Caution and Conservatism in Monetary Policymaking

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Abstract

This paper introduces a new concept of conservatism into a game of monetary policy. Multiplicative instrument uncertainty is shown to induce caution or 'Brainard-conservatism' in the behaviour of a rational policymaker. The analysis reveals that Brainard-conservatism does make a difference for various components of social welfare. The inflationary bias problem of monetary policy is reduced, whereas inflation and output variability generally increase. As a result, the overall effect on social welfare is ambiguous. The model also has implications for the link between poor credibility and volatile inflation, and can be extended to address the transparency of operating procedures and the free lunch result of central bank independence.
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1. Introduction

Many practitioners and observers in the field of central banking hold the view that the formulation of monetary policy is characterized by a degree of 'conservatism'. Consider for example the following quote:

'An important reason to expose central bankers to elected officials is that, just as the latter may have an inflationary bias, the former may easily develop a deflationary bias. Shielded as they are from public opinion, cocooned within an anti-inflationary temple, central bankers can all too easily deny that cyclical unemployment can be reduced by easing monetary policy.' (Fischer, 1994)

The theoretical literature on monetary policy models has led to a variety of concepts of conservatism, a common feature of many being that they somehow proxy a tendency towards a more restrictive stance of policy. 'Weight-conservatism', for example, refers to a strong preference for low inflation variability as compared to output variability and has received most of the attention since Rogoff's (1985) paper. Another concept is that of a 'conservative desired inflation rate' (as introduced by Svensson, 1995), which captures a suboptimally low inflation target. Finally, 'discount-rate conservatism' derives from a low rate of time preference, which increases the time horizon of the policymaker and makes policy more subject to long-term considerations and less activism.

In this paper, we introduce into a model of monetary policy yet another form of conservatism, which we have termed 'Brainard-conservatism'. Brainard-conservatism refers to a more cautious stance of policy as a rational response to uncertainty. To give an example, pervasive instrument uncertainty (that is, regarding the relationship between instruments and targets) will make a policymaker want to err on the side of caution.

This general principle was established long ago by Brainard (1967), who showed that uncertainty regarding the policy multipliers in a simple aggregate demand model leads to a more modest setting of the instrument. It is however important to note that these results are crucially dependent on the nature of the uncertainty. If uncertainty were merely additive, certainty equivalence would hold.

1For a recent overview of the concept of weight-conservatism, see Herrendorf and Lockwood (1996).
and the setting of the instrument would not be affected.\textsuperscript{2} Only if uncertainty is\textit{ multiplicative} does the Brainard-result hold and is the setting of policy affected.

In the recent strand of theory which applies game-theoretic ideas to models of monetary policymaking under rational expectations, Brainard-conservatism has never actually been an issue. This is simply because most models do not feature multiplicative instrument uncertainty. Instead, it is often assumed that the monetary authorities can control their target(s), for example the inflation rate, either perfectly or up to an additive innovation.

In what follows, we present a monetary policy game in which multiplicative instrument uncertainty does play a role. The analysis formally shows that such uncertainty makes a difference: it raises the degree of caution exercised in the setting of the instrument and it affects the well-known credibility/flexibility trade-off. Generally, credibility improves at the expense of flexible output stabilization, and, provided that the credibility problem of monetary policy is sufficiently high, Brainard-uncertainty can even lead to a net gain in social welfare.\textsuperscript{3}

The remainder of the paper is organized as follows. Section 2 develops the benchmark model featuring additive and multiplicative instrument uncertainty. Section 3 states propositions relating Brainard-uncertainty to the credibility/flexibility trade-off, and discusses implications for social welfare. We also claim that weight-conservatism and Brainard-conservatism may be difficult to disentangle empirically. Section 4 presents two possible extensions of the model: we discuss the role of transparency in the central bank’s decision-making process from the perspective of Brainard-uncertainty and argue that Brainard-uncertainty offers a new theoretical justification for the free lunch result of central bank independence. All this is followed by concluding remarks in section 5.

\textsuperscript{2}Throughout we maintain that objective functions are quadratic.

\textsuperscript{3}As Brainard-conservatism and caution are so closely related, they will often be treated as equivalent terms. Similarly, Brainard-uncertainty and multiplicative uncertainty will be regarded as synonyms.
2. Brainard-conservatism in monetary policymaking

The purpose of this section is to introduce multiplicative instrument uncertainty into the Barro-Gordon (1983a,b) model of monetary policy. This modification will yield a better understanding of the consequences of more general forms of instrument uncertainty while at the same time keeping things analytically tractable.

2.1. The setup

First, preferences and technology are outlined. Next, the stochastic structure of the model is described. Finally, the players, the payoff functions, the timing and the information structure of the monetary policy game are identified.

2.1.1. Preferences and technology

Assume that the central bank’s objective function and the social welfare function are represented by the following one-period loss functions

\[
L = A\pi^2 + (y - ky^*)^2 \quad \text{(2.1)}
\]

\[
L^s = \alpha\pi^2 + (y - ky^*)^2 \quad \text{(2.2)}
\]

where \(\pi\) denotes the inflation rate, \(y\) and \(y^*\) are the logs of the current and natural level of output, and \(ky^*\) (with \(k > 1\)) indicates the first-best rate of output. First-best output exceeds its natural level due to, for example, labour market distortions.\(^4\) Deviations of inflation from its desired level, which we set here equal to zero \((\pi^* = 0)\), and deviations of output from its first-best rate both enter the loss function quadratically and are weighed by the non-stochastic parameters \(A\) and \(\alpha\), which measure the level of weight-conservatism of the central bank and that of society respectively. For analytical convenience we will assume at this stage that \(A\) equals \(\alpha\) (as opposed to Rogoff, 1985) and leave the issue of monetary policy delegation to an independent authority as a topic for further research.

\(^4\)The fact that policymakers have an incentive to boost current output to its first-best rate (by adopting other than first best policies aiming at the removal of the distortions themselves) drives the celebrated inflationary bias result of monetary policymaking. Also a balance-of-payments motive or a seignorage-revenue motive could produce an inflationary bias (Cukierman, 1992).
The supply side of the economy is given by a standard short-term Lucas supply function

\[ y = y^* + b(\pi - \pi^*) + \varepsilon, \quad (2.3) \]

which postulates a temporary (postcontracting) relation between output and unanticipated inflation. Parameter \( b \) captures the marginal benefit of unanticipated inflation. Economy-wide aggregate supply disturbances (e.g., productivity shocks) are proxied by \( \varepsilon \). Inflationary expectations, \( \pi^e \), are assumed to be rational:

\[ \pi^e = E[\pi | I_{PS}], \quad (2.4) \]

i.e. the (subjective) expectation of the inflation rate (\( \pi^e \)) coincides with its (objective) mathematical expectation conditional on the information set available at the time of the projection. \( I_{PS} \) is the private sector’s information set at that time.

We assume that inflation is determined as follows:

\[ \pi = s_1^{IP} + s_2. \quad (2.5) \]

What this says, is that the central bank is to some extent able to influence the inflation rate by varying its instrument \( \pi^p \), which represents, for example, (normalized) planned money growth or nominal short-term interest rates. A crucial feature of this relationship is that it exhibits instrument uncertainty of a general form, that is, control is subject to both multiplicative (\( s_1 \)) and additive (\( s_2 \)) shocks.\(^5\)

2.1.2. Stochastic structure

Let us be more explicit now about the stochastic specification of the productivity and control shocks. We assume that

\[
\begin{pmatrix}
\varepsilon \\
\sigma_1 \\
\sigma_2
\end{pmatrix} \sim N_3 \left( \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 & 0 \\
0 & \sigma_2^2 & \sigma_{12} \\
0 & \sigma_{12} & \sigma_2^2 \end{pmatrix} \right). \quad (2.6)
\]

Thus, the productivity and control shocks follow a multivariate normal distribution. The productivity shock is independent from the control shocks, but the

\(^5\)Multiplicative instrument uncertainty (\( s_1 \)) could for example originate from uncertainty with respect to the interest elasticities of money demand and real aggregate demand. Caplin and Leahy (1996) endogenize the difficulty that a central bank has in controlling a particular policy target. In their interesting model, instrument uncertainty originates from the policymaker’s ignorance concerning agents’ reactions to policy initiatives.
control shocks are not necessarily independent of each other (possibly $\sigma_{12} \neq 0$). 6
Productivity disturbances ($\varepsilon$) are assumed to have mean 0 and variance $\sigma^2_2$. Multiplicative shocks ($\tilde{s}_1$) have mean 1 and variance $\sigma^2_1$, whereas additive shocks ($\tilde{s}_2$) have mean 0 and variance $\sigma^2_2$. All variances are assumed to be strictly positive and finite.

2.1.3. Description of the game

We have two players: the private sector and the central bank. The strategy of the private sector is the formulation of inflationary expectations ($\pi^e$) and its payoff function$^7$ equals $-(\pi^e - \pi)^2$. Thus, the private sector only derives utility from minimizing its forecast errors regarding the future inflation rate. The strategy of the central bank consists of the formulation of monetary policy, i.e., the setting of the level of its instrument ($i^p$). Its payoff function equals $-A\pi^2 - (y - ky^*)^2$, which has already been described.

The timing of the game consists of four consecutive events. First, the private sector moves by formulating inflationary expectations. It is assumed, though not explicitly modelled here, that inflationary expectations will be embedded into nominal wage contracts which cannot be renegotiated afterwards. This assumption is crucial to delivering the inflationary bias result. Second, a productivity shock realizes. Third, the central bank sets the optimal stance of monetary policy. Fourth, control shocks realize and both inflation and output are determined.

We have symmetric information. Both players know the structure of the model. The information set of the private sector at time 1 (denoted by $I^1_{PS}$) does not include the productivity shock. The information set of the central bank at time 3 (denoted by $I^3_{CB}$) includes the productivity shock and the private sector's strategy made at time 1. Both information sets exclude the control shocks.

---

6The distributional assumption of normality is merely for convenience. A lognormal distribution may, however, be more appropriate as this would ensure a positive support. Nevertheless, in what follows, we stick to the assumption of normality and assume that the variance is not too large so that negative realizations are sufficiently unlikely.

7As we want to confine our attention to the consequences of Brainard-uncertainty, this payoff function will do. For more general payoff functions see Goodhart and Huang (1996).
2.2. Discretionary equilibrium

We are looking for a time-consistent equilibrium (i.e. a subgame-perfect Nash equilibrium) in the absence of a credible commitment device (hence the qualification ‘discretionary’). First, the central bank’s optimal feedback rule is derived. Then, the private sector’s inference problem is solved. Finally, equilibrium is described.

2.2.1. The central bank’s optimal feedback rule

At date three, the central bank determines its optimal strategy by minimizing the expected value of the loss function given its information set at that time ($I^3_{CB}$):

$$\min_{\{p\}} E \left[ A \pi^2 + (y - ky^*)^2 \mid I^3_{CB} \right]$$

subject to

$$y = y^* + b(\pi - \pi^e) + \varepsilon$$
$$\pi = s_1 p + s_2.$$ (2.7)

Substituting the two constraints into the objective function yields

$$\min_{\{p\}} E \left[ A (s_1 p + s_2)^2 + (y^* + b(s_1 p + s_2 - \pi^e) + \varepsilon - ky^*)^2 \mid I^3_{CB} \right].$$

Setting first derivatives with respect to $p$ equal to zero produces the first-order condition:

$$E \left[ 2A (s_1 p + s_2) s_1 + 2 ((1 - k)y^* + b(s_1 p + s_2 - \pi^e) + \varepsilon) bs_1 \mid I^3_{CB} \right] = 0.$$

After rearranging and evaluating expectations we have

$$[(1 + \sigma^2) (A + b^2)] p = zb - \sigma_{12} (A + b^2) + b \pi^e - b \varepsilon,$$

where we have made use of $E [s_1^2] = 1 + \sigma^2$. Also, we have defined $z \equiv (k - 1)y^*$, which reflects the difference between the first-best and the natural rate of output and provides the central banker with a temptation to inflate systematically. In other words, the strictly positive parameter $z$ creates the credibility problem of monetary policy.  

---

8Setting $z = 0$ is analogous to assuming that monetary policy faces no time-inconsistency problem of the kind considered in this paper. If feasible, this approach would yield the first-best solution. Alternatively, while maintaining $z > 0$, one could assume that the time-inconsistency problem can be solved by mechanisms, such as a constitutional rule without escape clauses, a Walsh-contract (Walsh, 1995) or an inflation contract with a conservative-desired inflation rate (Svensson, 1995). Ideally, this would yield the second-best solution. Note, however, that these approaches may in practice be infeasible or not renegotiation-proof. As a result, we would ultimately end up in a third-best situation featuring an inflationary bias. (The fourth-best solution, which is simple discretion, would correspond to the highest inflationary bias.)
Rearranging yields
\[ \vartheta = z b - \sigma_{12} (A + b^2) \frac{b^2}{(1 + \sigma^2_1)(A + b^2)} \pi^e - \frac{b}{(1 + \sigma^2_1)(A + b^2)} \varepsilon, \] (2.8)
which is the central bank's reaction function.

2.2.2. The private sector's inference problem

Let us now focus on the inference problem of the private sector at date one. Knowing the structure of the problem, the individuals understand that the central bank has an incentive to spring an inflationary surprise on them. Consequently, a myopic strategy based on zero inflationary expectations would clearly violate the assumption of a rationally optimizing individual.\(^2\) As a zero inflation strategy is time-inconsistent, the individual could do better by using his knowledge about the structure of the model (in particular, the preferences of the central bank) in deriving the optimal predictor of the inflation rate (cf. Kydland and Prescott, 1977). This optimal predictor is found by taking expectations over the central bank's reaction function conditional on the information available to the private sector at date one \((I_{P,1})\):

\[ \hat{\vartheta}_{P,1} = \frac{z b - \sigma_{12} (A + b^2)}{(1 + \sigma^2_1)(A + b^2)} + \frac{b^2}{(1 + \sigma^2_1)(A + b^2)} \hat{\vartheta}_{P,1}, \]

that is,

\[ \hat{\vartheta}_{P,1} = \frac{z b - \sigma_{12} (A + b^2)}{A + \sigma^2_1 (A + b^2)}, \] (2.9)

where we have made use of \(\pi^e = \hat{\vartheta}_{P,1}\). Expression (2.9) is commonly referred to as the inflationary bias of monetary policy. Note that the private sector would be able to make a better prediction by conditioning its strategy on the productivity shock, if it was not already locked into nominal contracts. Note also that the private sector's expectations of the inflation rate and of the level of the instrument coincide as the control shocks disappear in expectations.

\(^2\)We consider here a world in which commitment is not an option because it is not credible.
2.2.3. Equilibrium outcomes

Combining now the optimal strategies of both players, we can compute the equilibrium outcomes of the instrument level, the inflation rate and the output level. Substituting the private sector's expectation of the level of the instrument into the central bank's reaction function produces the actual stance of monetary policy:

\[ i^* = \frac{zb - \sigma_{12}(A + b^2)}{A + \sigma_T^2(A + b^2)} - \frac{b}{(1 + \sigma_T^2)(A + b^2)} \varepsilon, \]  

in which \( s_2 \) itself does not play a role. As a result, additive instrument uncertainty does not affect the stance of monetary policy. This is simply Theil's certainty equivalence result under quadratic objective functions.

At date four, the multiplicative and additive control shocks realize. Actual inflation equals

\[ \pi = s_1 \left( \frac{zb - \sigma_{12}(A + b^2)}{A + \sigma_T^2(A + b^2)} - \frac{b}{(1 + \sigma_T^2)(A + b^2)} \varepsilon \right) + s_2. \]  

(2.11)

In order to find equilibrium output, we first need to calculate unanticipated inflation. Subtracting equation (2.9) from (2.11) yields

\[ \pi - \pi^e = (s_1 - 1) \left( \frac{zb - \sigma_{12}(A + b^2)}{A + \sigma_T^2(A + b^2)} - \frac{b}{(1 + \sigma_T^2)(A + b^2)} s_1 \varepsilon \right) + s_2. \]  

(2.12)

Substituting unanticipated inflation into the supply function and rearranging yields

\[ y = y^* + b(s_1 - 1) \frac{zb - \sigma_{12}(A + b^2)}{A + \sigma_T^2(A + b^2)} + \frac{(1 + \sigma_T^2)(A + b^2) - b^2 s_1}{(1 + \sigma_T^2)(A + b^2)} \varepsilon + bs_2, \]  

(2.13)

which is the equilibrium output supply reaction function.
3. Implications

First, we discuss the implications of Brainard-uncertainty for the equilibrium rate of inflation. Second, the effects on equilibrium output are examined. Third, we elaborate on the social welfare consequences. Finally, we claim that Brainard-and weight-conservatism may be difficult to disentangle empirically.

3.1. Inflation

The inflation rates with and without instrument uncertainty are respectively

\[ \pi = s_1 \left( \frac{zb - \sigma_{12}(A + b^2)}{A + \sigma_1^2(A + b^2)} - \frac{b}{(1 + \sigma_1^2)(A + b^2)} \right) + s_2 \]  \hspace{1cm} (3.1)

\[ \pi^{\text{no}} = \frac{zb}{A} - \frac{b}{A + b^2} \varepsilon, \]  \hspace{1cm} (3.2)

where the superscript 'no' will subsequently refer to the case without instrument uncertainty.

Taking unconditional expectations, the inflationary biases are respectively

\[ E[\pi] = \frac{zb - \sigma_{12}(A + b^2)}{A + \sigma_1^2(A + b^2)} \]  \hspace{1cm} (3.3)

\[ E[\pi^{\text{no}}] = \frac{zb}{A}. \]  \hspace{1cm} (3.4)

With respect to the variance of inflation, we have

\[ Var[\pi] = \left( \frac{zb - \sigma_{12}(A + b^2)}{A + \sigma_1^2(A + b^2)} \right)^2 \sigma_1^2 + \left( \frac{b}{A + b^2} \right)^2 \frac{\sigma_1^2}{(1 + \sigma_1^2)} + \sigma_2^2 + 2 \left( \frac{zb - \sigma_{12}(A + b^2)}{A + \sigma_1^2(A + b^2)} \right) \sigma_{12} \]  \hspace{1cm} (3.5)

\[ Var[\pi^{\text{no}}] = \left( \frac{b}{A + b^2} \right)^2 \sigma_2^2, \]  \hspace{1cm} (3.6)

where we have made use of \( Cov[s_1, s_1\varepsilon] = Cov[s_2, s_1\varepsilon] = 0 \) and \( Var[s_1\varepsilon] = (1 + \sigma_1^2)\sigma_2^2 \).
Inspection and comparison of equations (3.1)-(3.6) yields the following propositions:

**Proposition 3.1.** Brainard-uncertainty improves the credibility of monetary policy but deteriorates the scope for flexible output stabilization.

With regard to the *credibility problem*, it is clear from (3.3) that Brainard-uncertainty improves the inflationary bias problem as

\[
\frac{\partial E[\pi]}{\partial \sigma_1^2} < 0.
\]

Intuitively, this result is quite straightforward: the private sector rationally predicts that the central bank will adopt a more cautious stance of monetary policy when \( \sigma_1^2 > 0 \). Hence, inflationary expectations will be set at a lower rate and the inflationary bias problem will be reduced.\(^{10}\)

With regard to the *flexibility problem*, inspection of the slope of the feedback rule in (2.10) shows that Brainard-uncertainty reduces the stabilization role of the instrument. Multiplicative instrument uncertainty makes it optimal for a central banker to exercise greater caution in setting monetary policy or, to put it differently, to adopt a more neutral (or less activist) stance of monetary policy, which we have termed 'Brainard-conservatism'. This reduces the scope for flexible output stabilization (for a formal statement, see proposition 3.5).

**Proposition 3.2.** A negative (positive) covariance between additive and multiplicative control shocks increases (decreases) the inflationary bias.

This is simply because

\[
\frac{\partial E[\pi]}{\partial \sigma_{12}} < 0.
\]

The intuition is as follows. A negative covariance makes an activist policy stance relatively more attractive, because the negative relationship between additive and multiplicative shocks provides the central bank with a useful hedging device. The resulting higher propensity to inflate will be rationally predicted. As a result, a negative covariance compounds the inflationary bias problem. A positive covariance, on the other hand, makes the outcome of activist monetary

\(^{10}\)Devereux (1987) arrives at a similar conclusion based on a mechanism of endogenous wage indexing. He claims that greater monetary variability leads to greater wage indexing, reducing the authorities' tendency for inflationary surprises and thereby lowering the inflationary bias of monetary policy. Swank (1994) proposes a similar model which also hinges on multiplicative instrument uncertainty. His conclusion is that imperfect control of money growth reduces the incentive of a policymaker to create a surprise.
policy more unstable as both types of control shocks are likely to reinforce each other. Therefore, the central bank will find it more desirable to adopt a rather cautious stance of policy. As the private sector rationally understands this, the inflationary bias will be reduced.\footnote{As there are no a priori reasons which pin down the sign of the covariance, the issue is ultimately an empirical matter. We will from now on therefore assume that $\sigma_{12} = 0$.}

**Proposition 3.3.** Brainard-uncertainty has ambiguous effects on inflation volatility. The higher the credibility problem (flexibility problem) of the central bank and the lower (higher) its degree of weight-conservatism, the more likely the introduction of multiplicative instrument uncertainty increases (decreases) the variance of inflation.

Whereas additive instrument uncertainty increases the variance of inflation unambiguously (cf. the third term in (3.5)), multiplicative instrument uncertainty has ambiguous effects on the inflation rate. First of all, there is the direct effect of multiplicative instrument uncertainty on inflation volatility for a given stance of monetary policy. Multiplicative instrument uncertainty simply adds to inflation volatility by enhancing the unpredictability of the consequences of a particular stance of monetary policy for inflation (cf. the first term in (3.5)). Second, there is the indirect effect on inflation volatility through the Brainard-mechanism: greater uncertainty leads to higher Brainard-conservatism. Consequently, the inflation rate is less often influenced for stabilization purposes, which leads to lower inflation volatility (cf. the second term in (3.5)).

In order to assess the relative importance of these two opposing effects, let us evaluate the impact of the introduction of a small amount of multiplicative uncertainty on the variance of the inflation rate:

\[
\frac{\partial \text{Var}[\pi]}{\partial \sigma_1^2} \bigg|_{\sigma_{12} = 0} = \left(\frac{zb}{A}\right)^2 - \left(\frac{b}{A + b^2}\right)^2 \sigma_\varepsilon^2,
\]

which is positive if and only if

\[
\frac{z}{\sigma_\varepsilon} > \frac{1}{1 + \frac{b^2}{A}}.
\]

This last condition has an interesting interpretation: if the credibility problem of monetary policymaking ($z$) is sufficiently large relative to its flexibility problem ($\sigma_\varepsilon$), then Brainard-uncertainty can be expected to increase the volatility of inflation; if on the other hand the flexibility problem is the major issue, then the volatility of inflation is more likely to decrease.
Note also that a lower degree of weight-conservatism \((A)\) decreases the right-hand side of (3.7), making the inequality more likely to be satisfied for a given credibility and flexibility problem. In other words, lower weight-conservatism makes the Brainard-conservatism effect on inflation volatility less important and the increase in the variance of inflation more likely.

**Proposition 3.4.** Brainard-uncertainty explains why poor credibility of monetary policy is associated with more volatile inflation.

In contrast to the case without instrument uncertainty in (3.6) the variance in (3.5) includes the size of the inflationary bias. This establishes a positive correlation between the credibility problem of the central bank and the variance of inflation through the introduction of multiplicative instrument uncertainty. Note that this latter link also provides a rationale behind the positive correlation between the level of inflation and its variance, which is due to the fact that the inflationary bias is simply the unconditional expectation of the inflation rate.

### 3.2. Output

Let us first repeat the expressions for the output levels with and without instrument uncertainty:

\[
y = y^* + b(s_1 - 1)\frac{z b - \sigma_{12}(A + b^2)}{A + \sigma_2^2(A + b^2)} + \frac{(1 + \sigma_2^2)(A + b^2) - b^2 s_1}{(1 + \sigma_2^2)(A + b^2)} \varepsilon + b s_2
\]

\[
y^{\sigma} = y^* + \frac{A}{A + b^2} \varepsilon.
\]

Setting the control errors equal to their expected values and the variance and covariance terms equal to zero, it is straightforward to see that (3.8) is a special case of (3.9).

Equation (3.8) reveals that both additive and multiplicative control errors affect output. This is a direct consequence of the wedge they introduce between inflation and anticipated inflation. In expectations, however, output is in both cases predicted to remain at its natural rate,

\[
E[y] = E[y^{\sigma}] = y^*.
\]
With respect to the variance of output we have the following expressions:

\[
\begin{align*}
\text{Var} [y] &= \left( \frac{zb - \sigma_{12}(A + b^2)}{A + \sigma_1^2(A + b^2)} \right)^2 b^2 \sigma_1^2 + b^2 \sigma_2^2 + \frac{A^2 + \sigma_1^2}{(1 + \sigma_1^2)(A + b^2)^2} \sigma_\varepsilon^2 + \\
&\quad \quad 2 \left( \frac{zb - \sigma_{12}(A + b^2)}{A + \sigma_1^2(A + b^2)} \right) b^2 \sigma_{12} \\
\text{Var} [y^{no}] &= \left( \frac{A}{A + b^2} \right)^2 \sigma_\varepsilon^2.
\end{align*}
\]

(3.10) (3.11)

Inspection and comparison of equations (3.10)-(3.11) yields the following propositions:

**Proposition 3.5.** Introducing Brainard-uncertainty unambiguously raises the variance of output.

Taking the partial derivative of \( \text{Var} [y] \) with respect to \( \sigma_1^2 \), evaluating at \( \sigma_1^2 = \sigma_{12} = 0 \) and rearranging yields

\[
\left. \frac{\partial \text{Var} [y]}{\partial \sigma_1^2} \right|_{\sigma_1^2=\sigma_{12}=0} = \frac{z^2 b^4}{A} + \frac{b^4 + 2b^2 A}{(A + b^2)^2} \sigma_\varepsilon^2 > 0.
\]

Intuitively, there are again two mechanisms at work. First of all, the variance of output is increased because of the unexpected inflation triggered by the control errors. Second, as the central bank exercises greater caution when formulating monetary policy, the extent of output stabilization is reduced. Hence, output volatility will be greater.

**Proposition 3.6.** Poor credibility exacerbates the volatility of output in the presence of Brainard-uncertainty.

Equation (3.10) shows that the inflationary bias enters the expression as a squared coefficient on the variance of the multiplicative control error. Intuitively, this is due to unexpected inflation being a function of the inflationary bias. Hence, Brainard-uncertainty is predicted to have larger and more volatile output effects in countries with a serious inflationary bias problem.
3.3. Social welfare

Until now we have only examined the effect of Brainard-uncertainty on individual components of the (central bank's) objective function: the inflationary bias, the variance of inflation and the variance of output.\textsuperscript{12}

To look at the overall implications for social welfare, we take unconditional expectations over society's objective function (2.2), which can be rewritten in the following useful way:

\[
E[L^s] = \alpha E[n^2] + E[(y - ky')^2] = \alpha (E[\pi])^2 + \alpha \text{Var}[\pi] + z^2 + \text{Var}[y].
\]

Assuming for analytical convenience that \( A = \alpha \) (cf. 2.1 and 2.2)\textsuperscript{13}, tedious algebra delivers the following compact expression for the expected loss in social welfare under both regimes:

\[
E[L^s] = \frac{(1 + \sigma^2_1) (A + b^2)}{A + \sigma^2_1(A + b^2)} z^2 + \frac{A + \sigma^2_1(A + b^2)}{(1 + \sigma^2_1)(A + b^2)} \sigma^2_\varepsilon + (A + b^2) \sigma^2_\varepsilon.
\]

\[
E[L^{s,n_0}] = \frac{A + b^2}{A} z^2 + \frac{A}{A + b^2} \sigma^2_\varepsilon.
\]

Let us now try to sign the overall effect of Brainard-uncertainty on social welfare. Differentiating the equilibrium value of social welfare with respect to the variance of multiplicative control shocks yields:

\[
\frac{\partial E[L^s]}{\partial \sigma^2_1} = \frac{b^2}{(1 + \sigma^2_1)(A + b^2)} \frac{(A + b^2) b^2}{(A + \sigma^2_1(A + b^2))^2} z^2,
\]

which attains a negative value if and only if

\[
\frac{z}{\sigma_\varepsilon} > 1 - \frac{b^2}{(1 + \sigma^2_1)(A + b^2)}.
\]

\textsuperscript{12}Note that Brainard-uncertainty does not affect the deadweight loss of output.

\textsuperscript{13}If \( A \) were to exceed \( \alpha \), then we could explore how delegation of monetary policy to an independent agent can alter the credibility and flexibility problem. In order not to complicate matters any further now, this issue is left as an exercise for further research.
Proposition 3.7. Brainard-uncertainty may be advantageous for social welfare as long as the credibility problem of monetary policymaking is sufficiently large relative to the flexibility problem.

The rationale behind this surprising result, which can be derived from (3.14), is as follows. On the one hand, when the credibility problem is relatively large compared with the flexibility problem, Brainard-uncertainty increases social welfare through a reduction in the inflationary bias. On the other hand, the possible increase in the volatility of inflation and output (due to Brainard-uncertainty in combination with poor credibility) lowers social welfare. The algebra shows that the overall effect on social welfare can be positive as long as the credibility/flexibility ratio $z/\sigma_c$ is sufficiently high.

This conclusion extends the findings of Swank (1994), who established that multiplicative instrument uncertainty unambiguously increases social welfare. Our analysis suggests that this conclusion is only correct under the conditions specified in (3.14). Similarly, the intuitively more plausible statement that multiplicative instrument uncertainty triggers a drop in social welfare is a special case of our proposition. As such, the results of our model are also supported by the findings of Letterie and Lippi (1997), who state that the welfare effects of 'transmission uncertainty' are ambiguous and depend on the size of the credibility problem.

The finding that uncertainty may increase welfare has also been confirmed in other contexts. Bijffinger, Hoeberichts and Schaling (EHS, 1997), for example, find that preference uncertainty (regarding the degree of weight-conservatism of the monetary authorities) on the behalf of the private sector may be beneficial for social welfare. Their conclusion is, however, dependent on the entirely opposite condition that the credibility problem should be relatively small compared to the flexibility problem. It thus seems that multiplicative preference uncertainty and multiplicative instrument uncertainty lead to entirely different conclusions. Also, the components of social welfare seem to behave in different ways under multiplicative preference uncertainty. EHS predict that such preference uncertainty yields a higher inflationary bias, a higher variance of inflation and ambiguous effects on the variance of output. The model in this paper predicts that greater multiplicative instrument uncertainty yields a lower inflationary bias, an ambiguous effect on the variance of inflation and an increase in the variance of output.

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14 Other papers which allow for a stochastic preference parameter include Cukierman and Meltzer (1988), Lewis (1990), Briault et alli (1996) and Nolan and Schaling (1996).
To sum up, it is important to closely examine the underlying causes of monetary policy uncertainty, as different types of uncertainty generate different predictions for the equilibrium behaviour of inflation and output. Moreover, the impact of one such type of uncertainty on social welfare is generally ambiguous and dependent on the credibility/flexibility trade-off in that particular economy.

3.4. Disentangling weight- and Brainard-conservatism

Changes in weight-conservatism ($A$) and changes in Brainard-conservatism (via $\sigma^2$) both have the same qualitative effect on the setting of the instrument.\footnote{\textit{It} is not yet clear whether the similarity between weight-conservatism and Brainard-conservatism is just an artefact of the two-period structure of the model. In order to address this issue a full-fledged multiple-period version of the model is necessary. It would be a worthwhile exercise for future research to examine to which extent the two concepts are substitutable (for example, along the lines of Frattiani and Huang's (1995) analysis of the substitutability between weight-conservatism and reputation).} This can easily be seen from:

$$i^p = \frac{zb}{A + \sigma^2(A + b^2)} - \frac{b}{(1 + \sigma^2)(A + b^2)^{\varepsilon}}. \tag{3.15}$$

To put it differently, an apparently restrictive stance could be the outcome of two possible scenarios: the emphasis of the central banker on low inflation variability may have increased; alternatively, the central banker may have become more cautious in the implementation of monetary policy as a reaction to increased perceived instrument uncertainty.

Obviously this does not imply that the two concepts of conservatism are identical. In fact, the distinction of Drazen and Masson (1994) and Cukierman and Tomassi (1996) between the credibility of policies and the credibility of policymakers also applies here. In this setting, policy credibility refers to adopting policies with the right amount of caution (as rationally expected by the private sector) in the presence of instrument uncertainty, whereas policymaker credibility refers to the policymaker’s degree of weight-conservatism.

Nevertheless, even though a differentiation between both concepts is necessary, disentangling them empirically may not be straightforward, especially in a richer setting featuring incomplete information on the behalf of the private sector.\footnote{The difficulty the public has in discovering changes in Brainard-uncertainty is greater when the monetary operating procedures of the central bank lack sufficient transparency, in which case the realisations of the control shocks could be considered as the private information of the central bank.}
Taking this idea one step further, criticisms addressed at central banks for adopting overly restrictive policies are sometimes misleading as the policy outcome could equally have been the result of a socially optimal response to uncertainty.\textsuperscript{17}

\textsuperscript{17}Also, one could ask to which extent a central bank is able to be a statistical outlier by adopting policies which are too weight-conservative compared to society’s preferences. Ultimately, no central bank can claim to enjoy full goal independence or can afford to alienate legislators who can infringe on its independence. (Lohmann, 1992; Cukierman, 1996)
4. Extensions

In what follows, we present two extensions of the model of the previous sections. First, we discuss the role of transparency in the central bank's decision-making process from the perspective of Brainard-uncertainty. Second, we argue that Brainard-conservatism helps to explain the free lunch result of central bank independence.

4.1. The transparency of decision-making

Recently, there has been a marked tendency towards greater transparency and greater accountability in the operating procedures of many central banks. With regard to preference uncertainty, greater openness in the decision-making process of the central bank enables the public to make more accurate predictions about the future inflation rate, which is also conducive to a lower inflationary bias (Nolan and Schaling, 1996).

In the context of Brainard-uncertainty, a case in favour of transparency can be made as well. First of all, if the central bank has private information, for example, about the control shocks (as in Canzoneri, 1985), then it may be favourable to disclose it, so that the subsequent stabilization of the control shock is not mistakenly interpreted by the private sector as a change in weight-conservatism. Secondly, transparency about the decision-making process of the central bank may yield better insights into how the central bank perceives the uncertainty. Greater transparency with regard to the central bank's perception of Brainard-uncertainty could be achieved, for example, by an in-depth explanation of the reasons behind a particular policy change.

4.2. Credibility, flexibility and the free lunch debate

Empirical studies have often suggested that granting more independence to a central bank is like a free lunch, as inflation can be lowered without any harmful implications for the real economy. Alesina and Summers (1993) were the first to discover that neither the level nor the variability of real variables such as growth, unemployment and real interest rates are significantly affected by central bank independence (CBI).

In a sense, these empirical findings are somewhat surprising, because at the theoretical level we would have expected the opposite. The Rogoff approach

\footnote{See for example DeBelle and Fischer (1994), Posen (1994) and Fischer (1995).}
predicts that inflation can be lowered but only at the cost of higher output variability (the so-called credibility-flexibility trade-off). Alesina and Gatti (1995) have subsequently tried to make the theory consistent with the facts by introducing political uncertainty. Their point is that granting independence to the central bank increases 'economic variability' due to greater passivity with regard to the stabilization of output shocks\textsuperscript{19}, but at the same time reduces 'political variability' by insulating monetary policy from political pressures. Hence the overall effect is ambiguous and may well be insignificant, which would be consistent with the data.

The concept of Brainard-conservatism allows for an additional explanation of the small effect of CBI on overall output volatility. One could argue that a central bank involved into day-to-day policymaking has a better understanding of the monetary policy transmission mechanism than the government has. If this is correct, then the increase in output volatility due to the higher weight-conservatism of the central bank could be offset by a decrease due to lower Brainard-conservatism as a result of less instrument uncertainty. Consequently, a transfer of authority from government to central bank should not necessarily be associated with increased output volatility. This constitutes another theoretical justification for the free lunch result.

\textsuperscript{19} Actually, one should carefully differentiate between CBI and weight-conservatism. Increased economic variability is, in fact, only a consequence of increased weight-conservatism. Increasing CBI per se does not deliver the higher output variability result.
5. Conclusions

This paper proposes a new concept of conservatism in monetary policymaking, which follows from the response of a rational policymakers to uncertainty. It has been shown that multiplicative instrument uncertainty triggers caution or 'Brainard-conservatism' on the behalf of the monetary authorities. As the private sector rationally understands the central bank's reaction function, inflationary expectations will be lower in equilibrium, improving the credibility problem of monetary policy. Inflation variability on the other hand generally increases (unless initial credibility is low) and so does output variability (unambiguously). The overall implication of the theoretical exercise is that Brainard-uncertainty generally decreases social welfare, unless the credibility problem of monetary policy is sizable.

Furthermore, the model explains why poor credibility is so often associated with volatile inflation. We have also claimed that restrictive policies do not necessarily result from increased weight-conservatism. Instead, they could merely derive from a socially optimal way of responding to uncertainty. Finally, Brainard-uncertainty can constitute a case in favour of transparent decision-making and provides a theoretical justification for the free lunch result of central bank independence.

A possible limitation of the paper relates to the Barro-Gordon framework in which we have cast the analysis. Alternative foundations for the sources of the inflationary bias, for example, could be more realistic but would give little additional insight into the mechanism of Brainard-conservatism itself. Further, a distinction between the extent of weight-conservatism of the central bank and of the government may be necessary to explicitly address the issue of delegation. Finally, a multiple-period model featuring output persistence would add to realism but again at the expense of analytical tractability. We expect however that the main point of this paper should extend to a more dynamic setting and leave this as a task for further research.
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