Transport externalities and optimal pricing and supply decisions in urban transportation: a simulation analysis for Belgium

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ABSTRACT

In this paper, we study the joint optimisation of transport prices and supply decisions of urban transport services, taking into account all relevant external effects of transportation. A simple theoretical model is developed that clearly identifies the different channels through which demand and supply factors affect the marginal social costs of transport services. Optimal pricing and supply rules are derived, both in the presence and in the absence of a budget restriction on the public transport sector. A detailed simulation analysis for Belgian urban areas illustrates the empirical implications of the optimality rules. We calculate optimal prices of car and public transport in peak and off-peak periods under various assumptions, we consider optimal public transport supply of vehicle-kilometers in both periods as well as optimal fleet size, and we look at the optimal supply of road infrastructure.
INTRODUCTION

The purpose of this paper is to study the problem of jointly optimising prices and supply characteristics of urban transport services in the presence of externalities, using a simple theoretical model as well as a detailed simulation analysis for Belgian urban areas. We numerically determine welfare-optimal prices for private and public urban transport in both peak and off-peak periods, and we calculate optimal levels for a variety of supply variables (public transport supply in both periods, number of buses to be used in public transport, lane capacity of the road system). The analysis takes all relevant private and external costs (due to, e.g., congestion, air pollution, noise, and accident risks) into account.

The literature has produced a number of studies dealing with the joint optimisation of prices and transport service levels. Seminal papers include Mohring (1972), Turvey and Mohring (1975), and Keeler and Small (1977). The former two studies analyse optimal bus fares and supply under first-best conditions. Optimal fares are shown to equal marginal social costs, consisting of marginal boarding and alighting time costs, marginal waiting time costs, and marginal operating costs. Optimisation of supply requires equality between marginal cost and marginal benefit of extra vehicle-kilometres. Note that, as these models deal with public transport only, congestion is not explicitly modeled. Keeler and Small (1977) on the other hand present a model that jointly determines optimal tolls, capacities, and service levels for urban highways. The model assumes there are T subperiods and that cross-price elasticities of demand between different periods are zero. Congestion is captured through a formal speed-flow relation. Not surprisingly, the study finds that optimal prices in each period should reflect marginal social cost, and that lane capacity should be expanded to the point where the marginal cost of an extra unit of capacity equals the marginal value of the user cost savings induced by that investment.

Unfortunately, these early analyses do not incorporate modal choices, and they ignore the presence of important environmental costs associated with urban transport. More recently, a second strand of literature has focused on the design of optimal policies towards transport

1 A very recent paper in the first-best tradition is Jansson’s (1993) analysis of public transport pricing and supply characteristics.
externalities in a multi-modal framework. Existing studies include detailed theoretical and empirical analyses of optimal congestion pricing (see, e.g., Glaister and Lewis (1978), Small (1983), Cohen (1987), Nolan (1994)), cost-benefit analyses investigating the desirability of introducing new pricing technologies (Kraus (1989)), and models dealing with a number of specific policies to reduce emissions (Koopman (1995), Eskeland (1994)). However, none of these papers explicitly considers optimal supply decisions with respect to infrastructural policies, public transport supply, etc.

In a former paper (De Borger et al. (1994)), we studied optimal pricing rules of urban passenger transport services in Belgium, taking into account all relevant external costs (congestion, pollution of various emissions, accident risks, noise). The model included two transport modes, viz. private car and public transport, and two periods. The demand for each mode and each period depends on all transport prices and on the average speed in the corresponding period. However, a serious drawback of the model was that it assumed fixed occupancy rates for public transport, implying that rolling stock is adjusted according to demand variations, and a fixed road capacity. The purpose of this paper is to build upon Viton (1983) in order to extend the former model by endogenising both prices and important supply characteristics, including the supply of vehicle-kilometre by the public transport authority in both peak and off-peak periods, the number of vehicles to be used in public transport, and the supply of road infrastructure².

The structure of this paper is as follows. In Section 1 we present a theoretical model that clearly identifies the channels through which extra demand for passenger-kilometre and extra supply of vehicle-kilometre affect the marginal social costs of transport. Welfare-optimal pricing rules are then derived both in the presence and in the absence of a formal budget constraint on the public transport sector in Section 2. The implementation of the model for simulation purposes is explained in Section 3. We consecutively discuss the specification of the demand and supply sides of the model, and we provide some information on the data.

² As far as we know, Viton (1983) is the only study that explicitly incorporates externalities other than congestion in a multi-modal framework that allows joint optimisation of transport prices and supply characteristics.
used in the application. Section 4 contains the empirical results of the simulation exercises. Finally, a summary of the major findings is presented in Section 5.

1. THE THEORETICAL MODEL

The model we develop extends the partial equilibrium analysis of the urban passenger transport market presented in De Borger et al. (1994). It incorporates two transport modes, viz. the private car and public transport, and two periods, peak and off-peak. There is neither interference with freight transport nor with other regions. The localisation of households is assumed to be exogenously given. Traffic is assumed to take place on one network link; in other words, there is no spatial disaggregation. The notation used can be summarized as follows:

<table>
<thead>
<tr>
<th>Superscript</th>
<th>Transport service</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>private car - peak</td>
</tr>
<tr>
<td>2</td>
<td>private car - off-peak</td>
</tr>
<tr>
<td>3</td>
<td>public transport - peak</td>
</tr>
<tr>
<td>4</td>
<td>public transport - off-peak</td>
</tr>
</tbody>
</table>

1.1. The behavior of households

There are H households. Utility of household h is assumed to depend on the quantity of a composite numeraire good \( x_h \), on the household's use of the four types of transport services \( x_{ih} \) (the number of kilometres individual \( h \) travels by transport service \( i \) (\( i=1, ..., 4 \))), and on other variables that are introduced to incorporate all major external effects associated with transport services in peak and off-peak periods. Specifically, the utility function is given by

\[
U_h = U_h \left( x_h, x_{1h}, ..., x_{4h}, y^1, ..., y^4, E, CA^1, ..., CA^4, w^3, w^4, X^3, X^4 \right) \forall h
\]
where $y^i$ is the average speed of transport service $i$, $E$ is an indicator of the level of environmental pollution, $CA^i$ measures the number of accidents associated with transport service $i$, and $w^i$ $(i=3,4)$ is waiting time associated with public transport mode $i$. Finally, $X^i$ $(i=3,4)$ is the total number of passenger-kilometres travelled using the public mode. Note that all these variables are taken as exogenously given by each household.

The inclusion of $X^3$ and $X^4$ in the utility function reflects the effect an increase in public transport use has on total boarding or alighting times; ceteris paribus, both variables have a negative impact on utility. All other externality-related variables are indirectly a function of traffic volumes and infrastructure levels. First, average speed during peak and off-peak periods is given by

\[
y^i - y^i (Q^1, Q^3, I) \quad \text{for } i=1,3
\]

\[
y^i - y^i (Q^2, Q^4, I) \quad \text{for } i=2,4
\]

Speed is assumed to decrease with the number of vehicle-kilometres travelled by car and by public transport in the period under consideration, $Q^i$, and to increase with $I$, the capacity of the road infrastructure. Second, the level of environmental pollution $E$ is defined as

\[
E = a + \sum_{i=1}^{4} CE^i (Q^i)
\]

where $CE^i$ represents total environmental pollution emitted by transport service $i$; it is assumed to be positively related to $Q^i$. Third, the number of accidents associated with transport service $i$ during peak and off-peak periods is given by

\[
CA^i - CA^i (Q^1, Q^3) \quad \text{for } i=1,3
\]

\[
CA^i - CA^i (Q^2, Q^4) \quad \text{for } i=2,4
\]

Accidents are assumed to be positively related to traffic volumes. Finally, the waiting time $w^i$ associated with public transport mode $i$ is written as
\[ w^i - w^i(Q^i) \quad \text{for } i=3,4 \]  

More public transport supply \((Q^3, Q^4)\) implies a decrease in waiting times.

Of course, in practice the \(X^i\) and the \(Q^i\) are not independent. For passenger cars, we specify the general relation

\[ Q^i - Q^i(X^i) \quad \text{for } i=1,2 \]  

In the empirical part of the paper we will simply assume fixed occupancy-rates for passenger cars. With respect to public transport, we treat both fares \((p^3, p^4)\) and the supply of vehicle-kilometres \((Q^3, Q^4)\) as policy variables. Although this could easily be incorporated, in the theoretical model no formal restriction was imposed on the relation between passenger- and vehicle-kilometres for public transport.

Using all the above structural relations, we can write a reduced form of the individual’s utility function. This is given by\(^3\)

\[ u_h - u_h(x_h, x^1_h, \ldots, x^4_h, X^1, \ldots, X^4, Q^3, Q^4, I) \quad \forall h \]  

where, as suggested before, \(X^i, Q^i\), and I are taken as exogenous parameters by the individual. More passenger-kilometres travelled by car \((X^1, X^2)\) implies an increase in congestion \([(2), (3) \text{ and } (8)]\), environmental pollution \([(4) \text{ and } (8)]\), and in the number of accidents \([(5), (6) \text{ and } (8)]\). More passenger-kilometres travelled by public transport \((X^3, X^4)\) implies that the stops will take longer. An increase in the number of public transport vehicle-kilometres supplied \((Q^3, Q^4)\) contributes to decreasing speed \([(2) \text{ and } (3)]\), lower waiting times \([(7)]\), and an increase in environmental pollution \([(4)]\) and accident risks \([(5) \text{ and } (6)]\). The level of road infrastructure, I, has an impact on individual h’s utility via the speed relations.

\(^3\) Note that we use capital \(U\) for the extensive form and the lowercase \(u\) for the reduced form. A similar notation will be used to distinguish \(V\) from \(v\) in the case of the indirect utility function.
Assuming sufficient differentiability, we can define demand functions and a reduced-form indirect utility function \( v \)

\[
v_h = v_h (P, p^1, ..., p^4, Y_h, X^1, ..., X^4, Q^3, Q^4, I) \quad \forall h
\]  

(10)

with corresponding extensive form \( V \)

\[
V_h = V_h (P, p^1, ..., p^4, Y_h, y^1, ..., y^4, E, CA^1, ..., CA^4, w^3, w^4, X^3, X^4) \quad \forall h
\]  

(11)

where \( P \) is the price of the composite commodity, the \( p^i \) represent the price of transport service \( i \), and \( Y_h \) is individual \( h \)'s income. Inversion of equation (10) leads to the following expenditure function

\[
g_h = g_h (P, p^1, ..., p^4, X^1, ..., X^4, Q^3, Q^4, I, u_h) \quad \forall h
\]  

(12)

from which the individual's compensated demand function for transport service \( i \) can be derived, viz.

\[
x^i_h = x^i_h (P, p^1, ..., p^4, X^1, ..., X^4, Q^3, Q^4, I, u_h) \quad \forall i, h
\]  

(13)

1.2. The definition of marginal external effects

In this subsection we clearly specify the marginal external effects of private and public transport. Following Glaister and Lewis (1978), and using the duality results for public goods derived by King (1986), we can define the marginal external cost \( (\text{mec}_h^i) \) associated with an increase in the number of passenger-kilometres travelled by transport service \( i \) and imposed on individual \( h \) as

\[
\text{mec}_h^i = \frac{\partial g_h (P, p^1, ..., p^4, X^1, ..., X^4, Q^3, Q^4, I, u_h)}{\partial X^i} = - \frac{\partial v_h}{\partial Y_h}
\]  

(14)
Using the extensive form of the indirect utility function (see (11)) the different components of marginal external costs can be clearly identified. For example, the marginal external cost associated with an increase in passenger-kilometres travelled by private car in the peak period can be written as

\[
    mec_h^1 = - \frac{1}{\frac{\partial V_h}{\partial Y_h}} \left[ \sum_{i=1,3} \frac{\partial V_h}{\partial y_i} \frac{\partial y_i}{\partial Q^1} + \frac{\partial V_h}{\partial E} \frac{\partial E}{\partial C E^1} \frac{\partial C E^1}{\partial Q^1} + \sum_{i=1,3} \frac{\partial V_h}{\partial C A^i} \frac{\partial C A^i}{\partial Q^1} \frac{\partial Q^1}{\partial X^1} \right] 
\]

(15)

An additional car user in the peak period will affect the utility of household \( h \) through three different channels: traffic speed for private and public transport in the corresponding period declines, extra pollution is generated, and accident risks increase. An analogous expression holds for increases in \( X^2 \).

Similarly, the marginal external cost caused by an increase in public transport passenger-kilometres in the peak is easily shown to be given by

\[
    mec_h^2 = - \frac{1}{\frac{\partial V_h}{\partial Y_h}} \left[ \frac{\partial V_h}{\partial X^3} \right] 
\]

(16)

The term between brackets reflects the impact on utility of additional passenger-kilometres due to the implied changes in boarding and alighting times for public transport vehicles. A similar relation holds for an increase in \( X^4 \).

An increase in public transport supply generates both marginal external costs and benefits. To see this, note that

\[
    \frac{\partial g_h( P, p^1, \ldots, p^4, X^1, \ldots, X^4, Q^3, Q^4, I, u_h )}{\partial Q^i} = - \frac{\partial v_h}{\partial Y_h} \quad \text{for } i=3,4 
\]

(17)

Taking an increase in public transport supply in the peak period as an example we can rewrite the corresponding relation for \( Q^3 \) as
\[
\frac{\partial g_h}{\partial Q^3} = -\frac{1}{\partial V_h} \left[ \sum_{i=1,3} \frac{\partial V_h}{\partial y^i} \frac{\partial y^i}{\partial Q^3} + \frac{\partial V_h}{\partial E} \frac{\partial E}{\partial CE^3} \frac{\partial CE^3}{\partial Q^3} \right] + \sum_{i=1,3} \frac{\partial V_h}{\partial CA^i} \frac{\partial CA^i}{\partial Q^3} + \frac{\partial V_h}{\partial x^3} \frac{\partial x^3}{\partial Q^3} \right] 
\]  

(18)

This result clearly identifies the different costs and benefits. The first three terms between brackets capture external costs: more peak-period supply contributes to congestion by lowering travel speed, it generates extra pollution, and it increases the risk of accidents. The fourth term reflects the benefit of increased supply; viz. the decrease in waiting time for public transport passengers.

A similar relation holds for Q'. Finally, investment in infrastructure affects travel speeds. Indeed, the external effect of extra infrastructural capacity is given by

\[
\frac{\partial g_h}{\partial I} = -\frac{1}{\partial V_h} \left[ \sum_{i=1}^4 \frac{\partial V_h}{\partial y^i} \frac{\partial y^i}{\partial I} \right] 
\]  

(19)

1.3. Organisation of the transport sector

Both public transport and road infrastructure is provided by the government. Total costs of road infrastructure are denoted \(C_t(I)\). Total costs of public transport consist of variable operating costs, capacity costs associated with rolling stock, and fixed costs \(FC\).

The capacity cost of rolling stock is generally specified as a function of the number of public transport vehicles \(B\), \(C_v(B)\). Variable operating costs of private and public transport are given by

\[
C^i = C^i \left( Q^i \right) \quad \text{for } i=1,2 
\]  

(20)

\[
C^i = C^i \left( Q^i, X^i \right) \quad \text{for } i=3,4 
\]  

(21)
The supply of vehicle-kilometres by public transport ($Q^i$, $i=3,4$) is constrained by the number of available public transport vehicles $B$. Specifically, the following capacity restrictions are imposed

$$Q^i \leq \kappa^i B \quad \text{for } i = 3,4$$

where the $\kappa^i$ may reflect operating conditions (e.g., commercial speed), maintenance policies, etc.

1.4. Formulation of the optimisation problem

Two cases can be distinguished. First, if there is no budget restriction imposed on the public transport firm, the model searches for optimal prices, optimal supply of public transport, optimal number of buses, and optimal road infrastructure so as to solve the following problem:

$$\begin{align*}
\text{MAX} & \quad W \left[ v_1, \ldots, v_k, \ldots, v_H \right] \\
& + (1+\lambda) \left[ \sum_{i=1}^{4} \left( p^i X^i - C^i \right) - C_B \left( B \right) - C_I \left( I \right) - FC \right] \\
\text{s.t.} & \quad Q^k \leq \kappa^k B \quad \text{for } k = 3,4 \quad (\mu_k) \\
& \quad p^1 \geq 0, \ p^2 \geq 0, \ p^3 \geq 0, \ p^4 \geq 0, \\
& \quad Q^3 \geq 0, \ Q^4 \geq 0, \ I \geq 0, \ B \geq 0
\end{align*}$$

where $W(.)$ is the relevant social welfare function, and $(1+\lambda)$ is the marginal cost of public funds. In other words, we assume the government maximises an objective function consisting of social welfare defined over individual utilities plus government net revenues, evaluated at the marginal cost of public funds. Allowing for a marginal cost of public funds $(1+\lambda)$ larger than one has been suggested to account for existing distortions in the economy (e.g., due to the impossibility of lump-sum taxation) in a partial equilibrium model (Laffont and Tirole, 1990).
Second, if the public transport authority does face a budget constraint, we add the restriction

\[ \sum_{i=3}^{4} C^i + C_\theta(B) + FC \leq \sum_{i=3}^{4} p^i X^i + D \]  

(24)

to the above optimisation problem, where D is the maximum allowed deficit.

2. OPTIMALITY PRICING RULES AND SUPPLY DECISIONS

In this section we present optimal pricing and supply rules derived from the formulated model. Note, however, that the presence of fixed costs and externalities implies non-convexities. It is well known that in this case the first-order conditions may be insufficient and that corner solutions could be optimal (Guesnerie, 1980 and Börs, 1985). We restrict ourselves to a discussion of non-corner solutions. We further assume that \( \kappa^i \) is constant for the theoretical derivations (but not in the simulation model, see below).

2.1. No formal budget constraint

Using Roy's identity, the duality results of King (1986), and defining \( \sigma_h \) as the marginal social utility of income, the first-order conditions with respect to prices are given by

\[ \sum_h \sigma_h \left( x_{ij} + \sum_{i=1}^{4} \frac{\partial g_{ik}}{\partial x_i} \frac{\partial x_i}{\partial p^j} \right) + (1+\lambda) \]

\[ \left( X^j + \sum_{i=1}^{2} \left( p^i - \frac{\partial C^i}{\partial q^i} \frac{\partial q^i}{\partial x_i} \right) \frac{\partial x_i}{\partial p^j} + \sum_{i=3}^{4} \left( p^i - \frac{\partial C^i}{\partial x_i} \right) \frac{\partial x_i}{\partial p^j} \right) = 0 \]  

for \( j=1,...,4 \)  

(25)

These relations trade off the relevant costs and benefits of marginal changes in prices. Similarly, the optimality conditions with respect to public transport supply compare marginal social benefits and costs of supply changes. They are given by
\[
\left[ \sum_h \sigma_h \left( \frac{\partial g_h}{\partial Q^j} + \sum_{i=1}^{4} \frac{\partial g_h}{\partial X^i} \frac{\partial X^i}{\partial Q^j} \right) \right] - \left[ (1 + \lambda) \left( \frac{\partial C_j}{\partial Q^j} \right) \right] + \left[ \sum_{i=1}^{4} \left( \frac{\partial C_i}{\partial Q^j} \right) \frac{\partial X^i}{\partial Q^j} \right] + \left[ (1 + \lambda) \sum_{i=1}^{4} p_i \frac{\partial X^i}{\partial Q^j} \right] - [\mu_j] = 0
\]

for \( j = 3,4 \)

The first term in (26) captures individuals’ marginal willingness to pay for more public transport supply and the marginal external costs of all traffic volume changes induced by the increase in supply. The second term reflects the effect on the costs of the various transport services. The third term expresses the effect of supply changes on the revenues of the transport sector. Finally, the fourth term captures the shadow value of the extra capacity requirement induced by a supply increase.

With respect to optimal infrastructure levels a fairly similar expression is obtained

\[
\left[ \sum_h \sigma_h \left( \frac{\partial g_h}{\partial I} + \sum_{i=1}^{4} \frac{\partial g_h}{\partial X^i} \frac{\partial X^i}{\partial I} \right) \right] - \left[ (1 + \lambda) \left( \frac{\partial C_f(I)}{\partial I} \right) \right] + \left[ \sum_{i=1}^{4} \left( \frac{\partial C_i}{\partial I} \right) \frac{\partial X^i}{\partial I} \right] + \left[ (1 + \lambda) \sum_{i=1}^{4} p_i \frac{\partial X^i}{\partial I} \right] = 0
\]

The first term represents the marginal willingness to pay for more infrastructure and the marginal external costs associated with the changes in traffic, which are caused by variations in the level of infrastructural investments. The second part of equation (27) is the sum of infrastructural investment costs and the effect of extra road infrastructure on the money costs of private and public transport. The last term expresses the effect of additional infrastructure on revenues in the transport sector.

The first-order condition with respect to the number of vehicles reads

\[
\mu_3 \kappa^3 + \mu_4 \kappa^4 - (1 + \lambda) \frac{\partial C_g(B)}{\partial B} = 0
\]

The first two terms reflect the shadow values of extra capacity. The third term captures the
cost of extra vehicles. To facilitate the interpretation of this condition, suppose that the capacity restriction is binding in the peak period but not in the off-peak so that \( \mu_3 > 0 \) and \( \mu_4 = 0 \). Then (28) suggests that the social value of extra capacity should equal the extra costs, valued at one plus the shadow cost of public funds.

Finally, the capacity restrictions imply

\[
\delta^t B - Q^j \geq 0 \quad \text{for } j = 3,4 \tag{29}
\]

\[
\mu_j [ \delta^t B - Q^j ] = 0 \quad \text{for } j = 3,4 \tag{30}
\]

\[
\mu_3 \geq 0, \mu_4 \geq 0 \tag{31}
\]

It is easily shown that at the optimum at least one of the capacity restrictions will be binding. To see this, consider equation (28) and note that the marginal cost of extra vehicles is always positive.

It is instructive to reformulate the above optimality conditions in terms of familiar economic concepts. Define the marginal social costs \( S^i \) associated with an additional unit of \( X^i \) as:

\[
S^i = \sum_h \sigma_h mec_h^i + \frac{\partial C^i}{\partial Q^i} \frac{\partial Q^i}{\partial X^i} \quad \text{for } i=1,2 \tag{32}
\]

\[
S^i = \sum_h \sigma_h mec_h^i + \frac{\partial C^i}{\partial X^i} \quad \text{for } i=3,4
\]

The marginal social cost associated with an extra unit of traffic of type \( i \) equals the weighted sum over all individuals of the marginal external cost of additional traffic of type \( i \) plus marginal private costs. Using (32) and the following elasticities

\[
\eta^i_j = \frac{\partial X^i}{\partial p^i} \frac{p^i}{X^i}, \quad \epsilon^i_{Q^i} = \frac{\partial X^i}{\partial Q^i} \frac{Q^i}{X^i}, \quad \epsilon^i_j = \frac{\partial X^i}{\partial I} \frac{I}{X^i} \tag{33}
\]

equation system (25) can be rewritten as\(^4\)

\(^4\) More details can be found in Appendix 1.
\[
\begin{bmatrix}
\eta_1^1 & \eta_1^2 & \eta_1^3 & \eta_1^4 \\
\eta_2^1 & \eta_2^2 & \eta_2^3 & \eta_2^4 \\
\eta_3^1 & \eta_3^2 & \eta_3^3 & \eta_3^4 \\
\eta_4^1 & \eta_4^2 & \eta_4^3 & \eta_4^4 \\
\end{bmatrix}
\begin{bmatrix}
(S^1+\lambda \frac{\partial C^1}{\partial Q^1} \frac{\partial Q^1}{\partial X^1} -(1+\lambda)p^1)X^1 \\
(S^2+\lambda \frac{\partial C^2}{\partial Q^2} \frac{\partial Q^2}{\partial X^2} -(1+\lambda)p^2)X^2 \\
(S^3+\lambda \frac{\partial C^3}{\partial Q^3} \frac{\partial Q^3}{\partial X^3} -(1+\lambda)p^3)X^3 \\
(S^4+\lambda \frac{\partial C^4}{\partial Q^4} \frac{\partial Q^4}{\partial X^4} -(1+\lambda)p^4)X^4 \\
\end{bmatrix}
- \begin{bmatrix}
(\sum_h \sigma_h \frac{\partial \lambda}{\partial X^1} -1-\lambda)p^1X^1 \\
(\sum_h \sigma_h \frac{\partial \lambda}{\partial X^2} -1-\lambda)p^2X^2 \\
(\sum_h \sigma_h \frac{\partial \lambda}{\partial X^3} -1-\lambda)p^3X^3 \\
(\sum_h \sigma_h \frac{\partial \lambda}{\partial X^4} -1-\lambda)p^4X^4 \\
\end{bmatrix}
\]

Similarly, (26) and (27) can be rewritten as

\[
\begin{bmatrix}
e_1^1 & e_1^2 & e_1^3 & e_1^4 \\
e_2^1 & e_2^2 & e_2^3 & e_2^4 \\
e_3^1 & e_3^2 & e_3^3 & e_3^4 \\
e_4^1 & e_4^2 & e_4^3 & e_4^4 \\
\end{bmatrix}
\begin{bmatrix}
(S^1+\lambda \frac{\partial C^1}{\partial Q^1} \frac{\partial Q^1}{\partial X^1} -(1+\lambda)p^1)X^1 \\
(S^2+\lambda \frac{\partial C^2}{\partial Q^2} \frac{\partial Q^2}{\partial X^2} -(1+\lambda)p^2)X^2 \\
(S^3+\lambda \frac{\partial C^3}{\partial Q^3} \frac{\partial Q^3}{\partial X^3} -(1+\lambda)p^3)X^3 \\
(S^4+\lambda \frac{\partial C^4}{\partial Q^4} \frac{\partial Q^4}{\partial X^4} -(1+\lambda)p^4)X^4 \\
\end{bmatrix}
- \begin{bmatrix}
(\sum_h \sigma_h \frac{\partial \lambda}{\partial X^1} -1-\lambda)p^1X^1 \\
(\sum_h \sigma_h \frac{\partial \lambda}{\partial X^2} -1-\lambda)p^2X^2 \\
(\sum_h \sigma_h \frac{\partial \lambda}{\partial X^3} -1-\lambda)p^3X^3 \\
(\sum_h \sigma_h \frac{\partial \lambda}{\partial X^4} -1-\lambda)p^4X^4 \\
\end{bmatrix}
\]

These results show that, if distortionary taxation is assumed (i.e., \(\lambda > 0\)), optimal pricing and supply decisions depend in a complex way on the shadow cost of public funds, on all own and cross-elasticities, and on the distributional characteristics of the different transport services. However, more insight in optimal pricing and supply behavior can be obtained if one is willing to ignore distributional issues and assume zero cross-elasticities of demand, both with respect to prices and supply of vehicle-kilometres (i.e., assume \(\eta_i = e_{ij} = 0\) except for \(i=j\)). Under those conditions the optimal pricing rules reduce to

\[
p^i - \frac{1}{1+\lambda} S^i - \frac{\lambda}{1+\lambda} \frac{\partial C^i}{\partial Q^i} \frac{\partial Q^i}{\partial X^i} = - \frac{\lambda}{1+\lambda} \frac{1}{\eta_i} \quad \text{for } i=1,2
\]
\[
\frac{p^i - \frac{1}{1+\lambda} S^i - \frac{\lambda}{1+\lambda} \frac{\partial C^i}{\partial X^i}}{p^i} - \frac{\lambda}{1+\lambda} \frac{1}{\eta^i} \quad \text{for } i=3,4
\] (37)

These expressions show that optimal pricing requires a markup of price over a weighted average of marginal private and social cost; the markup is inversely proportional to the price elasticity of demand. These findings are consistent with the findings of, e.g., Sandmo (1975), Oum and Threteway (1988). With respect to optimal public transport supply it is easily shown that

\[(1+\lambda) \left[ p^j \frac{\partial X^j}{\partial Q^j} - \frac{\partial C^j}{\partial X^j} \frac{\partial X^j}{\partial Q^j} - \frac{\partial C^j}{\partial Q^j} \right] - \sum_h \frac{\partial g_h}{\partial Q^j} + \sum_h \frac{\partial g_h}{\partial X^j} \frac{\partial X^j}{\partial Q^j} + \mu_j \quad \text{for } j=3,4 \] (38)

This amounts to equality between appropriately defined marginal social benefits and marginal social costs of extra public transport output. Note that private revenues and costs are valued at the marginal cost of public funds to reflect the social value of government net revenues.

Finally, the interpretation of the optimality conditions (34), (35) and (28) to (31) can be further simplified if the government can use first-best instruments. This amounts to assuming \(\lambda = 0\) and ignoring redistributional concerns by setting all welfare weights equal to 1. In that case the solution to (34) an (35) is given by well-known first-best rules:

\[
S^i = p^i \quad \forall i, \quad \sum_h \frac{\partial g_h}{\partial Q^3} + \frac{\partial C^3}{\partial Q^3} + \mu_3 = 0,
\]

\[
\sum_h \frac{\partial g_h}{\partial Q^4} + \frac{\partial C^4}{\partial Q^4} + \mu_4 = 0 \quad \text{and} \quad \sum_h \frac{\partial g_h}{\partial I} + \frac{\partial C_I}{\partial I} = 0
\] (39)

Equation system (39) shows that the optimum combines marginal social cost pricing with public transport supply levels such that the marginal social cost of extra vehicle-kilometers, including the shadow price of the induced capacity expansion, equals marginal social benefit. Similarly, levels of infrastructure are optimal when marginal costs equal marginal benefits.

---

5 Alternatively, these relations imply a markup of price over private cost plus a fraction of external cost. The markup is not over the full external cost. The reason is that an increase in price over marginal private cost generates revenues for the transport sector while at the same time reducing the level of external effects.
2.2. Imposing a budget constraint on the public transport sector

To analyse the implications of the imposition of a budget restriction on the public transport sector for optimal transport fares and supply levels, we add the restriction

$$\sum_{i=3}^{4} C^i + C_B(B) + FC \leq \sum_{i=3}^{4} p^i X^i + D$$

(40)

to problem (23). Again, we restrict the analysis to non-corner solutions. The resulting first-order conditions describing optimal prices and optimal supply levels can be rewritten as:

$$\begin{bmatrix}
\eta_1^1 \eta_1^2 \eta_1^3 \eta_1^4 \\
\eta_2^1 \eta_2^2 \eta_2^3 \eta_2^4 \\
\eta_3^1 \eta_3^2 \eta_3^3 \eta_3^4 \\
\eta_4^1 \eta_4^2 \eta_4^3 \eta_4^4
\end{bmatrix}
\begin{bmatrix}
(S^1 + \lambda \frac{\partial C^1}{\partial Q^1} - (1 + \lambda)p^1)X^1 \\
(S^2 + \lambda \frac{\partial C^2}{\partial Q^2} - (1 + \lambda)p^2)X^2 \\
(S^3 + (\lambda + \phi)\frac{\partial C^3}{\partial X^3} - (1 + \lambda - \phi)p^3)X^3 \\
(S^4 + (\lambda + \phi)\frac{\partial C^4}{\partial X^4} - (1 + \lambda - \phi)p^4)X^4
\end{bmatrix}
= \begin{bmatrix}
\left(\sum_{h} \sigma_h \frac{x^1_h}{X^1} - (1 - \lambda)p^1 X^1\right) \\
\left(\sum_{h} \sigma_h \frac{x^2_h}{X^2} - (1 - \lambda)p^2 X^2\right) \\
\left(\sum_{h} \sigma_h \frac{x^3_h}{X^3} - (1 - \lambda - \phi)p^3 X^3\right) \\
\left(\sum_{h} \sigma_h \frac{x^4_h}{X^4} - (1 - \lambda - \phi)p^4 X^4\right)
\end{bmatrix}

(41)

$$\begin{bmatrix}
e_1^1 e_1^2 e_1^3 e_1^4 \\
e_2^1 e_2^2 e_2^3 e_2^4 \\
e_3^1 e_3^2 e_3^3 e_3^4 \\
e_4^1 e_4^2 e_4^3 e_4^4
\end{bmatrix}
\begin{bmatrix}
(S^1 + \lambda \frac{\partial C^1}{\partial Q^1} - (1 + \lambda)p^1)X^1 \\
(S^2 + \lambda \frac{\partial C^2}{\partial Q^2} - (1 + \lambda)p^2)X^2 \\
(S^3 + (\lambda + \phi)\frac{\partial C^3}{\partial X^3} - (1 + \lambda + \phi)p^3)X^3 \\
(S^4 + (\lambda + \phi)\frac{\partial C^4}{\partial X^4} - (1 + \lambda + \phi)p^4)X^4
\end{bmatrix}
= \begin{bmatrix}
\left(\sum_{h} \sigma_h \frac{\partial g^1_h}{\partial Q^3} + (1 + \lambda + \phi)\frac{\partial C^3}{\partial Q^3} + \mu_3\right)Q^3 \\
\left(\sum_{h} \sigma_h \frac{\partial g^2_h}{\partial Q^4} + (1 + \lambda + \phi)\frac{\partial C^4}{\partial Q^4} + \mu_4\right)Q^4 \\
\left(\sum_{h} \sigma_h \frac{\partial g^3_h}{\partial I} + (1 + \lambda)\frac{\partial C_B(B)}{\partial I}\right)I
\end{bmatrix}

(42)

$$\mu_3 \kappa^3 + \mu_4 \kappa^4 - (1 + \lambda + \phi) \frac{\partial C_B(B)}{\partial B} = 0

(43)
\[ \kappa^j B - Q^j \geq 0 \quad \text{for } j = 3,4 \]  
(44)

\[ \mu_j [ \kappa^j B - Q^j ] = 0 \quad \text{for } j = 3,4 \]  
(45)

\[ \sum_{i=3}^{4} p^i X^i + D - \sum_{i=3}^{4} C^i - C_B(B) - FC \geq 0 \]  
(46)

\[ \phi [ \sum_{i=3}^{4} p^i X^i + D - \sum_{i=3}^{4} C^i - C_B(B) - FC ] = 0 \]  
(47)

where \( \phi \) represents the multiplier associated with the budget restriction. Note that (46) and (47) are new and that (41), (42) and (43) differ from (34), (35) and (28) by the inclusion of \( \phi \) in the optimality conditions for public transport.

For completeness sake, let us see how the results are simplified if the government can use first best redistributive taxation instruments (all \( \sigma_b \) equal 1 and \( \lambda = 0 \)). We find under those circumstances

\[
\begin{bmatrix}
\eta_1^1 \\ \eta_1^2 \\ \eta_1^3 \\ \eta_1^4 \\
\eta_2^1 \\ \eta_2^2 \\ \eta_2^3 \\ \eta_2^4 \\
\eta_3^1 \\ \eta_3^2 \\ \eta_3^3 \\ \eta_3^4 \\
\eta_4^1 \\ \eta_4^2 \\ \eta_4^3 \\ \eta_4^4 \\
\end{bmatrix}
\begin{bmatrix}
(S^1-p^1)X^1 \\
(S^2-p^2)X^2 \\
(S^3+\phi \frac{\partial C^3}{\partial X^2}-(1+\phi)p^3)X^3 \\
(S^4+\phi \frac{\partial C^4}{\partial X^4}-(1+\phi)p^4)X^4 \\
\end{bmatrix} = 
\begin{bmatrix}
0 \\
0 \\
\phi p^3 X^3 \\
\phi p^4 X^4 \\
\end{bmatrix}
\]  
(48)

\(^6\) Of course, one might wonder why budgetary restrictions should be imposed if the government has first-best tax instruments available. However, they are often justified based on incentive arguments (Laffont and Tirole (1990)).
\[
\left[ \begin{array}{c}
\epsilon_1^1 \\
\epsilon_2^1 \\
\epsilon_3^1 \\
\epsilon_4^1 \\
\epsilon_1^2 \\
\epsilon_2^2 \\
\epsilon_3^2 \\
\epsilon_4^2 \\
\epsilon_1^3 \\
\epsilon_2^3 \\
\epsilon_3^3 \\
\epsilon_4^3 \\
\epsilon_1^4 \\
\epsilon_2^4 \\
\epsilon_3^4 \\
\epsilon_4^4 \\
\end{array} \right]
\times
\left[ \begin{array}{c}
(S^1-p^1)X^1 \\
(S^2-p^2)X^2 \\
(S^3+\phi \frac{\partial C}{\partial X^3}-(1+\phi)p^3)X^3 \\
(S^4+\phi \frac{\partial C}{\partial X^4}-(1+\phi)p^4)X^4 \\
\end{array} \right]
- \left[ \begin{array}{c}
\sum_h \frac{\partial g}{\partial Q^3} + (1+\phi) \frac{\partial C}{\partial Q^3} + \mu_3 Q^3 \\
\sum_h \frac{\partial g}{\partial Q^4} + (1+\phi) \frac{\partial C}{\partial Q^4} + \mu_4 Q^4 \\
\sum_h \frac{\partial g}{\partial I} + \frac{\partial C}{\partial I} I \\
\end{array} \right]
= 0
\]

\[
\mu_3 \kappa^3 + \mu_4 \kappa^4 - (1+\phi) \frac{\partial C_g(B)}{\partial B} = 0
\]

These expressions can be further simplified by assuming zero cross-price elasticities of demand. Note that even in this case, marginal social cost pricing of public transport is no longer optimal. We find the following pricing rules

\[
p^i - S^i
\]

for \(i=1,2\)

\[
p^i - \frac{1}{1+\phi} S^i - \frac{\phi}{1+\phi} \frac{\partial C}{\partial X^i}
\]

\[
p^i = \frac{1}{1+\phi} \eta^i
\]

for \(i=3,4\)

In other words, social cost pricing remains optimal for private car transport, whereas for public transport a markup over a weighted average of private and social cost should be charged that is inversely proportional to the price elasticity of demand. Note that for public transport the resulting pricing structure is essentially the same as the one obtained in the case without budget restriction but where the marginal cost of funds exceeded one. Optimal rules for public transport supply are also analogous; we find

\[
(1+\phi) \left[ p^j \frac{\partial X^j}{\partial Q^j} - \frac{\partial C}{\partial X^j} \frac{\partial X^j}{\partial Q^j} - \frac{\partial C}{\partial Q^j} \right] = \sum_h \frac{\partial g}{\partial Q^j} + \sum_h \frac{\partial g}{\partial X^j} \frac{\partial X^j}{\partial Q^j} + \mu_j
\]

for \(j=3,4\)
3. IMPLEMENTATION OF THE MODEL

In what follows, we apply a simplified version of the model presented in the theoretical section. Specifically, we look for optimal transport prices and optimal supply using Belgian data on urban transport. Due to current restrictions on data availability, the illustration is necessarily limited to a highly stylised application. It is based on the use of aggregate data and therefore ignores distributional considerations. Moreover, we assume throughout that the government can use first best tax instruments.

In this section we provide information on the specification of the demand and supply sides of the simulation model, and briefly review the construction or the sources of the necessary data. We consecutively deal with the specification of the demand functions, the structure of marginal social costs of additional private and public transport, the marginal social effects of additional infrastructure, and the construction of the budget constraint. To improve the readability of the paper, we provide a brief summary of the procedures used to collect the data and other required inputs for the model in the main body of the paper, and refer for more details to a series of appendices.

3.1. Specification of the demand functions

The model has to be interpreted as reflecting an 'aggregate' Belgian urban area, consisting of all Belgian cities offering public urban transport. For the two transport modes considered in the application, viz. private car and an aggregate public transport\(^6\) mode, a distinction is made between peak and off-peak periods. The peak period is assumed to cover five hours a day, the off-peak period covers the remaining seventeen hours (see STRATEC (1992) and NIS (1985)).

The aggregate demand functions for the different transport services are taken to be loglinear functions of the transport service prices, of the supply levels of public transport, of the level of road infrastructure and of average speed.

\(^6\) The aggregate public transport mode consists of both bus and tram. Metro transport, which is offered only in Brussels, has been left out of the analysis. For more details, see De Borger et al. (1994).
\[ X^i = a^i \exp \left( \sum_{j=1}^{4} \eta^i_j \ln (p^j) + \sum_{j=5}^{4} \varepsilon^i_{Q^j} \ln (Q^j) + \varepsilon^i_I \ln (I) + \tau^i \ln (y^i) \right) \]  

To calibrate these demand functions, information is needed on elasticity values \((\eta, \varepsilon, \tau)\) and on levels of all relevant variables \((X, p, Q, B, I, y)\) in the reference situation. A careful discussion of the data used is provided in Appendix 2\(^7\).

### 3.2. Marginal social costs and benefits associated with additional traffic

To streamline the discussion we first present the social costs caused by additional traffic by car, and then turn to the discussion of the marginal costs and benefits associated with additional public transport.

#### 3.2.1. Marginal social costs associated with additional car use \((S^1 \text{ and } S^2)\)

Consistent with De Borger et al. (1994) we assumed throughout that the occupancy rate of private cars was constant at 1.7 persons per vehicle. As a consequence, a direct relation exists between the marginal social costs of an additional car-kilometre and those of an extra passenger-kilometre. They consist of external costs and private money costs (see (32)). The former include marginal congestion costs, marginal environmental costs, and marginal accident costs (see (15)).

**a. Marginal congestion costs**

The procedure for calculating marginal congestion costs associated with additional car users is fully described in Appendix 3. It basically consists of two steps. First, we determine marginal congestion, i.e., the time loss suffered by road users due to an extra kilometre travelled by car. Use is made of a relation describing how the average time needed to drive one kilometre by car depends on the total hourly traffic volume, expressed in passenger car

\(^7\) Some of the data used in this paper were updated from our previous study (De Borger et al. (1994)). The current paper also required a substantial amount of new data; more information is provided below as well as in a number of appendices.
equivalent unit kilometres (PCUkm/h$^8$), and on the level of road infrastructure (I). It is assumed that the ratio of the average speed of public transport vehicles relative to car speed is constant at approximately 0.77, reflecting the relative speeds in the current situation. This information allowed us to calculate marginal congestion. The result was then combined with information on traffic composition and on respective values of in-vehicle time of car and public transport users to estimate the marginal external congestion costs.

b. Marginal external accident and environmental costs

Due to the limited data available, the marginal external costs other than congestion are assumed to be independent of traffic levels. The relevant information was taken from Mayeres (1993); the methodology is briefly reviewed in Appendix 4. The results yielded marginal external accident costs equal to 0.6489 BF per passenger-kilometre in both periods. With respect to the marginal environmental costs, we make an explicit distinction between marginal air pollution costs and marginal noise costs. The marginal air pollution costs associated with a car passenger-kilometre were calculated to be 0.354 BF, while the marginal noise costs associated with car use were found to be negligible (Mayeres, 1993).

c. Marginal private money costs

The marginal private money costs associated with an additional car-kilometre include expenses on fuel, tyres, oil and maintenance. The average variable private money costs, exclusive of taxes$^9$, were used as an approximation of the relevant marginal costs. Based on Cuijpers (1992), Zierock et al. (1989), and NIS (1990), the average variable private money costs$^{10}$ were estimated at 2.5525 BF per vehicle-kilometre or 1.5014 BF per passenger-kilometre. Note that these costs were assumed not to depend on traffic levels. In other words,

---

$^8$ 1 passenger car (PC) = 1 passenger car equivalent unit (PCU), and 1 public transport vehicle (PTV) = 2 passenger car equivalent units

$^9$ These marginal private money costs can be interpreted as the 'producer' price. The difference between the optimal price (to be determined) and this producer price is the optimal tax to be levied on the corresponding transport service.

$^{10}$ The variable private money costs are weighted by the proportion of total vehicle-kilometres travelled with gasoline, diesel and LPG cars.
the nonzero but empirically small effect of traffic levels on energy consumption was ignored.

3.2.2. **Marginal social costs and benefits associated with additional public transport**

When discussing the marginal social costs and benefits associated with extra public transport, it is useful to distinguish between supply and demand effects. The former consider the impact of extra supply at a given level of demand for passenger-kilometre. They include the extra monetary expenditures (on personnel, energy, materials, and reparations), marginal external costs (more congestion, more environmental pollution and more accident costs), and external benefits (a decrease in waiting time for the public transport passengers). In subsection a. we report how they were determined. Demand effects on the other hand look at extra costs associated with an increase in the number of passenger-kilometre for a given supply of vehicle-kilometre. Following Mohring (1972) and Viton (1983), it is assumed that an additional public transport passenger causes no additional congestion, environmental or accident costs. As a consequence, the relevant costs simply consist of marginal boarding and alighting time costs and marginal private money costs. The procedure to calculate these costs is discussed in subsection b.

a. **Marginal costs and benefits associated with additional public transport supply**

*Marginal private money costs*

The marginal private money costs of public transport supply, denoted by $\partial C/\partial Q^t$ in the theoretical model, were approximated by the average variable money costs, consisting of expenditures on drivers, energy (insofar as related to rolling stock), materials and reparations. De Borger et al. (1994, Appendix 5) provide more details on the calculation methods and the data.

The expenditures per kilometre on energy for rolling stock were calculated to be 8.6 BF per vehicle-km, those on materials and reparations amounted to 7.6 BF per vehicle-km. These expenditures were assumed to be independent of traffic volumes and the same for peak and off-peak periods. The average expenditures on drivers per kilometre are variable, however.
For peak and off-peak, the model calculates the time needed to drive one kilometre. This number is multiplied by the average gross hourly wage for drivers, viz. 751 BF.

**Marginal external effects**

As indicated by equation (18), the marginal external effects associated with additional public transport supply (denoted as $\sum a \partial g/a \partial Q$) consist of marginal external costs and marginal external benefits. The former include marginal congestion costs of a public transport vehicle-kilometre, marginal environmental costs, and marginal accident costs. The latter reflects the marginal reduction in waiting times.

The procedure for calculating the marginal congestion costs associated with additional public transport vehicle-kilometres is similar as that used for determining the marginal congestion costs associated with additional car use. First the time loss suffered by road users due to an extra public transport vehicle-kilometre is calculated, next the monetary valuation is estimated. For details we refer again to Appendix 3.

As before, all marginal external costs other than congestion costs are assumed to be constant. The marginal external environmental costs consist of air pollution and noise costs. Marginal external air pollution costs associated with an additional bus-kilometre are estimated to be 2.346 BF. Marginal external noise costs amount to 0.55 BF in the peak period and 1.88 BF in the off-peak period. The marginal external accident costs equal 2.5188 BF in both periods. The method of calculation is the same as before, see Appendix 4.

The external benefit caused by an additional public transport vehicle-kilometre consists of the decrease in waiting time for all public transport passengers. To calculate the marginal benefit we make use of a relation describing how average waiting time for public transport is influenced by the number of vehicle-kilometres supplied. Appendix 5 has all the details. Taking the derivative with respect to vehicle-kilometres supplied, we obtain the decrease in waiting time for one passenger. This waiting time decrease is then combined with the number of public transport passengers and their values of waiting time to find an estimate of the relevant marginal benefit. Based on this procedure and using information reported in Bradley
Gunn (1991) and Boniver (1993), we found that the value of waiting time for public transport users equals 218 BF per peak hour and 208 BF per off-peak hour.

b. Marginal social costs associated with additional public transport passenger-km ($S^3$ and $S^4$)

As previously suggested the marginal social costs associated with additional public transport passenger-kilometres simply consist of marginal boarding and alighting time costs and marginal private money costs for the transport companies. The marginal boarding/alighting time for an additional passenger was taken to be 2.5 seconds (Jansson (1984), Glaister (1985)). Since the average distance travelled by a public transport passenger in Belgian urban areas is approximately 4 kilometres (Boniver (1992)), the marginal boarding/alighting time for an additional passenger-kilometre equals 0.625 seconds. Of course, all bus travellers -on average there are $X^i / Q^i$ people aboard- experience this delay\(^\text{11}\). Their value of in-vehicle time was calculated to be 151 BF per peak hour and 137 BF per hour in the off-peak period. Using this information yields the marginal boarding and alighting time costs associated with an additional public transport passenger. More details are given in Appendix 6.

The private money costs for the transport companies caused by an additional passenger are estimated to be in the range of 0.35 to 1.1 BF\(^\text{12}\). Using the fact that the average distance travelled by a public transport passenger in Belgian urban areas is approximately 4 kilometres, we used 0.2 BF as a crude estimate of the marginal private money cost.

3.3. Marginal effects of additional road infrastructure

Following Viton (1983), we assume that the location and length of the road is given, such that the decision variable $I$ has to be interpreted as the number of lane-kilometres supplied.

\(^{11}\) By boarding or alighting the passenger will cause some people further along the route to wait longer. Other people will have to wait for a shorter time, since they would have had to wait for the next bus if this one had not been delayed. Following Turvey and Mohring (1975), we assume that these two effects neutralise each other.

\(^{12}\) These cost information is based on interviews with Mr. De Vos, Mr. Fys and Ms. Vinck of the study centre of De Lijn. The marginal private money costs are the material costs associated with providing passengers with tickets.
We estimated that in the reference year 1989 there were approximately 15 000 lane-kilometres in the relevant urban areas\textsuperscript{13}.

Additional road infrastructure involves costs and benefits. First, the average construction, maintenance and expropriate costs were used as an approximation of the marginal infrastructure costs\textsuperscript{14}. These costs were determined at 6 248 BF per lane-kilometre and per day. Second, when more lane-kilometres are available, traffic can move faster, resulting in a decreasing time needed to travel one kilometre. The monetary value of this time saving can be calculated as follows. For each transport service, we multiply the marginal time saving by the number of passenger-kilometres and by their values of in-vehicle time. The sum over the transport services gives the monetary value of the time saving due to the increase in infrastructural investments. Appendix 8 summarizes the calculations and the data used.

3.4. The budget constraint

To illustrate the impact of budgetary constraints, we also analyse a model with a budget restriction on the public transport sector. The constraint implies that the sum of all variable private costs of public transport, the capacity costs of rolling stock, and the fixed costs should be covered by public transport revenues plus a maximum allowed deficit (or surplus). Fixed costs, FC, were calculated in De Borger et al. (1994, Appendix 5) and amount to 16 765 086 BF per day. Appendix 9 provides more details on the determination of daily capacity costs of rolling stock, calculated to be 5 207 BF per vehicle.

4. SOME SIMULATION RESULTS FOR BELGIUM

In this section, we report some simulation results obtained using the non-linear optimisation

\textsuperscript{13} More information can be found in Appendix 2.

\textsuperscript{14} In Appendix 7 the relevant data and the calculation method are discussed. Please note that in our analysis, $\Sigma \partial g_i / \partial I$ and $\partial C_s / \partial I$ only capture monetary costs.
program GAMS/MINOS (Brooke et al. (1992))\(^{15}\). We first carefully discuss the results of the basic model, in which no budgetary restrictions are imposed. We look at optimal prices, optimal public transport supply, optimal level of road infrastructure and corresponding traffic levels, consider marginal social costs and average speed at the optimum, and compare these optimal values with the observed values of the initial situation. Second, we analyse optimal prices, optimal public transport supply and optimal infrastructural investment for models with a budget constraint for the public transport sector\(^{16}\).

4.1. Basic model: no budget constraint

In the absence of budgetary constraints and assuming that the government can use first-best instruments, optimal prices were shown to equal marginal social costs, and optimal supply levels were found to be such that the marginal social cost of extra supply, including the shadow price of extra vehicles, equals the marginal social benefit (see (39)). Relevant simulation results for this case are given in Table 1. For each relevant variable, the table contains the observed value in the reference situation, the optimal value, and the percentage change.

First, consider the optimal prices of the basic model. For car transport, optimal prices turn out to be substantially higher than current price levels. Specifically, internalisation of externalities yields car prices in the peak that are 86% above current prices, whereas off-peak prices have risen by 12% compared to reference prices. Since, conditional on capacity and quantity of vehicle-kilometres supplied, extra passengers imply a relatively low marginal cost (i.e., consisting of marginal boarding and alighting time costs and the marginal costs

\(^{15}\) Of course, the model uses a large number of estimated parameters and data from a variety of sources. In order to test the sensitivity of the results with respect to the most important demand and cost parameters a substantial number of additional simulations were carried out. Not surprisingly, we found the numerical results to be somewhat sensitive to the assumed own price elasticities, to variations in the marginal private cost of public transport, and to the initial level of infrastructure. However, the qualitative conclusions to be derived from these exercises were in no way affected.

\(^{16}\) Note that, in the simulations, we leave the assumption that \( \kappa' \) is constant. \( \kappa' \) is here defined such that the number of vehicle-kilometres supplied in the peak and off-peak period \( (Q) \) is bounded by the distance B buses are able to travel during the relevant period. This involves some adjustments in the definition of the marginal social costs and in the first-order optimality conditions.
### Table 1: Simulation results: optimal pricing and supply results

<table>
<thead>
<tr>
<th></th>
<th>INITIAL SITUATION</th>
<th>BASIC OPTIMUM without budget constraint Optimal values</th>
<th>% change w.r.t. initial situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRICES (BF per passenger km)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Car</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak</td>
<td>2.665</td>
<td>4.968</td>
<td>96.42%</td>
</tr>
<tr>
<td>Off-peak</td>
<td>2.665</td>
<td>2.986</td>
<td>12.05%</td>
</tr>
<tr>
<td>Bus &amp; tram</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak</td>
<td>3.460</td>
<td>1.345</td>
<td>-51.13%</td>
</tr>
<tr>
<td>Off-peak</td>
<td>3.460</td>
<td>0.550</td>
<td>-84.10%</td>
</tr>
<tr>
<td>SUPPLY</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Infrastructure (lane km)</td>
<td>15 000</td>
<td>19 850</td>
<td>32.33%</td>
</tr>
<tr>
<td>Bus &amp; tram (vehicle km a day)</td>
<td>66 651</td>
<td>75 432</td>
<td>13.17%</td>
</tr>
<tr>
<td>Off-peak</td>
<td>129 382</td>
<td>199 817</td>
<td>54.44%</td>
</tr>
<tr>
<td>Off-peak</td>
<td>577</td>
<td>506</td>
<td>-12.34%</td>
</tr>
<tr>
<td>MARGINAL SOCIAL COSTS of PASSENGER KM (BF per passenger km)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Car</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak</td>
<td>11.635</td>
<td>4.968</td>
<td>-57.30%</td>
</tr>
<tr>
<td>Off-peak</td>
<td>3.244</td>
<td>2.986</td>
<td>-7.95%</td>
</tr>
<tr>
<td>Bus &amp; tram</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak</td>
<td>3.781</td>
<td>1.345</td>
<td>72.22%</td>
</tr>
<tr>
<td>Off-peak</td>
<td>0.415</td>
<td>0.550</td>
<td>32.53%</td>
</tr>
<tr>
<td>MARGINAL SOCIAL EFFECTS of PUBLIC TRANSPORT VEHICLE KM (BF per vehicle km)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marginal Social Costs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak</td>
<td>85.175</td>
<td>55.100</td>
<td>-35.31%</td>
</tr>
<tr>
<td>Off-peak</td>
<td>47.008</td>
<td>45.472</td>
<td>-3.27%</td>
</tr>
<tr>
<td>Marginal Social Benefits</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak</td>
<td>52.899</td>
<td>90.093</td>
<td>70.31%</td>
</tr>
<tr>
<td>Off-peak</td>
<td>44.808</td>
<td>45.472</td>
<td>1.48%</td>
</tr>
<tr>
<td>TRAFFIC FLOW (mio passenger km a day)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Car</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Peak</td>
<td>47.313</td>
<td>40.968</td>
<td>-13.41%</td>
</tr>
<tr>
<td>Off-peak</td>
<td>48.695</td>
<td>45.608</td>
<td>-8.34%</td>
</tr>
<tr>
<td>Total</td>
<td>96.008</td>
<td>86.576</td>
<td>-9.82%</td>
</tr>
<tr>
<td>Bus &amp; tram</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak</td>
<td>1.544</td>
<td>3.369</td>
<td>118.14%</td>
</tr>
<tr>
<td>Off-peak</td>
<td>1.297</td>
<td>3.140</td>
<td>142.05%</td>
</tr>
<tr>
<td>Total</td>
<td>2.842</td>
<td>6.509</td>
<td>129.06%</td>
</tr>
<tr>
<td>Total passenger km (mio a day)</td>
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<td>93.085</td>
<td>-5.83%</td>
</tr>
<tr>
<td>Total vehicle km (mio a day)</td>
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<td>51.202</td>
<td>-9.65%</td>
</tr>
<tr>
<td>AVERAGE SPEED (km/h)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Car</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak</td>
<td>30.000</td>
<td>38.735</td>
<td>29.12%</td>
</tr>
<tr>
<td>Off-peak</td>
<td>45.271</td>
<td>46.702</td>
<td>3.16%</td>
</tr>
<tr>
<td>Bus &amp; tram</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak</td>
<td>23.100</td>
<td>29.826</td>
<td>29.12%</td>
</tr>
<tr>
<td>Off-peak</td>
<td>34.859</td>
<td>39.961</td>
<td>3.16%</td>
</tr>
<tr>
<td>SURPLUS (BF a day)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transport sector as a whole</td>
<td>-72 657</td>
<td>62 054 400</td>
<td>-85 507.80%</td>
</tr>
<tr>
<td>Private transport sector</td>
<td>17 994 400</td>
<td>39 727 100</td>
<td>376.41%</td>
</tr>
<tr>
<td>Public transport sector</td>
<td>-18 067 070</td>
<td>-23 672 650</td>
<td>31.03%</td>
</tr>
<tr>
<td>MULTIPLIERS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>capacity constraint peak μ3</td>
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<td></td>
<td>34.916</td>
</tr>
<tr>
<td>capacity constraint off-peak μ4</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>budget restriction μ</td>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>
associated with handling tickets), the optimal prices for public transport combined with optimal supply decisions are in both periods considerably below current prices. Public transport prices decrease by 61% in the peak, and by 84% in the off-peak period. Finally, note that optimal prices per passenger-kilometre are higher for private transport than for public transport, unlike current prices.

The optimal level of infrastructure is about 32% higher than the estimated current level. Optimal supply of public transport in the peak period is 13% higher than the 1989 supply level; for the off-peak period, the increase even amounts to 54%. The reason for the substantial increase in off-peak public transport supply is due to the observation that reductions in waiting time can be realised at very low extra costs. Since the capacity of vehicles is determined by transport needs in the peak period the marginal cost of extra vehicle-kilometres in the off-peak is small.

The optimal fare and supply structure causes 13% and 6% reductions in peak and off-peak car traffic, respectively. The negative impact of the own price increases clearly dominate cross-price effects and the positive demand effect of the increase in average speed. Not surprisingly, the percentage changes in public transport use are much more pronounced. We note an increase by 118% and 142% for the peak and the off-peak period, respectively. These results are mainly induced by the reduction in fares and the increase in public transport supply. Of course, the absolute magnitude of the percentage changes should be interpreted in view of the small share of public transport in the reference situation. In 1989, about 97% of people travelling were using private car.

The ultimate result is a 6% decrease in the total number of passenger-kilometres. Total car traffic decreases by 10%, whereas public transport use increases by 129%. The share of public transport rises from 3% to some 7%. This shift is consistent with results found in Viton (1983). He found extreme transit-favoring modal splits when all pricing and investment decisions are made correctly\(^\text{17}\).

\(^{17}\) Note that when non-monetary costs of road infrastructure would be considered, the optimal level of road infrastructure would be lower, implying less private car use and more public transport use.
The substantial increase in the number of passenger-kilometres travelled by public transport involves a substantial increase in occupancy rates; in 1989 there were on average 23 people on a public transport vehicle in the peak and 10 in the off-peak, whereas in the optimum there are on average 44 people aboard in the peak and 15 in the off-peak. In terms of the environmental impact it is interesting to note that the number of vehicle-kilometres travelled declines by 10%.

Increased use of public transport, the reduction in vehicle-kilometres by cars, and provision of more road infrastructure all contribute to reduced congestion and imply an increase in average speed for both modes relative to the initial situation. The rise in speed has a somewhat surprising side-effect. Indeed, compared to the initial situation, the optimum yields a 12% decrease in the number of public transport vehicles B despite the increase in public transport supply. Higher average speed (an increase of almost 30% in the peak) allows public transport companies to provide more services with a somewhat smaller fleet. Note, by the way, that the capacity constraint is binding in the peak period and non-binding in the off-peak period, as was expected.

In comparison with the initial situation, marginal social costs of car use have been reduced, whereas those associated with public transport use have been increased. For private car and public transport, the change in marginal social costs associated with peak traffic is larger than the one associated with off-peak traffic. In both periods, the percentage change in marginal social costs is larger for the public transport mode than for the private mode. Note that the changes are actually quite substantial.

A final remark relates to the transport sector deficit: with optimal prices and supply levels, the public transport deficit\(^{18}\) increases by 31%. This is however more than compensated by the increase in government revenues due to heavier taxation of car use. Overall, the deficit of the transport sector is transformed into a sizeable surplus.

\(^{18}\) The public transport sector deficit equals total (private, fixed and capacity) costs minus public transport revenues: \(C^3 + C^4 + FC + C_B(B) - (p^*X^3 + p^*X^4)\). The private transport sector surplus is calculated as: private car tax revenues minus infrastructural costs; \(p^*X^1 + p^*X^2 - C^5 - C^6(B)\). The surplus of the transport sector as a whole is defined as: private transport surplus minus public transport deficit; \(p^*X^1 + p^*X^2 + p^*X^3 + p^*X^4 - C^5 - C^6 - C^7 - C^8(B)\).
4.2. **Introduction of a budget constraint on the public transport sector**

To illustrate the impact of budgetary constraints, we analyse a model with a budget restriction on the public transport sector. We implicitly assume for the purpose of these simulation that the fixed costs of public transport remain at their 1989 level\textsuperscript{19}.

Table 2 reports some results. The first column repeats the results of the basic optimum without budget constraint as a basis for comparison. The second column gives the results for the case where the maximum public transport deficit allowed is the observed deficit for 1989. Columns 3 and 4 report simulation results for more restrictive deficit constraints; they refer to a 10% and a 30% reduction in the observed deficit, respectively.

In order to remain within the observed 1989 budget all optimal prices have to rise in comparison with the unconstrained optimum (see column 2). This is not surprising, because the 1989 transport deficit is substantially (some 24%) below the level at the unconstrained optimum. Public transport prices increase by 105% in the peak and 75% in the off-peak. The budget restriction on the public transport sector also makes car use more expensive. With nonzero cross price elasticities, the increasing public transport prices imply increasing congestion. To counteract this negative effect optimal car prices rise as well, viz. by 8% in the peak and 2% in the off-peak.

Interestingly, imposition of a budget restriction implies a decrease in optimal public transport supply in the off-peak period, but an increase in peak-period supply as well as in the optimal number of public transport vehicles. These seemingly strange results can be explained as follows. First consider peak-period supply of vehicle-kilometres. Extra supply induces more people to use public transport. However, as public transport prices exceed marginal social costs of extra passenger-kilometres at the optimum (and therefore certainly marginal private

\textsuperscript{19} Strictly speaking, fixed costs (which consist of expenditures on non-driving personnel, costs of energy for infrastructure, of materials, reparations and deliveries with respect to infrastructure, of insurance and of depreciation expenses other than those for rolling stock) may deviate from the 1989 level. Therefore, the budget restriction imposed is used as an example to illustrate the impact of budgetary constraints.
| Table 2: Simulation results: imposing a public transport sector budget restriction |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|
|                                 | BASIC OPTIMUM   | OPTIMUM with budget constraint 1 |
|                                 | Optimal values  | D = initial deficit | Optimal % change w.r.t. basic optimum | D = 90% of initial deficit | Optimal % change w.r.t. basic optimum | D = 70% of initial deficit | Optimal % change w.r.t. basic optimum |
|                                 | Optimal values  | D = initial deficit | Optimal % change w.r.t. basic optimum | D = 90% of initial deficit | Optimal % change w.r.t. basic optimum | D = 70% of initial deficit | Optimal % change w.r.t. basic optimum |
| PRICES (BF per passenger km)    |                 |                 |                 |                 |                 |                 |                 |
| Car                             |                 |                 |                 |                 |                 |                 |                 |
| Peak                            | 4.958           | 5.352           | 7.73%           | 5.492           | 10.55%          | 5.786           | 16.47%          |
| Off-peak                        | 2.956           | 3.041           | 1.84%           | 3.059           | 2.44%           | 3.095           | 3.65%           |
| Bus & tram                      |                 |                 |                 |                 |                 |                 |                 |
| Peak                            | 1.345           | 2.754           | 104.76%         | 3.096           | 130.33%         | 3.612           | 165.55%         |
| Off-peak                        | 0.550           | 0.961           | 74.73%          | 1.118           | 103.27%         | 1.479           | 165.91%         |
| SUPPLY                          |                 |                 |                 |                 |                 |                 |                 |
| Infrastructure (lane km)        |                 |                 |                 |                 |                 |                 |                 |
| Bus & tram (vehicle km a day)   |                 |                 |                 |                 |                 |                 |                 |
| Peak                            | 19,859          | 19,859          | 0.04%           | 19,836          | -0.07%          | 19,781          | -0.35%          |
| Off-peak                        | 75,432          | 111,714         | 48.10%          | 127,873         | 69.52%          | 163,029         | 116.13%         |
| Bus & tram (buses a day)        |                 |                 |                 |                 |                 |                 |                 |
|                                 | 199,817         | 174,328         | -12.76%         | 169,333         | -15.26%         | 161,911         | -19.36%         |
|                                 | 506             | 748             | 47.79%          | 855             | 66.98%          | 1,087           | 114.93%         |
| MARGINAL SOCIAL COSTS OF PASSENGER KM (BF per passenger km) |                 |                 |                 |                 |                 |                 |                 |
| Car                             |                 |                 |                 |                 |                 |                 |                 |
| Peak                            | 4.953           | 4,975           | -0.14%          | 4,978           | 0.20%           | 4,985           | 0.36%           |
| Off-peak                        | 2.956           | 2,964           | -0.07%          | 2,985           | -0.03%          | 2,985           | -0.03%          |
| Bus & tram                      |                 |                 |                 |                 |                 |                 |                 |
| Peak                            | 1,345           | 1,132           | -15.84%         | 1,091           | -18.86%         | 1,034           | -23.12%         |
| Off-peak                        | 0.550           | 0.520           | -5.45%          | 0.510           | -7.27%          | 0.490           | -10.91%         |
| MARGINAL SOCIAL EFFECTS OF PUBLIC TRANSPORT VEHICLE KM (BF per vehicle km) |                 |                 |                 |                 |                 |                 |                 |
| Marginal Social Costs           |                 |                 |                 |                 |                 |                 |                 |
| Peak                            | 55,100          | 55,021          | -0.14%          | 54,987          | -0.21%          | 54,914          | -0.34%          |
| Off-peak                        | 45,472          | 45,470          | 0.00%           | 45,473          | 0.00%           | 45,461          | 0.02%           |
| Marginal Social Benefits        |                 |                 |                 |                 |                 |                 |                 |
| Peak                            | 90,093          | 49,769          | -44.74%         | 41,628          | -53.79%         | 30,639          | -55.99%         |
| Off-peak                        | 45,472          | 47,933          | 5.54%           | 47,932          | 5.41%           | 47,417          | 4.28%           |
| TRAFFIC FLOW (mio passenger km a day) |                 |                 |                 |                 |                 |                 |                 |
| Car                             |                 |                 |                 |                 |                 |                 |                 |
| Peak                            | 40,968          | 40,617          | -0.86%          | 40,390          | -1.41%          | 39,879          | -2.56%          |
| Off-peak                        | 45,606          | 45,814          | 0.45%           | 45,851          | 0.53%           | 46,002          | 0.64%           |
| Total                           | 86,574          | 86,430          | -0.17%          | 86,241          | -0.33%          | 86,902          | -0.92%          |
| Bus & tram                      |                 |                 |                 |                 |                 |                 |                 |
| Peak                            | 3,359           | 4,084           | 21.21%          | 4,474           | 32.76%          | 5,352           | 58.85%          |
| Off-peak                        | 3,140           | 2,523           | -19.57%         | 2,377           | -24.30%         | 2,128           | -32.23%         |
| Total                           | 6,499           | 6,607           | 1.49%           | 6,851           | 5.25%           | 7,480           | 14.52%          |
| Total passenger km (mio a day)  | 93,085          | 93,037          | -0.50%          | 93,091          | 0.01%           | 93,251          | 0.19%           |
| Total vehicle km (mio a day)    | 51,202          | 51,127          | -0.15%          | 51,027          | -0.34%          | 50,783          | -0.52%          |
| AVERAGE SPEED (km/h)            |                 |                 |                 |                 |                 |                 |                 |
| Car                             |                 |                 |                 |                 |                 |                 |                 |
| Peak                            | 38.735          | 38,817          | 0.21%           | 38,859          | 0.32%           | 38,951          | 0.56%           |
| Off-peak                        | 38.732          | 46,936          | -2.01%          | 46,690          | -0.33%          | 46,878          | -0.05%          |
| Bus & tram                      |                 |                 |                 |                 |                 |                 |                 |
| Peak                            | 29,828          | 29,889          | 0.21%           | 29,921          | 0.32%           | 29,992          | 0.56%           |
| Off-peak                        | 38,961          | 35,955          | -8.02%          | 35,951          | -0.33%          | 35,942          | -0.05%          |
| SURPLUS (BF a day)              |                 |                 |                 |                 |                 |                 |                 |
| Transport sector as a whole     | 62,054          | 84,794          | 36.65%          | 92,416          | 48.93%          | 107,791         | 73.70%          |
| Private transport sector        | 85,727,100      | 102,862,000     | 19.99%          | 108,677,000     | 26.77%          | 120,438,000     | 40.49%          |
| Public transport sector         | -23,672,650     | -18,067,070     | -23.58%         | -16,260,360     | -31.31%         | -12,645,950     | -46.58%         |
| MULTIPLIERS                     |                 |                 |                 |                 |                 |                 |                 |
| capacity constraint peak μ3     | 34,915          | 38,626          | 11.35%          | 38,972          | 73.1%           | 39,315          | 0%              |
| capacity constraint off-peak μ4 | 0               | 0               | 0               | 0               | 0               | 0               | 0               |
| budget restriction              | 0               | 0.109           | 0.12            | 0               | 0               | 0               | 0               |
costs), the extra revenues that are generated by far exceed the extra variable operating costs associated with serving more passengers. This revenue increase more than offsets the marginal private cost of extra supply of vehicle-kilometres (including the cost of extra capacity), resulting in an increase in profit for the public transport company. The requirement to cut the deficit is therefore consistent with both more peak public transport supply and an increase in the number of public transport vehicles operated.

Despite a higher capacity of available vehicles, supply in the off-peak period declines due to more stringent budgetary restrictions. To understand this, note that price increases are much less pronounced than in the peak. As a consequence, the revenue increase generated by the extra demand for off-peak travel induced by more off-peak supply is insufficient to cover the extra direct (associated with extra supply) and indirect (associated with supply-induced extra passengers) operating costs. Therefore, the imposition of a budget constraint leads to a decrease in the number of off-peak vehicle-kilometres supplied.

Imposing a budget restriction affects the modal composition of transport flows, but, as can be seen in Table 2, total transport demand is hardly affected. Similarly, the impact on average speed is also very small. Finally, the results do yield higher occupancy rates for public transport vehicles: imposing the 1989 deficit as a restriction implies average occupancy rates of 36 and 14 passengers in the peak and off-peak periods, respectively.

Columns 3 and 4 in Table 2 indicate that, as expected, more stringent budget restrictions lead to even higher prices for all modes. Moreover, it results in an increase in peak public transport supply, a decrease in off-peak public transport supply, an increase in the number of buses used, and a decrease in the number of lane-kilometres available. Average speed of the different modes hardly changes. These trends imply small changes in private car use and more substantial changes in public transport use. However, the total number of passenger-kilometres and vehicle-kilometres travelled remains approximately constant.

A final remark is in order. The exercises reported in this paper clearly suggest that socially optimal pricing and supply decisions may require adjustments in current prices and public transport supply that are so large as to be politically unacceptable. However, the purpose of
this paper was not to come up with an immediately applicable pricing structure. Its primary purpose was precisely to provide information on the magnitude of the deviations of the current pricing structure and public transport supply levels from their socially optimal levels, determined by taking all relevant externalities into account. The most desirable path towards this social optimum was not explicitly studied in this paper.

5. CONCLUSIONS

In this paper, we analysed the introduction of social cost considerations in the pricing of urban transport and in the determination of the optimal supply levels of both public transport and road infrastructure. The model was applied to study optimal transport prices and optimal supply levels using Belgian data. Due to limitations on data availability the empirical analysis uses aggregate data and therefore ignores distributional considerations. It does capture all relevant private and external costs (associated with, e.g., congestion, air pollution, noise, accident risks, and boarding and alighting times) as well as external benefits associated with additional public transport and road infrastructure.

Ignoring distributional and budgetary considerations, optimal prices were found to be well above the 1989 level for private transport (86% in the peak period and 12% in the off-peak period). For public transport, the model yields optimal prices that are much lower than the 1989 fares (61% in the peak and 84% in the off-peak). Optimal prices are higher in the peak than in the off-peak, and in both periods optimal transport prices are higher for private car use than for public transport use. Optimal supply of public transport is substantially above the actual supply level for the peak and off-peak period. Furthermore, there is a 32% increase in lane capacity of the road system. When all prices and supply levels are optimally determined, we observe a 6% decrease in the number of passenger-kilometres travelled. The number of vehicle-kilometres is reduced by 10%. The increase in lane capacity and the decrease of vehicle-kilometres travelled involve an increase in average speed for both modes and periods. It turns out that, with respect to the initial situation, less buses are needed to provide the optimal level of public transport vehicle-kilometres. The public transport sector deficit increases by 31%.
A number of simulations were performed to study the effects of imposing a budget restriction on the public transport sector. Not surprisingly, prices of all modes and in all periods rise with respect to the basic model. Interestingly, for the peak period, there is an increase in public transport supply as compared to the unconstrained optimum. Additional supply encourages people to use public transport, which, at optimal prices that by far exceed private marginal costs, yields an increase in revenues that more than compensates for the increase in costs. In the off-peak period, the opposite is true. Imposing a public transport sector budget restriction leads to less public transport supply in the off-peak.
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APPENDICES

Appendix 1. Rewriting the first-order conditions with respect to prices in matrix notation

The first-order conditions with respect to prices are given by equation (25)

\[-\sum_{h} \sigma_{h} (x'_{h} + \sum_{i=1}^{4} \frac{\partial g_{h}}{\partial x^{i}} \frac{\partial X^{i}}{\partial p^{j}}) + (1+\lambda)(X' + \sum_{i=1}^{2} (p^{i} - \frac{\partial C^{i}}{\partial q^{i}} \frac{\partial Q^{i}}{\partial x^{i}}) \frac{\partial X^{i}}{\partial p^{j}} + \sum_{i=3}^{4} (p^{i} - \frac{\partial C^{i}}{\partial x^{i}}) \frac{\partial X^{i}}{\partial p^{j}}) = 0\]

for \ j=1,...,4

Using equation (14) and rearranging terms, these optimality conditions can be rewritten as

\[-\sum_{i=1}^{4} \sum_{h} \sigma_{h} m_{i}c_{h}^{i} + (1+\lambda) \left( \sum_{i=1}^{2} (p^{i} - \frac{\partial C^{i}}{\partial q^{i}} \frac{\partial Q^{i}}{\partial x^{i}}) + \sum_{i=3}^{4} (p^{i} - \frac{\partial C^{i}}{\partial x^{i}}) \right) \frac{\partial X^{i}}{\partial p^{j}} \]

\[-\sum_{h} \sigma_{h} x'_{h} -(1+\lambda)X^{j} \]

for \ j=1,...,4

Introducing \( S_{i} \) in these formulas gives

\[-\sum_{i=1}^{4} S^{i} - \lambda \left( \sum_{i=1}^{2} \frac{\partial C^{i}}{\partial q^{i}} \frac{\partial Q^{i}}{\partial x^{i}} + \sum_{i=3}^{4} \frac{\partial C^{i}}{\partial x^{i}} \right) + (1+\lambda) \sum_{i=1}^{4} p^{i} \right) \frac{\partial X^{i}}{\partial p^{j}} \]

\[-\sum_{h} \sigma_{h} x'_{h} -(1+\lambda)X^{j} \]

for \ j=1,...,4

Multiplying both sides by \(-(p/X)(X/p)^{i}\) leads to

\[\sum_{i=1}^{4} S^{i} + \lambda \left( \sum_{i=1}^{2} \frac{\partial C^{i}}{\partial q^{i}} \frac{\partial Q^{i}}{\partial x^{i}} + \sum_{i=3}^{4} \frac{\partial C^{i}}{\partial x^{i}} \right) - (1+\lambda) \sum_{i=1}^{4} p^{i} \right) \frac{\partial X^{i}}{\partial p^{j}} \frac{p^{i}}{X^{i}} \frac{X^{i}}{p^{j}} \]

\[-(\sum_{h} \sigma_{h} x'_{h} -(1+\lambda)X^{j}) \]

for \ j=1,...,4

Introducing \( \eta_{i}^{j} \) and matrix notation and rearranging terms leads to equation (34).
Appendix 2. Data needed to calibrate the aggregate demand functions

In this appendix we summarize relevant information to calibrate the parameters in equation (54). First, we discuss the elasticity values: \( \eta_p, \epsilon_{q_t}, \epsilon_{t} \) and \( \tau_{t} \). Second, we summarize daily traffic levels and prices for the different transport services, \( X_t \) and \( p_t \), and look at the supply variables associated with public transport, \( Q_t \) and \( B \). Finally, we explain how infrastructural investments, \( I \), influence the aggregated demand. All data refer to the year 1989\(^{20}\).

a. Elasticity values

(1) Based on empirical literature (e.g. Oum, Waters and Yong (1992), Goodwin (1992), ’t Hoen, Kuik and Poppelaars (1991), Webster (1977), Ruitenberge (1983), and Glaister and Lewis (1978)) we composed the following matrix of price elasticities, \( \eta_{ij} \):

<table>
<thead>
<tr>
<th>price/demand</th>
<th>peak car</th>
<th>off-peak car</th>
<th>peak pub.tr.</th>
<th>off-peak pub.tr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>peak car</td>
<td>-0.3</td>
<td>0.048</td>
<td>0.57</td>
<td>0</td>
</tr>
<tr>
<td>off-peak car</td>
<td>0.05</td>
<td>-0.6</td>
<td>0</td>
<td>0.57</td>
</tr>
<tr>
<td>peak pub.tr.</td>
<td>0.02</td>
<td>0</td>
<td>-0.2</td>
<td>0.03</td>
</tr>
<tr>
<td>off-peak pub.tr.</td>
<td>0</td>
<td>0.02</td>
<td>0.03</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

(2) The matrix of supply elasticities (elasticities with respect to the number of scheduled vehicle-kilometres and with respect to the infrastructural investments) is given by:

<table>
<thead>
<tr>
<th>supply/demand</th>
<th>peak car</th>
<th>off-peak car</th>
<th>peak pub.tr.</th>
<th>off-peak pub.tr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>peak pub.tr.</td>
<td>-0.005</td>
<td>0</td>
<td>0.7</td>
<td>0</td>
</tr>
<tr>
<td>off-peak pub.tr.</td>
<td>0</td>
<td>-0.005</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>infrastructure</td>
<td>0.02</td>
<td>0.02</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Based on empirical literature (Webster (1977), Van de Voorde (1981), Ruitenberge (1983), Varrii Auctores (1980) and Lago et al. (1981)), we assume that the vehicle-kilometres elasticity for public transport equals 0.7 in the peak and 0.2 in the off-peak period. We further assume that a change in public transport supply in the peak has no impact on traffic levels in the off-peak

---

\(^{20}\) In 1989, 1 US$ corresponded to 39.43 BF and 1 pound sterling was equal to 64.55 BF.
and visa versa. Since we were unable to obtain reliable empirical estimates of the other supply elasticities, we used values that we believe to be reasonable.

(3) Demand elasticities with respect to speed, $\eta$, were taken from Van de Voorde (1981), Webster (1977) and Ruitenbergh (1983). Relevant values used amounted to 0.8 for public transport and to 0.2 for private car use. We were unfortunately unable to find time elasticities that differentiated between the peak and off-peak periods.

b. Daily traffic levels and prices

Table A1 recapitulates the data on traffic levels and prices (see De Borger et al. (1994) for sources and further information).

c. Daily supply of public transport

The number of vehicle-kilometres supplied by public transport in the different cities under consideration is taken from "Verkeer en Vervoer in België" (Belgium, Ministerie van verkeer en infrastructuur (1990)). Dividing this annual number by 365, leads to the average daily supply. As the public transport companies supply more vehicle-kilometres during a peak hour than in an off-peak hour, we tried to differentiate between peak and off-peak supply. Based on Dorssomont (1984) and MIVB (1990) we calculated that 34% of the vehicle-kilometres is supplied in the peak period, while 66% is supplied in the off-peak period. Using these percentages leads to the values for $Q^3$ and $Q^4$ in Table A2.

d. Number of public transport vehicles

The definition of B used in the paper deviates from the observed capacity of vehicles. In the initial situation, we defined B as the number of vehicles that are necessary to produce the observed number of vehicle-kilometres in the peak period, given observed commercial speed. In other words, using data on speed and the supply of vehicle-kilometres, B was calculated for the reference situation by solving $Q^3 = \alpha^3 B$. In the initial situation B equals 577. This number does not exactly correspond to observed vehicle capacity. Indeed, in practice some vehicles
remain unused because of maintenance operations, or as a back-up in case of mechanical defects. These vehicles were not taken into account in our model; our procedure implicitly assumes that this excess capacity remains constant.

**e. Infrastructural investments: number of lane-km supplied in the Belgian urban area**

Since we were not able to find accurate data on the number of lane-kilometres supplied in the relevant Belgian urban areas, they were approximated as follows. According to NIS (1988), 10.8% of the surface of Belgium (= 329 594.5 ha) is urbanised. The cities under consideration have a total urbanised surface of 28 249 ha, which is approximately 8.5% of the figure for Belgium. Based on "Verkeer en vervoer in België" (Belgium, Ministerie van verkeer en infrastructuur (1990)) and Spilstijns (1992), we calculated that in Belgium there are 242 594.7 lane-kilometres, of which we left highway lane-kilometres as well as 'gewestwegen' out of the analysis. Applying the 8.5% mentioned before yielded approximately 15 000 lane-kilometres as a rough estimate.

**Appendix 3. Determination and valuation of marginal external congestion costs**

**a. Average time needed to drive one kilometre**

The relation describing how the average time needed to drive one kilometre by car depends on the number of passenger car equivalent unit kilometres per hour and on the level of road infrastructure, was based on several crude 'observations' on average speed and traffic levels in the initial situation. For peak and off-peak, the average time needed to drive one kilometre by car is given by:

---

21 It is assumed that travel speed of freely flowing traffic is 50 kilometres per hour, that average speed at the current peak traffic level is approximately 30 kilometres per hour, and that average speed drops to 10 kilometres per hour with a traffic level of approximately 7 000 000 PCU per hour. More detail can be found in De Borger et.al. (1994).
\[ t^i = \frac{-10464.204}{[ -138926.12 - (138926.12^2 + 20928.408 \cdot 6133949 - 20928.408 \cdot 15000 \cdot \frac{PCU\text{km/h}^i}{I} )^{1/2} ]} \]

for \( i=1,2 \)

where \( PCU\text{km/h}^i \) equals \((Q^1 + 2*Q^3)/5\), \( PCU\text{km/h}^2 \) equals \((Q^2 + 2*Q^4)/17\) and \( I \) is the level of road infrastructure. Average speed of public transport vehicles amounts to approximately 77\% of average car speed in the current situation, so that

\[ t^3 = \frac{t^1}{0.77} \quad \text{and} \quad t^4 = \frac{t^2}{0.77} \]

b. Value of in-vehicle time

The value of in-vehicle time (IVT) for car and public transport users depends on several factors. One of those factors is the trip purpose. Bradley (1990) provides us with a whole range of monetary valuations of in-vehicle time for car and public transport users. He makes a distinction between three journey purposes: business, commuting and other motives. According to Boniver (1993), the values of in-vehicle time presented in Table A3 are most representative for Belgian urban areas. Table A4 presents data on the relative importance of trip purpose for Belgian cities (see Boniver (1993)). Combining these data, we can calculate the following weighted average values of in-vehicle time for car and public transport users:

<table>
<thead>
<tr>
<th></th>
<th>Peak</th>
<th>Off-peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car</td>
<td>252 BF/hour</td>
<td>297 BF/hour</td>
</tr>
<tr>
<td>Bus/Tram</td>
<td>151 BF/hour</td>
<td>137 BF/hour</td>
</tr>
</tbody>
</table>

c. Marginal external congestion costs

To calculate marginal external congestion costs, we combine marginal congestion with relevant information on traffic flows and values of in-vehicle time. Marginal congestion is defined as the marginal effect of an additional kilometre travelled on the average time needed to drive one kilometre, which can be calculated using the relation between time needed to travel one kilometre and vehicle flows described above. The marginal external congestion costs of a peak
car passenger-kilometre are defined as:

\[
MECC_1 = \frac{\partial t^1}{\partial X^1} \cdot [X^1 \cdot IVT^1] + \frac{\partial t^3}{\partial X^1} \cdot [X^3 \cdot IVT^3]
\]

where IVT is the value of in-vehicle time for transport service i (BF per hour).

By analogy, marginal external congestion costs associated with additional car passenger-kilometres in the off-peak period is given by:

\[
MECC_2 = \frac{\partial t^2}{\partial X^2} \cdot [X^2 \cdot IVT^2] + \frac{\partial t^4}{\partial X^2} \cdot [X^4 \cdot IVT^4]
\]

For the marginal costs of peak and off-peak public transport vehicle-kilometres we find

\[
MECC_3 = \frac{\partial t^1}{\partial Q^3} \cdot [X^1 \cdot IVT^1] + \frac{\partial t^3}{\partial Q^3} \cdot [X^3 \cdot IVT^3]
\]

\[
MECC_4 = \frac{\partial t^2}{\partial Q^4} \cdot [X^2 \cdot IVT^2] + \frac{\partial t^4}{\partial Q^4} \cdot [X^4 \cdot IVT^4]
\]

respectively.

Appendix 4. Determination and valuation of the marginal external costs other than congestion costs

a. Marginal air pollution costs

The marginal external air pollution costs of urban transport were determined in two consecutive steps. First the emissions per car-km were calculated; in a second step the monetary valuation of these emissions was estimated. Due to data limitations, we had to limit the analysis to the following air pollutants: nitrogen oxides (NOx), sulphur dioxide (SO2), carbon dioxide (CO2) and hydrocarbons (HC). The emission factors per vehicle-km are based on CONCAWE (1986), Cuijpers (1992), Econotec (1990) and Zierock et al. (1989).
For the monetary evaluation of the emissions per vehicle-km we extensively used the results of Mayeres (1992, 1993). These studies point at two important problems associated with the direct measurement of the marginal costs of pollution. First, ideally evaluation of the effects of marginal emissions would require information on the impact of emissions on the concentration levels of the different primary and secondary air pollutants concerned. This necessitates the use of dispersion models to predict the spread of the pollutants and transformation models that describe the interaction of different pollutants to form secondary pollutants. Unfortunately, such quantitative models are not yet fully operational. Second, the existing international literature on the monetary evaluation of air pollution (see, e.g., Nordhaus (1991)) produces a wide range of numerical results, which in addition cannot easily be transferred to the Belgian situation. Therefore, two different, indirect approaches were used to approximate the monetary value of marginal emissions of air pollutants. The first one was used for the monetary valuation of NOx, SO2 and HC emissions, whereas the second one concerns the valuation of CO2 emissions. The difference in approach is motivated mainly by a difference in available information.

The air pollution problems associated with the emissions of SO2, NOx and HC have an important and non-negligible international dimension: emissions in Belgium do not only have an impact on the Belgian territory but also affect other countries. When trying to evaluate the damages from air pollution, two attitudes are possible. Either one approaches the problem from the point of view of a non-cooperative country which only takes into account the damages on its own territory, or one also considers the damages for the other countries. In the latter case we can speak of a cooperative approach. Considering the fact that Belgium has adhered to international conventions on the control of SO2, NOx and SO2, the “cooperative” approach seems to be called for and therefore has been used in our analysis. The methodology for evaluating SO2, HC and NOx emissions starts from the emission reduction objectives for the different air pollutants, based on existing international agreements which Belgium has signed (see Mayeres (1993) for more details). Using information on different abatement techniques, their abatement potential and their unit reduction costs, it is calculated what the effect on the abatement costs is if one has to reach these objectives and an extra vehicle-kilometre generates additional emissions. This difference in abatement costs is then used as a proxy for the marginal external air pollution costs. Note that this approach makes a number of important assumptions. It assumes that there are no indivisibilities in the emission abatement possibilities and that the
abatement techniques are used in a cost-effective way, i.e. the cheapest technologies are used first. Moreover, it is assumed that the air pollution damage does not depend on the place or time of the emissions. Finally, it is also clear that the results are to a large extent affected by the emission reduction objectives put forward.

The methodology for evaluating CO2 emissions is different from the one described previously, but also looks at the problem in a cooperative way. CO2 emissions are at the basis of world-scale environmental problems. One therefore has to decide whether one takes account of the increased damages at world level, EC level or national level. The approach opted for in Mayeres (1993) values increases in CO2 emissions at the total marginal damage at the EC level. A similar approach is used by Proost and Van Regemorter (1995). The energy carbon tax of $10 per barrel of oil proposed by the EC, is interpreted as the marginal willingness to pay of the EC for a reduction in CO2 emissions (also see Nordhaus (1991)). The resulting marginal air pollution costs associated with the use of private car are estimated to be 0.6018 BF per kilometre. The corresponding figure for public transport is 2.346 BF per vehicle-km.

b. Marginal noise costs

Concerning the marginal external noise costs, the exercise only takes into account those caused by public transport. The values are taken from Boniver (1993). She reports 0.55 BF and 1.88 BF for the peak and the off-peak period respectively. She first determines the effect of an additional public transport vehicle-km on the noise level, and then expresses the change in the noise level in monetary terms based on a variety of existing hedonic price studies for traffic noise (see, e.g., Nelson (1982), Pearce & Markandya (1989) and Alexandre & Barde (1987)). The values with respect to car use are negligible (Mayeres (1993)).

c. Marginal accident costs

Estimates for the marginal external accident costs have been based on results of Mayeres (1993) and Boniver (1993). They apply the methodology proposed by Jones-Lee (1990) to car and public transport in Belgium. Three types of possible marginal external accident costs are discerned: (i) those associated with the risk of death or injury to the occupants of an additional
car or public transport vehicle, (ii) those associated with the increased risk to other motorized road users and (iii) those associated with the increased risk to pedestrians and cyclists. The second category has to be included if an additional vehicle-km changes the probability that other motorized road users are involved in accidents. In our paper we have assumed that this is not the case and that the ratio of the marginal to the average accident ratio equals unity. An important input in the calculation of the marginal external accident costs is the value of a statistical life or statistical injury. Our results were obtained using the findings of Jones-Lee (1990) and O'Reilly et al. (1992). This finally yields a marginal accident cost for car use of 0.6489 BF per passenger-kilometre. The external accident costs caused by an additional public transport vehicle-kilometre are calculated to be 2.5188 BF. These marginal accident costs contain no allowance for the pain, grief and suffering experienced by relatives and friends.

Appendix 5. Marginal external benefits associated with additional public transport vehicle-kilometres

The marginal external benefits associated with additional public transport vehicle-kilometres equal the monetary value of the decrease in waiting time for all public transport passengers due to additional public transport supply. To calculate the monetary value, we combine the decrease in waiting time with the number of transit passengers and with their values of waiting time.

a. Average waiting time function

We assume the average waiting time for a public transport vehicle (WT) to be proportional to the inverse of frequency (see Mohring (1972), Jansson (1984), Van De Voorde (1981)),

\[ WT^i \propto \frac{1}{Freq^i} \]

An indicator of frequency is obtained by dividing the number of public transport vehicle-kilometres during the period considered (\( Q^i \)) by the public transport network length (NL),

\[ Freq^i = \frac{Q^i}{NL} \]
In "Verkeer en vervoer in België" (Belgium, Ministerie van verkeer en infrastructuur (1990)), we find the total length of the public transport network in the Belgian urban areas considered, viz. NL = 1595.3 kilometres. We assume that the length of this network remains constant. Using this information we calculate that the inverse of frequency, i.e., the average headway, is 7.18 minutes in the peak and 12.57 minutes in the off-peak.

To calculate the proportionality factors of the average waiting time function, we started from the observation that "The greater the headway of the service, the bigger the proportion of passengers who will arrive to catch a particular trip - above about 15 minutes headway, this probably applies to most passengers" (Nash (1982)). We assumed therefore that commuters and students heading for school know the public transport timetable, and that, on average, they wait one third of the headway. Other passengers are assumed to have somewhat less information on the time schedule of public transport; they have an average waiting time of half the headway. Based on these assumptions, and using data on the relative importance of trip motives (Table A4), we found the proportionality factors to be 0.35 for the peak and 0.41 for the off-peak period. This implies that the average waiting time is 2.51 minutes in the peak and 5.15 minutes in the off-peak.

b. Value of waiting time for public transport users

Values of waiting time for public transport users were derived from Visser and van der Mede (1988) and especially Gunn (1991). The surveys they conducted supported the expectation that, in general, the value of out-of-vehicle time is higher than the value of in-vehicle time. Moreover, Gunn (1991) found that the value of waiting time (VOWT) is related to the value of in-vehicle time (IVT) as follows:

- Business: \[ VOWT = 1.4 \times IVT \]
- Commuting: \[ VOWT = 1.3 \times IVT \]
- Other motives: \[ VOWT = 1.7 \times IVT \]

\[ ^{22} \text{We find average waiting times that are close to the ones obtained by Van der Waard (1988), especially in the off-peak period.} \]
Based on these results and on the information given in Table A3 and Table A4, we obtained the following values of waiting time for public transport users:

Peak \( VOWT = 218 \text{ BF/hour} \)

Off-peak \( VOWT = 208 \text{ BF/hour} \)

c. Marginal external waiting time benefits

The monetary valuation of the decrease in waiting time due to an additional public transport vehicle-kilometre can be found by differentiating the average waiting time function w.r.t \( \text{Q} \) and multiplying the result with the number of public transport passengers and their values of waiting time. Using the observation that the average distance travelled by public transport is 4 kilometre Boniver (1992), so that \((X^i/4, i=3,4)\) corresponds to the number of public transport passengers in period \( i \), the marginal external waiting time benefit associated with extra public transport supply can be written as

\[
MEWB_i = -\frac{\partial WT^i}{\partial Q^i} \frac{X^i}{4} VOWT^i \left(\frac{\text{hours}}{\text{period}}\right)^i \quad \text{for } i=3,4
\]

were \((\text{hours/period})^i\) equals 5 and 17 for the peak and off-peak, respectively.

Appendix 6. Marginal boarding and alighting time costs

Marginal boarding and alighting time costs associated with an additional public transport passenger (MBATC') can be written as\(^{23}\)

\[
MBATC^i = MBAT \cdot \frac{X^i}{Q^i} \cdot IVT^i \quad \text{for } i=3,4
\]

where MBAT is the marginal boarding/alighting time for an extra passenger-kilometre (0.625/3600 hours).

\(^{23}\) Remember that we assumed that an additional public transport passenger causes no additional congestion, environmental, or accident costs.
Appendix 7. Marginal infrastructure costs

We assume that one lane-kilometre equals 3 000 m$^2$. The costs for supplying one more lane-kilometre consist of construction costs, expropriation costs, and maintenance costs. The construction costs for one lane-kilometre were estimated at 4 500 000 BF. Expropriation costs are estimated at approximately 21 000 000 BF for one lane-kilometre. Some 15 years after construction 1 050 000 BF has to be paid for maintenance; the same amount has to be paid 25 years after construction. Using an annual interest rate of 0.08, we find a daily cost of 6 248 BF per lane-kilometre.

Appendix 8. Marginal benefits associated with additional infrastructure

When more lane-kilometres become available, a given traffic flow can move faster, resulting in less congestion. The monetary value of the resulting time saving (MB$^t$) is given by:

\[ MB^t = \sum_{i=1}^{4} \frac{\partial t_i}{\partial I} \cdot X^i \cdot IVT^i \]

Appendix 9. Capacity costs of public transport vehicles

The cost of capital of rolling stock was calculated using an expected lifetime of public transport vehicles and applying interest and depreciation charges (see De Borger et al. (1994) for more details). The cost of capital associated with rolling stock is calculated at 1 900 717 BF per vehicle and per year. The daily capacity cost of public transport vehicles therefore amounts to 5 207 BF per vehicle.

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24 The data presented in this appendix are based on interviews with Van Gorp's moderne wegenbouw N.V., Ir. Nijs (Departement voor Werken, Gemeentebestuur Antwerpen) and Mr Clayé (Ministerie Vlaamse Gewest, Wegeninfrastructuur en Verkeer).
### TABLE A1 : Daily traffic levels and prices (1989)

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>VALUE</th>
<th>DIMENSION</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X^1$</td>
<td>47 312 533</td>
<td>passenger-km</td>
<td>demand for peak car</td>
</tr>
<tr>
<td>$X^2$</td>
<td>48 695 041</td>
<td>passenger-km</td>
<td>demand for off-peak car</td>
</tr>
<tr>
<td>$X^3$</td>
<td>1 544 494</td>
<td>passenger-km</td>
<td>demand for peak bus &amp; tram</td>
</tr>
<tr>
<td>$X^4$</td>
<td>1 297 262</td>
<td>passenger-km</td>
<td>demand for off-peak bus &amp; tram</td>
</tr>
<tr>
<td>$p^1$</td>
<td>2.665</td>
<td>BF per passenger-km</td>
<td>price for peak car</td>
</tr>
<tr>
<td>$p^2$</td>
<td>2.665</td>
<td>BF per passenger-km</td>
<td>price for off-peak car</td>
</tr>
<tr>
<td>$p^3$</td>
<td>3.46</td>
<td>BF per passenger-km</td>
<td>price for peak bus &amp; tram</td>
</tr>
<tr>
<td>$p^4$</td>
<td>3.46</td>
<td>BF per passenger-km</td>
<td>price for off-peak bus &amp; tram</td>
</tr>
</tbody>
</table>

### TABLE A2 : Daily supply of public transport (1989)

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>VALUE</th>
<th>DIMENSION</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q^3$</td>
<td>66 651</td>
<td>vehicle-km</td>
<td>supply of public transport in the peak</td>
</tr>
<tr>
<td>$Q^4$</td>
<td>129 382</td>
<td>vehicle-km</td>
<td>supply of public transport in the off-peak</td>
</tr>
</tbody>
</table>

### TABLE A3 : Values of in-vehicle time by purpose and mode

<table>
<thead>
<tr>
<th>Mode</th>
<th>Purpose</th>
<th>Guilder (1988) per hour</th>
<th>BF (1989) per hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car</td>
<td>Business</td>
<td>42.27</td>
<td>810.65</td>
</tr>
<tr>
<td></td>
<td>Commuting</td>
<td>11.44</td>
<td>219.40</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>9.05</td>
<td>173.56</td>
</tr>
<tr>
<td>Bus/Tram</td>
<td>Business</td>
<td>37.4</td>
<td>717.26</td>
</tr>
<tr>
<td></td>
<td>Commuting</td>
<td>9.54</td>
<td>182.96</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>5.64</td>
<td>108.16</td>
</tr>
</tbody>
</table>
### TABLE A4: Relative importance of trip purpose in peak and off-peak periods in Belgian cities

<table>
<thead>
<tr>
<th>Mode</th>
<th>Purpose</th>
<th>Peak</th>
<th>Off-peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car</td>
<td>Business</td>
<td>0.08</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>Commuting</td>
<td>0.60</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>0.32</td>
<td>0.50</td>
</tr>
<tr>
<td>Bus/Tram</td>
<td>Business</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>Commuting</td>
<td>0.49</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>Other (education)</td>
<td>0.50 (0.40)</td>
<td>0.68 (0.21)</td>
</tr>
</tbody>
</table>
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