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Preference characterisation and indirect Allais coefficients*

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Abstract To assess the way commodities interact with one another, economists are used to look at the signs and values of the Slutsky or Antonelli substitution effects. These effects—which correspond to the elements of the hessian matrix of the expenditure and distance function, respectively—have the desirable property of being ordinal. Following a suggestion by Allais (1943), Barten (1990) has shown that, after ordinalisation, the Hessian of the direct utility function provides a very intuitive third classification rule. But under standard regularity conditions, the indirect utility function is also capable of completely describing the consumer's preference ordering. In this paper I show that the hessian of this function, after a similar ordinalisation, contains relevant information to infer the degree of substitutability/complementarity among commodities. I will relate this matrix, called the *indirect Allais matrix*, to the three other matrices. In particular, I will argue that it is related in a positive (i.e. non-inverse) way to the Slutsky matrix, but at the same time has the desirable property of not being liable to the bias towards substitutability. The four classifications are illustrated using an empirical demand system for foodstuffs and beverages of the 16th century beguinage of Lier estimated by Schokkaert & Van der Wee (1988).

Keywords: preference structure, consumer behaviour, Allais coefficients

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1 Introduction

'When can we claim whether two commodities are each others complement or substitute, and how can we infer this from market data?'. This is an old question in economics, and most members of the profession will agree by now that the answer should be an ordinal one, that it should reflect features of the consumer's preference ordering rather than of one particular representation of that ordering. Two classifications which satisfy this criterion are the signs and absolute values of the Slutsky and Antonelli substitution effects—what Hicks (1956, ch 16) called the degree of *p*- and *q*-complementarity, respectively. Both the Slutsky and Antonelli coefficients have the property that they stem from the hessian matrix of the expenditure and distance function, respectively (see e.g. Deaton & Muellbauer, 1980, ch 2). The French economist Allais (1943, p 147) also provided a way of classifying commodities, by what he called the intrinsic marginal desirability. In fact, what Allais suggests is to look at the elements of the direct utility function's hessian, after this matrix has been 'ordinalised'. Despite the effort of Charette & Bronsard (1975) to draw attention to Allais' idea, it got somehow lost in the literature until it was recently reintroduced by Barten (1990). Using the apparatus of modern consumer theory, Barten makes explicit the ordinalisation procedure and shows how the matrix of Allais coefficients—which I shall henceforth call the *direct Allais matrix*—can easily be retrieved from the Rotterdam parameterisation of either the Slutksy and Antonelli matrix. He also shows that weak and strong separability assumptions on the direct utility function easily translate into a simplifying structure for these interaction coefficients.

These three types of classification all have in common that they are based on the (transformed) hessian of a function which completely describes the preference ordering. In this paper, it is my intention to explore a fourth characterisation of preferences, which closes the circle in a natural way. Indeed, under standard regularity assumptions, also the indirect utility function completely describes the consumer's preference order. Of course, this function has a cardinal character and the elements of its hessian are not invariant to different cardinalisations of that function. But, as I shall argue, this cardinality can be removed by exactly the same type of transformation which Allais (1943) and Barten (1990) applied on the hessian of the direct utility function. The ensuing matrix—which I shall coin the *indirect Allais matrix*—invites for a fourth classification and can be considered of as the dual to the direct Allais matrix.[1]

[1] These four classifications are by no means exhaustive. Madden (1991), for instance,

Of these three 'established' classification rules, the Slutsky rule is probably the oldest one and the most widespread in use. Still, this rule has a number of deficiencies, the most important being the inherent bias towards substitutability. Indeed, with every diagonal element of the Slutsky matrix being negative, the homogeneity condition implies positivity of at least one off-diagonal element. Thus, under the Slutsky rule, every commodity interacts with at least one other commodity in a substitutable way. In this paper, I will show that the indirect Allais matrix is positively (i.e. not inversely) related to the Slutsky matrix, but that it is *not* liable to this bias towards substitutability.

The outline of the paper is as follows. First I shall give a very brief overview of the direct Allais interaction coefficients: their definition, interpretation and relation to both the Slutsky and Antonelli matrix. Next, in section 3, I shall define the indirect Allais coefficients, stress their ordinal character, and argue that these coefficients partially control for changes in the welfare loss a consumer experienses because of a price increase. The relationship of the matrix of indirect interaction coefficient to the Slutsky and Antonelli matrices is analysed in section 4. This analysis suggests that the direct and indirect Allais matrices are inversely related to one another, and also this inverse relationship will be made explicit. In section 5, I show the implications of indirect separability assumptions for the structure of the indirect Allais matrix and claim that such assumptions have the same simplifying implications for the Antonelli substitution effects as direct separability assumptions have for the Slutsky substitution effects. In section 6, I pay attention to the class of homothetic preferences. Under homotheticity, the relationship between direct and indirect Allais coefficients simplifies considerably and with the tools introduced in earlier sections, it is shown that direct and indirect separability assumptions imply one another. An empirical illustration of the four different criteria to classify commodities is provided in section 7. For this I use a demand system for foodstuffs estimated by Schokkaert & Van der Wee (1988) on data of the 16th century Lier Beguinage. Concluding remarks are collected in section 8.

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presents other classifications, indexed on the set of rationed commodities—the *R*-classifications. The idea is that when a consumer faces a quantity constraint w.r.t. some commodities, the compensated price effects on the quantities purchased of the unrationed commodities and the compensated quantity effects on the virtual prices of the rationed commodities may be used to classify the interactions. If the subset of constrained goods is empty, the *R*-classification coincides with the Hicksian *p*-classification, while the *q*-classification emerges when the quantities of all but one good are fixed.

2 Allais coefficients and Rotterdam parameterisation—a brief reminder

I shall assume that preferences are representable by a direct utility function $u(\cdot):R^n_+\to R$, which is strongly quasi concave and differentiable at least two times. The vector of commodities will be denoted as $q\in R^n_+$.[2] To obtain the Allais interaction coefficients, one starts by singling out a standard pair of commodities, commodities r and s, say. The Allais interaction coefficient between commodities i and j, is then defined as (subscripts with u denote partial derivatives):

$$a_{ij} =_{\text{def}} \mu(q)\frac{u_{ij}}{u_i u_j} - \alpha(q), \tag{2.1}$$

where $\mu(q) =_{\text{def}} \Sigma_h\, u_h\, q_h$ and $\alpha(q) =_{\text{def}} \mu(q)\cdot u_{rs}/(u_r u_s)$. It is not difficult to verify that these coefficients are invariant to any monotone increasing transformation of the utility function. The *direct Allais matrix* is then defined as the $n\times n$ symmetric matrix $A =_{\text{def}} (a_{ij})$.

Let the consumer's opportunity set be defined by the price vector $p\in R^n_+$ and exogenous income m and let the vector of normalised prices be denoted as π, that is $\pi =_{\text{def}} \frac{1}{m}p$. The consumer's problem is then to find the commodity bundle which solves the problem $\max_q\{u(q)\,|\,p'q=m\}$. For an interior solution—which I shall assume throughout in the paper—the optimal bundle $q^*(p,m)$ must necessarily satisfy the first order condition $u_q=\lambda^*(p,m)p$, where u_q is the vector of marginal utilities evaluated at the bundle $q^*(p,m)$, and $\lambda^*(p,m)$ is optimal value for the Lagrange multiplier to the budget constraint. Because the demand functions are homogenous of degree zero in p and m, it is true that $q^*(\pi,1)= q^*(p,m)$, and no information will be lost when working with the system of demand functions expressed in terms of the normalised price vector π, which I shall denote as $q(\pi) =_{\text{def}} q^*(\pi,1)$. Similarly, because $\lambda^*(\cdot)$ is homogenous of degree -1 in (p,m), $\lambda^*(\pi,1)=m\lambda^*(p,m)$; so I shall define $\lambda(\pi) =_{\text{def}} \lambda^*(\pi,1)$.

Taking advantage of the first order condition and making use of (2.1), the total (logarithmic) differential of the marginal utility of commodity i can be written as

$$\text{dln}u_i = \Sigma_j a_{ij} w_j \text{dln}q_j + \frac{\alpha(q)}{\mu(q)}\text{d}u, \tag{2.2}$$

where $w_j =_{\text{def}} \pi_j q_j (=p_j q_j/m)$, the budget share of commodity j. In Barten's (1990) words, this expression "shows that the relative change in the marginal utility of

[2] A vector is always a column vector. Row vectors will be transposed column vectors. The transposed of the column vector x will be denoted as x'.

good i can be decomposed in a part $\alpha(q)/\mu(q)$ du which is general and not invariant, and a part which is invariant and specifically involves the i,j interactions." This specific interaction is measured by the coefficient a_{ij}, weighted by the importance of commodity j in the consumer's budget. A positive coefficient is then associated with complementarity, and a negative one with substitutability; if the coefficient is zero, commodities i and j are said to be independent. Since $du=\Sigma_j u_j dq_j$, and making use of the first order condition, the general effect can also be written as $\alpha\Sigma_j w_j d\ln q_j$. Thus, the general effect works through the change in the Divisia quantity index. By an appropriate change in this quantity index, utility is kept constant, and therefore the elasticity $a_{ij}w_j$ is a compensated elasticity. As Barten (1990) shows, standard assumptions imply no real restrictions on the direct Allais matrix, except for the fact that

$$A=A' \text{ and } y'Ay < 0,\ \forall y\text{: } \iota'y = 0. \tag{2.3}$$

The Rotterdam parameterisation of the regular demand system $q^*(p,m)$ is well known. Using a ` above a vector to denote the diagonal matrix with the vector elements on its diagonal, this system may be written in differential form as

$$\grave{w}d\ln q = b[d\ln m - w'd\ln p] + Sd\ln p, \tag{2.4}$$

where $b=\grave{p}\cdot\partial q^*/\partial m$ is the vector of marginal propensities to spend on the different commodities and S is a symmetric matrix whose typical element s_{ij} equals $(p_i p_j/m)\times$the Slutsky substitution effect of a marginal change in p_j on q_i. Commodities i and j are then said to be Slutsky (or Hicksian, or p-) substititutes/ complements when s_{ij} is positive/negative. The substitution and income effects share the properties that:

$$\iota'b = 1,\ S=S',\ S\iota = 0, \text{ and } y'Sy < 0,\ \forall y\text{: } y\neq\theta\iota,\ \theta \text{ a real scalar.} \tag{2.5}$$

The last two properties are precisely the ones generating the bias towards substitutability, mentioned in the introduction. The vector of income elasticities and the matrix of compensated price elasticities are given by $\grave{w}^{-1}b$ and $\grave{w}^{-1}S$, respectively. Since $d\ln\pi=d\ln p-\iota d\ln m$, another way of writing (2.4) is

$$\grave{w}d\ln q = -b[w'd\ln\pi] + Sd\ln\pi, \tag{2.6}$$

which is the differential Rotterdam form of the system $q(\pi)$. It relates changes in demand to changes in the Divisia index of normalised prices ($w'\text{dln}\pi$) and changes in normalised prices. This system will be used later on.

An inverse demand system expresses the consumer's willingness to spend on the different commodities in terms of the quantities consumed.[3] A measure of the willingness to spend on a commodity is provided by the normalised price of that commodity. It can be shown that in differential form, the Rotterdam parameterisation of the inverse demand system is very close to (2.6) (see e.g. Barten & Bettendorf, 1989, eq 13):

$$\hat{w}\text{dln}\pi = g[w'\text{dln}q]+H\text{dln}q. \tag{2.7}$$

The vector g is a vector of scale effects, measuring how the willingness to pay is affected by the change in the Divisia quantity index, while the (i,j)-element of the symmetric matrix H, h_{ij}, equals $q_i q_j \times$the Antonelli substitution effect of a marginal change in consumption of commodity j on the marginal willingness to spend on commodity i. When h_{ij} takes on a positive/negative sign, commodities i and j are said to be Antonelli (or q-) complements/substitutes. These scale and substitution effects satisfy the following set of restrictions:

$$\iota' g = -1,\ H=H',\ H\iota = 0,\ \text{and}\ y'Hy < 0,\ \forall y{:}y\neq\theta\iota,\ \theta\ \text{a real scalar} \tag{2.8}$$

When (2.2) is written in matrix notation, one gets the following 'demand system':

$$\text{dln}u_q = \alpha\iota[w'\text{dln}q]+A\hat{w}\text{dln}q, \tag{2.9}$$

At first sight, since the LHS variables of this system are unobservable, one is tempted to argue that neither is the matrix of Allais coefficients, A. But this is refuted by the following proposition due to Barten (1990), stating that the direct Allais coefficients may be retrieved from parameters which are in principle observable from market behaviour:

Proposition 1 (Barten, 1990): *The direct Allais matrix relates to the parameters of the regular and inverse Rotterdam demand system in the following way:*

$$\begin{pmatrix} A-(^1/\varphi_A)\iota\iota' & \iota \\ \iota' & 0 \end{pmatrix}\begin{pmatrix} S & b \\ b' & 0 \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & 1 \end{pmatrix}, \tag{2.10}$$

[3] See Anderson (1980) for an extensive discussion of inverse demand systems.

$$\begin{pmatrix} H & g \\ g' & 0 \end{pmatrix} = \begin{pmatrix} \Omega & -w \\ w' & 1 \end{pmatrix} \begin{pmatrix} A-\rho_A \iota\iota' & \iota \\ \iota' & 0 \end{pmatrix} \begin{pmatrix} \Omega & w \\ -w' & 1 \end{pmatrix}, \tag{2.11}$$

where $\varphi_A =_{\text{def}} \iota' A^{-1} \iota$, $\rho_A =_{\text{def}} 2 + w'Aw$ *and* $\Omega =_{\text{def}} (\hat{w} - ww')$.

Since $\begin{pmatrix} \Omega & -w \\ w' & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \hat{w}^{-1} & \iota \\ -\iota' & 0 \end{pmatrix}$, expression (2.11) can also be rearranged as

$$\begin{pmatrix} A-\rho_A \iota\iota' & \iota \\ \iota' & 0 \end{pmatrix} = \begin{pmatrix} \hat{w}^{-1} & \iota \\ -\iota' & 0 \end{pmatrix} \begin{pmatrix} H & g \\ g' & 0 \end{pmatrix} \begin{pmatrix} \hat{w}^{-1} & -\iota \\ \iota' & 0 \end{pmatrix}. \tag{2.12}$$

From expressions (2.10) and (2.12), it transpires that the parameters of either the regular or inverse demand system determine the Allais interaction coefficients up to a constant (φ_A or ρ_A). But this indeterminacy is related to the selection of a standard pair of commodities. Once the interaction coefficient of that standard pair is fixed, the direct Allais matrix is fully determined. Loosely speaking, (2.10) and (2.12) show that the Allais matrix is 'inversely' related to the Slutsky matrix, but is 'positively' related to the Antonelli matrix. A nice example of how the Allais interactions are retrieved from an inverse demand system is provided by Barten & Bettendorf (1989).

3 Interactions based on the indirect utility function: the indirect Allais matrix

Inserting the optimal commodity bundle $q(\pi)$ in the direct utility function yields the indirect utility function, $v(\pi)$. Because the direct utility function is strongly quasi concave, this indirect utility function will be strongly quasi convex.[4] Its vector of first derivatives is

$$v_\pi =_{\text{def}} \frac{\partial v(\pi)}{\partial \pi} = -\lambda(\pi) q(\pi) \tag{3.1}$$

and thus $\pi' q = 1$ implies that $\pi' v_\pi = -\lambda$. The hessian of the indirect utility function w.r.t. normalised prices, $v_{\pi\pi}$, is clearly not invariant to monotonous increasing transformation of that function. For instance, after transformation of the indirect utility function to $\bar{v}(\pi) = f(v(\pi))$, $df(\cdot)/dv > 0$, its hessian is transformed into

[4] Because preferences are *strictly* convex, and not just convex or even non-convex, the indirect utility function $v(\cdot)$ will be differentiable everywhere, or, to put it differently, the associated indifference curves in the space of normalized prices will not have 'kinks'. Because $u(\cdot)$ is *strongly* quasi concave, the indifference surface in the space of normalized prices will not have 'flat' regions (see Deaton & Muellbauer, 1980, pp 47-9).

$$\tilde{v}_{\pi\pi} = \frac{d^2 f(v(\pi))}{dv^2} v_\pi v_\pi{}' + \frac{df(v(\pi))}{dv} v_{\pi\pi}. \tag{3.2}$$

If $f(\cdot)$ is sufficiently concave, the transformation may change the sign of the hessian elements.

If, in analogy with the direct Allais interaction coefficients, a standard pair of commodities, the pair (r,s), say, is selected, it is not difficult to convince oneself that

$$\pi' \tilde{v}_\pi \left(\frac{\tilde{v}_{ij}}{\tilde{v}_i \tilde{v}_j} - \frac{\tilde{v}_{rs}}{\tilde{v}_r \tilde{v}_s} \right) = \pi' v_\pi \left(\frac{v_{ij}}{v_i v_j} - \frac{v_{rs}}{v_r v_s} \right). \tag{3.3}$$

Let me therefore define the *indirect Allais interaction coefficient* between commodities i and j as

$$c_{ij} =_{\text{def}} \eta(\pi) \frac{v_{ij}}{v_i v_j} - \beta(\pi), \tag{3.4}$$

where $\eta(\pi) =_{\text{def}} \pi' v_\pi$ and $\beta(\pi) =_{\text{def}} \eta(\pi) \cdot v_{rs}/(v_r v_s)$; the indirect Allais matrix is then defined as $C = (c_{ij})$. Before interpreting these interaction coefficients, note that the second order partial derivative of the indirect utility function, v_{ij}, can now be written as

$$v_{ij} = \frac{1}{\eta(\pi)} v_i v_j c_{ij} + \frac{\beta(\pi)}{\eta(\pi)} v_i v_j. \tag{3.5}$$

Consider then the total differential of $(-v_i)$, the welfare loss a consumer experiences due to a marginal increase in π_i: $d(-v_i) = \Sigma_j (-v_{ij}) d\pi_j$. Using (3.5), dividing through by $(-v_i)$ and exploiting the fact that $\eta(\pi) = -\lambda(\pi)$, one obtains

$$\begin{aligned} d\ln(-v_i) &= \Sigma_j c_{ij} w_j d\ln\pi_j + \frac{\beta(\pi)}{\eta(\pi)} \Sigma_j v_j d\pi_j \\ &= \Sigma_j c_{ij} w_j d\ln\pi_j + \frac{\beta(\pi)}{\eta(\pi)} dv. \end{aligned} \tag{3.6}$$

According to (3.6), the relative change in the utility loss of a marginal increase in π_i due to a change in the normalised price π_j is controlled by a commodity specific effect and a general effect. The former is ordinal and equals the indirect Allais interaction coefficient c_{ij} weighted by the importance of commodity j in the consumer's budget, w_j. The general effect has a cardinal nature, and since $(\beta/\eta) \cdot dv = \beta \Sigma_j w_j d\ln\pi_j$, this effect works through the impact of the price change of commodity j on the Divisia normalised price index. Again, by an appropriate change in this index, utility is kept constant, and the elasticity of v_i w.r.t. π_j reduces to $c_{ij}w_j$.

The fact that a consumer's welfare loss due to a marginal increase of commodity i's normalised price equals λq_i suggests the following classification rule: if $c_{ij}>0$ (<0) commodity i is a substitute for (complement of) commodity j and if $c_{ij}=0$ the two commodities are independent; here substitute for (complement of) should be read as "a better substitute (a better complement) for ... then commodity r is for commodity s". It also implies that

$$\frac{\partial \ln(q_i/q_j)}{\partial \ln \pi_k} = (c_{ik}-c_{jk})w_k. \tag{3.7}$$

In other words, the difference between the uncompensated price elasticities of commodities i and j w.r.t. π_k, is a fraction of the difference in the indirect Allais interactions (i,k) and (j,k), the fraction being the importance of commodity k in the budget.

The indirect Allais matrix C must satisfy the following two properties[5]:

$$C=C', \text{ and } y'Cy < 0, \ \forall y\colon \iota' y = 0. \tag{3.8}$$

The symmetry of C immediately follows from the symmetry of $v_{\pi\pi}$. The second property looks like a negativity property but is much weaker.[6] In particular, the negativity of the diagonal elements is not implied because the elements of the unit vector (0,...,1,...0)' do not add up to zero. There is therefore no *a priori* reason for the indirect Allais classification to be biased towards substitutability.[7]

Writing (3.6) in matrix notation gives rise to a fourth demand system:

$$d\ln(-v_\pi) = \beta\iota[w' d\ln\pi] + C\hat{w} d\ln\pi. \tag{3.9}$$

Again the LHS variables are unobservable, but it is possible to relate the indirect Allais matrix C to parameters which in principle can be estimated. This is the subject of the next section.

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[5] Because $C=\eta(\pi)\hat{v}_\pi^{-1}v_{\pi\pi}\hat{v}_\pi^{-1}-\beta(\pi)\iota\iota'$, the first property follows from the symmetry of $v_{\pi\pi}$. Moreover, strong quasi convexity of the indirect utility function $v(\pi)$, $x'v_{\pi\pi}x>0$, $\forall x{:}x'v_\pi=0$, implies the second property (choose $y=\hat{v}_\pi x$) since $\eta(\pi)=\pi' v_\pi=-\lambda(\pi)<0$.

[6] Indeed, the set of vectors for which pre- and postmultiplication of the matrices S and H will results in a negative number is given by $Y_1=_{\text{def}}\{y \mid y \not\propto \iota\}$. For the matrices C and A, this set is given by $Y_2=\{y \mid y'\iota=0\}$. Compared with Y_1, Y_2 is infinitesimally small.

[7] For similar reasons, the direct Allais matrix does not *a priori* favour complementarity.

4 The relation between the indirect Allais matrix and the Slutsky, Antonelli and direct Allais matrices

The next proposition relates the indirect Allais matrix to the parameters of the regular and inverse demand systems. The proposition thereafter gives the relation w.r.t. the direct Allais matrix. Proofs of both propositions are relegated to the paper's appendix.

Proposition 2: *The relation between the indirect Allais matrix and the Slutsky and Antonelli matrix is as follows:*

$$\begin{pmatrix} S & b \\ b' & 0 \end{pmatrix} = \begin{pmatrix} -\Omega & w \\ w' & 1 \end{pmatrix} \begin{pmatrix} C-\rho_C \iota\iota' & \iota \\ \iota' & 0 \end{pmatrix} \begin{pmatrix} -\Omega & w \\ w' & 1 \end{pmatrix}, \tag{4.1}$$

$$\begin{pmatrix} C-(^1/\varphi_C)\iota\iota' & -\iota \\ -\iota' & 0 \end{pmatrix} \begin{pmatrix} H & g \\ g' & 0 \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & 1 \end{pmatrix}. \tag{4.2}$$

where ρ_C *is a shorthand for* $2+w'Cw$ *and* φ_C *is a shorthand for* $\iota' C^{-1}\iota$.

Thus, the indirect Allais matrix relates to the Antonelli matrix in a very similar way as the direct Allais matrix does to the Slutsky matrix, and vice versa. Since $\begin{pmatrix} -\Omega & w \\ w' & 1 \end{pmatrix}^{-1} = \begin{pmatrix} -\hat{w}^{-1} & \iota \\ \iota' & 0 \end{pmatrix}$, an equivalent way of writing (4.1) is,

$$\begin{pmatrix} C-\rho_C \iota\iota' & \iota \\ \iota' & 0 \end{pmatrix} = \begin{pmatrix} -\hat{w}^{-1} & \iota \\ \iota' & 0 \end{pmatrix} \begin{pmatrix} S & b \\ b' & 0 \end{pmatrix} \begin{pmatrix} -\hat{w}^{-1} & \iota \\ \iota' & 0 \end{pmatrix} \tag{4.3}$$

In particular,

$$C-\rho_C \iota\iota' = \hat{w}^{-1} S \hat{w}^{-1} - \iota b' \hat{w}^{-1} - \hat{w}^{-1} b \iota'. \tag{4.4}$$

Thus, if estimates are available for S and b, one can determine the indirect Allais interactions in the following way: first one solves for ρ_C by constraining $c_{rs}=0$ ($\rho_C = -s_{rs}/(w_r w_s) + b_r/w_r + b_s/w_s$); next one adds this value to each element of the RHS matrix in (4.4). And in an analogous way the estimates of the inverse demand system, can be used to compute the indirect Allais interaction coefficients by postmultiplying (4.2) through by the inverse of the second LHS matrix, look for the value of $1/\varphi_C$ for which $c_{rs}=0$, and add this value to every element of the NW-block of this inverse matrix.

The above analysis suggests that the direct and indirect Allais matrices are inversely related to one another. I will now show that this is indeed the case. In particular, I will demonstrate that the direct Allais matrix stands to the indirect one in exactly the same way as the indirect Allais matrix stands to the direct one. Before embarking on this, it is useful to derive a relationship between the scalars ρ_A, ρ_C, φ_A and φ_C.

Multiplying out the RHS of (4.1) gives that $b=(I-\Omega C)w$ (because $\Omega\iota=0$). But multiplying out the LHS of (2.10) one also has that $Ab=(^1/\varphi_A)\iota$. Whence,

$$Aw-\frac{1}{\varphi_A}\iota = A\Omega Cw. \tag{4.5}$$

On the other hand, $Cg=-(^1/\varphi_C)\iota$ from (4.2), while (2.11) implies that $g=-(I-\Omega A)w$. Therefore,

$$Cw-\frac{1}{\varphi_C}\iota = C\Omega Aw. \tag{4.6}$$

Multiplying both (4.5) and (4.6) through by w', subtracting the latter expression from the former, and making use of the definitions of ρ_A and ρ_C, one gets that

$$\rho_A-\frac{1}{\varphi_A} = \rho_C-\frac{1}{\varphi_C}. \tag{4.7}$$

The relation between the direct and indirect Allais matrices is now given in proposition 3 (the proof is relegated to the appendix):

Proposition 3: *The direct and indirect Allais matrices relate in the following way:*

$$A-\frac{1}{\varphi_A}\iota\iota' = \hat{w}^{-1}\left[C^{-1}+\rho_C ww'-\frac{1}{\varphi_C}\left(C^{-1}\iota+w\right)\left(C^{-1}\iota+w\right)'\right]\hat{w}^{-1}. \tag{4.8}$$

$$C-\frac{1}{\varphi_C}\iota\iota' = \hat{w}^{-1}\left[A^{-1}+\rho_A ww'-\frac{1}{\varphi_A}\left(A^{-1}\iota+w\right)\left(A^{-1}\iota+w\right)'\right]\hat{w}^{-1} \tag{4.9}$$

By constraining a_{rs} to zero, the direct Allais matrix A can be written as a function of the indirect Allais matrix C by subtracting from each element of the RHS matrix in (4.8) the (r,s)-element of the same matrix. Let this matrix function be denoted by $\Lambda[\cdot]$, then $A=\Lambda[C]$. But by constraining the indirect Allais interaction coefficient c_{rs} to zero, the matrix C can also be written as a function of the matrix A. The similarity between the expressions (4.8) and (4.9) then implies that $C=\Lambda[A]$. In this respect, the direct and indirect Allais matrices are inversely related to one another: the same function which maps the A into C also maps C into A. Pre- and postmultiplying both expressions through by $\hat{w}$, and making use of (4.7), one also has that

$$\hat{w}A\hat{w}-\rho_A ww' = C^{-1}-\frac{1}{\varphi_C}(C^{-1}\iota\iota'C^{-1}+C^{-1}\iota w'+w\iota'C^{-1}), \qquad (4.10)$$

$$\hat{w}C\hat{w}-\rho_C ww' = A^{-1}-\frac{1}{\varphi_A}(A^{-1}\iota\iota'A^{-1}+A^{-1}\iota w'+w\iota'A^{-1}). \qquad (4.11)$$

Either of this pair of expressions may be used to trace out the implications of certain assumptions on the interaction matrix of one type for the interactions of the other type. This will be the subject of the two following sections. (The reader interested only in the empirical application of section 7 can, without risk of loss, skip sections 5 and 6.)

5 Implications and meaning of indirect separability

Indirect separability has not received much attention in the literature. For instance, in the Deaton & Muellbauer (1980) text, this type of separability is mentioned only once, in an exercise to their chapter 'Restrictions on preferences'. Nevertheless, in applied work, one often implicitly assumes a form of indirect separability by starting of from a manageably specified indirect utility function. In this section, I will first explore the implications of indirect separability assumptions for the indirect interaction coefficients. Next, I will trace out the consequences for the inverse demand system, and finally I will study what indirect separability means in terms of the direct Allais coefficients.

Suppose that the n commodities can be partitioned into N mutually exclusive groups, with the typical group F containing n_F commodities ($\Sigma_{F=1}^{N} n_F=n$). The preference ordering is then said to exhibit *weak indirect separability* when the indirect utility function $v(\pi)$ can be written as

$$v(\pi) = \psi[v_A(\pi_A),v_B(\pi_B),...,v_N(\pi_N)]. \qquad (5.1)$$

On the other hand, the preference ordering is *strongly indirectly separable* when the indirect group indices $v_F(\cdot)$ enter $\Psi(\cdot)$ in an additive way, viz

$$v(\pi) = \psi[v_A(\pi_A)+v_B(\pi_B)+...+v_N(\pi_N)]. \tag{5.2}$$

In analogy with the direct case, one can say that preferences exhibit *indirect additivity* when there is strong indirect separability and in addition every group consists of only one commodity.[8]

Let me first concentrate on indirect weak separability. In that case, for commodities $i \in F$ and $j \in G$, $F \neq G$,

$$v_i = \frac{\partial\psi}{\partial v_F}\frac{\partial\psi}{\partial\pi_i}, \text{ and } v_{ij} = \frac{\partial^2\psi}{\partial v_F \partial v_G}\frac{\partial v_F}{\partial \pi_i}\frac{\partial v_G}{\partial \pi_j} = \kappa_{FG} v_i v_j, \tag{5.3}$$

where

$$\kappa_{FG} =_{\text{def}} \frac{\partial^2\psi}{\partial v_F \partial v_G}\bigg/\frac{\partial\psi}{\partial v_F}\frac{\partial\psi}{\partial v_G}. \tag{5.4}$$

Therefore, the indirect Allais interaction coefficient for any two commodities $i \in F$ and $j \in G$, $F \neq G$, depends only on the indirect *group* interaction:

$$c_{ij} = \eta(\pi)\kappa_{FG} - \beta(\pi) =_{\text{def}} \theta_{FG} \quad (i \in F,\ j \in G,\ F \neq G).$$

Weak indirect separability thus allows for the following simplified structure for the indirect Allais matrix:

$$C = \begin{pmatrix} C_A & 0 & \cdots & 0 \\ & C_B & \cdots & 0 \\ & & \ddots & \vdots \\ & & & C_N \end{pmatrix} + \begin{pmatrix} J_A & 0 & \cdots & 0 \\ & J_B & \cdots & 0 \\ & & \ddots & \vdots \\ & & & J_N \end{pmatrix} \begin{pmatrix} 0 & \theta_{AB} & \cdots & \theta_{AN} \\ & 0 & \cdots & \theta_{BN} \\ & & \ddots & \vdots \\ & & & 0 \end{pmatrix} \begin{pmatrix} J_A' & 0 & \cdots & 0 \\ & J_B' & \cdots & 0 \\ & & \ddots & \vdots \\ & & & J_N' \end{pmatrix}$$

$$= C_d + J_d T J_d' . \tag{5.6}$$

[8] When the direct utility function has a structure similar to (5.1)/(5.2), I will speak of weak/strong *direct* separability. *Direct* additivity prevails when under strong direct separability every group consists of only one commodity. The terminology 'direct' and 'indirect' additivity is due to Houthakker (1960). Sometimes, such preference orderings are said to exhibit complete direct (or indirect) independence.

Here C_F is an $n_F \times n_F$ matrix of within group interactions, and J_F is an $n_F \times 1$ vector with all elements equal to unity. The definition of the matrices C_d, J_d and T should be obvious.

In the case of indirect strong separability, the coefficient κ_{FG} is identical for all pairs of subgroups F,G: $\kappa_{FG}=\kappa$. If the standard pair is also selected out of two different subgroups, $\beta(\pi)=\eta(\pi)\kappa$, and $c_{ij}=0$, $\forall i \in F$, $\forall j \in G$, $F \neq G$. In that case, the matrix C simplifies to a block diagonal matrix, the blocks containing the indirect interactions for commodities within the same group: $C=C_d$.

Now, from (4.2), it may be established that $H=C^{-1}\text{-}C^{-1}\iota(\iota' C^{-1}\iota)^{-1}\iota' C^{-1}$ and $g=\text{-}C^{-1}\iota(\iota' C^{-1}\iota)^{-1}$ (see also the appendix, (A8)). The special structure imposed by weak indirect separability on the matrix C can then be shown to simplify the structure of the H matrix in a way indicated by the following proposition.

Proposition 4: *Under indirect weak separability, the Antonelli matrix obtains the following simplified structure (with obvious definition of the matrices H_d, g_d and Φ):*

$$H = \begin{pmatrix} H_A & 0 & \cdots & 0 \\ & H_B & \cdots & 0 \\ & & \ddots & \vdots \\ & & & H_N \end{pmatrix} + \begin{pmatrix} g_A & 0 & \cdots & 0 \\ & g_B & \cdots & 0 \\ & & \ddots & \vdots \\ & & & g_N \end{pmatrix} \begin{pmatrix} \Phi_{AA} & \Phi_{AB} & \cdots & \Phi_{AN} \\ & \Phi_{BB} & \cdots & \Phi_{BN} \\ & & \ddots & \vdots \\ & & & \Phi_{NN} \end{pmatrix} \begin{pmatrix} g_A' & 0 & \cdots & 0 \\ & g_B' & \cdots & 0 \\ & & \ddots & \vdots \\ & & & g_N' \end{pmatrix}$$

$$= H_d + g_d \Phi g_d' \qquad (5.7)$$

In this expression, g_F is the $n_F \times 1$ vector of scale effects $(g_i)_{i \in F}$, while ϕ_{FG} is a scalar.

The proof of this proposition has been left out because it proceeds along exactly the same lines as Barten's (1990, section 6) proof to show that *direct* weak separability imposes a structure on the matrix of Slutsky substitution effects, S, of the same kind as in (5.7), with the scale effects, g_F, replaced by income effects, b_F.

Expression (5.7) shows that the Antonelli substitution effect of a quantity change in commodity $j \in G$ on the marginal willingness to pay for commodity $i \in F$ $(F \neq G)$ is proportional to the product of the respective scale effects, with a factor of proportionality ϕ_{FG} that is identical for any pairs of commodities chosen from these two groups: $h_{ij}=\phi_{FG} g_i g_j$ $(i \in F, j \in G, F \neq G)$.

One can push the argument even a bit further, since the matrix $g_d\Phi g_d'$ can also be written as $g_d\hat{z}^{-1}(\hat{z}\Phi\hat{z})\hat{z}^{-1}g_d'$; here z is an $N\times 1$ vector containing for each group F the scale effect of a change in the quantity index w'dln$q$ on the marginal willingness to spend on that group as a whole, i.e. $z=(z_F)$, where $z_F=J_F'g_F$. Defining $\Sigma =_{\text{def}} \hat{z}\Phi\hat{z}$, it follows that $J_F'h_{FG}J_G=\sigma_{FG}$ $(F\neq G)$, where σ_{FG} is the typical element of the $N\times N$ matrix Σ. Let us then consider the effect on the marginal willingness to spend on commodities belonging to group F when the quantities in which the commodities of group $G(\neq F)$ are consumed all change with one percentage, viz dln$q_G=J_G$. From (2.7) it follows that these effects are given by the vector equation

$$\hat{w}\text{dln}\pi_F = g_F[w_G'J_G]+h_{FG}J_G . \tag{5.8}$$

Premultiplying through by J_F', it appears that the compensated effect of such a proportional change of the quantities in group G on the willingness to spend on commodities in group F as a whole is precisely σ_{FG}. For instance, if F groups freshwater fish and G groups saltwater fish, the compensated effect on the average auction price of freshwater fish because of an exceptional catch by offshore fishers is σ_{FG}.

Under strong indirect separability, the matrix T vanishes, and $C=C_d$. In that case, it can be shown that $\sigma_{FG}=\sigma$, $\forall F,G$ (the proof is again the same as in Barten (1990)).

We may now wonder which implications indirect separability has for the direct Allais matrix. That is, if we regard the Allais coefficients as the closest to one's intuition about how commodities interact, in which way should these coefficients be structured for indirect separability to prevail? Still in other words, what kind of preferences gives rise to indirect separability? In principle, this question can be answered, since we know the structure which is placed by indirect separability on the C-matrix and we also know how the direct Allais matrix is related to this matrix. The strong non-linearity of the latter relationship, however, does not allow for a straightforward answer, and this should already be a first indication that indirect separability is a concept which is not easy to grasp. Let me therefore concentrate on strong indirect separability.

Under strong indirect separability, the matrix C reduces to the block diagonal matrix C_d. The direct Allais interactions between commodities F and G, $F\neq G$, for which the indirect interaction is constrained to be zero, must then satisfy the relationship (see (4.10)):

$$\dot{w}_F A_{FG} \dot{w}_G - \rho_A w_F w_G' = -\frac{1}{\varphi_C}\left(C_F^{-1} J_F J_G' C_G^{-1} + C_F^{-1} J_F w_G' + w_F J_G' C_G^{-1}\right). \tag{5.9}$$

In particular, the interaction between commodity $i \in F$ and $j \in G$, $F \neq G$, is given by

$$a_{ij} - \rho_A = -\frac{1}{\varphi_C}\left((\Sigma_{l\in F}\frac{c_F^{il}}{w_i})(\Sigma_{k\in G}\frac{c_G^{kj}}{w_j}) + \Sigma_{l\in F}\frac{c_F^{il}}{w_i} + \Sigma_{k\in G}\frac{c_G^{kj}}{w_j}\right). \tag{5.10}$$

In this expression, c_F^{il} denotes the coefficient on row i and column l in the inverse of C_F, and similarly for c_G^{kj}. Thus, the direct Allais interaction coefficient between i and j will even under strong indirect separability typically involve *all* indirect interaction coefficients for commodities belonging to both groups F and G. In the extreme case of indirect additivity, (5.10) reduces to

$$a_{ij} - \rho_A = -\frac{1}{\varphi_C}\left(\frac{1}{c_{ii}w_i}\frac{1}{c_{jj}w_j} + \frac{1}{c_{ii}w_i} + \frac{1}{c_{jj}w_j}\right). \tag{5.11}$$

Choosing the same pair (r,s) as the one for which indirect Allais interaction is zero, the direct interaction between the pair (i,j), relative to the pair (r,s) is given by

$$a_{ij} = \frac{1}{\varphi_C}\left((\frac{1}{c_{rr}w_r}\frac{1}{c_{ss}w_s} - \frac{1}{c_{ii}w_i}\frac{1}{c_{jj}w_j}) + (\frac{1}{c_{rr}w_r} - \frac{1}{c_{ii}w_i}) + (\frac{1}{c_{ss}w_s} - \frac{1}{c_{jj}w_j})\right). \tag{5.12}$$

Now, $^1/(c_{kk}w_k)$ is the inverse of the compensated elasticity of $(-v_k)$ w.r.t. π_k. Thus, (5.12) shows that sign and magnitude of the direct Allais coefficient a_{ij}, hinges on the inverse of these elasticities, not only for commodities i and j, but also for the commodities making up the standard pair. These very restrictive assumptions on the indirect utility function will therefore not imply in general a trivial direct interaction between the two commodities i and j. The direct Allais interaction coefficient is the sum of three terms with ambiguous sign; a zero direct interaction can but does not have to be implied. However, there is one class of preferences which will imply a very simple relationship between the direct and indirect Allais matrices, this is the class of homothetic preference orderings to which I will turn next.

6 Homothetic preferences

Although of not much relevance from an empirical point of view, homothetic preferences have a theoretical interest on their own. When preferences are homothethic, the direct utility function can be written as some monotonously increasing transformation of a function which is homogenous of degree one. As explained by Samuelson (1965), homotheticity of the direct utility function implies and is implied by homotheticity of the indirect utility function.[9] In that case—and only in that case—all Engel curves will be straight lines through the origin and all expenditure elasticities will equal unity. Formally, this means that $\hat{w}^{-1}b=\iota$ or $b=w$. But as $b=(I-\Omega C)w$, homotheticity requires that $\Omega Cw=0$. And since $\Omega=\hat{w}-ww'$ is a matrix of rank $(n-1)$ whose null space is spanned by the vector of units, ι, it must be true that $Cw=\gamma\iota$, $\gamma\neq0$. Multiplying through by C^{-1}, gives $w=\gamma C^{-1}\iota$, and adding-up shows that $1/\gamma=\iota' C^{-1}\iota=\varphi_C$; whence

$$Cw = \frac{1}{\varphi_C}\iota, \tag{6.1}$$

or $w=C^{-1}\iota\,(1/\varphi_C)$. But earlier one it was argued that the vector of scale effects of the inverse demand system must satisfy the relation $Cg=-\iota\,(^1/\varphi_C)$. Homotheticity therefore requires that $w=-g$. Imposing constancy of the vector g (as the Rotterdam parameterisation of the inverse demand system does) and the assumption of homotheticity thus gives rise to a very simple system of regular demand equations:

$$q_i = (-g_i)\frac{m}{p_i}. \tag{6.2}$$

Furthermore, in view of (6.1), it follows from (4.5) that

$$Aw = \frac{1}{\varphi_A}\iota, \tag{6.3}$$

Combining this with (6.1) then shows that the difference in direct and indirect Allais matrices is a singular matrix whose null space is spanned by the vector of budget shares:

$$\left(\varphi_C C-\varphi_A A\right)w = 0. \tag{6.4}$$

To see how homotheticity simplifies the relationship between the direct and indirect Allais matrices, one goes back to (4.8) and (4.9) and substitutes $C^{-1}\iota$ and

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[9] More precisely stated, $u(\cdot)$ is homothetic iff $-v(\cdot)$ is homothetic, i.e. iff $v(\cdot)$ is a monotonous transformation of a function which is homogenous of degree -1.

$A^{-1}\iota$ for $\varphi_C w$ and $\varphi_A w$, respectively. Rearrangement then yields

$$A = \hat{w}^{-1}C^{-1}\hat{w}^{-1} - (\varphi_C - \frac{1}{\varphi_A})\iota\iota', \text{ and} \tag{6.5}$$

$$C = \hat{w}^{-1}A^{-1}\hat{w}^{-1} - (\varphi_A - \frac{1}{\varphi_C})\iota\iota'. \tag{6.6}$$

Consider (6.5). Constraining a_{rs} to zero requires that $-{}^{1}/\varphi_A = (\hat{w}^{-1}C^{-1}\hat{w}^{-1})_{rs} - \varphi_C$. If indirect additivity prevails, C is a diagonal matrix, and so is C^{-1}. Thus, $(\hat{w}^{-1}C^{-1}\hat{w}^{-1})_{rs}$ must be zero. But this means that ${}^{1}/\varphi_A = \varphi_C$. Therefore, A must be a diagonal matrix as well, and direct additivity must hold. And a similar argument can be used on (6.6) to show that under homotheticity, direct additivity implies indirect additivity. These results are summarised in proposition 5:

Proposition 5 (Samuelson, 1965): *When preferences are homothetic, direct (indirect) additivity implies indirect (direct) additivity.*

Alternatively, if both direct and indirect additivity prevail, the preference order must be homothetic. To see this, start from the relation (4.9) and rewrite it as

$$C = \hat{w}^{-1}A^{-1}\hat{w}^{-1} + (\rho_A + \frac{1}{\varphi_C})\iota\iota' - \frac{1}{\varphi_A}\hat{w}^{-1}(A^{-1}\iota + w)(A^{-1}\iota + w)'\hat{w}^{-1}. \tag{6.7}$$

Since both the LHS matrix and the first matrix on the RHS are diagonal by assumption, it must be true that the rest of the RHS vanishes:

$$(\rho_A + \frac{1}{\varphi_C})ww' = \frac{1}{\varphi_A}(A^{-1}\iota + w)(A^{-1}\iota + w)'. \tag{6.8}$$

If this is the case, pre-(post-) multiplication of (6.7) by w' (w) implies that $\rho_C\text{-}2 = \varphi_A$. And because $\rho_A + {}^{1}/\varphi_C = \rho_C + {}^{1}/\varphi_A$, (6.8) can be written as

$$(1+\varphi_A)^2 ww' = (A^{-1}\iota + w)(A^{-1}\iota + w)'. \tag{6.9}$$

Whence, a necessary condition is that $A^{-1}\iota + w = (1+\varphi_A)w$, that $({}^{1}/\varphi_A)A^{-1}\iota = w$ or that $b = w$. Since unitary budget elasticities are a necessary and sufficient condition for homothetic preferences, it has been shown that

Proposition 6 (Houthakker, 1960): *If preferences are both directly and indirectly additive, they are also homothetic.*

The next corollary then immediately follows:

Corollary: *Any two out of the following three properties of the preference order imply the third one: direct additivity, indirect additivity, homotheticity.*

7 Identifying complements and substitutes in foodstuff and beverages consumed by the Beguinage of Lier in the 16th century.

A few years ago, Schokkaert & Van der Wee (1988, S&W hereafter) carried out an econometric study of the consumption of foodstuff by the infirmary of the beguinage of Lier (a town 25 km south of Antwerp, Belgium) during the 16th century. Disposing of the accounts of the infirmary for the period 1526-1575, and of excellent data on prices of food and beverage items for the same period, the authors described the allocation of the budget for foodstuffs over ten different items by estimating a regular Rotterdam demand system. The estimation of this allocation process is of high quality for several reasons. First, because the price data show a lot of variation over the period, a huge number of coefficients have been estimated with relatively high precision. Second, because on average no more than ten nuns inhabited the infirmary, one should expect that aggregation problems are insignificant, if not absent. From the point of view of classifying commodities, this study is inviting as one would expect that among ten items of food and beverages both strong interactions of substitutability as well as complementarity arise. Moreover, it is possible to verify such interactions with one's intuition and knowledge about eating and cooking habits—something which is much harder when trying to explain the interactions among the ten main consumption categories of the national accounts.

The ten foodstuff categories studied by S&W, and the average share they occupied in the budget spent on food by the infirmary are: white bread (or wheat) (.075), rye bread (.37), beer (.19), wine (.015), fish (.037), sugar & spices (.045), vinegar & oil (.009), salt (.012), meat (.237) and cheese & butter (.01). The large share of beer is striking, but it should be noted that the beer in that time was very light hopbeer. For a detailed description of all categories, the reader is referred to the S&W study.

The remainder of this section is organised in the following way. First, I present S&W's estimation results of the regular Rotterdam system as well as the coefficients of the implied inverse Rotterdam demand system, and the direct and indirect Allais matrices. Next, I normalise these four types of interaction coefficients so as to make them comparable. Finally, I discuss how the four classifications compare to one another.

- *Estimation results and implied interactions*

Table 1 reproduces the estimation results of the regular Rotterdam system. The first column is the vector of marginal propensities to consume, b, while the symmetric matrix is the matrix S containing the substitution effects. b and S were estimated subject to the restrictions set out in (2.5).

---

insert table 1 here

Before discussing the interactions suggested by these estimates, I will present the direct and indirect Allais matrices, and the Rotterdam parameterisation of the Antonelli matrix. To start with the former, I computed the inverse of the bordered Slutsky matrix, $\begin{pmatrix} S & b \\ b' & 0 \end{pmatrix}^{-1}$, and singled out its NW-block. This gives the direct Allais matrix up to a constant (see (2.10)). To remove this indeterminacy, I restricted a_{37} to zero. In other words I choose beer & oil as the standard pair, in the first place because one would regard these two commodities neither as strong substitutes nor as strong complements, and in the second place because the Slutsky substitution effect is small (s_{37}=.0057 with standard error .0038). The matrix of direct Allais interactions is then as given by table 2. Notice that a_{33} and a_{99} are positive. The reason is that the element (3,7) of $\begin{pmatrix} S & b \\ b' & 0 \end{pmatrix}^{-1}$ is more negative than the 3th and 9th diagonal elements of that matrix; subtracting the former from the latter gives positive numbers. But, as argued in section 2, this is not ruled out by the restriction (2.3).

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insert table 2 here

The indirect Allais matrix was computed according to (4.4), using the average budget shares. Again the indeterminacy is resolved by choosing ρ_C such that $c_{37}=0$. It is given in table 3.

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insert table 3 here

Finally, I made use of the fact that $h=\Omega Aw-w$ and $H=\Omega A\Omega$ (see (2.11)), to calculate the vector of scale effects and the matrix of Antonelli substitution effects (again using the average budget shares). The Rotterdam version of the inverse demand system, as implied by the estimates by Schokkaert & Van der Wee is presented in table 4. It satisfies all the set of restrictions given by (2.8).

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insert table 4 here

- *Normalisation of coefficients*

To get a clearer insight in the degree of complementarity/substitutability implied by one type of classification and to allow comparison across classification, it is necessary to transform the interaction coefficients thus obtained. One possibility would be to express the interactions as elasticities;[10] these are dimensionless and a standard rule is to say that an elasticity is high (low) when in absolute value it exceeds (falls short of) unity. But elasticities have the undesirable property that they are not symmetric. Another way of normalisation was put forward by Allais (1943) himself. He suggested to transform a_{ij} into $a_{ij}/\surd(a_{ii}a_{jj})$. But the possibility to carry out this normalisation may depend on the selection of the standard pair. Indeed, if the selection of that pair results in a positive value for one or more of the diagonal direct Allais coefficients (as it did above for the 3th and 9th diagonal direct Allais coefficients) one runs into problems. So I have recourse to a third normalisation procedure which is at the same time symmetric, dimensionless and always feasible. It goes as follows: first, I calculate for each matrix the difference between the largest and smallest off-diagonal element. Next, I express all off-diagonal elements as a percentage of this difference. For instance, the normalised Slutsky effect between commodities i and j ($j \neq i$) is defined as

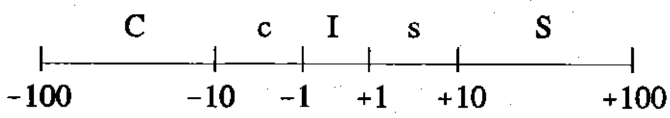
$$\frac{s_{ij}}{M_S - m_S} \cdot 100, \tag{7.1}$$

where

$$\begin{aligned} M_s &= \max\{s_{ij} \mid i,j=1,...,n, \text{ and } i \neq j\}, \text{ and} \\ m_s &= \min\{s_{ij} \mid i,j=1,...,n, \text{ and } i \neq j\}. \end{aligned} \tag{7.2}$$

[10] This would require division of s_{ij} and h_{ij} by w_j and multiplication of a_{ij} and c_{ij} by w_j.

In a similar way, I compute the normalised off diagonal elements on *C*,*A* and *H*, except that for the latter two the sign is reversed; in that way a positive (negative) sign indicates substitutability (complementarity) in all four classifications. I then use the following rules to label interactions:



where

C (S) = strong complementarity (substitutability);
c (s) = weak complementarity (substitutability); and
i = independence.

The interaction coefficients thus obtained and the classifications implied are displayed in tables 5-8. Ten commodities give rise to 45 interactions.

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insert tables 5-8 here

- *Discussion of the results*

Close inspection of tables 5-8 reveals several regularities. These will now be discussed. A first feature is that the number of weak and strong substitutabilities is the highest under the Slutsky criterion (29 or 65% out of 45), lowest under indirect Allais (10 or 22%), and in between under Antonelli (15 or 33.3%) and direct Allais (12 or 26.7%). See table 9.

Table 9. Frequency of interactions

	Slutsky	Antonelli	Dir Allais	Ind Allais
Substitutability	.65	.33	.27	.23
strong	.38	.09	.11	.07
weak	.27	.24	.16	.16
Independence	.04	.20	.42	.13
Complementarity	.31	.47	.31	.64
strong	.13	.20	.09	.11
weak	.18	.27	.22	.53

As a more detailed comparison, I have calculated the number of times that the interactions implied by one classification resulted in the same or adjacent category implied by another classification. Expressed as a percentage out of 45, this gives an idea about how close classifications on the basis of two different criteria are. These percentages are given in table 10 below. Thus, Antonelli and direct Allais classify 96% of the interactions in the same or in an adjacent category, while the largest mismatch is between Slutsky and direct Allais (only 47% similarities), a little bit worse than the mismatch between Slutsky and Antonelli (49% similarities). The Antonelli matrix is of course positively related to the direct Allais matrix. But the same is true for the Slutsky and indirect Allais matrix. Still, these latter two criteria classify only 62% of the interactions in the same or adjacent category.

Table 10 Fraction of interactions classified in the same or adjacent category by the different classification rules.

	Antonelli	Dir Allais	Ind Allais
Slutsky	.49	.47	.62
Antonelli		.96	.73
Dir Allais			.36

This ranking of similarities is confirmed by the ordinary and rank correlation coefficients between the off-diagonal normalised matrix elements. These correlations are reported in table 11 above and below the diagonal, respectively. None of the correlation coefficients are negative, despite the inverse relationship between one matrix and two of the other matrices. This is reassuring, since a negative correlation would mean that exploiting different information about the same preference ordering results in a totally opposite classification of commodities. The highest rank correlation obtains between direct Allais and Antonelli (.91), followed by Slutsky and indirect Allais (.70). Again, Slutsky and direct Allais differ widely (.18). The ordinary correlation coefficients take into account both the relative position of the interactions and the intensity with which these interactions differ.[11] Except for Dir Allais-Ind Allais, the ordinary

[11] Note that the rankings of the interactions within both the direct and indirect Allais classification is independent of the selection of the standard pair: selecting a new standard pair means subtracting the coefficient pertaining to that pair of the old interaction matrix. This is a monotone transformation (a linear one). None of the rank correlation coefficients is therefore affected by the choice of a different standard pair, nor are the ordinary correlation coefficients.

correlation coefficients are all below the corresponding rank correlation coefficients. For instance, this means that although Antonelli and direct Allais rank pairs of commodities on the complements-substitutes spectrum in a very similar way, there is much less agreement w.r.t. to the intensities of pairs being complements/substitutes.

Table 11. Ordinary (above diagonal) and rank (below diagonal) correlation coefficients among the different classifications.

	Slutsky	Antonelli	Dir Allais	Ind Allais
Slutsky		.14	.07	.28
Antonelli	.20		.48	.13
Dir Allais	.18	.91		.34
Ind Allais	.70	.32	.27	

It is interesting to identify the pairs of commodities for which the type of interaction is robust w.r.t. the classification rule. There are 13 pairs (29%) which under all four criteria are classified as either substitutes or complements; they are listed in table 12, along with their degree of interaction. According to all four classification rules, salt interacts with spices, oil and cheese in a substitutable way, while the relationship w.r.t. wine and meat is one of complementarity. Salt "was an important ingredient in the kitchen of the Middle Ages and the Early Modern Times. Salting perishable foodstuffs such as meat, fish and butter was the most common way at that time to preserve them" (S&W, p 143). The category "oil" comprises both oil and vinegar. The use of the latter for flavouring purposes explains the substitution possibilities with salt. Also spices were used both as flavourings and as a preservative for perishable products, but they served also as drugs (S&W, p 142). This explains the substitutability of spices with salt. The fact that the category cheese consists of both cheese and butter and that butter, as S&W observe, was salted in those days for preservation purposes, nicely explains the strong degree of substitutability among salt and the category cheese. An odd relationship is the complementarity between salt and wine, which is quite strong according to both Allais classifications. Wheat and cheese being substitutes (although not in a very strong way) looks peculiar, but probably this is due to the fact that both white bread, cheese and butter were regarded as luxury products in the 16th century: two luxury products having to fit into the same budget can give rise to substitutability, even though from a diet point of view they are complementary. Whence, if cheese and/or butter were consumed, this was probably in combination with the much cheaper rye bread.

Table 12. Robust interactions

Robust substitutes	Slutsky	Antonelli	Dir Allais	Ind Allais
wheat & oil	10.47	5.51	4.23	4.46
wheat & cheese	15.26	8.76	6.03	6.33
fish & wine	9.77	3.75	3.51	4.86
salt & spices	12.85	11.55	11.11	8.64
salt & oil	7.23	9.14	44.17	27.18
salt & cheese	9.91	13.62	59.24	33.54
Robust complements	**Slutsky**	**Antonelli**	**Dir Allais**	**Ind Allais**
oil & rye	-11.11	-12.54	-2	-3.27
cheese & rye	-16.33	-18.74	-2.71	-4.15
spices & wine	-1.87	-1.45	-1.17	-3.62
salt & wine	-5.35	-4.51	-13.17	-15.34
fish & oil	-2.68	-7.7	-12.1	-5.71
fish & cheese	-23.83	-13	-18.41	-30
salt & meat	-17.14	-5.7	-1.11	-4.53

Finally, I have checked for which pairs of commodities the Slutsky criterion implies an interaction which is different from the one implied by the other three criteria—see table 13. Deviant Slutsky substitutes are: wheat & wine, salt & rye, spices & beer. The first two of these pairs are classified as (weak) complements under the other three classifications. The use of both salt and rye to bake rye bread would indeed suggest these items to be complements. Spices & beer are according to Slutsky, strong substitutes, while according to the other three criteria, these items are independent from one another, which is also more in line with intuition. Deviant Slutsky independents are fish & wheat, and fish & salt. The other three criteria classify both pairs as complements. The preservative role of salt indeed suggests that fish and salt were used in a complementary way. Deviant Slutsky complements are not encountered. And indeed, deviant Slutsky substitutes are more likely to occur than deviant complements because of the bias towards substitutability inherent to the Slutsky criterion.

Table 13. Deviant Slutsky classifications

Deviant Slutsky substitutes	Slutsky	Antonelli	Dir Allais	Ind Allais
wheat & wine	2.81	-2.69	-1.3	-1.56
salt & rye	1.61	-52.5	-6.24	-1.17
spices & beer	29.18	0.21	-.03	-.027
Deviant Slutsky independents	**Slutsky**	**Antonelli**	**Dir Allais**	**Ind Allais**
wheat & fish	0.27	-22.02	-4.17	-2.21
fish & salt	-0.54	-34.59	-40.76	-2.58

8 Conclusion

In this paper, I have considered the transformed hessian of the indirect utility function as a fourth source to make inference about the way commodities interact with one another at the preference level. I have coined this the indirect Allais matrix because it results from the same type of ordinalisation procedure which Allais (1943) applied on the Hessian of the direct utility function. I showed that the indirect Allais is positively related to the Slutsky matrix, but inversely to the Antonelli and direct Allais matrices.

Indirect Allais coefficients can be given the interpretation that they measure the compensated elasticity of the marginal welfare loss a consumer experiences due to a price rise of one commodity w.r.t. a price rise of another commodity. A positive coefficient is then associated with substitutability, while a negative coefficient indicates complementarity. I argued that the indirect Allais classification does not exhibit the bias towards substitutability to which the Slutsky classification is liable.

Furthermore, I showed that indirect separability assumptions easily translate into a simplified structure for the indirect Allais matrix, and that they reduce cross Antonelli substitution effects in the same way as cross Slutsky substitution effects are affected by direct separability assumptions. In general, the relation between the direct and indirect Allais matrices is complex, but this is no longer true for the class of homothetic preferences.

Finally, I considered the demand system for foodstuff estimated by Schokkaert & Van der Wee (1988) using the accounts of the Lier beguinage's infirmary in the 16th century. Using their estimation results, I calculated the interactions between ten different food items according to the four criteria (Slutsky, Antonelli, direct and indirect Allais) and commented on the degree of congruence and mismatch among those criteria.

Even though the indirect Allais criterion is free from the substitution bias, its weakness is clearly the crucial dependence on the standard pair selected—a weakness shared by the direct Allais classification. Still, I believe that the indirect Allais matrix sheds light on aspects of the preference ordering which remain hidden when using either of the three other classification rules. So, rather than promoting the indirect Allais criterion as the best classification rule, I would suggest it as a useful additional instrument in analysing demand behaviour. In a first instance, and from a theoretical point of view, because it seems only natural to exploit the information contained in every function which completely represents the preference ordering, including the indirect utility function. Secondly, and from an empirical stance, complementing the three 'standard' criteria with the indirect Allais rule allows the econometrician to identify robust interactions among commodities. Verifying to which extent robust complements/ substitutes accord with one's intuition can be useful in assessing the empirical performance of a demand system.

In normative tax theory, the Slutsky and Antonelli matrices play a pivotal role in the interpretation of optimal linear indirect tax rules (see e.g. Stern, 1986). Although the taxation literature does not make any reference to direct Allais coefficients, these coefficients control for optimal non-linear indirect tax rates. Indeed, whether the marginal tax on commodity i should exceed or fall short of that on commodity j depends on the sign of $\partial \ln(u_i/u_j)/\partial \ell$ where ℓ denotes leisure (see Deaton, 1981, eq 24), and thus on the sign of $a_{i\ell}-a_{j\ell}$ (see (2.2)). It would be interesting to investigate how indirect Allais coefficients can be made use of in the interpretation of normative policy rules.

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Appendix

Proof of Proposition 2:
To obtain expression (4.1), one can start from the 'demand system' (3.10) and replace the LHS by $d\ln\lambda\iota + d\ln q$ (see (3.1)). Rearranging and multiplying through by $\hat{w}$ then yields:

$$\hat{w}\,d\ln q = \hat{w}C\hat{w}\,d\ln\pi + w(\beta w' d\ln\pi - d\ln\lambda). \tag{A1}$$

Premultiplying through by ι', and rearranging produces

$$(\beta w' d\ln\pi - d\ln\lambda) = w' d\ln q - w' C\hat{w}\,d\ln\pi. \tag{A2}$$

Substituting the round brackets term in (A1) for the RHS of (A2) results in

$$\hat{w}\,d\ln q = ww' d\ln q + \Omega C\hat{w}\,d\ln\pi, \tag{A3}$$

where $\Omega =_{def} \hat{w} - ww'$. Adding and substracting $bw' d\ln\pi$ to the RHS, making use of the fact that $w' d\ln q = -w' d\ln\pi$ and rearranging, one obtains

$$\hat{w}\,d\ln q = -bw' d\ln\pi + (\Omega C\hat{w} - ww' + bw')d\ln\pi. \tag{A4}$$

But since in differential form, the Rotterdam parameterisation of the regular demand system $q(\pi)$ is given by (2.6), the matrix in round brackets with which $d\ln\pi$ is premultiplied on the RHS in (A4) must equal the matrix S. And since $S\iota = 0$, it follows that $b = (I - \Omega C)w$ and therefore that $S = \Omega C \Omega$. These relationships can then be collected as in (4.1).

To see how the C matrix relates to the Antonelli matrix, one premultiplies (A1) through by $C^{-1}\hat{w}^{-1}$ and isolates $\hat{w}\,d\ln\pi$:

$$\hat{w}\,d\ln\pi = C^{-1} d\ln q - C^{-1}\iota(\beta w' d\ln\pi - d\ln\lambda). \tag{A5}$$

Premultiplying through by ι', the round brackets term can again be isolated:

$$(\beta w' d\ln\pi - d\ln\lambda) = (\iota' C^{-1}\iota)^{-1}(\iota' C^{-1} d\ln q - w' d\ln\pi). \tag{A6}$$

Taking advantage of this equality, another way of writing (A5) is then

$$\hat{w}\,d\ln\pi = [C^{-1} - C^{-1}\iota(\iota' C^{-1}\iota)^{-1}\iota' C^{-1}]d\ln q + C^{-1}\iota(\iota' C^{-1}\iota)^{-1} w' d\ln\pi. \tag{A7}$$

But since $w' d\ln\pi = -w' d\ln q$, this expression can also be written as

$$\hat{w}\,d\ln\pi = [-\frac{1}{\varphi_C} C^{-1}\iota]w' d\ln q + [C^{-1} - \frac{1}{\varphi_C} C^{-1}\iota\iota' C^{-1}]d\ln q, \tag{A8}$$

where φ_C is a shorthand for $\iota' C^{-1}\iota$.

In view of the Rotterdam parameterisation of the inverse demand system, it is then clear that $g = -C^{-1}\iota(1/\varphi_C)$ and $H = C^{-1} - \varphi_C gg'$. Since $Cg = -\iota(1/\varphi_C)$ and $CH = I + \iota g'$, it is not difficult to check that C, H and g relate as in (4.2). QED

Proof of Proposition 3:

Using (2.10), (4.1), and the fact that $\begin{pmatrix} -\Omega & w \\ w' & 1 \end{pmatrix}^{-1} = \begin{pmatrix} -\hat{w}^{-1} & \iota \\ \iota' & 0 \end{pmatrix}$, the 'bordered' direct Allais matrix can be related to the 'bordered' indirect Allais matrix in the following manner:

$$\begin{pmatrix} A-(^1/\varphi_A)\iota\iota' & \iota \\ \iota' & 0 \end{pmatrix} = \begin{pmatrix} -\hat{w}^{-1} & \iota \\ \iota' & 0 \end{pmatrix} \begin{pmatrix} C-\rho_C\iota\iota' & \iota \\ \iota' & 0 \end{pmatrix}^{-1} \begin{pmatrix} -\hat{w}^{-1} & \iota \\ \iota' & 0 \end{pmatrix}. \tag{A9}$$

Now let $D =_{def} (C-\rho_C \iota\iota')^{-1}$. Applying next the inversion formula for partitioned matrices yields:

$$\begin{pmatrix} C-\rho_C\iota\iota' & \iota \\ \iota' & 0 \end{pmatrix}^{-1} = \begin{pmatrix} D-D\iota(\iota'D\iota)^{-1}\iota'D & D\iota(\iota'D\iota)^{-1} \\ (\iota'D\iota)^{-1}\iota'D & -(\iota'D\iota)^{-1} \end{pmatrix}, \tag{A10}$$

so that

$$A-\frac{1}{\varphi_A}\iota\iota' = \hat{w}^{-1}\left[D-\frac{1}{\iota'D\iota}(D+\hat{w})\iota\iota'(D+\hat{w})\right]\hat{w}^{-1}. \tag{A11}$$

But using the Bartlett inverse formula,

$$\begin{aligned} D &= C^{-1}+\frac{\rho_C}{1-\rho_C\varphi_C}C^{-1}\iota\iota'C^{-1} \\ &= \frac{1}{1-\rho_C\varphi_C}[(1-\rho_C\varphi_C)C^{-1}+\rho_C C^{-1}\iota\iota'C^{-1}], \end{aligned} \tag{A12}$$

from which it follows that

$$D\iota = \frac{1}{1-\rho_C\varphi_C}C^{-1}\iota, \text{ and } \iota'D\iota = \frac{\varphi_C}{1-\rho_C\varphi_C}. \tag{A13}$$

Another way of writing (A11) is then

$$A-\frac{1}{\varphi_A}\iota\iota' = \hat{w}^{-1}\left[D-\frac{\left(C^{-1}\iota+(1-\rho_C\varphi_C)w\right)\left(\iota'C^{-1}+(1-\rho_C\varphi C)w'\right)}{\varphi_C(1-\rho_C\varphi_C)}\right]\hat{w}^{-1}. \tag{A14}$$

Putting the square bracket terms on the common denominator $\varphi_C(1-\rho_C\varphi_C)$, substituting D for the lower RHS of (A12), and rearranging then finally produces (4.8) in the text.

In a similar way, one can combine (4.2) and (2.11) to express the 'bordered' indirect Allais matrix as a function of the 'bordered' direct Allais matrix (since $\begin{pmatrix} \Omega & -w \\ w' & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \hat{w}^{-1} & \iota \\ -\iota' & 0 \end{pmatrix}$):

$$\begin{pmatrix} C-(^1/\varphi_C)\iota\iota' & \iota \\ \iota' & 0 \end{pmatrix} = \begin{pmatrix} \hat{w}^{-1} & -\iota \\ \iota' & 0 \end{pmatrix} \begin{pmatrix} A-\rho_A\iota\iota' & \iota \\ \iota' & 0 \end{pmatrix}^{-1} \begin{pmatrix} \hat{w}^{-1} & \iota \\ -\iota' & 0 \end{pmatrix}. \tag{A15}$$

Exactly the same type of manipulations then result in expression (4.9) of the text. QED

Table 1. Estimation results of the Rotterdam-model (1526-1575)

	b	*S*										R^2
		Wheat	Rye	Beer	Wine	Fish	Spices	Oil	Salt	Meat	Cheese	
Wheat (.075)°	.0911** (.0327)	-.0726** (.0157)	.0166** (.0148)	.0191* (.0136)	.0021 (.0066)	.0002 (.0066)	-.0018 (.0119)	.0076 (.0038)	.0012 (.0043)	.0162 (.0183)	.0114* (.0071)	.796
Rye (.37)	.2801** (.0684)		-.1059** (.0399)	.0416* (.0245)	.0067* (.0062)	.0034 (.0081)	-.0026 (.0129)	-.0083** (.0036)	.0037 (.0044)	.0569* (.0299)	-.0122* (.0067)	.546
Beer (.19)	.1821** (.0423)			-.1631** (.0303)	.0040 (.0067)	.0123* (.0084)	.0218* (.0139)	.0057* (.0038)	-.0039 (.0048)	.0512* (.0287)	.0114* (.0073)	.667
Wine (.015)	.0315** (.0139)				-.0151* (.0083)	.0073* (.0038)	-.0014 (.0074)	.0020 (.0033)	-.0040* (.0029)	-.0114 (.0123)	.0098* (.0054)	.423
Fish (.037)	.0537** (.0164)					-.0312** (.0055)	.0058 (.0072)	-.0020 (.0021)	-.0004 (.0025)	.0224* (.0115)	-.0178* (.0038)	.545
Spices (.045)	.0368* (.0271)						-.0455** (.0180)	-.0047* (.0043)	.0096* (.0049)	.0242* (.0230)	-.0064 (.0076)	.217
Oil (.009)	.0102* (.0080)							-.0073** (.0025)	.0054** (.0017)	.0113* (.0062)	-.0097** (.0030)	.421
Salt (.012)	.0099* (.0096)								-.0063** (.0025)	-.0128* (.0075)	.0074** (.0029)	.206
Meat (.237)	.286** (.061)									-.1864* (.0479)	.0285** (.0112)	.433
Cheese (.01)	.0184 (.0192)										-.0223* (.0071)	.495

Notes: Figures in parentheses are standard errors of the coefficients. The superscripts * and ** denote that the coefficient is larger than or larger than twice its standard error, respectively. ° denotes average budget share.
Source: Schokkaert & Van der Wee (1988, p 147)

Table 2. Implied direct Allais coefficients (divided by 100) ($a_{37}=0$)

	Wheat	Rye	Beer	Wine	Fish	Spices	Oil	Salt	Meat	Cheese
Wheat	-31.27	15.19	0.48	29.85	95.95	-24.26	-97.30	-317.42	3.31	-138.72
Rye		-5.55	1.08	-12.40	-41.20	12.31	45.90	143.52	-0.19	62.41
Beer			0.84	1.15	2.20	0.61	0	-2.98	0.92	-1.14
Wine				-33.45	-80.66	26.86	111.91	302.67	-1.62	113.79
Fish					-284.11	73.74	278.16	937.03	-6.02	423.26
Spices						-20.50	-85.45	-255.39	2.94	-103.35
Oil							-369.49	-1015.37	9.42	-385.60
Salt								-3189.18	25.58	-1361.76
Meat									0.65	10.69
Cheese										-631.92

Table 3. Implied indirect Allais coefficients ($c_{37}=0$)

	Wheat	Rye	Beer	Wine	Fish	Spice	Oil	Salt	Meat	Cheese
Wheat	-16.58	-2.62	-2.07	-2.69	-3.84	-3.81	7.67	-1.96	-2.75	10.9
Rye		-3.53	-2.37	-2.89	-3.2	-2.97	-5.62	-2.01	-2.56	-7.14
Beer			-7.68	-2.9	-1.9	-0.47	0	-4.75	-2.27	1.96
Wine				-72.55	8.36	-6.23	10.34	-26.41	-7.76	60.15
Fish					-26.93	-0.03	-9.83	-4.44	-1.35	-52.64
Spices						-24.85	-14.80	14.88	-1.00	-18.12
Oil							-93.63	46.78	1.71	-111.99
Salt								-45.98	-7.79	57.74
Meat									-6.98	7.74
Cheese										-228.92

Table 4. Implied coefficients of the Rotterdam inverse demand system

	g	*H*									
		Wheat	Rye	Beer	Wine	Fish	Spices	Oil	Salt	Meat	Cheese
Wheat	0.68	-18.20	39.50	-0.73	3.24	26.45	-8.52	-6.62	-28.71	4.12	-10.53
Rye	-1.62		-87.15	1.57	-7.41	-57.07	19.07	15.07	63.08	-9.18	22.52
Beer	-.01			-0.23	0.04	1.17	-0.25	-0.12	-1.02	0.08	-0.52
Wine	0.14				-0.78	-4.51	1.75	1.50	5.42	-0.93	1.68
Fish	-1.37					-38.92	12.19	9.25	41.55	-5.74	15.62
Spices	-0.02						-4.33	-3.49	-13.87	2.18	-4.72
Oil	-0.19							-3.00	-10.98	1.86	-3.48
Salt	0.71								-45.95	6.85	-16.37
Meat	-0.02									-1.40	2.16
Cheese	0.70										-6.34

Table 5. Normalised Slutsky interactions

	Wheat	Rye	Beer	Wine	Fish	Spices	Oil	Salt	Meat	Cheese
Wheat		22.22	25.57	2.81	0.27	-2.41	10.17	1.61	21.69	15.26
Rye	S		55.69	8.97	4.55	-3.48	-11.11	4.95	76.17	-16.33
Beer	S	S		5.35	16.47	29.18	7.63	-5.22	68.54	15.26
Wine	s	s	s		9.77	-1.87	2.68	-5.35	-15.26	13.12
Fish	i	s	S	s		7.76	-2.68	-0.54	29.99	-23.83
Spices	c	c	S	c	s		-6.29	12.85	32.4	-8.57
Oil	S	C	s	s	c	c		7.23	15.13	-12.99
Salt	s	s	c	c	i	S	s		-17.14	9.91
Meat	S	S	S	C	S	S	S	C		38.15
Cheese	S	C	S	S	C	c	C	s	S	

Table 6. Normalised Antonelli interactions

	Wheat	Rye	Beer	Wine	Fish	Spices	Oil	Salt	Meat	Cheese
Wheat		-32.88	0.61	-2.69	-22.02	7.09	5.51	23.9	-3.43	8.76
Rye	C		-1.31	6.17	47.5	-15.87	-12.54	-52.5	7.64	-18.74
Beer	i	c		-0.04	-.098	0.21	0.10	0.85	-0.07	0.43
Wine	c	s	i		3.75	-1.45	-1.25	-4.51	0.77	-1.40
Fish	C	S	i	s		-10.15	-7.70	-34.59	4.78	-13.00
Spices	s	C	i	c	C		2.90	11.55	-1.82	3.93
Oil	s	C	i	c	c	s		9.14	-1.55	2.90
Salt	S	C	i	c	C	S	s		-5.70	13.62
Meat	c	s	i	i	s	c	c	c		-1.79
Cheese	s	C	i	c	C	s	s	S	c	

Table 7. Normalised direct Allais interactions

	Wheat	Rye	Beer	Wine	Fish	Spices	Oil	Salt	Meat	Cheese
Wheat		-0.66	-0.02	-1.3	-4.17	1.06	4.23	13.81	-0.14	6.03
Rye	i		-0.05	0.54	1.79	-0.54	-2.00	-6.24	0.01	-2.71
Beer	i	i		-0.05	-0.10	-0.03	0	0.13	-0.04	0.05
Wine	c	i	i		3.51	-1.17	-4.87	-13.17	0.07	-4.95
Fish	c	s	i	s		-3.21	-12.1	-40.76	0.26	-18.41
Spices	s	i	i	c	c		3.72	11.11	-0.13	4.50
Oil	s	c	i	c	C	s		44.17	-0.41	16.77
Salt	S	c	i	C	C	S	S		-1.11	59.24
Meat	i	i	i	i	i	i	i	c		-0.47
Cheese	s	c	i	c	C	s	S	S	i	

Table 8. Normalised indirect Allais interactions

	Wheat	Rye	Beer	Wine	Fish	Spices	Oil	Salt	Meat	Cheese
Wheat		-1.52	-1.20	-1.56	-2.23	-2.21	4.46	-1.14	-1.60	6.33
Rye	c		-1.37	-1.68	-1.86	-1.73	-3.27	-1.17	-1.49	-4.15
Beer	c	c		-1.68	-1.10	-0.27	0	-2.76	-1.32	1.14
Wine	c	c	c		4.86	-3.62	6.01	-15.34	-4.50	34.94
Fish	c	c	c	s		-0.02	-5.71	-2.58	-0.78	-30.58
Spices	c	c	i	c	i		-8.60	8.64	-0.58	-10.53
Oil	s	c	i	s	c	C		27.18	1.00	-65.06
Salt	c	c	c	C	c	s	S		-4.53	33.54
Meat	c	c	c	c	i	i	i	c		4.49
Cheese	s	c	s	S	C	C	C	S	s	

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