions, the abbreviations *S*, *B* and *G* refer to the returns on stocks, bonds and gold, respectively. If the marginal tail probability *p* becomes small, the quantile function $Q(p)=(1-F)^{-1}(p)$ grows large (extreme right tail) whereas $Q(1-p)$ becomes very small (extreme left tail). Stocks are considered to be the riskiest asset and we want to assess whether the tendency of stock to co-crash with bonds or gold is stronger or weaker than selling of stocks and re-investing the proceeds into bonds or gold.

The results are summarized in Table 3. Just like the previous tables, the table is split into estimation results (left part) and testing results (right part). Horizontal panels distinguish between full sample outcomes (Panel A), recession outcomes (Panel B) and expansion outcomes (Panel C). The estimation results refer to all 6 conditional probabilities in (20)-(21). The probability estimates generally reveal that cross-asset co-movements (whether it be CO or FTQ) are present but highly heterogeneous (larger cross-asset pair and sample variation). Our main interest, however, lies in assessing whether FTQ probabilities exceed CO probabilities, for which type of asset pair this is the case and whether the gap between the two likelihoods is regime dependent. More specifically, one would expect that the CO-FTQ gap is widening during recessions as compared to expansions and the full sample results. The table only provides robust evidence for a dominance of FTQ over CO for pairs of stocks and bonds: FTQ probabilities exceed CO probabilities for the full sample and the recession sample; but the CO-FTQ asymmetry is strikingly bigger during the recession. The dominance of stock-bond FTQ is also reflected in the reported correlations: they are mostly negative and recession-based correlations are even lower. Moreover, the rise in CO-FTQ spreads during recessions is driven by a decrease in CO probability and an increase in FTQ probability. The testing outcomes (right panel) show that nearly all these CO-FTQ asymmetries are statistically significant for the full sample and the recession sample and to a lesser extent for the expansion sample. CO-FTQ asymmetries are also visible for other asset combinations but the testing panel shows that most of these asymmetries are not statistically significant.

[Insert Table 3 here]

4.5. Minimizing portfolio risk

As a final illustration of the role regimes can play in influencing the tail behavior of returns, we consider the impact of the business cycle phase on the potential for portfolio risk diversification. The regime-dependent correlations in the previous tables already shed some light on this issue and suggest that there is also a diversification meltdown when entering a recession. However, the traditional concept of diversification meltdown refers to a situation where correlations all jump to values close to 1 during high financial market volatility regimes. This implies that the potential for diversifying portfolio risk melts away during times it is most needed, see e.g. Boyer et al. (1999), Forbes and Rigobon (2002) or Ang and Chen (2002). Here we argue that also the real state of the economy seems to matter as well for the diversification potential in the financial sphere (and this regardless of the state of financial market volatility). We show that this is because asset return comovements (either correlations in the distributional centre or comovements in the tail) determining the potential for risk diversification depend on the business cycle.

In order to investigate the potential impact of business cycle regimes on portfolio risk diversification we opted for the most stylized portfolio setup thinkable: two stocks and a risk averse investor that does not care about realizing return. The investor is either measuring the portfolio risk of his two-asset portfolio either using the portfolio variance or with an EVT-based tail-VaR risk measure like in (13). Suppose the portfolio return of the two-asset equity portfolio is defined in the usual way as $R_p = wR_1 + (1-w)R_2$.

Minimizing the portfolio variance $\sigma_P^2 = w^2\sigma_1^2 + (1-w)^2\sigma_2^2 + 2w(1-w)\sigma_{12}$ renders the minimum variance (MV) portfolio:

$$w_{MV} = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}.$$

On the other hand, minimizing the Value-at-Risk of a portfolio for a given p-value (p) amounts to choosing the portfolio weight *w* such as to minimize the portfolio quantile estimator

$$\min_{w} \ \hat{x}_p(w) = R_{n-m,n}(w)\left(\frac{m}{pn}\right)^{1/\alpha}, \tag{22}$$

and with $R_{n-m,n}$ the (n-m)-th ascending order statistic of the portfolio return R_P. Notice that (22) is equivalent to (13) except that the quantile estimator is made dependent on the portfolio investment weight *w*.

The empirical application consists in calculating the risk-minimizing portfolio weights w^* as well as the corresponding minimized risk levels for the two risk measures (minimum standard deviation and minimum VaR). To that purpose we select the 8 largest companies (based on market cap) from the Dow Jones index which implies a total of 28 possible portfolios. Table 4 reports the outcomes of this stylized portfolio risk minimization exercise for the full sample, the recession sample and the expansion sample. Each of these panels further contains the correlation, the minimizing risk weight w^* as well as the corresponding minimized risk (and this for both risk measures: portfolio standard deviation and Value-at-Risk).

First, we observe that the correlations are in line with previous tables: the recession-based correlations dominate the expansion-based correlations. In line with that result, it is not surprising that the recession-based minimized risk (either measured by Value-at-Risk or standard deviation) always exceeds its expansion-based counterpart. This can be dubbed as an alternative form of diversification meltdown, i.e., portfolio risk is more difficult to diversify during recessions because correlations and tail dependencies move upward during that phase of the business cycle. It is important to realize that this outcome holds regardless the level of financial volatility. Last but not least, one observes that the risk minimizing portfolio weights strongly depend on the chosen risk measure but also on the regime.

[Insert Table 4 here]

5. Concluding remarks

In this paper we apply statistical techniques from univariate and multivariate extreme value analysis on subsamples of financial data. Our motivation for doing this is that (univariate) tail properties like the tail index and scaling constant or multivariate tail properties like the strength of the tail dependence may depend on regimes like e.g. the business cycle. The regime dependence then determines the sample partitioning. However, we argue this regime dependence is still reconcilable with stationary unconditional long-term return distributions.

The subsamples are determined according to the business cycle "regime", i.e., recession and expansion subsamples, in order to study whether univariate extreme risk (downside tail risk) or extreme return co-movements (co-crashes or flight-to-quality for stock and bond pairs, systemic risk for banks etc.) fluctuate across the business cycle. We consider different measures of tail risk and tail co-movements across the business cycle and find that regime dependence of these measures is quite general across many different types of assets or asset pairs.

In a first application, we establish that the tail indices and corresponding tail quantiles (downside tail-VaR) are higher during recessions than during expansions. This asymmetry seems to hold for a variety of assets (US bank stocks, G-5 stock and bond indices, US$ exchange rates and commodities). The observed regime dependence for bank stock tail risk is somewhat in line with the so-called "recession hypothesis" (see e.g. Gorton (1988) that states that recessions may trigger bank panics.[17] The regime

[17] An important difference with papers like Gorton (1988) is that we do not study whether there is a relation between *lagged* states of the business cycle and extreme financial returns. We leave lead-lag relationships between extremal behavior and the business cycle for future research.

dependence in tail risk for foreign exchange rate returns may be exploited to determine regime dependent trading limits for bank traders. Previous applications of this idea only considered (fully unconditional) full sample trading limits thereby neglecting the asymmetry between tail indices and scaling constants across business cycle phases.

In a second application, we establish regime dependence for market-based indicators of systemic risk (correlation, Marginal Expected Shortfall and tail-β) and find that systemic risk rises during recessions regardless the considered indicators. Moreover, the cross-regime differences are most of the time strongly significant, especially for the EVT-based tail-βs. The outcomes are in line with the earlier observed "procyclicality" of market-based systemc risk indicators which is undesirable because it basically signals the highest systemic risk during crises and the lowest prior to crises (when risks are supposed to build up despite low volatilities and correlations). One suggestion to deal with this may simply be to re-allocate the indicators of systemic risk to the other business cycle regime in order to establish anticyclical behavior. For example, on could think of using the recession-based MES to establish capital requirements during expansions or vice versa.

In a third application, we study to what extent flight to safety phenomena between stocks, bonds or gold become more or less likely depending on the state of the business cycle. We find that flight-to-quality from stocks into bonds for G-5 countries more strongly dominates stock/bond co-crashes during recession relative to expansion outcomes. The asymmetry also exists for the full sample but much smaller. But empirical evidence for flight to quality from stocks or bonds into gold is weak regardless whether one considers full samples or whether one conditions on recessions.

Finally, we establish that diversification meltdowns are not limited to periods of high financial volatility only. In fact, the potential for financial risk diversification is also reduced during recessions as compared to expansions. We illustrate this point by means of minimizing the portfolio variance vs. the portfolio Value-at-Risk for the recession and expansion sample separately. Also, the optimal portfolio weights seem to vary quite substantially across the regimes for a given minimized portfolio risk indicator.

References

Acharya, V., Pedersen, L., Philippon, T., Richardson, M., 2010. Measuring Systemic Risk, working paper, New York University.

Adrian, T., Brunnermeier, M.K., 2011. CoVaR. NBER Working Paper nr. 17454.

Ang, A., Bekaert, G., 2002. International asset allocation with regime shifts. Review of Financial Studies 15(4), 1137-1187.

Ang, A., Chen, J., 2002. Asymmetric correlations of equity portfolios. Journal of Financial Economics 63(3), 443-494.

Ang, A., Timmermann, A., 2012. Regime Changes and Financial Markets. Annual Review of Financial Economics 4, 313-337.

Bae, K., Karolyi, A., Stulz, R., 2003. A new approach to measuring financial contagion. Review of Financial Studies 16(3), 717-763.

Baele, L., Bekaert, G., Inghelbrecht, K., 2010. The determinants of Stock and Bond Return Comovements, Review of Financial Studies 23 (6), 2374-2428

Baele, L., Bekaert, G., Inghelbrecht, K. , Wei, M., 2013. Flights to Safety, National Bank of Belgium Working Paper No. 230.

Baele, L. 2005. Volatility Spillover effects in European Equity Markets. Journal of Financial and Quantitative Analysis 40(2), 373-401.

Baur, D.G., Lucey, B.M., 2009. Flights and contagion: an empirical analysis of stock-bond correlations. Journal of Financial Stability 5, 339-352.

Beber, A., Brandt, W., Kavajecz, K.A., 2009. Flight-to-quality Or flight-to-liquidity? evidence from the euro-area bond market. Review of Financial Studies 22, 925-957.

Beirlant, J., Dierckx, G., Goegebeur, Y., Matthys, G., 1999. Tail Index Estimation and an Exponential Regression Model. Extremes 2(2), 177-200.

Bekaert, G., Harvey, C.R., 1995. Time-Varying World Market Integration. Journal of Finance 50(2), 403-444.

Bekaert, G., Ehrmann, M., Fratzscher, M., Mehl, A., 2011. Global Crises and Equity Market Contagion. NBER Working Paper nr. 17121.

Bekaert, G., Hodrick, R.J., Zhang, X., 2009. International Stock Return Comovements. Journal of Finance 64(6), 2591-2626.

Bekaert, G., Harvey, C.R., Ng, A., 2010. Market Integration and Contagion. Journal of Business 78(1), 39-69.

Boyer, B., Gibson, M., Loretan, M., 1997. Pitfalls in tests for changes in correlation. International Finance Discussion Paper, no. 5-97, Board of Governors of the Federal Reserve System, Washington (DC), December.

Brière, M., Chapelle, A., Szafarz, A., 2012. No contagion, only globalization and flight to quality. Journal of International Money and Finance 31(6), 1729-744.

Brownlees, C. T., Engle, R., 2012. Volatility, Correlation and Tails for Systemic Risk Measurement. working paper.

Candelon, B., Straetmans, S., 2006. Testing for multiple regimes in the tail behavior of emerging currency returns. Journal of International Money and Finance 25, 1187-1205.

Cappiello, L., Gérard, B., Manganelli, S., 2005. Measuring comovements by regression quantiles. ECB Working Paper Series 0501, European Central Bank.

Cappiello, L., Engle, R.F., Sheppard, K., 2006. Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns. Journal of Financial Econometrics 4(4), 537-572.

Connolly, R., Stivers, C., Sun, L., 2005. Stock market uncertainty and the stock-bond return relation. Journal of Financial and Quantitative Analysis 40, 161-194.

Danielsson J., de Vries, C.G., 1997. Tail index and quantile estimation with very high frequency data. Journal of Empirical Finance 4, 241-257.

De Haan, L. de, Jansen, D.W., Koedijk, K., De Vries, C.G., 1994. Safety first portfolio selection, extreme value theory and long run asset risks, in J. Galambos, J. Lechner and E. Simiu (eds.), Extreme Value Theory and Applications (Dordrecht: Kluwer Academic Publishers), 471-487.

De Haan, L. de, de Ronde, J., 1998. Sea and wind: multivariate extremes at work. Extremes, 1, 7-45.

De Jonghe, O., 2010. Back to basics in banking? A micro-analysis of banking system stability. Journal of Financial Intermediation 19, 387-417.

Embrechts P., Haan L. de, Huang, X., 2000. Modelling Multivariate Extremes. In Extremes and Integrated Risk Management, Embrechts, P. (ed). Risk Books: London, 59-67.

Embrechts, P., Klüppelberg, C., Mikosch, T., 1997. Modelling Extremal Events. Springer-Verlag, Berlin.

Embrechts P., McNeil, A., Straumann, D., 1999. Correlation: Pitfalls and Alternatives. Risk 12(5), 69-71.

Engel, C., Hamilton, J.D., 1990. Long Swings in the Dollar: Are They in the Data and Do Markets Know It? American Economic Review 80(4), 689-713.

Feller, W., 1971. An Introduction to Probability Theory and its Applications (Volume II). John Wiley and Sons, Chicago.

Forbes, K., Rigobon, R., 2002. No contagion, only interdependence: measuring stock market co-movements. Journal of Finance 57, 2223-2261.

Galbraith, J.W., Zernov, S., 2004. Circuit breakers and the tail index of equity returns. Journal of Financial Econometrics 2, 109-129.

Goldie, C., Smith, R., 1987. Slow variation with remainder: Theory and applications. Quarterly Journal of Mathematics 38, 45-71.

Gorton, G., 1988. Banking panics and business cycles. Oxford Economic Papers 40, 751-781.

Guidolin, M., Timmermann, A., 2008. International asset allocation under regime switching, skew, and kurtosis preferences. Review of Financial studies 21(2), 889-935.

Gray, S.F., 1996. Modelling the conditional distribution of interest rates as a regime switching process. Journal of Financial Economics 42(1), 27-62.

Haeusler, E., Teugels, J., 1985. On asymptotic normality of Hill's estimator for the exponent of regular variation. Annals of Statistics 13, 743-756.

Hamilton, J.D., 1990. Analysis of Time Series Subject to Changes in Regime. Journal of Econometrics 45(1-2), 39-70.

Hamilton, J.D., Susmel, R., 1994. Autoregressive Conditional Heteroskedasticity and Changes in Regime. Journal of Econometrics 64, 307-333.

Hartmann P., Straetmans, S., Vries, C.G. de , 2003. A global perspective on extreme currency linkages. In Asset Price Bubbles: Implications for Monetary, Regulatory and International Policies, Hunter WC, Kaufman GG, Pomerleano M. (eds.) MIT Press: Cambridge (MA), 361-383.

Hartmann, P., Straetmans, S., de Vries, C.G., 2004. Asset market linkages in crisis periods. Review of Economics and Statistics 86(1), 313-326.

Hartmann, P., Straetmans, S., de Vries, C.G., 2006. Banking System stability: a cross-atlantic perspective, in: the Risk of Financial Institutions, Carey, M., Stulz R.M. (eds.), The University of Chicago Press (Chicago and London), 133-193.

Hill, B.M., 1975. A simple general approach to inference about the tail of a distribution. The Annals of Statistics 3(5), 1163-1173.

Hols, M.C.A.B., Vries, C.G. de, 1991. The limiting distribution of extremal exchange rate returns. Journal of Applied Econometrics 6, 287-302.

Huang, X., 1992. Statistics of Bivariate Extreme Values. Tinbergen Institute Research Series, Ph.D. thesis no. 22, Erasmus University, Rotterdam.

Jansen, D.W., de Vries, C.G., 1991. On the frequency of large stock returns: Putting booms and busts into perspective. Review of Economics and Statistics 73, 19-24.

Jondeau, E., Rockinger, M., 2003. Testing for differences in the tails of stock-market returns. Journal of Empirical Finance 10, 559-581.

King, M., Wadhwani, S., 1990. Transmission of volatility between stock markets. Review of Financial Studies 3, 5-33.

Koedijk, K., Schafgans, M., de Vries, C.G., 1990. The tail index of exchange rate returns. Journal of International Economics 29, 93-108.

Koedijk, K.G., Stork, P.A., de Vries, C.G., 1992. Foreign exchange rate regime differences viewed from the tails. Journal of International Money and Finance 11, 462-473.

Ledford, A., Tawn, J., 1996. Statistics for near independence in multivariate extreme values. Biometrika 83(1), 169-187.

Longin, F., 1996. The asymptotic distribution of extreme stock market returns. Journal of Business 69, 383-408.

Longin, F., Solnik, B., 2001. Extreme correlation of international equity markets. Journal of Finance 56, 649-676.

Mandelbrot, B., 1963. The variation of certain speculative prices. Journal of Business 36, 394-419.

Manganelli, S., Cappiello, L., Gerard, B. 2004. The Contagion Box: Measuring Co-Movements in Financial Markets by Regression Quantiles. Econometric Society 2004 Latin American Meetings 77, Econometric Society.

Pagan, A.R., Schwert, G.W., 1990. Testing for covariance stationarity in stock market data. Economics Letters 33, 165-170.

Poon, S.H., Rockinger, M., Tawn, J., 2004. Extreme Value Dependence in Financial Markets: Diagnostics, Models, and Financial Implications. Review of Financial Studies 17, 581-610.

Quintos, C., Fan, Z., Phillips, P., 2001. Structural change tests in tail behaviour and the Asian crisis. Review of Economic Studies 68, 633-663.

Straetmans, S., 1998. Extreme Financial Returns and their Comovements, Tinbergen Institute Research Series, Ph.D. thesis no. 181, Erasmus University, Rotterdam.

Straetmans S., 2000. Extremal spill-overs in equity markets, in Extremes and Integrated Risk Management, Embrechts P (ed). Risk Books: London; 187-204.

Straetmans, S., Candelon, B., 2013. Long Term Asset Tail risks in Developed and Emerging Markets. Journal of Banking and Finance 37(6), 1832-1844.

Straetmans, S., Verschoor, W., Wolff, C., 2008. Extreme US stock market fluctuations in the wake of 9/11. Journal of Applied Econometrics 23(1), 17-42.

Straetmans, S., Chaudhry, S., 2013. Tail risk and Systemic Risk for U.S. and Eurozone financial institutions in the Wake of the Global Financial Crisis. Working paper, Maastricht University.

Susmel, R., Engle, R.F., 1994. Hourly volatility spillovers between international equity markets. Journal of International Money and Finance 13, 3-25.

Werner, T., Upper, C., 2002. Time variation in the tail behaviour of Bund futures returns. Working Paper, European Central Bank.

White, H., Kim, T., Manganelli, S., 2013. VAR for VaR: Measuring Tail Dependence Using Multivariate Regression Quantiles. Working paper.

Zhou, C., 2010. Are banks too big to fail? Measuring Systemic Importance of Financial Institutions. International Journal of Central Banking 6(4), 205-250

Table 1 Extreme downside risk for different asset classes and the business cycle

Note: The table presents estimates and equality tests for the tail index and accompanying quantile estimates (p-value of 0.1%) for US bank stocks (Panel A), G-5 stock indices (Panel B), G-5 bond indices (Panel C), exchange rates (Panel D), and commodities (Panel E). The threshold m=250 is consistent with making Hill plots and applying the Beirlant et al. algorithm. The equality T-test evaluates whether expansion-based and recession-based estimates of tail characteristics differ in a statistically significant way.

	Estimations												Testing results	
	Full sample				Recession				Expansion				Equality test	
	μ%	σ%	α	q %	μ_R%	σ_R%	α_R	q_R%	μ_E%	σ_E%	α_E	q_E%	$a_R=a_E$	$q_R=q_E$
Panel A: bank stock returns (1/2/1985-31/12/2012)														
BANK OF AMERICA	0.025	2.6	2.3	19.1	-0.125	6.0	1.8	76.1	0.042	1.9	2.7	12.3	-3.3 ***	2.5***
BB&T	0.040	2.0	2.7	12.8	-0.033	4.0	2.4	29.8	0.048	1.7	3.1	9.2	-2.0 **	2.8***
BANK OF NEW YORK	0.040	2.3	2.8	13.7	-0.066	4.5	1.9	45.7	0.052	1.9	3.3	9.9	-3.9 ***	2.6***
COMERICA	0.037	2.1	2.5	14.8	-0.066	4.4	1.9	48.8	0.049	1.6	2.9	9.9	-2.9 ***	2.6***
HUNTINGTON BCSH.	0.024	2.9	2.0	24.8	-0.115	7.1	1.8	83.6	0.039	1.8	2.7	12.0	-2.9 ***	2.7***
JP MORGAN CHASE	0.032	2.4	2.8	14.4	-0.080	4.6	2.2	39.8	0.045	2.1	3.2	10.7	-3.0 ***	2.7***
KEYCORP	0.021	2.5	2.3	18.5	-0.164	5.8	1.8	69.5	0.042	1.7	2.8	11.1	-3.3 ***	***
NORTHERN TRUST	0.054	2.0	2.8	12.1	-0.016	3.7	2.1	33.6	0.062	1.7	2.9	9.9	-2.4 ***	2.5***
PNC FINL.SVS.GP.	0.035	2.2	2.7	13.2	-0.042	4.7	2.4	32.2	0.044	1.7	3.1	9.4	-1.9 *	2.9***
WELLS FARGO & CO	0.059	2.3	2.6	14.3	0.001	4.9	2.0	45.1	0.065	1.7	3.3	9.1	-3.5 ***	2.8***
M&T BANK	0.059	1.7	2.3	12.8	-0.023	3.4	2.2	30.8	0.068	1.4	2.5	9.2	-1.0	2.6***
REGIONS FINL.NEW	0.014	2.7	2.1	21.6	-0.169	6.4	2.1	58.5	0.035	1.9	2.8	11.8	-2.1 **	2.9***
SYNOVUS FINL.	0.027	3.0	2.5	22.0	-0.189	5.1	1.8	65.0	0.052	2.7	2.8	17.5	-3.1 ***	2.3***
STATE STREET	0.052	2.6	2.6	15.2	-0.031	5.8	2.0	46.2	0.062	1.9	3.0	10.9	-3.1 ***	2.6***
US BANCOR	0.055	2.1	2.5	14.4	-0.066	4.1	1.8	50.8	0.069	1.7	3.1	9.5	-3.9 ***	2.5***
ZIONS BANCORP.	0.031	2.6	2.2	20.6	-0.162	5.4	2.0	57.4	0.053	2.0	2.8	12.8	-2.3 ***	2.7***
Average	0.038	2.4	2.5	16.5	-0.084	5.0	2.0	50.8	0.052	1.8	2.9	10.9		
Panel B: Stock index returns of G-5 countries (1/2/1985-31/12/2012)														
US	0.039	1.1	2.7	7.41	-0.045	1.9	2.0	19.9	0.049	1.0	3.0	6.1	-2.8 ***	2.4***
GE	0.030	1.2	2.9	7.61	-0.033	1.4	2.5	10.2	0.050	1.1	2.8	7.4	-0.9	1.5
UK	0.038	1.1	2.6	7.15	0.017	1.3	2.5	9.9	0.043	1.0	2.7	6.3	-0.8	2.0**
FR	0.039	1.2	2.8	7.79	-0.047	1.7	2.4	13.0	0.050	1.2	2.9	7.0	-1.4	2.0**
JP	0.005	1.3	2.9	7.72	-0.035	1.5	2.9	8.9	0.021	1.1	2.6	7.7	0.7	0.9
Average	0.030	1.2	2.8	7.53	-0.028	1.6	2.5	12.4	0.043	1.1	2.8	6.9		
Panel C: Bond index returns (1/2/1985-31/12/2012)														
US	0.029	0.5	3.3	2.4	0.028	0.6	2.7	4.1	0.029	0.4	3.4	2.3	-1.6	2.0**
GE	0.026	0.3	3.1	1.9	0.036	0.3	2.3	2.6	0.023	0.3	3.2	1.9	-2.9 ***	1.4
UK	0.035	0.4	3.2	2.2	0.042	0.4	3.2	2.3	0.033	0.4	3.1	2.3	0.1	0.1
FR	0.032	0.4	3.1	2.0	0.052	0.4	2.6	2.4	0.029	0.4	3.1	2.0	-1.1	0.7
JP	0.018	0.3	2.3	2.5	0.026	0.3	2.2	2.6	0.015	0.3	2.3	2.6	-0.3	0.1
Average	0.028	0.4	3.0	2.2	0.037	0.4	2.6	2.8	0.026	0.4	3.0	2.2		
Panel D: exchange rate returns (1/2/1985-31/12/2012)														
US$/UK£	0.005	0.6	3.1	3.5	-0.034	0.8	2.5	6.5	0.010	0.6	3.2	3.2	-1.8 *	2.1**
US$/JPY	-0.010	0.8	3.1	4.5	-0.061	1.2	2.2	10.9	-0.004	0.7	3.3	3.9	-3.2 ***	2.4***
US$/SFR	-0.010	0.6	3.0	3.4	-0.036	0.8	2.2	7.5	-0.007	0.5	3.5	2.8	-3.3 ***	2.4***
Average	-0.005	0.7	3.1	3.8	-0.044	0.9	2.3	8.3	0.000	0.6	3.3	3.3		
Panel E: Commodity returns (1/2/1985-31/12/2012)														
OIL	0.019	2.4	2.8	14.4	-0.087	3.8	2.2	31.9	0.031	2.2	3.3	11.5	-2.9 ***	2.4***
SILVER	0.022	2.1	2.5	13.7	-0.046	2.4	2.0	23.8	0.029	2.0	2.5	13.2	-1.6	1.5
GOLD	0.023	1.0	2.7	6.6	0.018	1.5	2.4	12.7	0.024	0.9	2.9	5.6	-1.4	2.2**
Average	0.021	1.8	2.7	11.5	-0.038	2.6	2.2	22.8	0.028	1.7	2.9	10.1		

Table 2 Market-based indicators of systemic risk and the business cycle

Note: The table presents estimates (as percentage) and equality tests for three systemic risk measures: the correlation between individual bank stock returns and a banking market index, the tail-β (conditioned on the same banking index) and the Marginal Expected shortfall (MES). The MES is conditioned on 1% quantile loss exceedances of the banking index. Estimates are reported for the full sample, recession sample and expansion sample. The full sample threshold m=350 is consistent with making tail-β plots and selecting m in a horizontal range. The equality T-test evaluates whether expansion-based and recession-based estimates of tail-β's and MES differ in a statistically significant way. One-sided rejections at the 5%, 2.5% and 1% significance level are denoted with *,** and ***, respectively.

	Estimation results									Testing results	
	Full			Recession			Expansion			Equality test	
	Tail-β	MES	Cor	Tail-β_R	MES_R	Cor_R	Tail-β_E	MES_E	Cor_E	Tail-β_R=Tail-β_E	MES_R=MES_E
BANK OF AMERICA	63.7	5.3	86.4	74.0	14.5	91.5	62.9	3.8	81.0	1.7 *	5.9 ***
BB&T	54.0	3.5	71.1	62.3	7.7	83.2	46.3	2.6	61.9	3.2 ***	3.1 ***
BANK OF NEW YORK	52.0	4.0	74.8	58.4	8.1	78.4	54.3	3.3	73.0	0.7	2.7 ***
COMERICA	61.7	3.9	78.2	66.2	9.1	84.9	55.8	2.9	72.4	2.1 **	4.8 ***
HUNTINGTON BCSH.	52.3	4.5	67.0	58.4	14.0	71.3	48.2	2.9	62.2	2.0 **	5.0 ***
JP MORGAN CHASE.	62.9	4.7	83.0	72.7	9.3	89.5	58.3	3.8	79.6	2.7 ***	5.3 ***
KEYCORP	60.9	4.5	76.8	68.8	12.6	79.6	54.3	1	73.7	3.1 ***	3.4 ***
NORTHERN TRUST	53.7	3.4	69.6	62.3	6.9	78.1	50.9	2.6	64.8	2.1 **	4.2 ***
PNC FINL.SVS.GP.	58.9	3.9	79.0	59.7	8.8	84.2	53.7	2.9	74.2	1.1	2.3 ***
WELLS FARGO & CO	59.4	4.2	83.0	74.0	10.7	91.5	51.2	3.0	74.7	3.9 ***	4.6 ***
M&T BANK	54.3	2.8	68.8	62.3	7.1	82.8	45.1	2.0	57.9	3.4 ***	4.2 ***
REGIONS FINL.NEW	57.1	4.8	69.8	64.9	12.7	76.2	52.1	3.2	62.4	2.4 ***	3.6 ***
SYNOVUS FINL.	37.1	4.1	55.0	57.1	9.8	73.4	30.7	3.1	45.3	5.1 ***	2.8 ***
STATE STREET	52.0	4.0	68.2	61.0	10.5	71.2	48.5	3.0	65.2	2.3 ***	1.8 *
US BANCOR	51.4	3.6	71.2	67.5	8.7	83.4	46.0	2.6	62.0	3.7 ***	5.2 ***
ZIONS BANCORP.	50.3	4.3	65.3	61.0	11.7	80.2	43.9	2.9	51.9	3.4 ***	5.3 ***
Average	55.1	4.1	73.0	64.4	10.1	81.2	50.1	3.0	66.4		

Table 3 G-5 co-crashes versus flight-to-quality effects between stocks, bonds and gold and the business cycle

Note: The table presents estimates (as percentage) and equality tests for the conditional probability of co-crashes (“CO”) or flight-to-quality (“FTQ”) for pairs of stocks and bonds (SB), stocks and gold (SG), bonds and gold (BG). Estimates are reported for the full sample, recession sample and expansion sample. The threshold m=200 is consistent with making tail-β plots and selecting m in a horizontal range. The equality T-test evaluates whether estimates of co-crash and FTQ probabilities are equal. We compare CO and FTQ probabilities within the full sample, the recession sample and the expansion sample. One-sided rejections at the 5%, 2.5% and 1% significance level are denoted with *,** and ***, respectively.

	Estimation results									Testing results		
	P_{CO}^{SB}	P_{FTQ}^{SB}	Cor^{SB}	P_{CO}^{SG}	P_{FTQ}^{SG}	Cor^{SG}	P_{CO}^{BG}	P_{FTQ}^{BG}	Cor^{BG}	$P_{CO}^{SB} = P_{FTQ}^{SB}$	$P_{CO}^{SG} = P_{FTQ}^{SG}$	$P_{CO}^{BG} = P_{FTQ}^{BG}$
Panel A: Full												
US	7.0	22.5	-9.6	6.0	10.5	-5.3	4.0	9.0	-2.4	4.4***	1.7*	2.0 **
GE	5.0	18.0	-10.7	6.5	10.5	-4.2	6.5	5.0	4.1	4.3***	1.5	-0.7
UK	5.5	16.5	-6.6	10.0	10.0	-1.6	7.0	6.0	1.4	3.6***	0.0	-0.4
FR	6.0	10.0	0.1	8.0	9.5	-5.3	4.0	4.5	2.0	1.5	0.5	0.2
JP	6.0	9.0	-6.8	9.0	5.5	4.6	3.0	0.5	0.3	1.1	-1.3	-1.8 *
Average	5.9	15.2	-6.7	7.9	9.2	-2.4	4.9	5.0	1.1			
Panel B: Recession												
US	6.8	29.5	-25.2	9.1	22.7	-10.9	2.3	9.1	0.7	2.9***	1.6	1.1
GE	3.9	19.5	-18.5	10.4	14.3	-12.7	13.0	5.2	10.9	3.2***	0.7	-1.7
UK	4.2	36.6	-28.8	15.5	18.3	4.4	12.7	7.0	0.1	5.3***	0.5	-1.1
FR	8.3	18.8	-24.1	10.4	14.6	-13.1	14.6	12.5	9.4	1.5	0.6	-0.3
JP	1.1	9.2	-15.1	9.2	11.5	5.7	5.7	2.3	1.0	2.5***	0.5	-1.2
Average	4.9	22.7	-22.3	10.9	16.3	-5.3	9.7	7.2	4.4			
Panel C: Expansion												
US	8.6	17.6	-5.1	3.2	7.0	-3.3	3.2	8.0	-3.3	2.6***	1.7*	2.0 **
GE	4.8	19.6	-8.0	7.1	10.1	-0.9	7.7	4.8	-0.9	4.2***	1.0	-1.1
UK	7.6	7.6	2.1	8.1	7.0	-4.6	5.8	5.2	-4.6	0.0	-0.4	-0.2
FR	6.5	9.7	4.8	10.8	9.7	-3.2	2.2	5.4	-3.2	1.2	-0.3	1.6
JP	10.6	9.9	-3.0	8.1	5.6	4.0	2.5	0.6	4.0	-0.2	-0.9	-1.3
Average	7.6	12.9	-1.8	7.5	7.9	-1.6	4.3	4.8	-1.6			

Table 4 Minimum Standard Deviation and Value-at-Risk (VaR) portfolios across business cycles

Note: The table presents the optimal weights that minimize either the 99% Value-at-Risk (VaR) or standard deviation for 28 equity portfolios consisting of 2 stocks selected from the 28 stocks in the Dow Jones index with the largest Market Cap. We distinguish full sample portfolios, recession portfolios and expansion portfolios. Short selling is excluded implying nonnegative portfolio weights. The portfolio tail quantile (or Value-at-Risk) is calculated by letting the portfolio weight vary over a grid between 0 and 100%. The weight is selected such as to minimize the portfolio VaR. The minimum variance portfolio is determined using the classic “textbook” formula.

	Full					Recession					Expansion				
	Cor	Min VAR		Min Stdev		Cor	Min VAR		Min Stdev		Cor	Min VAR		Min Stdev	
		Weight	Min	Weight	Min		Weight	Min	Weight	Min		Weight	Min	Weight	Min
EXXON MOBIL & WAL MART	33.3	59.1	0.078	60.3	0.013	43.0	40.9	0.095	41.1	0.018	31.6	63.0	0.074	63.6	0.013
EXXON MOBIL & P & G	38.0	43.0	0.078	50.5	0.013	55.9	54.8	0.100	23.3	0.017	34.6	46.6	0.073	54.5	0.012
EXXON MOBIL & JOHNSON & JOHNSON	40.7	52.7	0.072	46.0	0.013	56.7	11.6	0.079	13.7	0.016	38.1	49.2	0.070	51.1	0.012
EXXON MOBIL & GENERAL ELECTRIC	45.0	61.9	0.101	64.1	0.014	47.8	100.0	0.144	79.0	0.022	44.2	58.9	0.095	58.6	0.013
EXXON MOBIL & INTERNATIONAL B.M.	34.4	59.9	0.089	60.4	0.013	51.7	46.7	0.112	38.0	0.018	31.1	61.0	0.085	63.6	0.013
EXXON MOBIL & JP MORGAN	35.5	76.9	0.092	82.0	0.015	40.5	100.0	0.119	95.3	0.022	34.1	71.2	0.086	75.6	0.013
EXXON MOBIL & PFIZER	38.0	62.7	0.084	61.2	0.014	54.0	62.1	0.101	41.3	0.019	34.8	68.5	0.080	64.1	0.013
WAL MART & P & G	36.9	31.2	0.084	39.5	0.014	57.3	45.4	0.096	33.1	0.017	33.8	29.5	0.080	40.2	0.013
WAL MART & JOHNSON & JOHNSON	38.5	37.8	0.080	35.1	0.013	53.4	13.4	0.076	24.5	0.015	36.4	32.0	0.079	36.3	0.013
WAL MART & GENERAL ELECTRIC	45.2	44.3	0.117	51.7	0.015	46.3	97.2	0.153	85.2	0.020	46.2	31.9	0.108	41.3	0.014
WAL MART & INTERNATIONAL B.M.	32.6	48.4	0.101	49.9	0.014	50.8	47.7	0.113	48.5	0.018	29.7	48.6	0.097	50.0	0.014
WAL MART & JP MORGAN	35.2	72.1	0.108	73.8	0.016	38.2	100.0	0.126	97.1	0.020	36.1	60.2	0.101	63.3	0.015
WAL MART & PFIZER	35.9	55.2	0.093	50.1	0.015	47.2	20.4	0.105	52.1	0.018	34.1	51.9	0.090	49.8	0.014
P & G & JOHNSON & JOHNSON	46.9	51.7	0.078	45.0	0.013	63.3	27.9	0.080	36.1	0.015	44.5	53.7	0.075	45.8	0.013
P & G & GENERAL ELECTRIC	42.6	67.3	0.099	63.0	0.014	47.7	99.2	0.133	94.0	0.018	42.6	56.8	0.092	53.2	0.013
P& G & INTERNATIONAL B.M.	28.8	64.6	0.088	59.2	0.013	53.0	60.2	0.107	63.9	0.016	25.1	60.1	0.084	58.7	0.013
P & G & JP MORGAN	31.4	78.8	0.088	80.1	0.015	43.9	97.3	0.110	100.0	0.018	29.3	78.1	0.081	70.5	0.014
P & G & PFIZER	39.3	67.9	0.085	60.9	0.014	58.0	61.9	0.102	69.5	0.017	36.4	72.0	0.082	60.0	0.013
JOHNSON & JOHNSON & GENERAL ELECTRIC	43.5	61.6	0.103	67.6	0.013	46.6	100.0	0.110	97.4	0.016	44.4	57.1	0.097	57.4	0.013
JOHNSON & JOHNSON & INTERNATIONAL B.M.	30.7	53.7	0.091	63.1	0.013	48.6	74.5	0.089	72.0	0.015	28.1	56.9	0.088	62.2	0.013
JOHNSON & JOHNSON & JP MORGAN	31.5	78.0	0.093	82.8	0.014	38.1	93.4	0.089	100.0	0.016	31.6	73.6	0.089	73.9	0.013
JOHNSON & JOHNSON & PFIZER	53.5	65.4	0.088	69.6	0.014	65.0	89.2	0.084	85.6	0.016	51.9	67.4	0.086	67.7	0.014
GENERAL ELECTRIC & INTERNATIONAL B.M.	45.5	58.4	0.114	48.2	0.015	55.8	4.9	0.143	7.6	0.020	44.2	54.9	0.105	58.4	0.014
GENERAL ELECTRIC & JP MORGAN	56.0	73.9	0.129	82.1	0.018	62.7	80.3	0.212	94.7	0.031	52.7	80.4	0.115	76.5	0.015
GENERAL ELECTRIC & PFIZER	43.6	50.7	0.114	48.4	0.015	49.5	12.5	0.148	14.4	0.020	43.2	51.7	0.108	58.0	0.014
INTERNATIONAL B.M. & JP MORGAN	39.7	80.9	0.115	75.5	0.017	54.0	99.1	0.133	100.0	0.020	37.5	67.8	0.108	63.5	0.015
INTERNATIONAL B.M. & PFIZER	30.8	49.0	0.102	50.2	0.014	45.1	37.2	0.110	53.3	0.017	28.6	53.0	0.098	49.8	0.014
JP MORGAN & PFIZER	34.0	25.4	0.105	26.7	0.016	44.8	5.3	0.122	-	0.021	32.3	34.3	0.098	37.3	0.015