Informational Feasibility, Decentralization and Public Finance Mechanisms

Fred Schroyen

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* SESO-UFSIA, University of Antwerp, Prinsstraat 13, B-2000 Antwerp
  (dse.schroyen.f@alpha.ufsia.ac.be)

Universitaire Faculteiten St.-Ignatius
Prinsstraat 13 - B 2000 Antwerpen

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Abstract The paper reviews the game form approach to redistribution policy. This approach provides a justification for studying tax systems as instruments to (re)allocate resources in a society made up of a heterogeneous population. The paper sketches the arguments for such justification in a unifying framework.

First, I draw attention to the very specific information structure implicitly assumed by the fundamental welfare theorem. Next, relying on concepts and results defined and obtained in the literature on mechanism design (Dasgupta, Hammond & Maskin, 1979, and Laffont, 1986), I explain the problem of decentralization for a general environment. The solution to this problem is then further refined for a specific environment—a large exchange economy with heterogeneous consumers—and shown to be equivalent to a specific form of decentralization defined by Hammond (1979). Finally, allowing for constant returns to scale production activities in this economy, it is shown that a tax system based on the net transactions taking place between consumers and the production sector can never be improved upon by other type of mechanisms relying on the same observational capacities.
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1. Introduction

Every society is made up of a variety of people and often also of a regulating authority. The economic relationships these agents enter into with one another take place within an economic environment. To characterize this environment, it suffices to specify the preferences and endowments of the constituting population, including those of the regulating authority—the planner, for short. The endowment of an agent specifies his initial wealth of resources, as well as what this agent knows about the environment he makes part of. This knowledge is summarized by what is called the information set. Typically, these information sets are very coarse w.r.t. some aspects of the environment, like the preferences and endowments of the other agents, while they can be very fine w.r.t. other aspects, usually one's own preferences and initial wealth.

In this economic environment, social states or outcomes—also called economic allocations—are only worthwhile considering when they respect the physical relationships imposed by the technology, consumption sets and initial stock of resources. Within this set of physically feasible social outcomes, the characterization of the Pareto efficient outcomes has received a lot of attention. For a long time, such attention seemed justified by the second fundamental welfare theorem. This theorem asserts that—subject to certain regularity conditions—every point on the Pareto frontier can be implemented by the competitive market mechanism after a suitable lump-sum redistribution of resources. The implementability, however, is conditional on a very specific information structure which is not likely to obtain.

As I just pointed out, the initial endowments also have an informational component. And just like physical feasibility of a social state is a *conditio sine qua non* for discussing its efficiency and equity properties, so is feasibility w.r.t. the information sets of agents and planner. In fact, the implicit assumption behind the theorem above is one of a very fine information set of the planner. This planner is endowed not only with the necessary authority to enforce contracts and to carry out redistribution of initial resources, but also with the necessary information to make this redistribution 'suitable'. In particular, it supposes the planner to be well versed about the individual characteristics of each agent, like his preferences, his consumption set, his endowment. This is a very strong assumption on which not everyone agreed, and some people engaged in questioning it openly. As Lerner (1944) was probably the first to observe, the problem with a utilitarian redistribution scheme is that
There is no way of discovering with certainty whether any individual's marginal utility of income is greater than, equal to, or less than that of any other individual.

[...T]hese things are not capable of being discovered. Every individual could declare that he has exceptionally high capacities for satisfaction and so should be given more income than anybody else if total satisfaction is to be maximized; and there is no way of testing the validity of such a claim. (p 28-29)

In other words, there may exist social states, physically feasible and even Pareto efficient, which are not implementable because of informational asymmetries. A first necessary step before judging economic allocations on their efficiency and equity properties therefore is to put them to the test of a criterion of informational feasibility. By introducing the concept of incentive compatibility, Hurwicz (1972) was probably the first to formalize such a criterion. Loosely speaking, an economic allocation is incentive compatible when the private information necessary to implement that allocation is willingly supplied by the owners of such information. But it turns out that the problem can be formulated more generally than the way Hurwicz did. What is needed is a framework without any reference to institutions at all, which defines informational feasibility by recognizing the desire on the part of the owners of private information to manipulate this information strategically. Strategic manipulation is to be understood in its most general form, i.e. strategic behaviour w.r.t. the one in request of the information—the planner—as well as w.r.t. the other owners of information. This suggest that we may define as informationally feasible, any outcome of an abstract non-cooperative game with incomplete information.

In such a game, both planner and agents participate as players, although the regulating role of the planner is acknowledged by giving this person the special status of a Stackelberg leader. This move consists of the design of a mechanism (also called game form, or incentive scheme) with which she subsequently confronts the other players.¹ In a mechanism, the planner endows each agent with an action or strategy set from which the agent is invited to select an action, often also called a message. At the same time, the planner announces publicly an outcome function. This is a rule which translates the messages of the agents into a physically feasible social state. Any outcome associated with the equilibrium point of such a non-cooperative game is then called informationally feasible: it can be sustained as the equilibrium outcome of strategic behaviour. Of course, the outcome function was announced in advance and the commitment

¹ In this essay, the planner is addressed as feminin; agents are masculin.
to this rule guarantees precisely why agents are prepared to reveal their private information through the act of choosing an action. This is the very notion of a Stackelberg game. If such commitment would not be possible, the planner's outcome rule would become incredible to the players who would adjust their strategic behaviour accordingly.

The notion of a planner inviting every agent to make a selection from the endowed strategy set arouses the idea of decentralization. Hayek (1945)—not quite a member of the same school of thought as Lerner—was also well aware that a lot of relevant information in an economy is of a dispersed and private nature. He observed that

"...[...] practically every individual has some advantage over all others in that he possesses unique information of which beneficial use might be made, but of which use can be made only if the decisions depending on it are left to him or are made with his active cooperation. (pp 521-2)"

This observation made him a strong proponent of the free market mechanism. Whether such a stance is completely justified by that observation is questionable, but this is not the issue here. Interesting, however, is Hayek's conviction that decentralized decision making can be instrumental to the efficient use of private information. It is not very difficult to convince oneself why this is the case. Indeed, although an agent may be just as ignorant as the planner w.r.t. the characteristics of the other agents, he has the additional information about his personal characteristic: he knows what kind of person he is—what his preferences and endowments look like. Consequently, when the agent is given the opportunity to make a selection from a set of actions, he will select in the capacity of the person he is: his choice will reflect what is optimal for someone with the kind of characteristics he shares, and not for a person of another type. This implies that the act of choosing a particular action may reveal information to the planner which would not have become available to her otherwise. By introducing some degree of decentralization, the planner can extract private information on which she can condition her own action—the selection of a physically feasible social outcome. By construction, this choice is informationally feasible. Incidentally, notice that this extraction of private information distinguishes the mechanism from a command system (a central planning system), where every outcome is decided upon the available public information, however coarse this may be.

Despite its frequent use in economic language, a definition of economic
decentralization does seldom appear explicitly. In the New Palgrave's entry on
the subject, Malinvaud skips the definition, while the entry in the International
Encyclopedia of Social Sciences is solely concerned with administrative
decentralization. In Dasgupta’s (1982) essay—mainly concerned with problems
of political philosophy—decentralization is defined as "the assignment of rights
to certain regions of economic decision making" (p 202). This definition clearly
encompasses the abstract game process sketched a moment ago. It will do for the
purpose of this paper.

The first task we have to face is the delineation of the set of physically
feasible social states that can be implemented, i.e. obtained as the outcome of a
decentralization process of the kind just described. This will provide us with the
set of informationally feasible states. Once this set is known, its elements may
be partially or even completely ordered according to the preferences of the social
planner. In particular, we may want to identify those informationally feasible
outcomes that are not susceptible to any Pareto improvement. This collection of
outcomes is called the informationally constrained Pareto frontier. By means of
a Bergsonian social welfare function, the planner may select the most preferred
outcome on this frontier.

But it is clear that such optimization exercises can only take place after a
careful identification of the social states which an imperfectly informed planner
can implement. Such an identification is the subject of this paper. Although my
ultimate aim is to determine what is informationally feasible in a large economy
with heterogeneous consumer/workers and a production sector, I start off in section
2 by defining the problem of implementation in a fairly general form: I will try
to explain what a mechanism is, how the information sets of planner and agents
look like (2.1) and how the content of the latter influence the equilibrium concept
of the game (2.2). Next, I show how the Revelation Principle allows us to
restrict attention to the class of direct mechanisms (whereby agents' action sets
coincide with the sets of possible characteristics) (2.3). A particular attractive
class of mechanisms is the one where truthful reporting of the private information
is a dominant strategy for every agent. Members of this class are called
straightforward mechanisms and their existence is investigated in section 2.4.

In section 3, I consider a particular environment, to wit a large exchange
economy (3.1). For this environment, the mechanism assigns physically feasible
economic allocations to individual agents; it is called a resource allocation
mechanism (3.2). After specifying the information set of the planner (3.3), I
discuss the anonymity condition which the resource allocation mechanism is
imposed to respect (3.4). This restriction, together with the assumption of a large economy, will guarantee the desirable property of straightforwardness (3.5).

Section 4 is about a particular form of decentralization, whereby the actions agents are allowed to choose from provide them directly with utility. This notion of decentralization is the one behind agents maximizing their utility over an opportunity set. Mechanisms respecting this particular definition of decentralization are defined in section 4.1—to emphasize this particular way of decentralization, I have called them strictly decentralizable. In the same subsection, I present a first version of the Hammond (1979) equivalence theorem between straightforward anonymous mechanisms and strictly decentralizable mechanisms for an exchange economy. Next, this equivalence result is discussed for large exchange economies (4.2). Finally, I consider the case of a large economy with a constant-returns-to-scale production sector and show the strictly decentralizable mechanisms are nothing else than the competitive market mechanism complemented with non-linear taxation: the so-called public finance mechanisms (4.3).

Some concluding remarks and examples are gathered in section 5.

Before turning to section 2, I should like to stress that with the ideas in this paper I do not want to make any claim to originality. Rather, I have tried to give an interpretation of the results skillfully obtained by public finance theorists like Dasgupta, Hammond and Guesnerie. Of course, any errors in interpretation are entirely mine.

2. The Problem of Implementation

In this section I shall try to explain how an imperfectly informed planner can overcome her ignorance by the use of a game form or mechanism. In order to focus entirely on the definitions and results of mechanism design theory, the economic environment is kept rather abstract and general. I will rely heavily on the treatment of the theory by Dasgupta, Hammond & Maskin (1979) and Laffont (1988). A non-technical overview with applications may be found in d'Aspremont & Gérard-Varet (1990) and Guesnerie (1992), while Laffont & Maskin (1980) present a more in-depth survey and define the problem at a level which is more general.
2.1. The Design of a Mechanism

Let there be $N$ agents in society, indexed $i \in I = \{1, 2, \ldots, N\}$, as well as a social planner. The set of physically feasible social states in this economy is denoted by $X$. Each agent is endowed with a characteristic $\theta_i$ which ties down the agent's preference order over the set $X$; let this ordering be given by the binary relation $R_i(\theta_i)$ and let it be complete. Often, I shall assume this relation is continuous as well, and consequently representable by a continuous utility function $u_i(\cdot; \theta_i)$. A priori it is known that $\theta_i \in \Theta_i$, the set of possible parameter values that may characterize agent $i$.

The profile of this society is defined as the ordered list of individual characteristics $\theta = (\theta_i)_{i \in I}$; this list is an element of the set of all possible profiles $\mathcal{Q} = \Theta_1 \times \Theta_2 \times \ldots \times \Theta_N$. Typically, the planner's most preferred outcome out of the set of feasible social outcomes is determined by the profile in society. In principle, the planner may regard more than one outcome as 'best' for a given society profile. An example is the planner in an exchange economy who only cares about efficiency, and thus is indifferent w.r.t. all points on the Pareto frontier. For simplicity, I assume the 'best' outcome to be unique in the sequel. The mapping which selects this unique most preferred physically feasible social state for a given profile is called the Social Choice Rule (SCR) and it is represented by the function $f(\cdot): \mathcal{Q} \to X$. Note that the outcome selected by the SCR should not necessarily be Pareto efficient. It could be any outcome belonging to the set $X$.

If the planner were completely informed about the true characteristic of each agent $i$, the enforcement of the SCR would be a trivial matter. In general, however, information in society is dispersed. In particular, the planner will be ignorant about the exact value of the individual characteristics. This incompleteness of information may be structured as follows:

**Agent's Information Set:** Agent $i$ ($1, \ldots, N$), knows the exact value of the parameter $\theta_i$—this is his private information. In addition, he holds beliefs w.r.t. the characteristics of the other agents; these beliefs may be of different degrees of accuracy, cf below;

**Planner's Information Set:** The planner is only aware of the fact that $\theta_i \in \Theta_i$ ($i=1, \ldots, N$); this information may be further augmented by beliefs on the set of possible profiles $\mathcal{Q}$. This is often called the public information.

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2 With a few obvious exceptions, all sets, variables and parameters that refer to concepts at the level of society but that are built up from their individual counterparts will appear in the text as underlined.
The theory of implementation is concerned with the following question: Does there exist an economic system, a mechanism, which can bring about the implementation or realization of the SCR despite the incompleteness of the planner’s information set?

The natural way to address this question is by conceiving any economic system as a game of incomplete information. Thus we may think of the planner as a person who frames the terms of a game to which all agents in society are invited to participate. The terms of such a game are called a mechanism or game form.

To construct a mechanism, the planner endows each agent $i$ with a message set $M_i$ and announces an outcome function $g(\cdot)$ which maps the list of messages to the set of feasible social outcomes. Thus, if $M=\bigtimes M_1 \times M_2 \times \ldots \times M_N$ denotes the message space then $g(\cdot) : M \rightarrow X$. The mechanism or game form is then defined by the ordered pair $\Gamma = (g, M)$. Having been informed about the rules of the game, every agent $i$ is asked to send a message $m_i$ from his personal message space to the planner, being well aware of the fact that the message profile $m = (m_i)_{i \in I}$ will entail the choice of the social state $g(m)$, thereby providing this agent with the utility level $u_i[g(m), m_i, \theta_i]$.\(^3\)

To characterize the equilibrium set of strategies of the different players, it is necessary to select an equilibrium concept for the game just described, like Nash, Bayesian Nash, or Dominant Strategy. This selection will be justified in the next section by making reference to the information set of the players. However, for the time being, assume that the equilibrium concept $c$ is justified, and that every player $i$ sends the message $m_i \in M_i$ as his optimal strategy. In other words, given the anticipated equilibrium behaviour of all other players, the message $m_i$ is the best response of player $i$. The message profile $m^c \in M$ is then the equilibrium of the non-cooperative game and it will depend on the characteristics of the different agents: $m^c = m^c(\theta)$.

The mechanism, or game form, $\Gamma$ is then said to implement the SCR $f(\cdot)$ for the equilibrium concept $c$, if it is true that

$$\forall \theta \in \Theta, \ g[m^c(\theta)] = f(\theta).$$

Schematically the implementation procedure may be represented by the

\(^3\) I assume the planner has the coercive power to make all agents participate in the game. Otherwise we would have an extra interim stage in the game where agents decide to accept or reject the mechanism (cf Fudenberg & Tirole, 1991, pp 244-5). In other words, I exclude the possibility of migration from society.
following diagram originally due to Reiter (1977):

Fig 1

2.2 Information and Equilibrium Concepts

The equilibrium outcome of this non-cooperative game will depend on the equilibrium concept adhered to. And this, in turn, will depend on the informational assumptions we make w.r.t. this society. So far nothing has been said about the information agents have about one another. Three scenarios naturally suggest themselves. Either agents are perfectly well aware about the personal characteristics of their fellow citizens (Complete knowledge); either they are imperfectly informed about these characteristics, but have formed some subjective beliefs on them (Statistical Knowledge); or agents are completely ignorant about the type of preferences their neighbours are endowed with (Ignorance).

The equilibrium strategy of agent \(i\) will be the transmission of a message \(m_i \in M_i\) which constitutes a best response against the anticipated equilibrium behaviour of all other agents. In the case of Complete Knowledge, every agent being fully informed about personal characteristics of the other players, \(m_i^*\) will be a best response against the equilibrium message profile played by the other players \((j \neq i)\), \(m_j^n\), if it is true that

\[
m_i^n = \text{argmax}_{m_i \in M_i} u_i[g(m_i, m_j^n); \theta_i], \quad \forall i \in I.
\]

This is the Nash Equilibrium concept \((n)\). The equilibrium message sent by player \(i\) will depend on the value of his personal characteristic, \(\theta_i\), as well as on the characteristics of all other players. We may therefore write \(m_i^* = m_i^*(\theta)\).

When agents have incomplete knowledge about each other, we must make precise how each agent anticipates the equilibrium behaviour of the other players. Following Harsanyi (1967-68), this is done by assuming that each agent \(i\) has a subjective probability distribution \(\mu_i(\cdot | \theta_i)\) on set of the other players' types, \(\Theta_{-i}\), which is conditional on his own type, \(\theta_i\). This probability distribution is well behaved in the sense that it respects the probability constraints

\[
0 \leq \mu_i(t_{-i} | \theta_i) \leq 1, \quad \forall t_{-i} \in \Theta_{-i}, \quad \text{and} \quad \int_{\Theta_{-i}} d\mu_i(t_{-i} | \theta_i) = 1.
\]
When agent $i$ anticipates that in equilibrium agent $j$ ($\neq i$) will send the message $m_i^j(\theta_j)$ in his capacity of type $\theta_j$, $i$'s optimal strategy will be given by

$$m_i^k(\theta_i) = \arg\max_{m_i \in M_i} \int_{\Theta_{-i}} u_i[g(m_i, m_{-i}^k(I_{-i})); \theta_i] \, d\mu_i(I_{-i} | \theta_i), \quad \forall i \in I,$$

where $m_i^k(\theta_i) = \text{def} [m_i^k(\theta_i)]_{\mu_i}$ and $u_i(\cdot; \theta_i)$ is now the von Neumann-Morgenstern utility function of agent $i$.

For each player to be able to compute the equilibrium point of the game on his own, a necessary condition is that all conditional probability distributions $\mu(\cdot | \theta_i)$ are common knowledge. This means that each agent $i$ knows that every other agent $j$ will select his best response on the basis of the distribution $\mu(\cdot | t_j)$ when his type happens to be $t_j$; and all agents know that all agents know this, etc. This condition will be satisfied when the subjective probability distributions are mutually consistent in the sense that the type profiles are drawn from an objective probability distribution $\mu(\cdot)$ over $\Omega$, and the subjective conditional probabilities are calculated according to Bayes' rule, as

$$\mu_i(\cdot | \theta_i) = \mu(\cdot | \theta_i) = \frac{\mu(\cdot, \theta_i)}{\int_{\Theta_{-i}} \mu(\cdot, \theta_i) \, dt_{-i}}, \quad \forall i \in I.$$

It is then natural to assume that $\mu(\cdot)$ makes up the planner's belief on the composition of society; it could for instance be established on the basis of a carefully designed and published census or population survey.

The set of messages given by $[m_i^k(\theta_i)]_{i \in I}$ constitutes a Bayesian Nash Equilibrium (b) sometimes also interpreted as a rational expectations equilibrium, because the anticipations each agent has w.r.t. the behaviour of his fellow players are confirmed in equilibrium. For a more in-depth discussion of Bayesian implementation theory, refer to Myerson (1983) and Harris & Townsend (1985).

A third equilibrium concept in non-cooperative game theory is the dominant strategy equilibrium (d). Such an equilibrium obtains when every agent $i$ has a best response message $m_i^b$, whatever the anticipated behaviour of the other players may be. Such a dominant strategy satisfies the following condition:

$$m_i^b(\theta_i) = \arg\max_{m_i \in M_i} u_i[g(m_i, m_{-i}); \theta_i], \quad \forall m_{-i} \in M_{-i}, \quad \forall i \in I.$$
In general, though, the existence of a dominant strategy mechanism to implement a certain SCR is not guaranteed (cf below). Nevertheless, it is clear that such a mechanism is the least demanding w.r.t. the agent’s capacities to anticipate the behaviour of their rivals in the game—whether in the deterministic or Bayesian sense. Especially when the information structure among agents is one of ignorance, domination is indeed a very desirable property. All that is needed is that the agent well understands his message space, and is able to pick out the best strategy, irrespective of what other players are up to. As Kreps (1990) argues

In cases where it may be unreasonable to expect players to find their way to a Nash equilibrium, it may be reasonable to expect them to recognize (and play) a dominant strategy. Even if the strategies are not strictly dominated (so there may be other equilibria in weakly dominated strategies), the mechanism designer may feel relatively secure in a prediction that players will settle on strategies that are dominant. (p 698)

2.3. Direct Mechanisms and the Revelation Principle

So far I considered the general case where no a priori restriction was imposed on the choice of the message space $M$. A natural choice, however, is to make every individual message set $M_i$ coincide with the corresponding characteristics set $\Theta_i$. This particular class of game forms are called direct mechanisms. Agents are then inquired immediately about their personal characteristics, but by no means they are compelled to send their true value. In analogy with the notation above, the best response functions under such a direct game form may be written as $\theta_i(\cdot)$.

When the message space coincides with the set containing the true characteristic of a player, it makes sense to talk about truthful reporting as a possible strategy. A very appealing idea for a direct mechanism is then to have truthful reporting as the equilibrium strategies, either under the Nash or the Bayesian Nash equilibrium concepts. In this case we have $\theta_{i}(\cdot) = \theta_{i}, \forall i \in I, c = n, b$.

Even more appealing is the case were an SCR can be implemented by means of a direct mechanism that has truthfulness as a dominant strategy, i.e. $\theta_{i}(\theta_{i}) = \theta_{i}, \forall i \in I$. A mechanism of this kind is called a straightforward mechanism. It derives its attractiveness from the fact that agents are now even no longer required to understand the consequences on their utility from announcing a
message from a possibly complex abstract message space. Rather, under no circumstances have agents any motivation to hide the truth. Hammond (1985) attributes the following virtues to this class of mechanisms:

First of all, such a mechanism has the advantage of being 'direct' in the sense that each individual is asked to signal a characteristic directly, either in full or perhaps in only a specified part. This avoids a great deal of confusion individuals may otherwise experience in trying to understand what exactly they are signalling and what their signal means. Second, straightforwardness requires that each individual's dominant strategy in the game is to announce his or her true characteristic. In no circumstances, then, is any individual encouraged to misrepresent his true characteristic. This has the obviously very desirable feature of not placing any disadvantage to those individuals who are too honest or too unsophisticated to indulge in some possibly rather complicated manipulative strategy that departs from the truth. Only individuals who do not know their characteristic fully may suffer, and even they should not be beyond help from suitable advisers. (p 415)

Suppose now the planner considers the implementation of a particular SCR with the help of a straightforward mechanism. Will she restrict herself by not considering any other, i.e. indirect, dominant strategy mechanisms? In other words, could the planner achieve a better outcome by allowing the agents to send more general messages? The answer to this question is negative, thanks to the Revelation Principle:

Theorem 1 (The Revelation Principle for dominant strategy mechanisms)(Dasgupta, Hammond & Maskin, 1979, Theorem 4.1.1, Laffont, 1988, p 114) Let \( \Gamma = (g, M) \) be a mechanism that implements the SCR for the dominant strategy equilibrium concept. Then there exists a straightforward mechanism \( \Sigma = (s, \Theta) \) by means of which this SCR can be implemented as well.

It is useful at this stage to note that the distinction between the SCR on the one hand, and the straightforward mechanism which takes care of its implementation becomes redundant. Let \( \Gamma = [g, \Theta] \) be the mechanism which implements the SCR \( f(\cdot) \) in a straightforward way. Then the following relationships must hold:

(i) \( \theta_i'(\theta_i) = \theta_i \) for each \( i \in I \); and

(ii) \( g[\theta_1'(\theta_1), \ldots, \theta_i'(\theta_i), \ldots, \theta_N'(\theta_N)] = f(\theta_1, \ldots, \theta_i, \ldots, \theta_N) \), for each \( \theta \in \Theta \).

Therefore it must be true that \( g(\theta) = f(\theta) \). This means that the verification whether an SCR is straightforwardly decentralizable may directly take place on the SCR itself. Hence, from now on I shall use the concepts 'SCR' and 'mechanism' interchangeably (for straightforward implementation).
The mechanism $f(\cdot)$ is then said to be straightforward if condition (i) above holds. It means that no agent has an incentive to hide his true characteristic, irrespective of the reporting behaviour of all other agents, i.e. when

$$
\forall i \in I, \forall \theta_i \in \Theta_i, \forall \theta_{-i} \in \Theta_{-i} : u_i[f(\theta_i, \theta_{-i}); \theta_i] \geq u_i[f(\eta_i, \theta_{-i}); \theta_i], \forall \eta_i \in \Theta_i.
$$

This set of inequality constraints are called the incentive compatibility or self-selection constraints, and very often, straightforward mechanisms are also called incentive compatible mechanisms.

2.4 Existence of Straightforward Mechanisms

It has already been mentioned briefly in section 2.2 that existence problems may arise when one insists on implementation in dominant strategies. As a special type of dominant mechanisms, straightforward mechanisms may therefore fail to exist as well for the implementation of a particular SCR. In fact, the search for a straightforward implementation is strongly discouraged by the Gibbard-Satterthwaite impossibility theorem:

**Theorem 2** (Dasgupta, Hammond & Maskin, 1979, Corollary 4.7.4, and Laffont, 1985, pp 117-9). *Let $X$ contain at least three elements. Let $f$ be an $N$-person SCR which is implementable by means of a straightforward mechanism and which has universal domain. Then $f$ is dictatorial.*

What this theorem asserts is that under the conditions specified, the only game forms that are straightforward must be dictatorial, that is the chosen outcome will be sensitive to the preferences of at most one individual, the dictator. Hence, if the SCR reflects the preferences of the individual players in any democratic way at all, the planner will run into difficulties if she insists on straightforward implementation.

Two elements in this theorem are important. The first concerns the finite number of agents in the game; I will come back to this restriction later. The second issue is concerned with the universal domain assumption. This assumption means that for every possible preference ordering agent $i$ may have over the outcome set $X$, there exist a value $\theta_i \in \Theta_i$ such that $R_i(\theta_i)$ corresponds to that ordering. For example, a subset of $\Theta_i$ may contain all $\theta_i$ generating Cobb-Douglas preferences; another subset may represent the class of preferences corresponding to quasi-linear utility functions, etc.

A particular case where the impossibility theorem does not apply is precisely
when \( \Theta_i \) coincides with this latter subset for all agents \( i \). Such orderings are often assumed in the literature on public goods, for instance

\[
u_i(x_i; \theta_i) = u_i(x_i; \theta_i) + y_i.
\]

Here \( x \) is the amount of the (single) public good and \( y_i \) denotes the transfer of the numéraire to agent \( i \) (the allocation of the private good). If the public good is of indivisible nature and \( x \) can take on the values 0 or 1, and adopting the normalization that \( v_i(0; \theta_i) = 0 \), an example of a SCR is:

\[
f(\theta) = [x(\theta), y_1(\theta), \ldots, y_n(\theta)], \text{ where } x(\theta) = \begin{cases} 1, & \text{if } \sum_{i,e} y_i(1, \theta_i) \geq 0, \\ 0, & \text{otherwise.} \end{cases}
\]

Groves (1973) and Clarke (1971) have proposed straightforward mechanisms that indeed succeed in implementing the above SCR.

Restricting the individual utility functions to quasi-linearity is therefore a convenient way of circumventing the negative result indicated by the theorem above. Whether it is empirically justifiable is a different issue.

3. Abstract Market Games in Large Exchange Economies

So far, I have discussed the implementation problem for general environments. The problem could be concerned with the election of a president from a number of candidates, or the assignment of a load of tulips to retailers at an auction, or the allocation of resources in a large exchange economy. It is on the last problem that I will concentrate in the present section which is in the spirit of Hammond (1979).

3.1. The Economic Environment

From now on, I concentrate on the following environment. The agents in society constitute a measure space \((I, \mathcal{I}, \pi)\). This means that \( \mathcal{I} \) is the sigma-field of the set \( I \) and that for every set \( F \in \mathcal{I}, \pi(F) \) denotes the proportion of agents in society whose name belongs to the set \( F \). One feature of the probability measure \( \pi(\cdot) \) is that \( \pi(I) = 1 \).

All agents \( i \in I \) have the same consumption set \( Q \subseteq R^*_+ \). Agents may however differ from one another by their initial endowment \( e_i \in R^*_+ \) and by their preferences. These, I represent by \( u_i(\cdot): Q \rightarrow R \), a function which is monotonously
increasing in all its arguments. Both preferences and endowments are unobservable by the planner. What the planner does observe are the net transactions agents carry out with one another: if $q_i$ is the final consumption vector by agent $i$, then $x_i=q_i-e_i$ is under control of the planner. Agent $i$'s feasible net transaction set and his preferences over this set are defined as $X_i = Q \backslash \{e_i\}$ and $u_i(\cdot) = u_i(\cdot - e_i) : X_i \rightarrow \mathbb{R}$, respectively. Moreover, I assume that both these concepts can be (possibly only partially) parameterized by a single characteristic $\theta_i$. Hence, the ordered pair $[X_i(\theta_i), u_i(\cdot ; \theta_i)]$ fully characterizes agent $i$. Notice that preferences are defined only over $X_i$, thus ruling out any altruistic attitudes. Other externalities, like public goods, are absent as well.

The planner is aware of the way the ordered pairs $[X_i(\theta_i), u_i(\cdot ; \theta_i)]$ depend on the parameter $\theta_i$, but is ignorant as to the exact value this parameter takes for agent $i$. An example may illustrate. Let the index $i$ be the name of a household. Every household has a Stone-Geary utility function of the form

$$u(q) = \sum_j \beta_j \log(q_j - \gamma_j).$$

The 'subsistence' levels $\gamma$ of the different commodities may be related to the number of children in the household. The planner knows this relationship and can observe this number. On the other hand, households may differ from one another by their initial endowment of the numéraire (commodity 0), their asset income, say. This asset income is not observable to the planner and it will play the role of $\theta_i$. The net transaction set and the utility over this set are then defined as

$$X_i(\theta_i) = X(\theta_i) \backslash \{0\} = \{\theta_i, x_i, 0_{i\neq i} \},$$

$$u_i(x; \theta_i) = \beta_0 \log(x_0 + \theta_i - \gamma_0) + \sum_{k=0}^n \beta_k \log(x_k - \gamma_k).$$

From now on, I shall assume that agents differ w.r.t. the value of their personal characteristic parameter, but are identical in all other respects. This means that the parameter $\theta_i$ now suffices to characterize the net transaction set and preferences of agent $i$ (i.e. the subscript $i$ in $X_i$, $R_i$ and $u_i(\cdot)$ may be dropped), and that the characteristic sets $\Theta_i$ now all coincide with the set $\Theta$, say. (In terms of the Stone-Geary example, all $\gamma_k$'s are identical across agents.)

### 3.2 Resource Allocation Mechanisms

The planner is concerned with the decentralization of an SCR. This rule assigns a transaction bundle $x_i$ to every agent $i$, depending on the characteristics profile in the economy. A possible rule would be the one derived from the
maximization of a Bergsonian Social Welfare Function that specifies a status quo in the case all agents happen to be identical, and advocates redistribution from rich towards less well endowed citizens.

An example may illustrate. Let there be two agents in society, Mr 1 and Mrs 2, who can either be of type \( R(\text{ich}) \) or \( P(\text{oor}) \). If both happen to be of the same type, the planner does not want to intervene in the economy; her policy choice is \( LF \) (Laisser Faire, or the mere implementation of the competitive market outcome). On the other hand, she wants to correct for skew income distributions by carrying out the transfer policy from the rich agent \( i \) to the poor agent \( j \): \( T_{i-j} \). The SCR is then defined by

\[
\begin{align*}
\text{Mrs 2} & \quad P & R \\
\text{Mr 1} & \quad LF & T_{2-1} \\
& \quad T_{1-2} & LF
\end{align*}
\]

To each policy then corresponds a resource allocation, that enters the utility functions of the agents.

In an exchange economy, a SCR is generally defined as a mapping \( f: I \times \Theta \to \mathbb{R}^n \) such that

\[
\forall i \in I, \forall \theta \in \Theta: f_i(\theta) \in X(\theta); \quad (\text{FI1})
\]

\[
\forall \theta \in \Theta: \int f_i(\theta) d\pi \leq 0. \quad (\text{FA1})
\]

That is, the transaction allocation must be \textit{physically feasible} both at the individual and aggregate level.

In order to decentralize this allocation, the planner has recourse to a direct game form which implements in dominant strategies. Using Gale’s (1982, p 295) terminology, we may call this game form an \textit{abstract market game}. The justification for relying on dominant strategy implementation will be given below in section 3.5. Taking this equilibrium concept for granted for the time being, we know by the Revelation Principle that there is no loss in restricting ourselves to straightforward mechanisms. If such a straightforward mechanism exist, we know it will coincide with the SCR, and it will have to satisfy the following incentive compatibility constraints:
\forall i \in I, \forall \theta_i \in \Theta, \forall \theta_{-i} \in \Theta_{-i} : u[f_i(\theta_i, \theta_{-i}); \theta_i] \geq u[f_i(\eta_i, \theta_{-i}); \theta_i], \ \forall \eta_i \in \Theta. \ \text{(IC1)}

where \( \Theta_{-i} = \Theta^{\times i} \). Hence we will speak of \( f_i(\cdot) \ (i \in I) \) as a feasible straightforward allocation mechanism if it satisfies conditions (FI1), (FA1), and (IC1).

3.3 The Planner’s Beliefs: Assignment Uncertainty

In the example above, the set of possible profiles is given by the following list of ordered pairs:

\[ \Theta = \text{def} \{ \theta_A = (P,P), \ \theta_B = (P,R), \ \theta_C = (R,P), \ \theta_D = (R,R) \} . \]

These four possible profiles may be considered as states of the world. As noted earlier, the planner—though imperfectly informed—may have some statistical a priori information in the form of the likelihoods with which each of those four states may occur. This statistical information is summarized in the distribution function \( \mu(\cdot) \) over \( \Theta \) (e.g. a historical frequency distribution). A special but interesting case is where the planner assigns zero probability to the profiles \( \theta_A \) and \( \theta_B \), and equal probability to states that can be obtained as a permutation of one another (like \( \theta_B \) and \( \theta_C \)). Such a priori information is called assignment uncertainty, a term introduced by Roberts (1984, p 182). It means that the planner has perfect knowledge about the distribution of characteristics, but is ignorant as to who is what type of person.

For future reference, it is useful at this stage to point out that to each profile in society, we can associate a characteristics measure space. Let \( \theta \in \Theta \) be a profile in the economy \( (I, \mathcal{F}, \pi) \). Then for this profile, the probability distribution of agents over the set of possible characteristics, \( \Theta \), may be defined as follows:

\[ \omega(E; \theta) = \text{def} \pi(\{i \in I \mid \theta_i \in E\}) , \]

where \( E \) is any subset from \( \Theta \). In other words, \( \omega(E, \theta) \) denotes the probability of running into a person with a characteristic belonging to the set \( E \). In a technical sense, \( E \) is a member from the sigma field \( \mathcal{B}(\theta) \) induced by the economy with profile \( \theta \). This sigma field is defined as

\[ \mathcal{B}(\theta) = \text{def} \{ E \in \Theta \mid \{ i \in I \mid \theta_i \in E \} \in \mathcal{F} \} . \]
In other words, for profile $\eta$ the measure space $[\Theta, \mathcal{B}(\Theta), \omega(\cdot; \eta)]$ is associated with the measure space $(I, \mathcal{F}, \pi)$. In the sequel, the distribution $\omega(\cdot; \eta)$ will simply be denoted as $\omega(\eta)$. The set of all probability distributions $\omega(\eta)$ on $\Theta$ will be denoted as $\Omega(\Theta)$.

A planner who is assignment uncertain, is aware that the characteristics distribution of the population is given by a particular $\omega^* \in \Omega(\Theta)$. The set of profiles which have the same likelihood to occur is given by

$$\Omega^*(\omega^*) \triangleq \{ \eta \in \Omega | \omega(\eta) = \omega^* \}.$$

To all other profiles, the planner attaches zero probability.

With these concepts in mind, it will be easier to highlight further properties of the allocation mechanism.

3.4 Anonymous Allocation Mechanisms

So far, the only constraints placed on the allocation mechanism were (individual and aggregate) feasibility and incentive compatibility. Besides these restrictions, the mechanism was allowed to work in an arbitrary way. In particular, the individual transaction allocations $f_i(\theta_i, \theta_{-i})$ were allowed to be conditioned on the agent’s name (the index $i$ in $f_i(\cdot)$), and could depend in an arbitrary way on the announcement of agent $j$ $(\neq i)$. At least two reasons may be put forward why such a treatment of agents is not desirable (cf Champsaur, 1989, pp 29-30). First, the planner’s a priori information was assumed to be at most symmetric: the planner is equally ignorant about the characteristic of any agent (because $\theta_i \in \Theta_i = \Theta$, for any $i$), and we abstracted from any observable differences between agents (like the number of children in the Stone-Geary example).

Consequently, the planner has no a priori information that might serve as a basis for a discrimination in favour or against a particular agent. Secondly, it has been assumed at the beginning of this section that both preferences and net transaction set of an agent are fully determined by his individual characteristic. So if the planner is concerned with horizontal equity then she should not give differential treatment to two agents that share the same characteristic. Furthermore, one may question why the planner should attach any particular significance to the announcement of a player with name $j$ in the assignment of an allocation to player $i$. Taking these arguments seriously leads to a further restriction on mechanism design, called anonymity.

A mechanism will be said to be anonymous when the following two
conditions are satisfied.⁴

(AR) Recipient Anonymity: The allocation for any agent \(i\) does not depend on the name of that agent, i.e. \(f_\theta(\theta_i, \_\_) = f_\theta(\theta, \_\_); \) and

(AI) Anonymity in Influence: The allocation to any agent \(i\) depends on the profile of announced characteristics only by way of the distribution of that profile, i.e. \(f_i(\theta_i, \_\_) = f_\theta(\theta, \omega(\theta)).\)

Anonymity thus requires that \(f_\theta(\theta_i, \_\_) = f_\theta, \omega(\theta)).\) It can be best explained as follows: if two agents, \(i\) and \(j\), permute their characteristics such that the distribution of characteristics in this society remains the same, then these two agents should receive the initial allocations but in a permuted way. As Hammond (1979, p 267) notes, this anonymity requirement is stronger than horizontal equity which in the present context would mean that if two agents share the same characteristic, the mechanism assigns them the same bundle.

3.5 Anonymous Mechanisms in Large Exchange Economies

With an anonymous mechanism, \(f_\theta, \omega\) will be the allocation received by an agent if he announces to be of the type \(\theta\), where \(\omega\) is the distribution of announced characteristics. In principle, a single agent's announcement has two effects on his allocation: a direct effect and an indirect effect through the aggregate distribution of announcements. I will now make precise the condition under which this second effect will vanish.

Suppose our exchange economy is large in the sense that every single agent is of negligible importance w.r.t. the entire economy. More precisely, assume that in the measure space of agents \((I, \mathcal{F}, \pi)\), every single agent forms a measurable set \(\{i\} \in \mathcal{F} \) of zero measure: \(\pi(\{i\}) = 0\). This means that the probability of meeting the agent bearing the name \(i\) out on the street is negligible. In such a society, the distribution of agents over types \(\omega(\theta)\) will not be affected when any single agent \(i\) is removed from the economy, or when he decides to announce a different characteristic; whence \(\omega(\theta) = \omega(\theta_i, \_\_) = \omega(\tilde{\theta}, \_\_).\) It is now immediately clear that we are only left with the direct effect of an announcement: what an agent reports will only be of importance to himself. Every agent will have a best reply strategy irrespective of the behaviour of the other agents. By the Revelation Principle, the planner can effectuate this best reply to be truthful.

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⁴ Here I follow the terminology of Guesnerie (1981, p 10). Hammond coins this restriction symmetry.
In other words, by assuming the economy consists of a continuum of agents, we rule out any strategic interaction among agents. If the planner has no a priori information about the distribution \( \omega \) of characteristics in society, she can always discover this distribution by organizing a census without identification of the agents. Because the economy is large, nobody will have an incentive to respond dishonestly.\(^5\)

The anonymous mechanism \( f: \Theta \times \Omega(\Theta) \to Q \) is then said to be a feasible and incentive compatible mechanism for a large economy when the following conditions are verified for all \( \omega \in \Omega(\Theta) \):

\[
\forall \theta \in \Theta: f(\theta, \omega) \in \mathcal{X}(\theta),
\]

\[(FI2)\]

\[
\int_{\Theta} f(\theta, \omega) d\omega \leq 0,
\]

\[(FA2)\]

\[
\forall \theta \in \Theta, \forall \eta \in \Theta: u[f(\theta, \omega); \theta] \geq u[f(\eta, \omega); \theta].
\]

\[(IC2)\]

In the case of assignment uncertainty, the distribution of the profile is fixed and known to the planner (\( \omega' \)); it may be subsumed in the function \( f \). The resulting self-selection constraints are the ones typically encountered in the adverse selection literature.

4. **Decentralization and Public Finance Mechanisms**

In the previous sections, the terms decentralization and implementation have been used interchangeably. Implementation takes place with a game form. And the outcome of the game will be determined by the agents themselves. That is, at the level at which the relevant information is situated. In this sense, a decentralization of the decision has been taken place: it is left to the agents' discretion which strategies to use. This notion of decentralization, of which

\(^5\) A dishonest report to the census is not a strictly dominated strategy, so agents may choose to lie as well as tell the truth. But since the effect of an individual report on census results are nil, lies will occur at random, and in a large economy, they will cancel out.

\(^6\) The construct of a continuum economy is only useful when results derived within that framework carry over to large but finite economies: Recent work by Dierker & Haller (1990) and Mas-Colell & Vives (1993) learns that the results from continuum analysis are indeed fairly robust w.r.t. finite approximations.
Reiters' figure is a diagrammatic illustration, conforms to Dasgupta's (1982) definition given in the introduction. In a game form, the actions undertaken by the agents, whether from an abstract strategy set or from a characteristics space, do not entail any immediate payoff to them; it is only indirectly, through the outcome function that a particular action provides the agent with utility. In other words, the actions have a merely communicating content, which explains the frequent use of the term message.

A more familiar notion of decentralization is the one whereby agents choose a member from an opportunity set which provides them directly with utility. In welfare economics, the outstanding theorem on decentralization is the second part of the fundamental welfare theorem which asserts that under a list of regularity conditions (especially convexity), every Pareto efficient allocation can be reached in a decentralized way by the announcement of a single price vector after a suitable lump-sum redistribution of real income. As argued by Dasgupta (1980, 1982), this is not the most interesting example of decentralization because the lump-sum redistribution requires the planner to be fully informed in which case the desired outcome could equally well be established by central quantitative planning. Nevertheless, this example reflects well the idea more often associated with decentralization, i.e. that of agents pursuing their own goals, settling on a desired outcome when they are confronted with appropriate (non-degenerate) opportunity sets. Let me call this decentralization in the strict sense, or strict decentralization for short. The remainder of this section will be demonstrate that this kind of decentralization and straightforward implementation are just two sides of the same coin.

4.1. Strict Decentralizable Mechanisms: Definition and Equivalence Result

In order to give a general definition of strict decentralizability of a mechanism, let us cast our minds back at the level of generality kept up in section 3 before the discussion of anonymous mechanisms in large economies. This will allow us to understand the proper meaning of this form of decentralization, independent of existence problems that may arise.

An allocation mechanism \( f(\theta) = [f(\theta)]_{i \in I} \) is said to be strictly decentralizable by the sets \( B_i(\theta) \) \( (i \in I, \theta \in \Theta) \) if the following two conditions are true (Hammond, 1979, p 266):

- (D1) \( \forall \theta \in \Theta, \text{ and } \forall i \in I, f(\theta) = \arg \max \{ u(x; \theta) \mid x \in B_i(\theta) \cap X(\theta) \} \); and

- (D2) \( B_i(\theta) \) is independent of \( \theta_i \).

Condition (D1) asserts that the trade bundle which the mechanism assigns
to agent $i$ is freely chosen by the agent when confronted with the opportunity set $B_i(\theta)$. This opportunity set is conditioned on the name of the agent because for the moment we have not yet ruled out any observable characteristics. However, what we certainly do want to rule out is any dependence of this set on the unobservable characteristic $\theta_i$, otherwise we might have $B_i(\theta) = \{f(\theta)\}$ which makes the decentralization degenerate. This explains why the definition is supplemented with the second condition which can also be stated as

$$(D2') \quad B_i(\eta, \theta, \underline{\theta}, \overline{\theta}) = B_i(\eta, \theta, \underline{\theta}, \overline{\theta}), \forall \eta \in \Omega, \theta_i \in \Theta, \forall \theta_i \in \underline{\theta}, \overline{\theta}.$$ 

Still in other words, condition $(D2')$ rules out any manipulation by the individual agent of the opportunity set he is allowed to choose from.

The following theorem, due to Hammond (1979), establishes the equivalence between incentive compatibility and strict decentralizability of a mechanism. The proof is included because its necessity part is constructive.

**Theorem 3** (Hammond, 1979, Theorem 1): *In an exchange economy, the allocation mechanism $f(\theta), \theta \in \Omega$, is incentive compatible if and only if there exist sets $B_i(\theta)$ ($i \in I$, $\theta \in \Theta$) by means of which $f(\theta)$ is strictly decentralized.*

**Proof**

(i) *Sufficiency.*

Let the mechanism be strictly decentralizable by the sets $B_i(\theta)$ ($i \in I$, $\theta \in \Theta$). Then by $(D1)$, $f(\theta) = \text{argmax} \{u(x;\theta) | x \in B_i(\theta) \cap X(\theta)\}, \forall i \in I, \forall \theta \in \Theta.$

Suppose now that agent $i$ announces any arbitrary characteristic $\eta_i \in \Theta$, different from his true characteristic $\theta_i$. He will then be assigned the bundle $f_i(\eta_i, \underline{\theta}, \overline{\theta})$ by the mechanism. But by definition $(D2)$, this false announcement will not affect the opportunity set this agent can choose from. Therefore $f_i(\theta_i, \underline{\theta}, \overline{\theta})$ will remain the maximizing argument of the constrained optimization problem, which means that

$$u_f(\theta_i, \underline{\theta}, \overline{\theta}) \geq u_f(\eta_i, \underline{\theta}, \overline{\theta}), \forall \eta_i \in \Theta.$$ 

It follows that the allocation mechanism must be incentive compatible.

(ii) *Necessity.*

Consider the allocation mechanism $f(\theta)$ ($i \in I$, $\theta \in \Theta$). By definition of incentive compatibility, this mechanism must satisfy the constraints

$$\forall i \in I, \forall \theta_i \in \Theta, \forall \theta_i, \theta_i' \in \Omega : u_f(\theta_i, \theta_i') > u_f(\eta_i, \theta_i'), \forall \eta_i \in \Theta.$$ 

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Next, construct the agent $i$'s opportunity set as $B_i(\theta) = \omega \{ f(\eta, \theta_d) \mid \eta \in \Theta \}$. By construction, this opportunity set is independent of the characteristic announced by agent $i$, thus satisfying (D2).

Consider now any arbitrary bundle $x_i \in B_i(\theta_i, \theta_d) \cap X(\theta_i)$. Then, there must exist a value $\eta_i$ such that $x_i = f(\eta_i, \theta_d)$. By the definition of incentive compatibility, it must be true that

$$u(f(\theta_i, \theta_d); \theta_i) \geq u(x_i; \theta_i), \forall x_i \in B_i(\theta_i, \theta_d) \cap X(\theta_i).$$

But this corresponds to condition (D1) of strict decentralizability. This proves the necessity part as well. QED

4.2. Strict Decentralization of Anonymous Mechanisms in Large Economies

The equivalence between incentive compatible and strictly decentralizable mechanisms, suggest that also the latter are in general subject to existence problems. As before, restricting ourselves to anonymous mechanisms in large economies, will circumvent these problems. This can be shown as follows.

First of all, the anonymity requirement rules out any conditioning of the opportunity sets on the names of the agents (cf AR): viz $B_i(\theta) = B(\theta)$. Second, the dependence of an agent’s opportunity set on the characteristics profile of the other agents $(\theta_d)$ is further restricted to the aggregate distribution of these characteristics, $\omega(\theta_d)$ (cf AI). Agent $i$’s opportunity set may thus be written as $B[\omega(\theta_d)]$. Next, recall that in a large economy, the effect of a single agent’s announcement on this distribution is negligible, so that $\omega(\theta_d) = \omega(\theta)$. Combining all these arguments justifies to write every agent’s opportunity set as $B[\omega(\theta)]$ or simply $B(\omega)$.

We are now in a position to state a second version of Hammond’s theorem for anonymous allocation models in large exchange economies. The proof is omitted because it goes along exactly the same lines as the previous one and because it will be given for a similar theorem in section 4.3.

**Theorem 4** (Hammond, 1979, Theorem 2) *In a large exchange economy, the anonymous allocation mechanism $f(\theta, \omega)$, $(\theta \in \Theta)$ is incentive compatible if and only if there exist budget sets $B(\omega)$ by means of which it is strictly decentralizable, for all $\omega \in \Omega(\Theta)$.*

The same 'large economy' argument will now ensure existence of strictly decentralizable mechanisms. The terms of trade at which an agent is able to transact are set by the common opportunity set $B(\omega)$. This set is designed by the planner using the information incorporated in the profile distribution $\omega$, which is
insensitive to manipulative behaviour of any single agent. Consequently, no agent is powerful enough to affect the terms of trade to his own advantage.

One example of such a strict decentralizable mechanism is the competitive mechanism where each agent is confronted with the opportunity set

$$B(\omega) \overset{\text{def}}{=} \{ x \in \mathbb{R}^n \mid p(\omega)^Ty \leq 0 \}.$$  

By the work of Aumann (1966), we know that under suitable regularity conditions such a mechanism exists.

These equivalence results are reminiscent of the other equivalence pointed out earlier between indirect mechanisms and the corresponding direct ones. Indeed, the equivalence theorem can be interpreted as an application of the Revelation Principle. According to this principle any indirect dominant strategy mechanism can be replaced by a straightforward game form. The strict decentralization of the mechanism can then be interpreted as an indirect mechanism where agents pick out net demands as strategies from the strategy set $B(\omega) \cap X(\theta)$. The outcome function which maps the message profile into the feasible outcome set is then just the identity function. Figure 2 illustrates.

Fig 2

Here $m_i(\cdot)$ is the utility maximizing bundle agent $i$ selects from the opportunity set, while the outcome function $g(\cdot)$ is the identity mapping. This figure may be compared with the Reiter diagram for a straightforward mechanism, as given below

Fig 3

This time, it is the set of best response functions which are identity mappings, and the outcome function maps information into a physically feasible allocation. We may therefore conclude that the sole difference between implementation and strict decentralization is the level at which important information is processed and translated into physical allocations.

4.3. Public Finance Mechanisms in a Large Economy with CRS Production

In this section, the economy is slightly modified by the introduction of a simple production sector. In particular, I shall assume that there is one public firm directly under control of the planner, which operates under a linear
technology. This technology is represented by the production function

\[ p'x \leq 0, \]

where \( x \) is the net output vector, and \( p \) is the (constant) vector of marginal rates of transformation/marginal costs. Since constant returns to scale prevail, the public firm makes no profits/losses. Alternatively, one could assume that a large number of private firms with the same technology exist. Since there are constant returns to scale, profits are zero and the ownership of these firms does not really matter.

The planner’s ignorance is of the assignment uncertainty type: she is aware of the fact that the characteristics in society are distributed according to \( \omega^* \), but is ignorant as to who is of what type. Agents now transact with the production sector, and these transactions are under control of the planner (i.e. she can observe them perfectly). Having modified the economic environment in this way opens up the possibility to discuss tax systems in a meaningful way.

To cope with production, the earlier stated aggregate feasibility condition on an allocation mechanism must be adapted. For the specific environment under consideration, condition (FA2) thus need to be replaced by

\[ p' f_\varnothing (\theta, \omega^*) \, d\omega^* < 0. \quad \text{(FA3)} \]

The mechanism \( f(\cdot; \omega^*) \) is a feasible anonymous straightforward allocation mechanism for the economy \( \omega^* \) if it respects the conditions (FI2), (FA3), (AR), (AI) and (IC2).

Next, I shall specify what a planner can achieve by relying on a non-linear tax mechanism. Since the planner can perfectly well observe all transactions (in both final goods and labour or capital services) which take place between consumers and the production sector, she is in the capacity to make the tax liability of a person dependent on these transactions. In implicit form, the budget constraint of agent \( i \) may be written as

\[ B(\tau) = \{ x \in \mathbb{R}^n \mid \tilde{\tau}(x) \leq 0 \}, \]

where \( \tilde{\tau}(\cdot) \) is the non-linear tax function. In the optimal taxation literature, one often finds the specification where the total tax liability is expressed in terms of the numéraire, the transactions in which are left untaxed, viz

\[ B(\tau) = \{ (x_0, x_{-0}) \in \mathbb{R}^n \mid x_0 + \tau(x_{-0}) \leq 0 \}. \]
Because no agent has any exogenous income from his own, both formulations are equivalent, and I shall use the latter.

The problem of an agent of type $\theta$ facing the tax schedule $\tau$, denoted as $P(\tau, \theta)$, is then given by

$$\begin{align*}
\text{Max}_x & \quad u(x; \theta) \\
\text{s.t.} & \quad x \in B(\tau) \cap X(\theta)
\end{align*}$$

the solution to which is denoted by the net demand function $x(\tau; \theta)$.\footnote{In a more rigorous formulation, $x(\cdot)$ should be a correspondence because the tax schedule may produce a non-convex budget set.}

A public finance mechanism is then defined by a tax schedule $\tau(\cdot)$ such that

$$p' \int_\Theta x(\tau; \theta) \ d\omega = 0. \quad \text{(FA$_d$)}$$

In other words, the resulting allocation must be physically feasible and efficient. When production takes place in a public firm, the equality in (FA$_d$) must be seen as an additional efficiency requirement. However, when a large number of private firms take care of production, equality will immediately follow from profit maximizing behaviour.

For this large economy with production, Hammond’s equivalence theorem reads as follows:\footnote{The theorem is adapted from Guesnerie (1981). See also Guesnerie (1982) for an informal discussion.}

**Theorem 5** The allocation $z(\theta) (\theta \in \Theta)$ can be obtained as the outcome of a public finance mechanism if and only if it can be obtained as the outcome of an anonymous straightforward allocation mechanism.

**Proof**

(i) **Necessity**

Let $z(\theta) (\theta \in \Theta)$ be obtainable from a public finance mechanism $\tau(\cdot)$. Then $z(\theta)$ must solve $P(\tau, \theta)$ for all $\theta \in \Theta$, and therefore $z(\theta) \in X(\theta)$ for all $\theta \in \Theta$. In addition, $z(\cdot)$ must satisfy the constraint $\int_\Theta z(\theta) d\omega^* = 0$. Consider then the allocation mechanism $f(\theta; \omega^*) = \omega^* \times z(\theta)$, $\forall \theta \in \Theta$. By construction, this is an anonymous and physically feasible mechanism. It is also incentive compatible, for suppose that an agent $\theta$ announces incorrectly an arbitrary characteristic $\eta \in N$ of another agent. Then either $f(\eta) \notin X(\theta)$, or $f(\eta) \notin X(\theta)$, in which case $f(\eta) \notin X(\theta) \cap B(\tau)$. However, because by construction $f(\theta)$ solves $P(\tau, \theta)$, it follows that...
\[ u[f(\eta); \theta] \leq u[f(\theta); \theta], \quad \forall \eta \in \Theta. \]

This means that the mechanism satisfies (IC2).

(ii) ** Sufficiency **

Assume that the mechanism \( f(\theta; \omega') \) is an anonymous straightforward allocation mechanism: it satisfies (AR), (Al), (F12), (FA3) and (IC2). Let us assume as well that this allocation mechanism is production efficient as well, i.e. that (FA3) holds with equality. Next, construct the opportunity set \( B = \{ f(\eta; \omega') \mid \eta \in \Theta \} \). Consider then the set \( B - R^*_\omega \), and define \( \tau(\cdot) \) as the function whose graph is the frontier of this set. [When there are only two commodities and with \( \Theta \) a discrete set, the collection of points in figure 4a is an example of \( B \); the shaded area in figure 4b corresponds to \( B - R^*_\omega \), and its frontier is the bold curve] Since \( f(\theta; \omega') \in B, f(\theta; \omega') \in B - R^*_\omega \), for all \( \theta \in \Theta \).

Fig 4

By definition \( x(\tau; \theta) \in B - R^*_\omega \), and by monotonicity of preferences, \( x(\tau; \theta) \in B \), for all \( \theta \in \Theta \). Now suppose that \( x(\tau; \theta) \neq f(\theta; \omega') \) for some \( \theta \in \Theta \). Then there must exist a characteristic \( \eta \in \Theta \), such that \( x(\tau; \theta) = f(\eta; \omega') \). But as the mechanism \( f(\theta; \omega') \) is incentive compatible, it is true that

\[ u[f(\theta; \omega); \theta] > u[f(\eta; \omega); \theta] - u[x(\tau; \theta); \theta], \]

which yields the contradiction that \( x(\tau; \theta) \) could not have solved the consumer programme \( P(\tau; \theta) \). QED

The import of this theorem is that a social planner who is assignment uncertain about the characteristics of the agents, will be able to implement the same resource allocations—and therefore welfare allocations—with the help of a nonlinear tax scheme, than the one a more 'sophisticated' planner could accomplish by designing a direct game form in which agents are interviewed about their personal characteristics, like preferences and initial endowments. This means that the optimal tax schedules obtained by Mirrlees (1971) and Atkinson & Stiglitz (1976) can be interpreted as optimal straightforward mechanisms which can never be improved upon by any other mechanism. This is a very fortunate conclusion, for it means that the theory of incentives has provided theoretical justification for much of the work of second-best theorists throughout the seventies. In his lecture on 'Information, Incentives and General Equilibrium', Champsaur (1989) makes the following interesting evaluation of the equivalence
results:

[E]conomists did not wait for such a systematic and explicit analysis of incentive-compatible mechanisms before guessing what allocations could be reached under information and incentives constraints. Second-best theory, especially when concerned with income redistribution, had to rely on such a guess since its development [...] preceded the formal analysis of incentive-compatible mechanisms. The latter theory has confirmed the intuitions of second-best theorists and determined under which assumptions they are right. (p 24)

Finally, it should be mentioned that I have implicitly ruled out any retrading of commodities after the net transactions with the production sector having taken place. If such retrading possibilities would exist, the game form would need to be followed by a walrasian market game for these commodities. In signalling their private information, agents will anticipate the outcome of this market game and adjust their strategic behaviour accordingly. As shown by Hammond (1987) and Guesnerie (1981), the equivalence theorems still go through provided that in the public finance mechanism tax schemes are restricted to be linear for retradeable commodities. In the second essay of this thesis, I will look for Pareto efficient taxation mechanisms in a two-class economy under different retradeability assumptions.

5. Concluding Remarks and Examples

Every economic environment can be characterized by specifying the preferences and endowments of its constituency and regulating authority. The endowment of an agent specifies his initial wealth of resources, as well as what this agent knows about the environment itself. This knowledge is summarized in his information set. Typically, these information sets are very coarse w.r.t. some aspects of the environment, like the preferences and endowments of the other citizens. Only w.r.t. one's own preferences and initial wealth do they provide accurate information.

Usually, social states or outcomes are only worthwhile considering, when they respect the physical relationships imposed by technology and initial wealth. Such a requirement seems almost too banal to spend much valuable time on—time which could otherwise be devoted to ascertain the efficiency and equity properties of a social state. But, as I have just pointed out, the initial endowments also have an informational component. In this essay I have tried to emphasize the
To illustrate the crucial importance of this concept, I would like to finish with a few examples. First, consider the two-class economy example of section 3.2 where a caring planner would like to redistribute resources when a skew income distribution prevails. Suppose the planner knows this is indeed the case, but is assignment uncertain as to who is of which type. Without imposing informational feasibility rich agents will always find it advantageous to pretend to be poor. The planner, who has committed herself to a transfer scheme, will then find out that she is unable to balance her budget. In fact in this static model, such a deficit is disastrous (it means a violation of the scarcity constraints), unless the planner can bring in resources from outside. But then, she might have found it more socially desirable to allocate these extra resources in an incentive compatible way in the first place.

Tax evasion and its legal counterpart, tax avoidance, are typical examples of activities whereby individuals attempt to hide their true characteristics, in casu income, by pretending to be somebody else. In western economies fiscal loopholes, whose existence has never been intended by the fiscal legislator, are spotted by accountants and opened by their clients in a lucrative way. Again, they are examples of small incentive incompatibilities which make it costly for the government to pursue its goals.

In Belgium, the scheme of scholarships for higher education operated by the Flemish Community is a good example of an incentive incompatible system. Such grants are conditioned on the taxable income of households, which seems at first sight a sensible thing to do. For employees, most of this income is closely monitored. Independent income earners and self-employed professionals, however, have the possibility to found a small corporation with limited liability under which they carry out their earning activities. As manager of such a corporation, it is then very easy for these people to impute themselves a small salary (small enough to qualify for a scholarship for their children) while the bulk of their earnings are received as profits (which are subject to the corporate tax law, and therefore do not appear as taxable income). This anomaly leads to paradoxical situations where the cheque of the grant is delivered to high income earning lawyers and doctors by the postman whose salary marginally exceeds the threshold to qualify for a scholarship for his children. This incentive incompatibility is not only very wasteful, its visible character often arouse feelings of exacerbation among the population. To remedy it, the late minister for education Mr Coens, decided to complement the taxable income criterion with a criterion on the property income of the dwellings a family rents or owns. A
recent study by the Higher Institute for Labour\textsuperscript{9} assesses the effect of this additional criterion on the target efficiency of scholarship awards. One of its main conclusions is that the additional criterion indeed disqualifies a large amount of prosperous applicants—aptly labelled as "wolves in sheepskin"—but at the same time excludes a number of "black sheeps". The reform has therefore strongly mitigated the incentive incompatibility of the system, but at the cost of giving in on the social objectives (and therefore the SCR).

Of course, one can always redefine an incentive incompatible resource allocation mechanism as an incentive compatible one w.r.t. a new social choice rule, but this is just turning a blind eye at the problem. 'Linear' policies, that is, policies which do not aim at differential treatment of individuals with the different characteristics, are typically incentive compatible. For instance, a poll tax, a constant marginal tax on income, the handing out of a universal food ration in a famine area, etc. do not give people incentives to disguise themselves as somebody else. But when the social choice rule urges the planner to target particular socio-economic groups, the reliance on linear policies can be inefficient, and in some cases even very costly in terms of human welfare and even human lives. To quote from Drèze & Sen (1989):

\begin{quote}
We have to accept the fact that when limited resources are spread uniformly and indiscriminately over the whole population in a famine situation, lives are to be lost. Even in the event where adequate resources are available, one may still wish to impart a redistributive element to the entitlement protection process by restricting or 'targeting' support to selected groups. (p 104)
\end{quote}

Policies which aim at targeting, and in case of asymmetric information at self-selection, are therefore often instruments which help the public authority achieve its objectives in an economic way. Of course, self-selection of individuals can take place in many ways, and not all of them seem as desirable. Victorian poor relief relied on workhouses with working conditions that were far from agreeable to screen the needy from the others (see Himmelfarb, 1984, p 165). Whether such a screening policy is optimal is a different issue. This essay was only concerned with the stage before optimization, i.e. feasibility. It has been shown that under certain circumstances, non-linear tax schemes can replicate any informationally feasible allocation of resources.

\textsuperscript{9} See Vos (1993).
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