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Redistribution and friends : on bilaterally incentive compatible tax schemes

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Abstract When side market trading is perfect, it is well known that only linear taxation of retradeable commodities survives attempts to arbitrage. The conditions for perfect side markets are, however, rather strong. In this paper, I discuss tax schemes when side trading is imperfect in the sense that commodities can only be re-exchanged within coalitions no larger than two people. The framework is that of a two-class economy. Coalitions which might have an incentive to form are identified and necessary conditions for the related incentive constraints to be binding are provided. Next, the Pareto efficient tax scheme is characterised. The formula for the optimal marginal tax rate turns out to be of a very simple form and strongly resembling the standard tax formula. Finally, a numerical exercise shows that the constraints imposed on policy by an imperfect side trading process are almost as tough as those imposed by perfect side trading.

1 Introduction

In modern welfare states, a substantial part of a citizen's tax liability is made contingent on the transactions which he or she undertakes with the production sector. In designing such tax schemes, policy makers have to compromise their desire to redistribute income with the distortions in the decisions of consumers and workers which those schemes bring about. In addition to transacting with the production sector, for some types of commodities consumers may be able to carry out trades among themselves, on formal but usually on less formal side markets. If this is the case, the design of tax schemes will be further constrained. Indeed, Guesnerie (1981, 1995 ch 1) and Hammond (1979) have shown that the existence of perfectly competitive side markets compels the government to reserve non-linear tax treatments to commodities which cannot be retraded. A similar result was obtained by Hammond (1987), who showed that if taxpayers are able to form coalitions of any finite size, the only feasible tax schemes on retradeable commodities are linear ones. Thus, the inability to monitor side trades, either within coalitions or on perfectly competitive side markets provides an explanation why some commodities should be taxed linearly and others non-linearly.

One can argue, however, that the perfect side trading processes considered by Guesnerie and Hammond are an extreme form of market arbitrage which one is not likely to encounter in real world economies. In the co-operative approach to side trading which Hammond takes, perfection requires that agents can form coalitions of any finite size, that they can make binding agreements on how they should co-ordinate their transactions with the production sector and on how to share the ensuing resources of the coalition among its members, and that these transactions and sharing take place in an efficient and individually rational way. These are very demanding conditions, for several reasons. Large coalitions will have problems in reaching a binding agreement, especially because formal enforcement mechanisms cannot be relied upon. In addition, if the members of potential coalitions differ in terms of endowments, preferences, needs,..., then asymmetry of information will typically prevent achieving an efficient and individually rational trade and sharing outcome (see e.g. Myerson & Satterthwaithe, 1983). Similar asymmetries will hinder an efficient outcome on the side markets which Guesnerie (1981) and Hammond (1979) consider.[1] In the worst case, side markets might fail to open, as Akerlof (1970) has convincingly argued.

The purpose of this paper is to analyse Pareto efficient tax policies when side market trading is possible but not perfect. The framework is that of a two class economy with wealthy and poor people, and side trading is taken to proceed in a co-operative way, as

[1] See, e.g. the discussions by Guesnerie (1992) and Stiglitz (1991).

in Hammond (1987) but with the restriction that coalitions cannot consist of more than two members. One can think of these two members as two friends, such that trust and reputation will function well as enforcement mechanisms, and informational imperfections are absent. In this framework, coalitions can be distinguished on their members' types as well as on the announcement these members make about their type to the government. A first result is that, besides a no-arbitrage condition, coalitions with one and only one honest member are the only ones with a potential interest to form (result 1). In that case, the standard (i.e. individual) incentive compatibility constraints will always be slack (result 2). Next, I derive necessary conditions on the tax scheme for a coalition with a certain type of honest member to have an incentive to form (result 3), and on preferences for a coalition with a certain type of dishonest member to form (result 4). Because these conditions are mutually exclusive, it is easy to identify which coalition will constrain the Pareto efficient redistributive tax policy. In fact, I will show that if compensated price effects (in absolute value) are positively related to income, it is the possibility of two rich people to collude rather than that of mixed coalitions, which will constrain the optimal policy. The characterisation formulae for this policy turns out to be of a very simple form, with close resemblance to those obtained in standard tax models (as e.g. in Stilglitz, 1982) and allowing for a tranparent cost-benefit interpretation (result 5).

One could express the hope that imperfections in side trade processes would mitigate the trade-off between redistribution and distortions--that the conclusions of Guesnerie and Hammond were too pessimistic. By means of a numerical example, I show that there is not much reason for such hope. This example shows that the amount of resources lost when the government would wrongly consider the side market mechanism as perfect, is very small. That is, the burden placed on the economy by side trading in the first place derives from bilateral coalition formation. Coalitions of higher order add very little to this burden.

In the next section the model is presented and the reader is briefly reminded of the optimal tax policy when side trade opportunities are fully absent. Section 3 allows for the possibility of bilateral coalition formation, identifies the coalitions which have an incentive to form and provides a set of mutually exclusive necessary conditions on the tax policy and on preferences for such formation to occur. The optimal bilaterally incentive compatible tax scheme is derived in section 4. Section 5 presents the numerical example, while concluding remarks are made in section 6.

2 The model

The economy which I consider has a large number of citizens. These people care about the consumption of two goods, x and y, and their (identical) preferences over these goods are representable by a monotonous and strictly concave utility function, $u(x,y)$ over the same feasible consumption set G. While every agent is endowed with e_x units of good x, the endowment of good y is unequally distributed: 50% of the agents (class 1) are endowed with only e_y^1, the other 50% (class 2) own $e_y^2 (> e_y^1)$. In the production sector both commodities can be transformed into one another at a constant marginal rate of transformation, the value of which is normalised to unity. If commodity y is selected as the *numéraire*, this means that the producer price of commodity x--and therefore its marginal social value--will be unity as well.

The net demand bundle by an agent of type i (=1,2) to the production sector will be denoted as $t^i = (t_x^i, t_y^i)$. Being endowed with a bundle $e^i = (e_x, e_y^i)$, this agent's final consumption bundle will be $t^i + e^i$. Feasibility of trade requires that $t^i \in F^i =_{\text{def}} G - \{e^i\}$.

An agent of type i's marginal willingness to pay for commodity x will be denoted by

$$MRS^i =_{\text{def}} -\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{\mathrm{d}u^i=0} = \frac{u_x(t_x^i + e_x, t_y^i + e_y^i)}{u_y(t_x^i + e_x, t_y^i + e_y^i)},$$

where subscripts denote partial derivatives. To place some further structure on preferences, I assume that both commodities are normal goods. Normality of x is equivalent to a marginal willingness to pay for this good which increases with the amount of y available. Consequently, the indifference curve of a wealthy endowed agent through any point in the (t_x, t_y)-space will always be steeper than that of a poorly endowed agent through the same point. This is the single crossing property, illustrated in figure 1.

--insert figure 1 here--

Without government intervention, every agent's budget set is given by $B = \{(t_x, t_y) \mid t_x + t_y = 0\}$. The competitive equilibrium in this economy will be reached when the conditions $MRS^i = 1$ and $t_x^i = -t_y^i$ hold for every person.

The government, knowing only the distribution of citizens over types, wants to improve upon the welfare of poor citizens by taxing the transactions with the production sector. By the revelation principle, the choice of such a tax policy is equivalent to the construction of an incentive compatible direct mechanism which allocates net trades to agents upon announcement of their type. The *individually* incentive compatibility (IIC) constraints impose that each agent i (i=1,2), when acting on her own, should have no incentive to disguise her type, i.e.

$$u(t_x^i + e_x, t_y^i + e_y^i) \geq u(t_x^j + e_x, t_y^j + e_y^i). \qquad (\lambda_i)$$

If the government wants to bring the poor type's living standard up to the level $\bar{u}^1$, the mechanism should also respect the constraint that

$$u(t_x^1 + e_x, t_y^1 + e_y^1) \geq \bar{u}^1. \qquad (\mu)$$

Furthermore, net trades should satisfy the resource constraint:

$$t_x^1 + t_y^1 + t_x^2 + t_y^2 \leq 0. \qquad (\gamma)$$

Ignoring a pooling equilibrium, the IIC constraints (λ_i) together with the single crossing property, impose that $t_x^2 > t_x^1$, and $t_y^1 > t_y^2$. Moreover, if redistribution takes place from class 2 to class 1, the net trades should satisfy the inequality $\frac{t_y^1 - t_y^2}{t_x^2 - t_x^1} > 1$.[2] The properties of an efficient IIC trade mechanism can be revealed by solving the following planning problem:

$$\max_{\{t_x^i, t_y^i\}} u(t_x^2 + e_x, t_y^2 + e_y^2)$$
$$\text{s.t.} \quad (\mu), (\lambda_1), (\lambda_2), \text{ and } (\gamma).$$

Because the IIC constraint (λ_1) will never be binding at the optimum, the main characterisation results of the optimal tax policy can be summarised as follows: rich persons face a zero marginal tax rate but a positive average rate, while poor persons are taxed at the margin of their net trade in x, but are net transfer receivers. Using the superscript '$i(j)$' to denote that an expression is evaluated when agent i announces to be of type j (e.g. $g^{i(j)} = g(e_x + t_x^j, e_y^i + t_y^j)$, these results can be written as

[2] Suppose not, then $t_y^1 + t_x^1 \leq t_x^2 + t_y^2$ or, using the resource constraint, $2(t_y^1 + t_x^1) \leq 0$. But then agent 1 is a net taxpayer, contradicting the assumption that redistribution goes from rich to poor.

$$MRS^{2(2)} = 1, \text{ and } MRS^{1(1)} - 1 = \frac{\lambda_2 u_y^{2(1)}}{\gamma}(MRS^{2(1)} - MRS^{1(1)}) > 0 \qquad (1)$$

where the inequality follows from single crossing: a rich person disguising as poor will have a higher marginal willingness to pay for commodity x than the poor person and this makes it efficient to tax the poor agent's net trade in this commodity at the margin (alternatively, to subsidise her sales at the margin). If the ambitions to redistribute are modest, the first best redistribution scheme will still be IIC ($\lambda_2=0$) and there is no need for distortionary taxation of poor people. These results are all very standard.

Suppose now that agents can trade both commodities with one another. To which extent is the second best tax scheme still incentive compatible? Clearly, if redistribution is sufficiently high such that $\lambda_2 > 0$, both types of agents face a different relative price for commodity x and will engage in a mutual improving side trade where agent 1 exchanges commodity y in return for commodity x. But such trading opportunities should not worry the planner too much since they make both agents better of when telling the truth. Much more worrying is the existence of Pareto improving side trades involving an untruthful agent. It is easy to see why. Consider the largest possible IIC lump sum redistribution scheme: the IIC constraint for agent 2 is binding, but only just, so that $\lambda_2=0$. If agent 2 reveals her identity, she will receive a trade bundle for which her *MRS* equals 1 and providing her with a utility level which she will be unable to improve upon by side trading. However, if she decides to disguise as a poor person, she is allocated a trade bundle which provides her with exactly the same utility level, but for which her *MRS* is larger than 1. By engaging *ex post* in a side trade with agent 1 (whose *MRS* equals 1) she will be able to raise her utility level above the level when telling the truth; the IIC redistribution scheme will no longer be incentive compatible. And by a continuity argument, the same conclusion applies to IIC first best schemes of which the redistributional aims are smaller but still in the neighbourhood of the one just considered.

In the next sections, I will consider other possible coalitions and give a complete characterisation of Pareto efficient tax schemes which survive collusive behaviour.

3 Bilaterally incentive compatible tax schemes

From now on, I will assume that agents can form small coalitions with one another without this being observed by the government. That is, agents can get together and commit themselves to an agreement that specifies which announcement strategy each of them will play, and how after the assignment by the trade mechanism of the trade bundles to the agents, the ensuing intermediate endowment is shared among them.

The type of coalitions I have in mind are small groups of people who know each other sufficiently well and who are socially related to one another in a way which fosters trust and reputation. A 'stylised' coalition would thus be one of two befriended people.[3] There are several reason for limiting myself to bilateral coalitions of this type. First, because I shall use what Tirole (1992, p 154-6) calls the "enforceable side contracts" approach and not go into modelling the mechanism by which agents are able to make a binding agreement. Because such agreements will thwart the redistribution efforts of the government, they will not be enforceable in front of the courtroom; mechanisms like trust and reputation then become very important in backing such agreements. A second reason is that I want the final allocation of the coalition's endowment across its members to be both Pareto efficient and individually rational. As Myerson & Satterthwaite (1983) have shown, imperfect information between two traders will typically rule out trades which share these two properties. Predicting the final allocation in the coalition would then ask for a description of the bargaining process and the relative bargaining strength of the coalition members. With perfect information, no such description is required.

Consider then the coalition consisting of one person of type i and another person of type j : $\{i,j\}$, $(i,j=1,2)$. A strategy of this coalition consists of (I) an announcement strategy $\{a,b\}$ $(a,b=1,2)$[4], and (II) a side trade strategy $s =_{\text{def}} (s^i,s^j)$--where $s^h =_{\text{def}} (s^h_x, s^h_y)$ denotes the net side trade bundle for agent $h=i,j$--which is both individually and bilaterally feasible: $t^a+s^i \in F^i$, $t^b+s^j \in F^j$ and $s^i+s^j \leq 0$. The allocation mechanism $\{t^1,t^2\}$ is then *bilaterally* incentive compatible (BIC) when for no coalition $\{i,j\}$ there exists a feasible strategy $(\{a,b\},s)$ such that

$$u(e_x + t^a_x + s^i_x, e^i_y + t^a_y + s^i_y) \geq u(e_x + t^i_x, e^i_y + t^i_y)$$
$$u(e_x + t^b_x + s^j_x, e^j_y + t^b_y + s^j_y) \geq u(e_x + t^j_x, e^j_y + t^j_y)$$

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[3] In addition to trust, such relationships will also arouse altruistic feelings. In the context of redistributional efforts by the government, it then becomes important to distinguish between intra and inter class friendships. In the latter case the need for such efforts would obviously be lessened, whether side market trading is possible or not. If, on the other hand, circles of friends consist primarily of people of the same social class, as sociologists argue, altruism would not change the need for redistribution across classes, but it might affect the need for social insurance since people will be prepared give financial assistance if one of their friends is temporarily hit by an adverse income or employment shock.

[4] Alternatively, one can think of the coalition members, facing a tax schedule, of agreeing on a trading strategy with the production sector.

with at least one inequality holding strictly (Hammond, 1987, p 402). With two types of agents, three types of coalitions can form, each with four possible announcement strategies. These 12 combinations are listed in table 1.

Table 1 Potential coalitions and their announcement strategies.

Type of coalition	Announcement strategy (·)
Poor coalition: {1,1}	{1(1),1(1)} {1(2),1(1)} or {1(1),1(2)} {1(2),1(2)}
Mixed coalition: {1,2}	{1(1),2(1)} {1(1),2(2)} {1(2),2(1)} {1(2),2(2)}
Rich coalition: {2,2}	{2(1),2(1)} {2(1),2(2)} or {2(2),2(1)} {2(2),2(2)}

i(*j*): agent *i* announcing to be of type *j*.

A number of combinations can be eliminated at first sight. When the government does not make use of random taxation, the truthful poor and rich coalitions ({1(1),1(1)} and {2(2),2(2)}) will not be able to engage in a Pareto improving side trade since the marginal rate of substitution of their members are equal. Also the deceitful poor and rich coalitions ({1(2),1(2)} and {2(1),2(1)}) will have no incentive to form if the IIC constraints are verified. This leaves us with six BIC constraints to consider.

With a strictly concave utility function, the BIC constraint preventing the truthful mixed coalition ({1(1),2(2)}) to effectuate a mutually improving side trade may be formulated as an equality of the marginal rates of substitution of the two types. Consider then the case where the members of the mixed coalition permute their types in the announcement ({1(2),2(1)}). This coalition will end up with the same aggregate endowment as when making truthful announcements. By IIC and if the government undertakes any redistribution, at least one of the members will be worse off than when acting on her own (at least agent 1). But since the side trading process has to be individually rational, this process will bring both agents back on their original indifference curves (and with strictly convex preferences back to the autarky point). The mixed coalition will therefore always be indifferent between announcing types truthfully or in a permuted way.

This leaves us with four coalitions each consisting of one and only one honest member: a poor coalition pretending to be half rich ({1(2),1(1)}), a mixed coalition pretending to be rich ({1(2),2(2)}) or to be poor ({1(1),2(1)}), and the rich coalition pretending to be half poor ({2(1),2(2)}). In the former two cases, the poor agent pretending to be rich will become initially worse off since she will be attributed the less favourable tax treatment destined for the rich. On the other hand, the subsequent side trade with the other coalition member will increase her utility level. In the latter two cases, the dissembling rich agent will loose to the extent that the IIC constraint (λ_2) is not binding, but will gain during the ensuing side trade process.

Result 1: *For coalition {1(1),2(2)} not to have an incentive to form, the two types should have the same marginal rates of substitution. Other coalitions with a potential interest to form consist of one and only one honest member.*

In the remaining of this section, I will present necessary conditions for any of the dishonest coalitions to thwart the tax scheme. In the following section, I will make use of these conditions to elicit the BIC constraint which will constrain the optimal redistributive policy and to derive the properties of such policy.

The characterisation of the feasible set of official trades is summarised in the next three results. The first one gives the relation between the IIC and BIC constraints. The next two results provide mutually exclusive necessary conditions for the BIC constraints to be binding.

Result 2: *If $\{t^1,t^2\}$ is an incentive compatible allocation, then the IIC constraints will not be binding.*

Result 2 conveys the very intuitive idea that if an agent is unable to exploit the tax scheme with the help of another person, she will certainly not be able to do so when acting on her own. Indeed, suppose that we have an allocation which is both IIC and BIC, and that one of the IIC constraints, (λ_i) say, is binding. In that case the coalition $\{i(j),j(j)\}$ ($j \neq i$) can form, and even before any side trade occurs these coalition members are on their reservation utility levels. But since $MRS^{i(j)} \neq MRS^{j(j)}$, Pareto improving side trades can take place.

Result 3: *Suppose $\{t^1,t^2\}$ is an incentive compatible allocation and let MRS denote the common marginal rate of substitution. Then a necessary condition for the BIC constraint related to a coalition with a dishonest rich (poor) person to be binding is that $\frac{t_y^1-t_y^2}{t_x^2-t_x^1} > MRS$ ($\frac{t_y^1-t_y^2}{t_x^2-t_x^1} < MRS$).*

Result 3 provides necessary conditions under which side trading opportunities will sufficiently compensate a deceitful person to make her as well of as when acting truthfully on her own. Its proof is given in the paper's appendix, but figure 2 below provides the graphical intuition. This figure shows the Edgeworth box for the truthful rich or mixed coalition. Let the lower left corner be the origin for either agent 1 (in the mixed coalition case) or agent 2 (in the rich coalition case), and let the upper right corner correspond to the origin for the other coalition member, an agent of type 2. Both agents have revealed their types and are provided by the mechanism the allocation A where their indifference curve are tangent to one another (with slope *MRS*). Suppose now that the second coalition member--the rich person--announces to be of type 1. She will then receive the trade bundle (t_x^1, t_y^1) which decreases her endowment of commodity x and increases that of commodity y, since $t_x^2 > t_x^1$, $t_y^1 > t_y^2$.. If $\frac{t_y^1 - t_y^2}{t_x^2 - t_x^1} >$ *MRS*, this rich person will end up in a point like D, providing a utility level $u^{2(1)}$ in stead of u^2. Because the aggregate dimensions of the Edgeworth box are no longer valid, it needs adjustment. This is carried out by shifting the upper right origin, and everything which is measured w.r.t. it, upwards in the direction indicated by the arrow DA. The indifference curves are now the dashed lines. Mutually advantageous side trade opportunities arise and it is clear that they can provide the dissembling rich agent with at least the utility level u^2 without making the other coalition member worse off (the shaded area). On the other hand, if $\frac{t_y^1 - t_y^2}{t_x^2 - t_x^1} \leq$*MRS*, D would be situated somewhere above the tangency line through A (e.g. point D'), and the original indifference curve with utility level u^2 would be projected *away* from the other coalition member's status quo indifference curve. A similar picture would show that the condition $\frac{t_y^1 - t_y^2}{t_x^2 - t_x^1} \geq$*MRS* rules out that a poor agent of a poor or mixed coalition can offset the initial utility loss when dissembling without making the other coalition member strictly worse off.

--insert figure 2 here--

Before discussing the third result, it is helpful to define the curvature of an indifference curve:

$$c^i =_{\text{def}} -\left.\frac{\mathrm{d}MRS^i}{\mathrm{d}x}\right|_{\mathrm{d}u^i=0} = -\left(\frac{\partial MRS^i}{\partial x} - \frac{\partial MRS^i}{\partial y} MRS^i\right).$$

This stricIty positive number measures by how much the *MRS* of an agent falls if we consider a small move along the indifference curve in South-East direction. Above, it was mentioned that for a trade mechanism to verify the BIC constraint related to the honest mixed coalition {1(1),2(2)}, both agents should receive a trade bundle such that their *MRS*s are equalised. This means that $x^2 > x^1$ and $y^2 > y^1$.[5] We can now distinguish between two situations, depending on whether the poor agent's indifference curve through the bundle (x^1, y^1) displays more or less curvature than the rich agent's indifference curve through bundle (x^2, y^2), as depicted in figures 3a and 3b, respectively.

--insert figures 3a and 3b here--

From now on, I will assume that the curvature of indifference curves at bundles on the same income expansion path (i.e. for which the indifference curves have identical slope) either increases or decreases monotonously if we move along this income expansion path. A third characterisation result is then the following:

Result 4: *Suppose that* $\{t^1,t^2\}$ *is an incentive compatible allocation. If the BIC constraint related to a coalition with an honest rich (poor) person is binding, then the curvature of indifference curves must fall (rise) with income along the same income expansion path.*

Result 4 can again easily be shown on the Edgeworth box. In figure 4a, the Edgeworth box portrays the allocation of resources granted to the coalition {2,2} upon having pretended to be {2,1}. The side process takes the members from A to B, after which both are back on their utility level when they act on their own (u^2), so the BIC constraint related to the coalition {2(1),2(2)} is binding. Suppose now, contrary to what result 4 asserts, that the curvature increases with income, and let us investigate the side trade opportunities when the dishonest rich person chooses an honest *poor* person as her coalition partner. For this purpose, I have added the poor person's indifference curve for utility level $\overline{u}^1$ to the picture as the dashed line (the lower left corner of the old Edgeworth box does not correspond to the origin of agent 1, but this is not important). Because in equilibrium rich and poor face the same marginal tax rate, in the new Edgeworth box the poor's indifference curve will be tangent to that of the honest rich in A. It then transpires that the allocation $\{t^1,t^2\}$ violates the BIC constraint w.r.t. coalition {1(1),2(1)}, which contradicts that this allocation is incentive compatible.

[5] $x^2 > x^1$ because $t_x^2 > t_x^1$, and because the two goods are normal, $MRS^1 = MRS^2$ implies that $y^2 > y^1$.

When the poor agent is the dishonest one, a similar argument can be constructed (see figure 4b), and the opposite arguments (to show that the curvature must rise when the BIC constraint w.r.t. a coalition with an honest poor person is binding) can be seen when regarding 1(1)'s indifference curve as the bold one 2(2)'s cuve as the dashed one.

--insert figures 4a and 4b here--

We have now at our disposal two different sets of necessary conditions for coalitions to form. The first set of conditions, discussed as result 3, allows to distinguish coalitions on the identity of the dishonest member: $\frac{t_y^1 - t_y^2}{t_x^2 - t_x^1} \geq (\leq) MRS$ rules out a coalition with a dishonest poor (rich) person. The second set of conditions, which result 4 refers to, enables to distinguish coalitions on the identity of their honest member: with falling curvature, the thwarting coalition cannot have an honest poor person, while with increasing curvature, it cannot have an honest rich person as a member. This information is summarised in table 2.

Table 2: Necessary conditions on the tax scheme and on preferences for dishonest coalitions to have an incentive to form.

	Curvature falls with income	Curvature rises with income
$\frac{t_y^1-t_y^2}{t_x^2-t_x^1} > MRS$	{2(2),2(1)}	{1(1),2(1)}
$\frac{t_y^1-t_y^2}{t_x^2-t_x^1} < MRS$	{1(2),2(2)}	{1(2),1(1)}

In the next section, I will first characterise the Pareto efficient tax policy when the BIC constraint w.r.t. either coalition with a dishonest rich person ({2(2),2(1)} or {1(1),2(1)}) is binding. Next, I will argue that if redistribution goes from rich to poor, the necessary condition for a coalition with a dishonest poor person ({1(2),2(2)} or {1(2),1(1)}) to constrain policy will never be satisfied.

4 Pareto efficient BIC tax rules

In characterising the optimal tax rules, I will distinguish between two cases, depending on whether the curvature coefficient falls or rises along the income expansion path. An assumption on the behaviour of this coefficient amounts to an ordinal restriction on the preference ordering. In terms of market behaviour, such an assumption imposes structure on the way compensated price effects change with income (or, more appropriately, with utility).[6] So it is related to the cross partial derivative of the Hicksian demand function, something on which standard convexity assumptions have little to say. I will first assume that the curvature coefficient falls as one moves along an income expansion path. This means that the compensated price effect is (in absolute value) stronger for a rich person than for a poor person, when these people face the same prices. To show that there is a large class of preferences for which the curvature coefficient falls, it is useful to relate this coefficient to the more conventional elasticity of substitution of preferences σ:[7] $c(x,y)=[y/x + MRS]/[\sigma(x,y)\cdot y]$. Writing $y(x,MRS)$ as the function whose graph is the income expansion path corresponding to the relative price MRS, a small movement along this path away from the origin yields the following effect on c:

$$\left.\frac{\mathrm{d}c}{\mathrm{d}x}\right|_{\mathrm{d}MRS=0} = -\frac{1}{\sigma\cdot y\cdot x}\left[\frac{x}{\sigma}\left.\frac{\mathrm{d}\sigma}{\mathrm{d}x}\right|_{\mathrm{d}MRS=0}\cdot\left(\frac{y}{x}+MRS\right)+\left(\frac{y}{x}+MRS\frac{\partial y}{\partial x}\frac{x}{y}\right)\right].$$

where the partial derivative $\partial y/\partial x$ is the slope of the expansion path, and is positive under normality of both commodities. Consequently, if the elasticity of substitution increases, remains constant (as with CES preferences) or does not decrease too fast along the income expansion path, the curvature measure c will fall as we consider indifference curves further away from the origin.

- *Declining curvature along the income expansion path*

With a declining curvature coefficient, of the two coalitions with a dissembling rich person, only coalition $\{2(1),2(2)\}$ may constrain policy. If this coalition forms, the maximal utility level which the dissembling first member can achieve without bringing the utility level of the honest second member below $u(t_x^2+e_x, t_y^2+e_y^2)$ is given by solving the following problem:

$$\begin{aligned}&\max_{\{s_x,s_y\}} v^{2(1)}(s_x,s_y)\\ &\text{s.t. } v^{2(2)}(-s_x,-s_y)\geq u(t_x^2+e_x,t_y^2+e_y^2) \quad (\alpha)\end{aligned}$$

[6] In a two commodity world, the curvature coefficient c is the inverse of the (own) Slutsky substitution effect of commodity x.

[7] $\sigma =_{\mathrm{def}} [\mathrm{d}MRS/MRS/\mathrm{d}(x/y)/(x/y)]^{-1}$ with $\mathrm{d}(x/y)$ taking place along the indifference curve.

where $v^{2(a)}(r_x,r_y) =_{\text{def}} u(t_x^a+e_x+r_x, t_y^a+e_y^2+r_y)$ (a=1,2) and α is the Lagrange coefficient related to the 'no worse off' constraint.[8] The necessary conditions for an interior solution $(\overset{*}{s}_x,\overset{*}{s}_y)$ to this problem are given by

$$mrs^{2(1)}(s_x^*,s_y^*) = mrs^{2(2)}(-s_x^*,-s_y^*), \text{ and } v_y^{2(1)}(s_x^*,s_y^*) = \alpha v_y^{2(2)}(-s_x^*,-s_y^*),$$

where $mrs^{2(a)}(\cdot)$ denotes the marginal rate of substitution of a rich agent when her intermediate endowment $(t_x^a+e_x, t_y^a+e_y^2)$ is complemented with the optimal side trade bundle. If the optimal value function of this problem is written as $\varphi^{2(1)}(t_x^1+e_x,t_y^1+e_y^2;t_x^2+e_x,t_y^2+e_y^2)$, the Envelope theorem ensures that

$$\frac{\partial\varphi^{2(1)}}{\partial t_x^1} = v_x^{2(1)},\ \frac{\partial\varphi^{2(1)}}{\partial t_y^1} = v_y^{2(1)},\ \frac{\partial\varphi^{2(1)}}{\partial t_x^2} = \alpha(v_x^{2(2)}-u_x^{2(2)}), \text{ and } \frac{\partial\varphi^{2(1)}}{\partial t_y^2} = \alpha(v_y^{2(1)}-u_y^{2(2)}).$$

The Pareto efficient BIC tax policy is then found by solving

$$\begin{array}{lll}
\max_{\{t_x^i,t_y^i\}} & u(t_x^2+e_x,t_y^2+e_y^2) & \\
\text{s.t.} & u(t_x^1+e_x,t_y^1+e_y^2) \geq \bar{u}^1 & (\mu) \\
& \varphi^{2(1)}(t_x^1+e_x,t_y^1+e_y^2;t_x^2+e_x,t_y^2+e_y^2) \leq u(t_x^2+e_x,t_y^2+e_y^2) & (\lambda_b) \\
& MRS(t_x^2+e_x,t_y^2+e_y^2) = MRS(t_x^1+e_x,t_y^1+e_y^1) & (\eta) \\
& t_x^1+t_y^1+t_x^2+t_y^2 \leq 0 & (\gamma).
\end{array}$$

Performing the operations foc(t_x^i) - foc(t_y^i) $\cdot$ $MRS^{i(i)}$, $(i = 1,2)$ and rearranging yields:

$$MRS^{1(1)} - 1 = \frac{\lambda_b v_y^{2(1)}}{\gamma}[mrs^{2(1)}(s_x^*,s_y^*) - MRS^{1(1)}] - \frac{\eta}{\gamma}c^1, \text{ and}$$

$$MRS^{2(2)} - 1 = \frac{\lambda_b \alpha v_y^{2(2)}}{\gamma}[mrs^{2(2)}(-s_x^*,-s_y^*) - MRS^{2(2)}] + \frac{\eta}{\gamma}c^2.$$

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[8] I am assuming here that the entire surplus of the coalition accrues to the dishonest person. Presumably, an equal split of the rent is more likely since the two members are identical. However, by imposing the allocation to be BIC, the existence of a positive surplus is ruled out. Therefore it does not matter how the surplus is split up.

Since (η) imposes that $MRS^{1(1)}$=$MRS^{2(2)}$ (=MRS, say), and because the optimal side trade ensures that $mrs^{1(1)}$=$mrs^{2(2)}$ (=mrs, say), and $v_y^{2(1)} = \alpha v_y^{2(2)}$, these two equations imply that η must be zero at an optimum, which means that the equalisation of marginal tax rates per se does not involve a social cost. Using this information in either of the two equations above yields the expression for the optimal (common) marginal tax rate:

$$MRS - 1 = \frac{\lambda_b v_y^{2(1)}}{\gamma}(mrs - MRS). \tag{2}$$

The sign of the RHS is positive, as can be seen as follows. After having announced to be of types {1,2}, but before engaging in side trading, the dissembling coalition member's marginal rate of substitution equals $mrs^{2(1)}(0,0)$ and the honest member it is $mrs^{2(2)}(0,0)$=$MRS^{2(2)}$ (=MRS). Since $t_x^2 > t_x^1$ and $t_y^1 > t_y^2$, 2(1) has less of commodity x but more of commodity y than 2(2), so that $mrs^{2(1)}(0,0)$> $mrs^{2(2)}(0,0)$. During the side trade process, 2(1) will buy x from 2(2) and sell y. This will increase the marginal willingness to pay for 2(2) and lower that of 2(1) up to the common level mrs. Consequently, mrs MRS.

Expression (2) resembles a lot the optimal marginal tax formula on the poor agent in the model without side trading (see (1)). The mimicker's marginal rate of substitution is now replaced by her marginal willingness to pay *after* optimal side exchanges have taken place. The tax rule can be given the following cost-benefit interpretation (and let us allow for the possibility that $MRS^{1(1)} \neq MRS^{2(2)}$). A small decrease in t_x^1, accompanied by increase in t_y^1 ($dt_y^1 = -MRS^{1(1)}dt_x^1$) keeps agent 1 on the utility level $\overline{u}^1$ while at the same time reducing the utility of the dishonest agent in the rich coalition with $v_y^{2(1)}(mrs - MRS^{1(1)})$ and increasing the deadweight loss of the transferring money to agent 1 by $MRS^{1(1)}$-1. Likewise, a small fall in t_x^2 accompanied by an increase in t_y^2 ($dt_y^2 = -MRS^{2(2)}dt_x^2$) does not affect the welfare level of rich agents--and therefore neither of the reservation values of the two coalition members. It does however increase the excess burden of taxing agent 2 by $MRS^{2(2)}$-1, and reduces the utility of the mimicking rich coalition member with $\alpha v_y^{2(2)}(mrs - MRS^{2(2)})$, which can also be written as $v_y^{2(1)}(mrs - MRS^{2(2)})$ because with optimal side trades $\alpha v_y^{2(2)} = v_y^{2(1)}$. At an optimum, the net marginal benefit of each instrument should be zero. But this can only be true if $MRS^{1(1)}$=$MRS^{2(2)}$. If the net marginal benefit for lowering t_x^1, say, were zero but $MRS^{1(1)}>MRS^{2(2)}$, then the net marginal benefit for lowering t_x^2 would still be positive, inviting for further lowering of this trade variable which will increase $MRS^{2(2)}$. Equality of the two marginal rates of substitution in equilibrium is therefore automatically guaranteed by the BIC constraint.

If the BIC constraint is not binding, the common marginal rate of substitution equals unity; this will happen when the redistributional aims are small such that the first best lump sum tax scheme is still BIC. People then have to choose from the common budget set $B=\{(t_x,t_y)\mid t_x+t_y=-T \text{ if } t_x\leq\tau_x \text{ and } t_x+t_y=T \text{ otherwise}\}$ where T is the lump sum tax, and τ_x is some predetermined transaction level in x. As mentioned earlier, the scope for first best BIC redistribution will be strictly smaller than that for IIC redistribution. The reason is that for sufficiently high values of T, two rich agents are able to convexify this stepwise budget set in a profitable way by forming a coalition.

- *Increasing curvature along the income expansion path*

If the curvature of indifference curves rises along the income expansion path, result 3 teaches that the relevant coalition to consider is $\{1(1),2(1)\}$. Using exactly the same procedure as for a decreasing curvature, it is possible to show that the optimal marginal tax rate should verify the condition (see the appendix)

$$MRS-1=\frac{\lambda_b v_y^{2(1)}}{\gamma}[mrs-MRS]\frac{2c^2}{c^1+c^2}. \tag{3}$$

As in the previous case, its sign is positive because after sending false information on her type, the rich coalition member will have the same amount of x but more of y at her disposal than the honest poor member. Whence $mrs^{2(1)}(0,0)>mrs>mrs^{1(1)}(0,0)=MRS$. Notice, however, that now the marginal tax rate is "scaled up" with the amount that the rich agent's curvature exceeds the average curvature. The reason is that the no arbitrage condition (η) is no longer automatically satisfied. This can be seen as follows. For t_x^1, the net marginal benefit of a compensated lowering is the same as in the previous case. On the other hand, a small compensated fall in t_x^2 now only entails a cost (the deadweight loss increases by MRS-1) but has no effect whatsoever on the BIC constraint (since no coalition member claims to be rich, no one receives the bundle (t_x^2,t_y^2)). The net marginal benefit of this reform is thus negative, which would be a reason to *raise* t_x^2. But doing so would make the rich person's MRS fall below that of the poor person. Equalisation of the marginal rates of substitution in equilibrium is thus no longer automatically verified. Tax formula (3) then shows that the effect of imposing this condition leads to a higher distortion.[9]

So far, I have characterised the solution taking the BIC constraints related to the {1(1),2(1)} and {2(2),2(1)} in account. Finally, I have to argue why solving the model subject to the BIC constraint w.r.t. coalitions {1(2),1(1)} and {1(2),2(2)} would be inappropriate. Suppose the latter two constraints were the proper side constraints. Then, under either assumption about the behaviour of the curvature coefficient, the solution to the planning problem would yield *MRS*<1 as a necessary condition (see the appendix for a proof). But if redistribution goes from rich to poor, $\frac{t_y^1-t_y^2}{t_x^2-t_x^1}>1$, so that $\frac{t_y^1-t_y^2}{t_x^2-t_x^1}>MRS$ must hold, which contradicts the necessary condition for either {1(2),1(1)} and {1(2),2(2)} to have an incentive to form. All these results are summarised as

Result 5: (i) *If the curvature of indifference curves falls along the same income expansion path, the optimal (common) marginal tax rate is characterised by expression* (2) *and will be strictly positive when the BIC constraint related to coalition* {2(2),2(1)} *is binding; otherwise it will be zero. This BIC constraint also ensures that marginal tax rates are equalised.*

(ii) *If the curvature of indifference curves increases along the same income expansion path, the optimal (common) marginal tax rate is characterised by expression* (3) *and will be strictly positive when the BIC constraint related to coalition* {1(1),2(1)} *is binding. In this case, equalisation of marginal tax rates bears an extra social cost.*

Earlier, I have argued in favour of a declining curvature coefficient. Because this implies that under redistributive taxation a mixed coalition will never have an incentive to form, it also follows that if people restrict their circle of friends to persons of the same social class, there would not arise more scope for redistribution--redistribution would not be possible with less efficiency losses.

5 How bad are linear tax schemes?

While marginal tax rates are equal under a BIC tax scheme, the lump sum components of the tax liability will still depend on the amount traded with the production sector. Globally, therefore, a BIC scheme is non-linear. When compared with a linear tax scheme, the administration of a BIC scheme entails

[9] Though one should keep in mind that (3) is a characterisation result, not an explicit tax rule.

additional costs because taxation at the source is ruled out. The question then rises how large these additional administration costs may grow before it becomes worthwhile to consider a switch to a fully linear tax scheme. As I will show on a numerical example: not large at all.

As Hammond (1987) has shown, a fully linear tax scheme is the only scheme which cannot be thwarted when agents can form coalitions of any finite size--it is the only *multilaterally* incentive compatible (MIC) scheme. Because the lump sum tax liability of an agent i facing a marginal tax rate of MRS^i-1 and trading the amounts (t_x^i, t_y^i) can be written as $-MRS^i t_x^i - t_y^i$, all MIC constraints can just be summarised as the constraint (η) (equality of marginal tax rates) together with

$$-MRS^1 t_x^1 - t_y^1 = -MRS^2 t_x^2 - t_y^2 \qquad (\lambda_m)$$

Substituting (λ_m) for the BIC constraint (λ_b) in the planning problem and solving gives the Ramsey rule for a two class economy (see appendix for derivation):

$$MRS - 1 = \frac{\lambda_m}{\gamma} \frac{1}{\frac{1}{c^1} + \frac{1}{c^2}} (t_x^2 - t_x^1) \qquad (4)$$

Since $1/c^1 + 1/c^2$ is the aggregate substitution effect (cf footnote 6), this is precisely the condition obtained by Mirrlees (1975, eq 9). Rewriting condition (λ_m) with the help of the resource constraint (γ) yields $(MRS\text{-}1)(t_x^2 - t_x^1) = 2(t_x^1 + t_y^1)$. If redistribution goes from rich to poor $t_x^1 + t_y^1 > 0$, and single crossing assures that $t_x^2 - t_x^1 > 0$. MRS-1, and therefore also λ_m, are positive numbers.[10]

I will now deal with the question raised earlier: how many resources can be thrown away under an optimally designed BIC tax scheme in order to reach the same utility allocation which an optimal fully linear tax scheme allows for. To address this question, I consider an economy where $e_x = e_y^2 = 5$, $e_y^1 = 1$ and consumers have Cobb-Douglas preferences: $u(x,y) = x^{1/2} y^{1/2}$. Evaluated at social value, the total endowment of a poor agent lies 40% below that of a rich agent. At

[10] The signing of the optimal marginal tax rate is easier than in Mirrlees (1975). The reason is that normality of commodity x has been assumed. This is equivalent to imposing single crossing. Since the MIC constraints encompass the IIC constraints, it must therefore be true that agent 2 trades more in x than agent 1. In addition, Mirrlees did no tmake any assumptions on the direction of redistribution.

the competitive equilibrium, the trade bundle of agent 1 is (-2,2) giving her a utility level of 3, while agent 2 reaches her highest utility level of 5 by not trading at all. Because Cobb-Douglas preferences belong to the CES class, the curvature coefficient falls along the income expansion path and the relevant BIC constraint to consider is {2(1),2(2)}. In figure 5, I have depicted the utility frontiers which correspond to the use of efficient BIC and linear tax schemes. For utility allocations in the neighbourhood of the competitive equilibrium, the BIC utility frontier coincides with the first best frontier. The largest living standard which poor people can be guaranteed without efficiency losses is 3.101. The Rawlsian solution with utilities 3.17 for the poor and 4.69 for the rich can be decentralised with the budget set $B=\{(t_x,t_y)|\ t_y=1.44\ (-t_x)-.58$ if $t_x\leq-2.35$, and $t_y=1.44\ (-t_x)-.93$ otherwise$\}$ from which poor and rich people select (-2.35,2.81) and (-1.09,.63), respectively. Both types are marginally subsidised on their sales of x at a rate of 44% but pay a different lump sum tax.

At the Rawlsian solution to the linear case, all citizens face a marginal subsidy rate of 52.2% on their sales of x and pay the lump sum tax of .96. Poor agents then choose to trade (-2.49,2.83) and wealthy agents select (-1.17,.83), providing them with a utility level of 3.10 and 4.72, respectively. If the government were now to implement this utility allocation with the help of a BIC tax scheme, it could disregard .249 units of the *numéraire*, which is less than 1.56% of the social value of total resources available in the economy (16 units). Thus, even though agents differ significantly from one another, the threat of bilateral coalition formation between two rich persons is large enough to destroy almost any improvement in targeting efficiency which the differentiation of the lump sum tax liability could bring about. Still in other words, the case for fully linear taxation is primarily made by the possibility to form bilateral coalitions, the possibility to form larger coalitions does not constrain the planner's policy much further.

--insert figure 5 here--

6 Concluding remarks

If citizens have access to informal markets on which transactions are hard to monitor by the government, further constraints are put on the mechanisms to redistribute income. Following different routes, Hammond (1987) and Guesnerie (1981) have shown that if such side markets are perfect, linear tax schemes on retradeable commodities are the best one can design. In the present paper, I have investigated within the simple framework of a two class economy the structure and efficiency properties of tax schemes when side trading is imperfect in the sense that such trading can only take place within bilateral coalitions. It turns out that of all incentive constraints, the one related to a rich coalition with one member disguising as poor is constraining the optimal policy. The characterisation rule for the optimal (common) marginal tax rate is very simple and resembles a lot the tax rule in the case where side trading is absent. A numerical example revealed, however, that imperfections in side market trading do not enhance in a significant way the possibilities to redistribute income. This means that if side market trading is possible but not perfect, the government would not loose much by acting as if it was perfect.

Clearly, the present framework can easily be adapted to analyse the profit maximising pricing strategy of a discriminating monopolist under limited retradeability of his product.

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Appendix

Proof of result 3.

Consider the Edgeworth box below where the lower left corner is the origin for the honest rich person, and the upper right corner is the origin for the other honest rich person.

–insert figure 6 here–

If $F^{\circ}(x)$ denotes the function whose graph is the indifference curve of the honest rich agent with the upper right corner as origin (for a utility level u^2), then $F^{\circ}(\cdot)$ is defined as

$$u[e_x + t_x^1 + e_x + t_x^2 - x, e_y^1 + t_y^1 + e_y^2 + t_y^2 - F^{\circ}(x)] = u^2$$

Let $G(x)$ be the indifference curve for utility level u^2 of the honest rich person with the lower left corner as origin. In point A with co-ordinates (x^2, y^2) both indifference curves are tangent to one another with common slope MRS: $F^{\circ}(x^2) = G(x^2)$ and

$$\forall x \neq x^2 : F^{\circ}(x) < -MRS(x - x^2) + y^2, \text{ and}$$
$$\forall x \neq x^2 : \ G(x) > -MRS(x - x^2) + y^2$$

If the rich person with the upper right corner as origin now announces that she is poor, she will receive the bundle (t_x^1, t_y^1) in stead of (t_x^2, t_y^2). She ends up in point D with co-ordinates, measured from the lower left origin, $(x^2 + (t_x^2 - t_x^1), y^2 - (t_y^1 - t_y^2))$. Projecting D to A means subtracting $(t_x^2 - t_x^1)$ from the x co-ordinate and adding $(t_x^1 - t_y^2)$ to each y co-ordinate. The indifference curve with utility level u^2 is therefore projected to the dashed line, which satisfies the following identity:

$$F[x - (t_x^2 - t_x^1)] = y + (t_y^1 - t_y^2), \ \forall y: F^{\circ}(x) = y, \ or$$
$$F[x - (t_x^2 - t_x^1)] = F^{\circ}(x) + (t_y^1 - t_y^2).$$

With this definition, it is easy to prove that $\frac{t_y^1 - t_y^2}{t_x^2 - t_x^1} > MRS$ is a necessary condition for the BIC constraint related to {2(1),2(2)} to be binding. Indeed, that would mean that $\exists\, x^* : F(x^*) = G(x^*)$. Now

$$
\begin{aligned}
F(x^*) &= F^{\circ}[x^* + (t_x^2 - t_x^1)] + (t_y^1 - t_y^2) \\
&< -MRS \cdot (x^* + (t_x^2 - t_x^1) - x^2) + (t_y^1 - t_y^2) + y^2 \\
&= -MRS \cdot (x^* - x^2) + y^2 + (t_x^2 - t_x^1) \cdot \left(\frac{(t_y^1 - t_y^2)}{(t_x^2 - t_x^1)} - MRS \right) \\
&< G(x^*) + (t_x^2 - t_x^1) \cdot \left(\frac{(t_y^1 - t_y^2)}{(t_x^2 - t_x^1)} - MRS \right)
\end{aligned}
$$

Since $F(x^*) = G(x^*)$, it follows that $\frac{t_y^1 - t_y^2}{t_x^2 - t_x^1} > MRS$. The same necessary condition would be obtained if the BIC constraint related to {1(1),2(1)} were binding. The person whose origin is now the lower left corner is an honest poor agent and the dimensions of the Edgeworth box are now different. If $G(x)$ is then defined as the indifference curve with utility level u^1 for the honest poor person, the same arguments can be applied.

A similar proof can be constructed for the second part of result 3. QED

Derivation of expression (3).
With an increasing curvature, the relevant coalition to consider is {1(1),2(1)}. The maximal utility level which the dishonest rich person can then achieve is given by

$$\max_{\{s_x,s_y\}} v^{2(1)}(s_x,s_y)$$
$$\text{s.t. } v^{1(1)}(-s_x,-s_y) \geq u(t_x^1+e_x,t_y^1+e_y^1) \quad (\beta)$$

with obvious definition of $v^{1(1)}(-s_x,-s_y)$ and with (β) being the individual rationality constraint for the honest poor coalition member. If $(\overset{\circ}{s}_x,\overset{\circ}{s}_y)$ is the solution, the following conditions should necessarily be satisfied:

$$mrs^{2(1)}(\overset{\circ}{s}_x,\overset{\circ}{s}_y) = mrs^{1(1)}(-\overset{\circ}{s}_x,-\overset{\circ}{s}_y), \quad \text{and}$$
$$v_y^{2(1)}(\overset{\circ}{s}_x,\overset{\circ}{s}_y) = \beta v_y^{1(1)}(-\overset{\circ}{s}_x,-\overset{\circ}{s}_y)$$

where $mrs^{1(1)}(\cdot\,)$ is the poor agent's marginal rate of substitution after optimal side trades have taken place. Writing the maximal utility function for 2(1) as $\phi^{2(1)}(t_x^1+e_x,t_y^1+e_y^2;t_x^1+e_x,t_y^1+e_y^1)$, the derivatives w.r.t. the policy variables are:

$$\frac{\partial\phi^{2(1)}}{\partial t_x^1} = v_x^{2(1)} + \beta(v_x^{1(1)} - u_x^{1(1)}) = 2v_x^{2(1)} - \beta u_x^{1(1)},$$
$$\frac{\partial\phi^{2(1)}}{\partial t_y^1} = v_y^{2(1)} + \beta(v_y^{1(1)} - u_y^{1(1)}) = 2v_y^{2(1)} - \beta u_y^{1(1)},$$
$$\frac{\partial\phi^{2(1)}}{\partial t_x^2} = 0, \text{ and } \frac{\partial\phi^{2(1)}}{\partial t_y^2} = 0.$$

Substituting $\phi^{2(1)}(\cdot)$ for $\varphi^{2(1)}(\cdot)$ in the LHS of the BIC constraint (λ_b) of the planning problem, and performing the same manipulations on the first order conditions gives

$$MRS^{1(1)} - 1 = \frac{2\lambda_b v_y^{2(1)}}{\gamma}[mrs^{2(1)} - MRS^{1(1)}] - \frac{\eta}{\gamma}c^1, \text{ and } MRS^{2(2)} - 1 = \frac{\eta}{\gamma}c^2,$$

with $v_y^{2(1)}$ and $mrs^{2(1)}$ evaluated at $(\overset{\circ}{s}_x,\overset{\circ}{s}_y)$. Writing MRS and mrs for the common marginal rates of substitution in and out of equilibrium, resp., we can solve for η/γ:

$$\frac{\eta}{\gamma} = \frac{\lambda_b v_y^{2(1)}}{\gamma}[mrs - MRS]\frac{2}{c^1+c^2}.$$

Using this in either of the two previous conditions yields expression (3).

Proof that the BIC constraints related to coalitions {1(1),1(2)} and {1(2),2(2)} cannot bind at the optimal redistributive policy.

(The proof is only given for the case of a declining curvature coefficient. The appropriate coalition to consider is then {1(2),2(2)}. In the opposite case, with coaltion {1(1),1(2)}, the proof goes along exactly the same lines and also concludes with a negative marginal tax rate.)

The maximal utility level which the dissembling first member of the coalition {1(2),2(2)}, can achieve without bringing the utility level of the honest second member below $u(t_x^2+e_x,t_y^2+e_y^2)$ is given by:

$$\max_{\{s_x, s_y\}} v^{1(2)}(s_x, s_y)$$
$$\text{s.t. } v^{2(2)}(-s_x, -s_y) \geq u(t_x^2 + e_x, t_y^2 + e_y^2) \quad (\beta)$$

where $v^{1(2)}(r_x, r_y) =_{\text{def}} u(t_x^2 + e_x + r_x, t_y^2 + e_y^1 + r_y)$, $v^{2(2)}(\cdot)$ is as defined in the text, and β is the Lagrange coefficient related to the 'no worse off' constraint. The necessary conditions for an interior solution $(\overset{*}{s_x}, \overset{*}{s_y})$ to this problem are given by

$$mrs^{1(2)}(s_x^*, s_y^*) = mrs^{2(2)}(-s_x^*, -s_y^*), \quad \text{and}$$
$$v_y^{1(2)}(s_x^*, s_y^*) = \beta v_y^{2(2)}(-s_x^*, -s_y^*)$$

with $mrs^{1(2)}(\cdot)$ denoting the marginal rate of substitution of the dissembling poor agent when her intermediate endowment $(t_x^2 + e_x, t_y^2 + e_y^1)$ is complemented with the optimal side trade bundle. If the optimal value function of this problem is written as $\varphi^{1(2)}(t_x^2 + e_x, t_y^2 + e_y^1; t_x^2 + e_x, t_y^2 + e_y^2)$, the Envelope theorem ensures that

$$\frac{\partial \varphi^{1(2)}}{\partial t_x^1} = 0, \quad \frac{\partial \varphi^{1(2)}}{\partial t_y^1} = 0,$$
$$\frac{\partial \varphi^{1(2)}}{\partial t_x^2} = v_x^{1(2)} + \beta(v_x^{2(2)} - u_x^{2(2)}) = 2v_x^{1(2)} - \beta u_x^{2(2)}, \quad \text{and}$$
$$\frac{\partial \varphi^{1(2)}}{\partial t_y^2} = v_y^{1(2)} + \beta(v_y^{2(2)} - u_y^{2(2)}) = 2v_y^{1(2)} - \beta u_y^{2(2)}$$

The planning problem may be written as

$$\max_{\{t_x^i, t_y^i\}} u(t_x^2 + e_x, t_y^2 + e_y^2)$$
$$\begin{array}{lll} \text{s.t.} & u(t_x^2 + e_x, t_y^2 + e_y^2) \geq \bar{u}^1 & (\mu) \\ & \varphi^{1(2)}(t_x^2 + e_x, t_y^2 + e_y^1; t_x^2 + e_x, t_y^2 + e_y^2) \leq u(t_x^1 + e_x, t_y^1 + e_y^1) & (\lambda_b) \\ & MRS(t_x^2 + e_x, t_y^2 + e_y^2) = MRS(t_x^1 + e_x, t_y^1 + e_y^1) & (\eta) \\ & t_x^1 + t_y^1 + t_x^2 + t_y^2 \leq 0 & (\gamma). \end{array}$$

Performing the operations $\text{foc}(t_x^i) - \text{foc}(t_y^i) \cdot MRS^{i(i)}$, $(i = 1,2)$ and rearranging yields:

$$MRS - 1 = -\frac{\eta}{\gamma} c^1, \quad \text{and}$$
$$MRS - 1 = \frac{2\lambda_b v_y^{1(2)}}{\gamma}[mrs^{1(2)} - MRS] + \frac{\eta}{\gamma} c^2,$$

Solving for η/γ yields

$$\frac{\eta}{\gamma} = \frac{2\lambda_b v_y^{1(2)}}{\gamma(c^1 + c^2)}(MRS - mrs)$$

and therefore

$$MRS - 1 = \frac{2\lambda_b v_y^{2(1)}}{\gamma(c^1 + c^2)}(mrs - MRS)c^1.$$

This expression has a negative sign. Before engaging in side trading, the dishonest poor person will have the same amount of x but less of y than the honest rich person. Hence, the former agent will have a smaller marginal willingness to pay for x than the latter (whose marginal willingness to pay before side trade is MRS). The result is that 1(2) will buy units of y from 2(2) and sell units of x. This will raise the marginal rate of substitution of 1(2) and lower that of 2(2) up to the common level mrs. Whence $mrs<MRS$. This means that $MRS<1$. By result 2(ii) this also implies that $\frac{t_y^1 - t_y^2}{t_x^2 - t_x^1}<1$, which contradicts the assumption that redistribution goes from rich to poor. QED

Derivation of expression (4).

Substituting (λ_m) for the BIC constraint (λ_b) in the planning problem, performing again the operations $\text{foc}(t_x^i) - \text{foc}(t_y^i) \cdot MRS$, $(i = 1,2)$ and rearranging produces

$$(MRS - 1) = -\frac{\lambda_m}{\gamma}(t_x^1 + \frac{\eta}{\lambda_m})c^1, \text{ and } (MRS - 1) = \frac{\lambda_m}{\gamma}(t_x^2 + \frac{\eta}{\lambda_m})c^2$$

so that

$$\frac{\eta}{\lambda_m} = -\left(\frac{c^1}{c^1 + c^2}t_x^1 + \frac{c^2}{c^1 + c^2}t_x^2\right).$$

Inserting this expression in either of the two equations above, and rearranging produces expression (4).

Figure 1

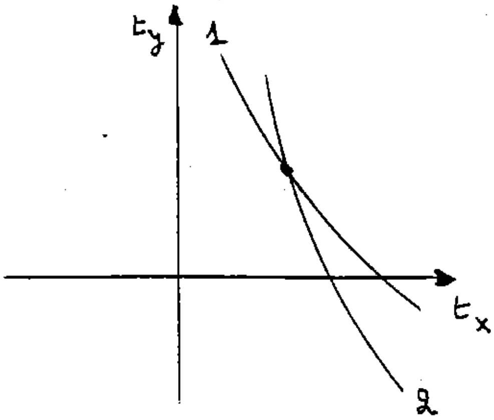


Figure 2

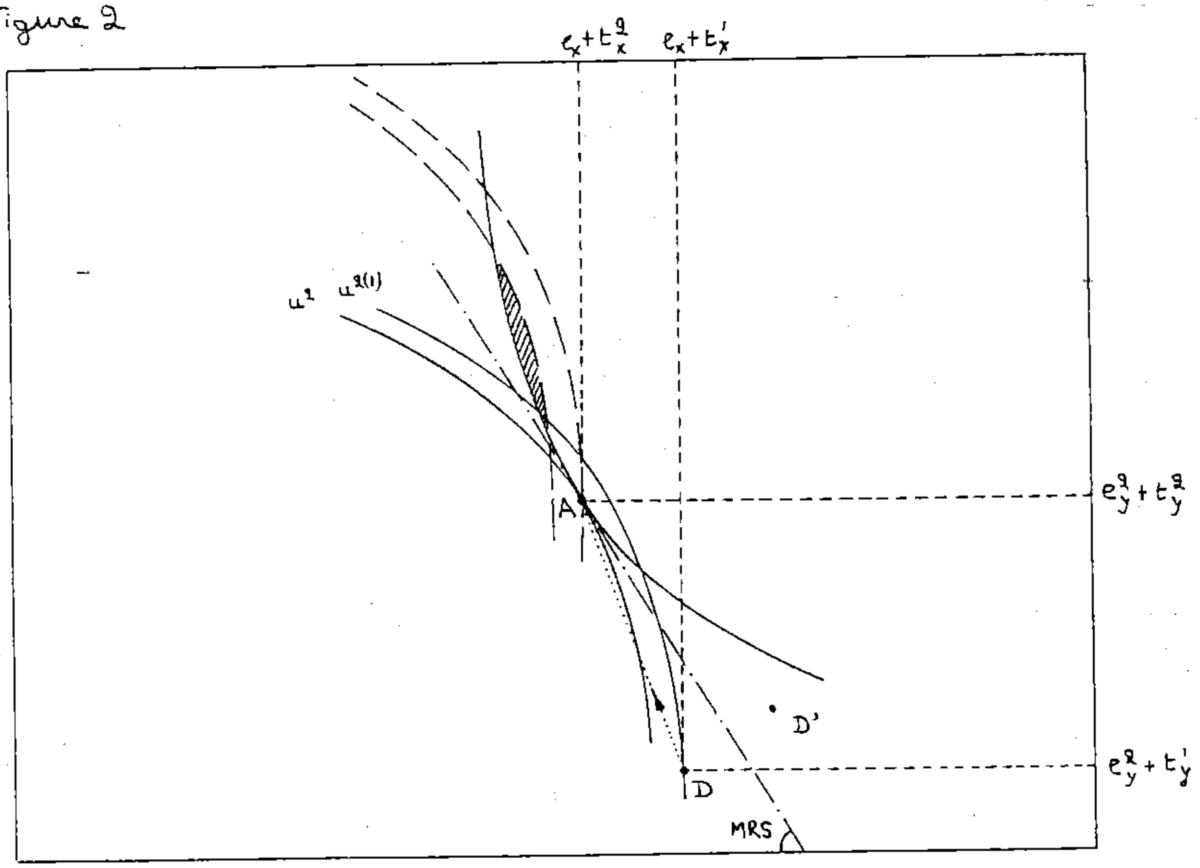


Figure 3a

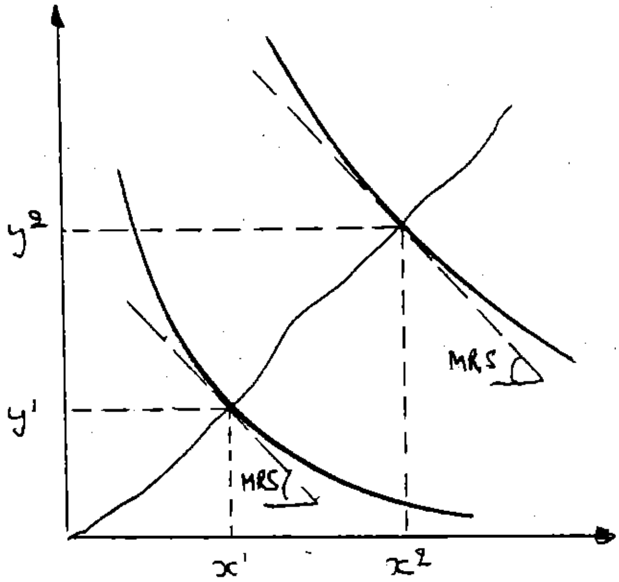


Figure 3b

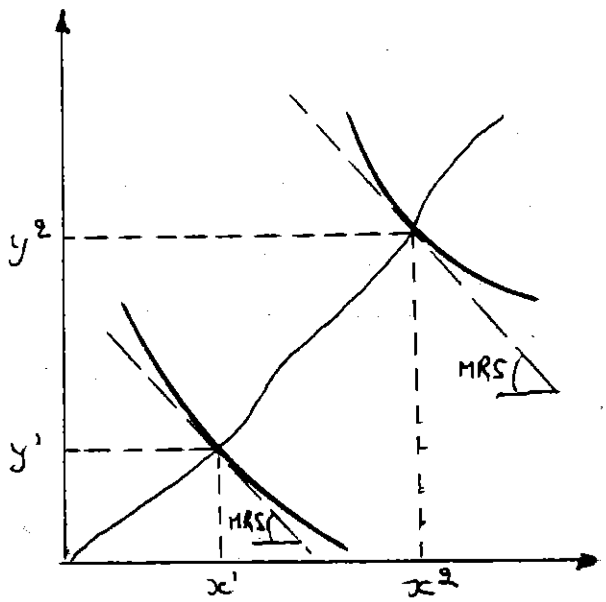


Figure 4a

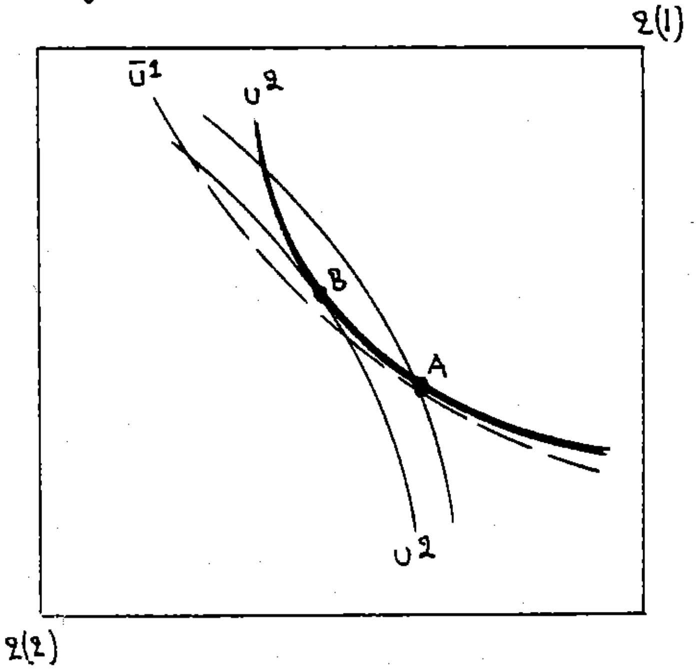


Figure 4b

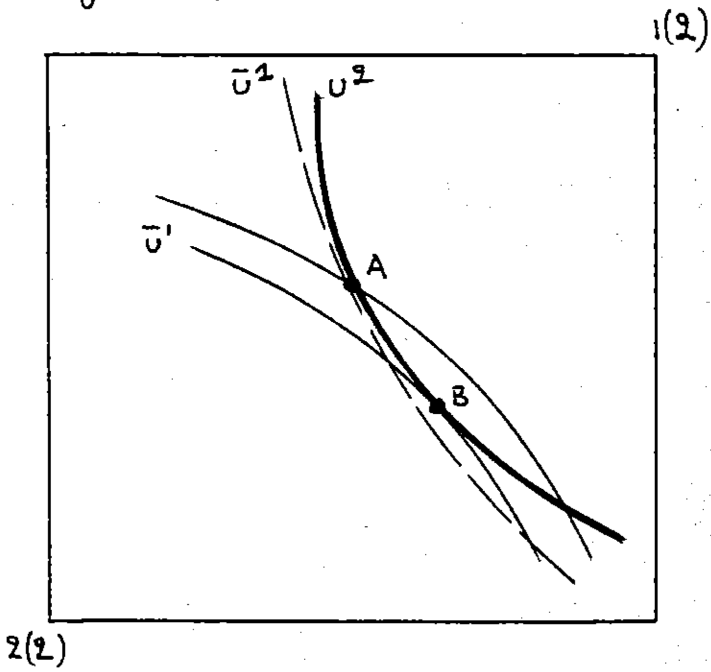


Figure 5

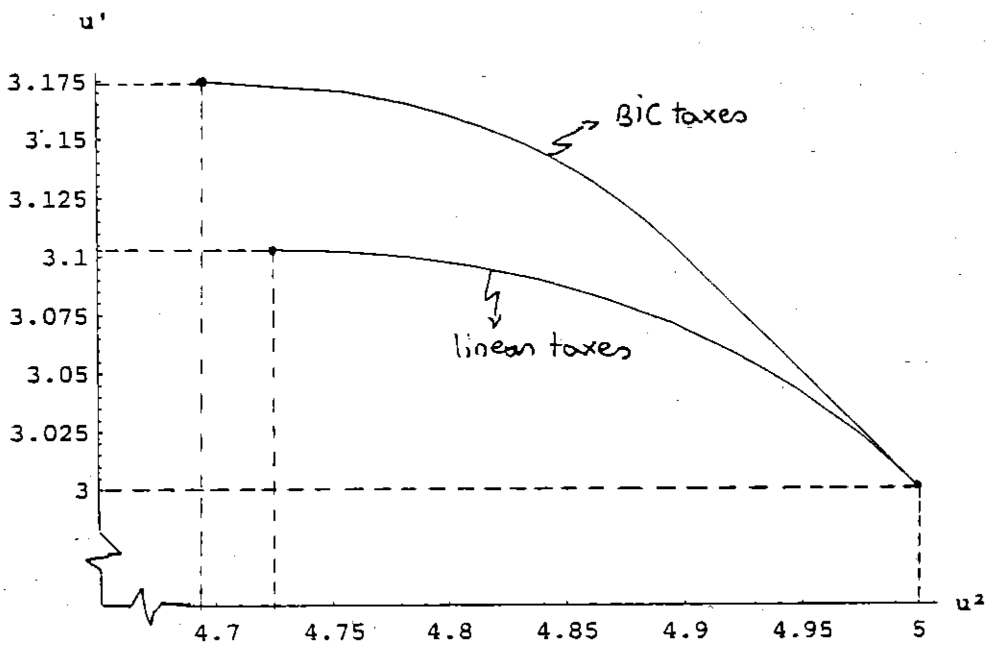
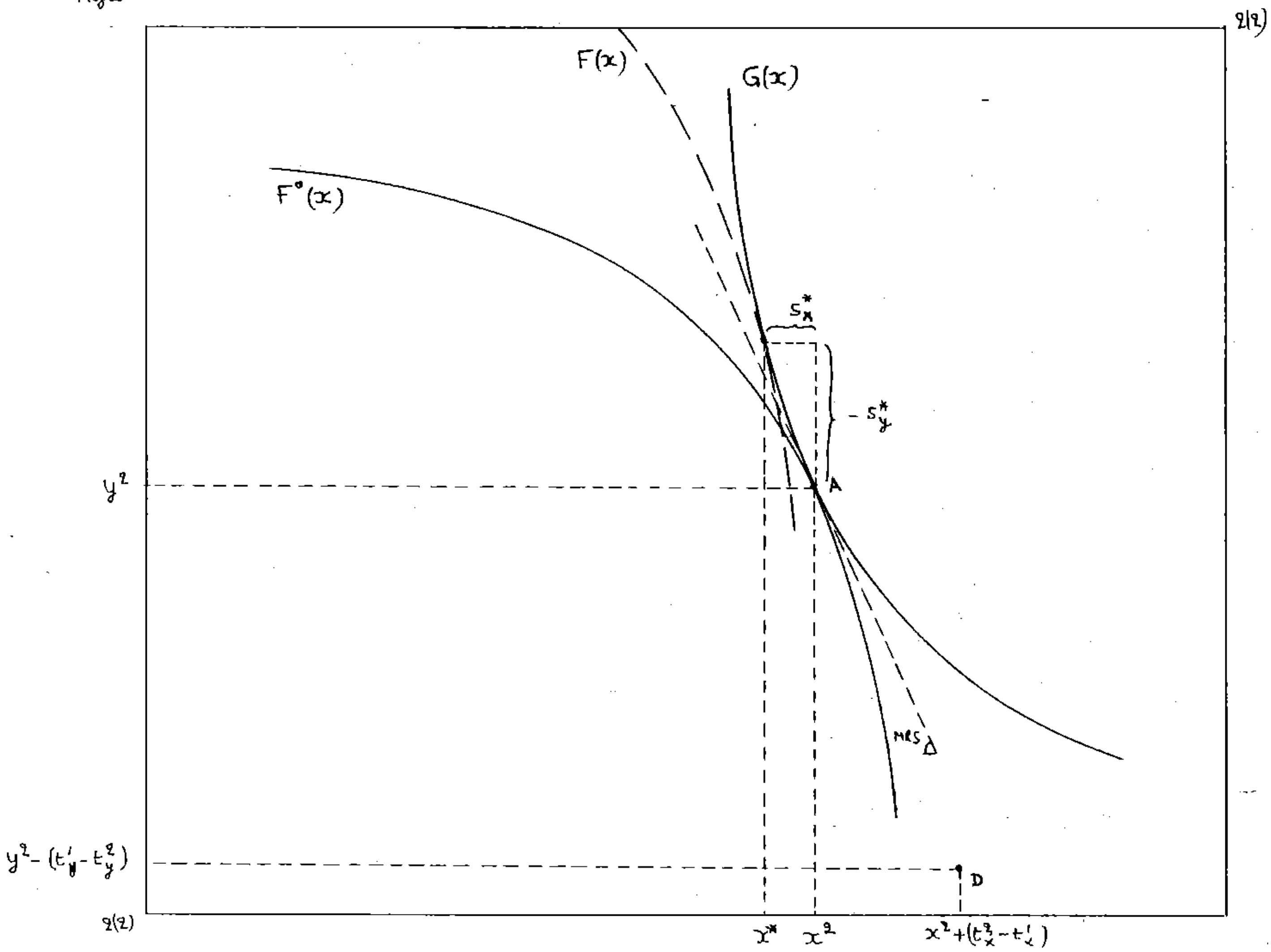


Figure 5



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