A Fallacy in Causality Research
on Growth and Capital Accumulation

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Abstract

In this paper we argue that recent causality research on economic growth and accumulation rates dismisses the constant returns neoclassical growth models based on a fallacy.

JEL-classification

E22, O11, O40.
A Fallacy in Causality Research on Growth and Capital Accumulation.

I. INTRODUCTION

The most striking fact in mainstream empirical literature on economic growth probably is that augmented versions of Solow's [1956] model can explain cross-sectional differences in the level and growth rates of the standard of living—as measured by per capita income—rather well. As it stands, this approach relies on the causality between the accumulation rates of different kinds of capital, and the growth of income per capita. In short, these models state that every period a fraction of income is forgone, and re-invested in economic fundamentals like physical or human capital, or technological know-how. All capital components are assumed to be subject to diminishing returns, so that investing a constant fraction of output will yield less additional output as time evolves. In the long run, the economy will therefore converge to a stable steady state in which the income level is determined by the rates of accumulation of the underlying growth fundamentals, as well as by the exogenous growth rate of the number of efficiency workers plus some technological parameters. Moreover the speed of convergence in these models can be approximated, and turns out to be a function of the capital shares, depreciation and the growth rates of labor and exogenous technology—roughly 2 percent a year based on reasonable assumptions for the parameters. Indeed, the expected close association between economic growth and the mentioned variables is found in the data, and it has been taken as a consistent empirical fact (see e.g. Barro and Sala-I-Martín [1992], Levine and Renelt [1992], Mankiw, Romer and Weil [1992] or Nonneman and Vanhoudt [1996] among others). These results are especially challenging for new growth theories (Rebello [1991], or Romer[1986], [1990]), for those theories have different predictions and implications. Yet time series tests seem not to confirm them (see for instance Jones [1995]).

Although it is difficult to imagine any growth mechanism that does not work through the increase of a stock of capital in one way or another, opponents now criticize the neoclassical approach based on its causality implication. Granger causality tests yield a counter-intuitive negative sign on the share of investment, they argue, and hence the 'mechanical link' running from investment to growth—as present in neoclassical models—fails to be confirmed by the data.

In this paper we show why this conclusion is false. The basic reason for this is that the used methodology and interpretations suffer from a fallacy. In order to prove these statements the paper is organized as follows. In section II the methodology and arguments in the recent causality research are briefly reviewed. Section II deals with the causality implications of both the neoclassical and Ak-growth theories and shows why the conclusions of the research are wrong. Section IV summarizes and concludes.

1 Because of the closed-economy assumption in the considered models we will use investment and saving interchangeably throughout the text.
II. RECENT CAUSALITY RESEARCH

The argument in recent causality research goes roughly as follows. In the textbook Solow model, and other things being equal, countries with higher investment shares should have higher economic growth—which is clearly confirmed by the significant cross-section correlation between these variables. Something closer to a policy question then is: can a country accelerate growth by raising capital formation, or, do changes in investment really precede changes in growth? Very few research has been undertaken to study this question. However, empirical studies now show that—if there is any causality running from investment to growth—the investment share always enters with a so called “counterintuitive” negative sign. In other words: increases in the investment share precede decelerations of the growth rate (see Blomström et al [1993], Carroll and Weil [1994] or Andrés et al [1996]). According to these authors this is evidence against—or at least very difficult to reconcile with—the constant returns neoclassical growth model.

Different approaches are used to come to this conclusion. All of them involve panel data estimations for a cross-section of different countries with time series consisting of five-year averages. Taking averages over a certain time span is usually motivated as follows. Granger Causality tests have been applied to time series—with yearly data—for different countries to test for causality between e.g. exports and growth (Jung and Marshall [1985], Bahmani-Oskooee et al [1991]) or defense spending and growth (Jordi [1986], LaCivita and Frederiksen [1991]). Using yearly data, however, this methodology is exposed to business cycle variations in the considered variables. Because growth theory focuses on the medium and long-run, cyclical influences should at least be diluted.

Blomström et al [1993] perform simple Granger causality tests on the growth rate ($\gamma$) of real per capita GDP and the investment share ($s$). The estimated equations are:

\[
\gamma_t = 0.661 + 0.227\cdot\gamma_{t-1} \\
[t = 7.0] [t = 3.7] [R^2 = 0.06 \ n = 303]
\]

\[
[1] \gamma_t = 0.660 + 0.228\cdot\gamma_{t-1} - 0.002\cdot s_{t-1} \\
[t = 7.0] [t = 3.7] [t = -0.01] [R^2 = 0.06 \ n = 303]
\]

From these regressions the authors conclude that they cannot reject the null-hypothesis that capital formation in the preceding period (with a period being five years) has no explanatory power with respect to growth in the current period, given the past history of growth in a country. Carroll and Weil [1994] use more refined panel data econometrics for analyzing the same question. They run fixed-effects regressions and allow for time effects:
\[ \gamma_t = 0.232 \cdot \gamma_{t-1} - 0.259 \cdot s_{t-1} \]
\[ (t = 2.6) \quad (t = -4.2) \]  \[ R^2 = 0.25 \quad n = 132 \quad (OECD) \]

\[ \gamma_t = -0.06 \cdot \gamma_{t-1} - 0.08 \cdot s_{t-1} + \text{time dummies} \]
\[ (t = -0.6) \quad (t = -11) \]  \[ R^2 = 0.46 \quad n = 132 \quad (OECD) \]

\[ \gamma_t = -0.06 \cdot \gamma_{t-1} - 0.12 \cdot s_{t-1} \]
\[ (t = -0.9) \quad (t = -3.0) \]  \[ R^2 = 0.15 \quad n = 353 \quad (Full \ Sample) \]

\[ \gamma_t = -0.24 \cdot \gamma_{t-1} - 0.01 \cdot s_{t-1} + \text{time dummies} \]
\[ (t = -3.7) \quad (t = -0.09) \]  \[ R^2 = 0.31 \quad n = 353 \quad (Full \ Sample) \]

The authors recognize the potential problem with these kind of regressions. It is known that fixed effect regressions with lagged endogenous variables on the right-hand side produce biased estimates in short panels. The bias results from a correlation between the residual for a given observation and the country fixed effect (see Hsiao, 1986). The problem can easily be solved by running the regressions in differenced form using the twice lagged difference for the dependent variable as an instrument for the once lagged difference. However, this correction does not change the basic conclusion very much. The Granger causality results in that case turn out to be:

\[ \Delta \gamma_t = -0.236 \cdot \Delta \gamma_{t-1} - 0.175 \cdot \Delta s_{t-1} \]
\[ (t = -1.0) \quad (t = -1.3) \]  \[ Root \ MSE = 0.018 \quad n = 88 \quad (OECD) \]

\[ \Delta \gamma_t = -0.610 \cdot \Delta \gamma_{t-1} - 0.002 \cdot \Delta s_{t-1} + \text{time dummies} \]
\[ (t = -3.5) \quad (t = -0.02) \]  \[ Root \ MSE = 0.014 \quad n = 88 \quad (OECD) \]

\[ \Delta \gamma_t = -0.071 \cdot \Delta \gamma_{t-1} - 0.058 \cdot \Delta s_{t-1} \]
\[ (t = -0.5) \quad (t = -0.9) \]  \[ Root \ MSE = 0.033 \quad n = 225 \quad (Full \ Sample) \]

\[ \Delta \gamma_t = -0.073 \cdot \Delta \gamma_{t-1} - 0.06 \cdot \Delta s_{t-1} + \text{time dummies} \]
\[ (t = -0.6) \quad (t = -1.0) \]  \[ Root \ MSE = 0.032 \quad n = 225 \quad (Full \ Sample) \]

Carol and Weil [1994] infer a similar verdict from their estimations as Blomström et al [1993]: "If there is any causality running from saving to growth, it is with a negative sign. This result is inconsistent with the common view that the reason cross-country regressions show a positive association between saving and growth is that high saving produces high growth via the mechanical link from saving to capital and capital to output" (p. 147). Yet it then is somehow surprising to read in a footnote "(..) we should emphasize that, despite the results we both still believe that an exogenous increase in the saving rate would lead to an increase in economic growth. The argument here is only that the observed pattern of data could not have been generated by a neoclassical model in which the primary shocks were exogenous changes in the saving rate" (p. 149).

Andrés et al [1996] follow a different and more fancy way to come to a similar conclusion. These authors start off from the 'conditional convergence' property of neoclassical growth models. Indeed, if one approximates the growth in the neighborhood of the long-run equilibrium (by means of a log-linearization evaluated at the steady state, see e.g. Mankiw, Romer and Weil [1992] or Barro and Sala-I-Martin [1995]), the cumulative growth of per capita income between time \( t_0 \) and \( t \) can be written as:
in which $y_s$ is the expression for the steady state level of per capita income — consisting of a weighted sum of accumulation rates in logs (investment, schooling, R&D and population growth as in e.g. Mankiw, Romer and Weil [1992] or Nonneman and Vanhoudt [1996]) — $y_0$ stands for the initial per capita income, and $\beta$ is the speed of convergence. They then view the convergence hypothesis as an adjustment process around a cointegration relationship, and the convergence equation in (4) as a non-fully specified error correction model — generalized to allow for adjustment costs and lags as:

$$\ln\left(\frac{y_t}{y_0}\right) = \beta\left[\ln(y_s) - \ln(y_0)\right] + \sum_i a_i \cdot \ln\left(\frac{y_{t-1}}{y_{t-i}}\right) + \sum_i b_i \cdot \Delta \ln(y_{t-i}) + u_t.$$

According to the authors, in this interpretation current accumulation rates should improve the forecast of future growth rates, based on the past history of economic growth. The authors report they fail to find support for this hypothesis for OECD data, conclude that their results provide strong evidence against the error correction model implied by the dynamics of the canonical exogenous growth models, and that endogenous growth models are preferable. Like in the previously mentioned studies — the crucial assumption still is that there should be positive Granger causality running from accumulation rates (investment shares in physical and human capital) to growth in neoclassical models.

III. CAUSALITY IN THE NEOCLASSICAL AND AK MODELS

A first natural question to ask is whether the neoclassical models theoretically indeed possess the Granger causality property as suggested and interpreted by these studies — namely that an increase in investment (and other accumulation rates) unequivocally precedes higher growth. In what follows we will show it has not.

Exogenous neoclassical growth theories imply that — in the long run — there is only a level-effect, and no growth-effect whatsoever, resulting from changes in accumulation rates. If the economy is on its balanced growth path and there is a shock — say an increase in the saving rate — the economy will experience immediately a boost of its growth rate. This growth rate will diminish gradually over time. In the long-run the economy will grow at exactly the same rate as in the original equilibrium (namely at the rate of exogenous technological change). Any recent standard textbook on macroeconomics will show this graphically.

As for causality, this means that there should not be causality in the Granger sense between changes in accumulation rates and long-run growth rates.

Nonetheless, there might be Granger causality between growth and accumulation rates in the medium-run. This causality can, however, either be positive or negative. To see this, consider the textbook exogenous Solow model and let us start off from a country which — for the sake of generality — is not in its steady state in the medium run. Chart 1 illustrates graphically.

<insert Chart 1>
The country is experiencing a certain non-steady state growth rate, denoted by $\gamma_0$, which belongs to the capital-labor ratio $k_0$ and is determined by the saving rate $s_0$, the growth rate of the working force, technology and depreciation $(n+x+\delta)$. The growth rate is given by the difference between the straight line $(n+x+\delta)$ and the downward sloping convex curve $s.f(k)/k$. Suppose now that there is an increase in the saving rate to $s_1$ while other things remain constant. Obviously, the growth rate will boost immediately, after which the economy slides down towards the new steady state. During this transition process the growth rate will eventually—at some time $\bar{t}$ and for some associated capital-labor ratio $\bar{k}$—be equally high as in the starting position viz. before the increase in the saving rate. Following the textbook Solow model, we can find this capital labor ratio by equating the expressions for the growth rates under the two investment regimes:

$$\gamma_0 = s_0 \cdot k_0^{(1-\alpha)} \cdot (n+x+\delta) = s_1 \cdot \bar{k}^{(1-\alpha)} \cdot (n+x+\delta)$$

from which follows:

$$\bar{k} = \left[ \frac{s_0}{s_1} \right]^{-\frac{1}{1-\alpha}} \cdot k_0.$$  

Accordingly the time period at which the growth rates are equal under both investment regimes, is endogenous as well. This will be shown next. The importance for granger causality tests of this all will be discussed thereafter.

We can solve the question how long it would have taken an economy to converge—under the new investment regime $s_1$—to this critical capital labor ratio $\bar{k}$ starting off from $k_0$. This can easily be calculated by using the time-path of $k$. Since we are assuming a Cobb-Douglas production function ($y = k^\alpha$), it is possible to get a closed-form expression for the exact time path of $k$ (see e.g. Barro and Sala-I-Martin [1995]):

$$k_t^{1-\alpha} = \frac{s_1}{n+x+\delta} + \left[ k_0^{1-\alpha} - \frac{s_1}{n+x+\delta} \right] \cdot e^{-(1-\alpha)(n+x+\delta)t}.$$  

Substituting the right-hand side of (7) for $k_t$ in (8) and taking logs afterwards yields the corresponding critical time value $\bar{t}$:

$$\bar{t} = -\frac{1}{(1-\alpha)(n+x+\delta)} \cdot \ln \left[ \frac{s_0}{s_1} \right]^{-\frac{1}{1-\alpha}} \cdot \frac{k_0^{1-\alpha} - \frac{s_1}{n+x+\delta}}{k_0^{1-\alpha} - \frac{s_1}{n+x+\delta}}$$

$$\bar{t} = -\frac{1}{(1-\alpha)(n+x+\delta)} \cdot \ln \left[ \frac{s_0}{s_1} \right]^{-\frac{1}{1-\alpha}} \cdot \frac{\frac{s_1}{y_0^{1-\alpha}} - \frac{s_1}{n+x+\delta}}{\frac{s_0}{y_0^{1-\alpha}} - \frac{s_1}{n+x+\delta}}.$$
Recall that diluting business cycle effects is usually done by taking averages over a certain period. The above ideas can easily be extended towards averaging the growth rate and investment share over m periods. Equation (6) then becomes:

\[ \gamma_0 = \left( \frac{1}{m} \sum_{i=1}^{m} s_{i+1} \right) \left( \frac{1}{m} \sum_{i=1}^{m} k_{i+1} \right)^{(1-\alpha)} - (n + x + \delta) \]

\[ = \left( \frac{1}{m} \sum_{i=1}^{m} s_{i+1} \right) \cdot k^{-\alpha} - (n + x + \delta) \]

from which

\[ \bar{k} = \left[ \left( \frac{\sum_{i=1}^{m} s_{i+1}}{\sum_{i=1}^{m} s_{i+1}} \right) \left( \frac{1}{m} \sum_{i=1}^{m} k_{i+1} \right)^{(1-\alpha)} \right]^{-\frac{1}{\alpha}} \]

which can be substituted into equation (8) to obtain:

\[ \tilde{i} = - \frac{1}{(1-\alpha)(n+x+\delta)} \ln \left[ \left( \frac{\sum_{i=1}^{n} s_{i+1}}{\sum_{i=1}^{n} s_{i+1}} \right) \left( \frac{1}{m} \sum_{i=1}^{m} k_{i+1} \right)^{(1-\alpha)} \right]^{-1} \]

\[ \tilde{i} = - \frac{1}{(1-\alpha)(n+x+\delta)} \ln \left[ \left( \frac{\sum_{i=1}^{n} s_{i+1}}{\sum_{i=1}^{n} s_{i+1}} \right) \left( \frac{1}{m} \sum_{i=1}^{m} y_{i+1} \right)^{(1-\alpha)} \right]^{-1} \]

So what?
In order to see the importance for Granger causality (i.e. precedence in time) note that to the left of the capital stock $\bar{k}$—viz. in the interval $[t, \bar{t}]$—(average) growth rates are higher than the initial one. To the right, the opposite holds. Henceforth, from a Granger-causality point of view, the neoclassical model theoretically implies that an increase in the investment share precedes higher growth rates to the left of $\bar{k}$, but that it leads to lower growth rates to the right. The idea that there should be at any time positive Granger causality running from accumulation rates to growth in neoclassical models thus is a fallacy. The correct inference from this type of growth models is that there might be positive or negative Granger causality in the medium run—depending on the considered time interval (compared to the critical time interval)—while there shouldn’t be causality in the long-run. None of the reported recent causality tests take this issue into account.

Of course, the point made in this paper can be mitigated if there would be evidence supporting the fact that the critical values would be rather high—if it would take a country, say, some 25 years before it converges to the pre-shock growth rate, we would indeed expect a very high probability of finding positive Granger-causality based on 5-year panel data. Using some plausible values for the variables reveals, however, it turns out there is no such support. Let us for instance follow Mankiw, Romer and Weil [1992]—among many others—by assuming one third for $n$ and six percent for $(n+x+\delta)$. Let us also approximate the investment regimes over the periods [1966-70], [1971-75], [1976-80], [1981-85] and [1986-90] by the average investment shares over those time intervals. We can now compute how long it would have taken a country to converge under the new investment regime (e.g. [1981-85]) from its original per capita income (e.g. 1980)—which was achieved under the old investment regime (e.g. [1976-80])—to the per capita income for which the growth rates under both investment regimes were exactly the same—other things being equal and in the absence of new shocks. Table I shows such calculations for some arbitrary chosen countries: Belgium, Germany, Japan, the UK and the USA, based on equations (9) or (12) with $m=5$.

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2 Data are taken from to the Penn-World Table mark 5.6
Table 1: \( t \) for five countries, under the textbook Solow assumptions

<table>
<thead>
<tr>
<th>Country</th>
<th>( y_0 ) (USD)</th>
<th>( i_0 \rightarrow i_t ) (%)</th>
<th>( t ) based on equation (9) USA (years)</th>
<th>( t ) based on equation (12) USA (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>12,963</td>
<td>22.18 → 21.42</td>
<td>0.87</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>13,682</td>
<td>21.42 → 21.46</td>
<td>0.05</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>15,295</td>
<td>21.46 → 21.04</td>
<td>0.49</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>16,570</td>
<td>21.04 → 21.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>JAPAN</td>
<td>7,307</td>
<td>35.62 → 38.32</td>
<td>1.83</td>
<td>2.30</td>
</tr>
<tr>
<td></td>
<td>8,381</td>
<td>38.32 → 34.94</td>
<td>2.30</td>
<td>2.26</td>
</tr>
<tr>
<td></td>
<td>10,072</td>
<td>34.94 → 32.00</td>
<td>2.20</td>
<td>2.13</td>
</tr>
<tr>
<td></td>
<td>11,771</td>
<td>32.00 → 35.64</td>
<td>2.69</td>
<td>2.74</td>
</tr>
<tr>
<td>BELGIUM</td>
<td>8,331</td>
<td>26.80 → 25.48</td>
<td>1.26</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>9,633</td>
<td>25.48 → 24.54</td>
<td>0.94</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>11,109</td>
<td>24.54 → 18.76</td>
<td>6.71</td>
<td>6.69</td>
</tr>
<tr>
<td></td>
<td>12,885</td>
<td>18.76 → 21.86</td>
<td>3.82</td>
<td>3.83</td>
</tr>
<tr>
<td>GERMANY</td>
<td>9,425</td>
<td>30.58 → 29.02</td>
<td>1.31</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>10,094</td>
<td>29.02 → 26.90</td>
<td>1.90</td>
<td>1.88</td>
</tr>
<tr>
<td></td>
<td>11,920</td>
<td>26.09 → 24.06</td>
<td>2.79</td>
<td>2.75</td>
</tr>
<tr>
<td></td>
<td>12,535</td>
<td>24.06 → 24.52</td>
<td>0.47</td>
<td>0.49</td>
</tr>
<tr>
<td>UK</td>
<td>8,537</td>
<td>19.78 → 19.04</td>
<td>0.95</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>9,312</td>
<td>19.04 → 17.78</td>
<td>1.71</td>
<td>1.68</td>
</tr>
<tr>
<td></td>
<td>10,167</td>
<td>17.78 → 16.06</td>
<td>2.54</td>
<td>2.52</td>
</tr>
<tr>
<td></td>
<td>11,237</td>
<td>16.06 → 18.54</td>
<td>3.59</td>
<td>3.63</td>
</tr>
</tbody>
</table>

Apparently, countries strongly differ with respect to this ‘critical’ times which are clearly rather short. Moreover they are typically not constant over time. This is of course 1. because the changes in investment shares differ in magnitude, and, perhaps more importantly, 2. because the distance to the steady state changes with changes in the investment share. This distance endogenously determines the speed of convergence (see Barro and Sala-I-Martin [1995]), and thus the interval \([t_0, t]\).

Note that new growth theories of the Ak-type have a totally different implication. In those theories, an increase in the saving rate will result in a permanent increase in the growth rate as illustrated in graph 2 (see Barro and Sala-I-Martin [1995]). These new-growth models thus clearly suggest that changes in the growth rate can only be due to preceding changes in accumulation rates, other things being equal.

<insert Chart 2>

Henceforth, results of the kind reported in recent causality studies (based on equations (1)–(4)) cannot be used to dismiss the validity of the neoclassical model. If they reveal anything, it rather is that the data do not support the causality implied by the new growth-Ak-theory. This reconciles with reported time series tests of the latter theories (e.g. Jones [1995]).
IV. CONCLUSION

In this paper we suggested that recent Granger causality research on economic growth and accumulation rates dismisses the validity of neoclassical growth models based on a fallacy. We showed that this kind of growth theory implies both positive and negative Granger causality between medium-run growth rates and investment shares, depending on the considered time interval. There is no Granger causality implied in the model between these variables in the long-run. Contrary to the conclusions of, and based on the estimation results in, recent causality research there is therefore no reason to reject the mechanical link between capital accumulation and growth which is present in this neoclassical approach, on the contrary. If one interprets the empirical results seriously, the conclusion would rather be that the 'new' growth theories of the Ak-type are rejected by the data based on their causality implications. This reconciles with earlier reported time series tests of the latter theories.
V. REFERENCES


VI. CHARTS

Chart 1

Chart 2
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