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V A K G R O E P P U B L I E K E E C O N O M I E

**SOCIAL COST PRICING OF
URBAN PASSENGER TRANSPORT
- With an Illustration for Belgium -**

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Abstract

Economists have often suggested to tackle the problems associated with increasing congestion and pollution levels in urban areas by appropriate pricing of all relevant transport modes. Under idealized conditions the usual prescription is to charge travellers the marginal social cost of their use of the transport system. This includes the value of time lost due to congestion, the valuation of increased pollution and noise, and the valuation of changes in accidents risks. More complicated pricing rules typically result if distributional considerations are taken into account, or if pricing restrictions impose additional constraints on policy makers.

The purpose of our research has been twofold: first, we have developed a simple theoretical model that conceptually determines optimal transport prices that take account of all private and external (congestion, pollution, noise, and accident risks) costs associated with the use of different transport modes.

Second, we have illustrated a simplified version of the model to study pricing policies for a hypothetical aggregate Belgian urban area, extensively using recent estimates of private and social marginal costs. The application determines optimal prices for private car traffic and public passenger transportation in both the peak and the off-peak periods of the day.

I. INTRODUCTION

Over the past decades increasing congestion and pollution levels in urban areas have widened the gap between private and social costs of urban traffic. This raises several questions. The first problem is the computation of all relevant marginal social costs at present traffic levels. This includes the valuation of time lost due to congestion, valuation of reduced pollution and noise, and valuation of changes in accident risks. Not surprisingly, these issues have been dealt with by several authors (for some recent contributions see, e.g., Newbery (1988, 1990) and Jones-Lee (1990)). A second question is how to determine optimal prices for automobile use and public transportation. Under idealized conditions economists' prescription is to make transport users pay the marginal social costs of their consumption. However, in practice the problem may be more complicated if distributional considerations have to be taken into account, or if pricing restrictions impose additional constraints on the optimal pricing problem. For example, price differentiation between peak and off-peak traffic may be technically or politically infeasible, or budget constraints may apply to the public transport authority. Finally, a third question relates to the numerical calculation of optimal prices. The level of the marginal social cost of congestion and other externalities is itself a function of the intensity of car and bus use. Consequently, an equilibrium optimum tax or price has to be computed which takes into account both demand and supply responses.

The aim of this paper is twofold. First, we analyze the introduction of social cost considerations in a theoretical pricing model of urban transport services. Although optimal prices under congested conditions have been studied by economic theorists as early as the sixties, other forms of external costs have not equally carefully been considered in theoretical models¹. The model we develop explicitly incorporates congestion, pollution, noise, and accident risks. Second, we illustrate a simplified version of the model to study optimal prices for Belgian urban areas.

The model builds upon Glaister and Lewis (1978) and Small (1983). The former presented

¹ Early contributions include Walters (1961), Johnson (1964) and Vickrey (1969). More recent analyses include Else (1981) and Newbery (1990).

an urban passenger transport model which includes three transport modes, viz. private car, bus and rail. For each of these modes they consider two periods, peak and off-peak. Demand for each mode and each period is a function of the prices of all modes in the two periods. A congestion cost is associated with car use in the peak period. The model is used to derive optimal bus and rail prices under the assumption that car prices cannot be varied. In some sense, Small (1983) studied the reverse problem. He designed an optimal toll on an urban highway which is also used by an express bus service. In his model the bus price is fixed and only the toll can be varied. Individual demand data are used so that the welfare distribution effects of different toll regimes can be studied.

In this paper we use an extended version of the Glaister and Lewis (1978) model which contains the Small (1983) model as a special case. There are four extensions. First, the welfare distribution dimension is discussed explicitly. Second, the social costs of both private and public transport modes are considered and include, besides congestion costs, also environmental and accident costs. Third, the prices of both private and public transport are treated as policy variables. Finally, we examine several types of pricing constraints such as the absence of discrimination possibilities between peak and off-peak periods, and the imposition of a constraint on the subsidies to the transport sector.

The structure of the paper is as follows. In section II the theoretical model is presented. Two alternative formulations are considered. In the first case there are no restrictions on pricing. In the second case pricing restrictions are imposed. Sections III and IV present the application of the model to the particular case of urban transport in Belgium. Here we make use of Mayeres (1993) who computed all relevant marginal social costs of car use in Belgium. The conclusions are summarized in section V.

It must be emphasized that all pricing models in this tradition have several important limitations. They are partial equilibrium models focusing on the passenger transport market. Moreover, they implicitly assume a given location of individuals, and given infrastructure. In this sense they are short-run models. This type of models is in general not spatially distributed so that the network infrastructure is represented by one link which is subject to congestion. Finally, use of these models implies that we limit ourselves to the pricing

instrument and do not consider other forms of rationing.

II. THE THEORETICAL MODEL

We use a static partial equilibrium model of the urban passenger transport market. It is assumed that there is no interference with freight transport, an assumption which is probably less problematic for urban than for interregional transport. The exclusion of interference with freight transport is necessary in order to use a partial equilibrium model. A partial approach requires that the prices of other commodities remain constant and this is impossible for goods with varying freight cost components.

Organisation of this section is as follows. We first present the structure of preferences and external costs. We then discuss the environment in which the transport authority operates. Finally, we proceed to the formulation of the optimal pricing problem and interpret the results.

A. The structure of preferences and external costs

The model incorporates two transport modes: private car and public transport. For each mode a distinction is made between peak and off-peak traffic. The notation used can be summarized as follows:

Superscript	Transport service
1	private car - peak
2	private car - off-peak
3	public transport - peak
4	public transport - off-peak

There are H households. In its extensive form, the utility of household h is written as a function of the quantity consumed of a composite numeraire good x_h , of its use of the four types of transport services x_h^i (the number of kilometres individual h travels by transport service i ($i=1, \dots, 4$)) and of other variables which are all quality or public good variables taken as exogenously given by each household:

$$U_h = U_h (x_h, x_h^1, \dots, x_h^4, X^3, X^4, I, y^1, \dots, y^4, E, CA^1, \dots, CA^4, w^3, w^4) \quad \forall h \quad (1)$$

The quality variables are introduced to identify all major external effects associated with transport services in peak and off-peak periods. They are defined as follows:

- X^i : the total number of passenger-kilometres travelled by transport service i
- I : road infrastructure
- y^i : average speed of transport service i
- E : an indicator of the level of environmental pollution
- CA^i : the number of accidents associated with transport service i
- w^i : waiting time associated with mode i

These quality variables are all indirectly a function of the traffic and infrastructure levels. To be specific, average speed during peak and off-peak periods is given by:

$$y^i = y^i (Q^1, Q^3, X^3, I) \quad \text{for } i=1,3 \quad (2)$$

$$y^i = y^i (Q^2, Q^4, X^4, I) \quad \text{for } i=2,4 \quad (3)$$

where Q^i gives the total number of vehicle-km travelled by transport service i. Average speed is assumed to decrease with Q^i and with the number of passenger-km travelled by public transport in the same time period. Vehicle- and passenger-kilometres are assumed to be related according to

$$Q^i = Q^i (X^i) \quad \forall i \quad (4)$$

In the empirical part of the paper we will simply assume fixed occupancy-rates for passenger cars and public transport vehicles. The implicit assumption is that the public transport firm adjusts supply of vehicle-kilometres in function of demand.

The level of environmental pollution E is defined as

$$E = a + \sum_{i=1}^4 CE^i (Q^i, y^i) \quad (5)$$

where CE^i gives total environmental pollution by transport service i , assumed to be positively related to Q^i . An increase in average speed can both increase or decrease CE^i .

The number of accidents associated with transport service i during the peak period, CA^i , is defined as

$$CA^i = CA^i (Q^1, Q^3, y^1, y^3, X^1, X^3, I) \quad \text{for } i=1,3 \quad (6)$$

where CA^i is assumed to increase with y^i and X^i . The influence of Q^i depends on the initial traffic volume. For the off-peak period the following definition holds:

$$CA^i = CA^i (Q^2, Q^4, y^2, y^4, X^2, X^4, I) \quad \text{for } i=2,4 \quad (7)$$

The waiting time w^i associated with mode i is written as

$$w^i = w^i (Q^i, X^i, y^i) \quad i=3,4 \quad (8)$$

It is assumed to decrease with Q^i and y^i and to increase with X^i . The latter effect can be explained as follows. As X^i increases, the number of stops will typically increase; at the same time, the stops will take longer as more passengers are boarding or alighting. Moreover, it becomes more likely that people will not be able to board because the vehicle is full so that they have to wait for the next vehicle.

Finally, as is shown in equation (1), the individual's utility is a direct function of X^3 and X^4 , the total number of passenger-kilometres travelled by public transport in respectively the peak and off-peak period. These two variables serve as an indicator of the comfort experienced by the individual when travelling by public transport. As more people use public transport, vehicles will become more crowded and the probability of finding a seat and of travelling comfortably decreases. Therefore, *ceteris paribus*, utility will decrease as X^3 and X^4 increase.

Using all the above structural relations, we can use a reduced form of the individual's utility function u (we use capital U for the extensive form and the lowercase u for the reduced form):

$$u_h = u_h (x_h , x^1_h , \dots , x^4_h , X^1 , \dots , X^4 , I) \quad \forall h \quad (9)$$

where all the X^i and I are taken as exogenous parameters by the individual. Assuming sufficient differentiability, we can define demand functions and an indirect utility function v :

$$v_h = v_h (P , p^1 , \dots , p^4 , Y_h , X^1 , \dots , X^4 , I) \quad \forall h \quad (10)$$

with corresponding extensive form V :

$$V_h = V_h (P , p^1 , \dots , p^4 , Y_h , X^3 , X^4 , I , y^1 , \dots , y^4 , E , CA^1 , \dots , CA^4 , w^3 , w^4) \quad \forall h \quad (11)$$

where:

- P : price of the composite commodity
- p^i : out of the pocket price of transport service i
- Y_h : full income of individual h

The individual's compensated demand function for transport service i can be written as:

$$x^i_h = x^i_h (P , p^1 , \dots , p^4 , X^1 , \dots , X^4 , I , u_h) \quad \forall i, h \quad (12)$$

Finally, individual expenditure functions can be written as

$$g_h = g_h (P , p^1 , \dots , p^4 , X^1 , \dots , X^4 , I , u_h) \quad \forall h \quad (13)$$

Using the duality results for public goods (see King (1986)) we can now define the marginal external cost (mec_h^i) of an increase in the traffic level i for individual h as:

$$mec_h^i = \frac{\delta g_h(P, p^1, \dots, p^4, X^1, \dots, X^4, u_h, I)}{\delta X^i} = - \frac{\frac{\delta V_h}{\delta X^i}}{\frac{\delta V_h}{\delta Y_h}} \quad (14)$$

The marginal external costs can be specified with the help of the extensive indirect utility function (11). For the marginal external costs of private car use during the peak period we obtain:

$$\begin{aligned} mec_h^1 = & - \left[\frac{\delta Q^1}{\delta X^1} \left(\sum_{i=1,3} \left(\frac{\delta V_h}{\delta y^i} + \frac{\delta V_h}{\delta E} \frac{\delta E}{\delta CE^i} \frac{\delta CE^i}{\delta y^i} + \sum_{j=1,3} \frac{\delta V_h}{\delta CA^j} \frac{\delta CA^j}{\delta y^i} \right) \frac{\delta y^i}{\delta Q^1} \right. \right. \\ & + \frac{\delta V_h}{\delta w^3} \frac{\delta w^3}{\delta y^3} \frac{\delta y^3}{\delta Q^1} + \left. \left(\frac{\delta V_h}{\delta E} \frac{\delta E}{\delta CE^1} \frac{\delta CE^1}{\delta Q^1} + \sum_{i=1,3} \frac{\delta V_h}{\delta CA^i} \frac{\delta CA^i}{\delta Q^1} \right) \right) \\ & + \left. \sum_{i=1,3} \frac{\delta V_h}{\delta CA^i} \frac{\delta CA^i}{\delta X^1} \right] \frac{1}{\frac{\delta V_h}{\delta Y_h}} \end{aligned} \quad (15)$$

This expression clearly shows that an additional car in the peak period will affect the utility of household h through several different channels. It will lower peak traffic speed, thereby increasing the time needed to complete peak car and peak public transport trips. In addition it affects pollution and accidents indirectly. Moreover, the additional car will directly generate extra pollution and increase the risks of accidents due to cars and public transport. A similar expression can be derived for public transport in the peak period:

$$\begin{aligned} mec_h^3 = & - \left[\frac{\delta V_h}{\delta X^3} + \sum_{i=1,3} \frac{\delta V_h}{\delta CA^i} \frac{\delta CA^i}{\delta X^3} + \frac{\delta V_h}{\delta w^3} \frac{\delta w^3}{\delta X^3} + \frac{\delta V_h}{\delta w^3} \frac{\delta w^3}{\delta y^3} \left(\frac{\delta y^3}{\delta X^3} + \frac{\delta y^3}{\delta Q^3} \frac{\delta Q^3}{\delta X^3} \right) \right. \\ & + \sum_{i=1,3} \left(\frac{\delta V_h}{\delta y^i} + \frac{\delta V_h}{\delta E} \frac{\delta E}{\delta CE^i} \frac{\delta CE^i}{\delta y^i} + \sum_{j=1,3} \frac{\delta V_h}{\delta CA^j} \frac{\delta CA^j}{\delta y^i} \right) \left(\frac{\delta y^i}{\delta X^3} + \frac{\delta y^i}{\delta Q^3} \frac{\delta Q^3}{\delta X^3} \right) \\ & + \left. \left(\frac{\delta V_h}{\delta E} \frac{\delta E}{\delta CE^3} \frac{\delta CE^3}{\delta Q^3} + \sum_{i=1,3} \frac{\delta V_h}{\delta CA^i} \frac{\delta CA^i}{\delta Q^3} + \frac{\delta V_h}{\delta w^3} \frac{\delta w^3}{\delta Q^3} \right) \frac{\delta Q^3}{\delta X^3} \right] \frac{1}{\frac{\delta V_h}{\delta Y_h}} \end{aligned} \quad (16)$$

Similar expressions hold for off-peak transport.

B. The transport authority

The variable costs of private and public transport are given by:

$$C^i = C^i(y^i, Q^i, I) \quad \forall i \quad (17)$$

The transport authority operates a given public transport network. The total costs of public transport are given by the sum of total variable costs (C^i , $i=3,4$) and the fixed costs of public transport FC.

The presence of high fixed costs in the operation of public transport firms can generate large deficits. On the other hand the use of the car mode is in general heavily taxed. The net effect of the transport sector on the government budget D is given by the following expression in which the fixed cost of road infrastructure is denoted by $C_1(I)$ ²:

$$D = -C_1(I) - FC - \sum_{i=3,4} C^i + \sum_{i=1,2} p^i X^i \quad (18)$$

For public transport ($i=3,4$) the last two terms represent the gross margin on public transport operations and for $i=1,2$ they represent the tax receipts from private car use.

C. The optimal pricing problem

Optimal pricing decisions derived in a partial equilibrium model of the transport sector are only "globally" optimal if there are no distortions in the other sectors of the economy. These distortions clearly exist and are difficult to take into account in a partial equilibrium model. One of the most important distortions is the impossibility of lump sum taxation and redistribution for the government³. In a partial equilibrium framework this can be translated

² We here assume that the whole transport sector is aggregated. Of course, one can also envisage specific institutional constraints on the deficit of the public transport authority. These can be motivated by management incentives (Laffont and Tirole, 1990).

³ The optimal tax problem in the presence of externalities has been dealt with in a general equilibrium framework by Bovenberg and Van der Ploeg, 1993. For a first general equilibrium application to the taxation of car use see Mayeres and Proost, 1994.

to a starting situation where the vector of after tax incomes does not guarantee the equality of the social marginal utilities of income and where there is a marginal cost of public funds $(1+\lambda)$ larger than one (Laffont and Tirole, 1990).

The choice of optimal transport prices and taxes can then be formulated as

$$\begin{aligned} & \text{MAX } W [V_1, \dots, V_h, \dots, V_H] \\ & + (1+\lambda) [\sum_{i=1}^4 (p^i X^i - C^i) - C_I(I_0) - FC] \end{aligned} \quad (19)$$

$$X^i \geq 0, p^i \geq 0$$

w.r.t. p^j

Differentiating w.r.t. p^j yields the following first-order necessary conditions for a maximum⁴

$$\begin{aligned} & \sum_h \sigma_h (-x_h^j - \sum_{i=1}^4 \frac{\delta g_h}{\delta X^i} X^i_j) \\ & + (1 + \lambda) [-\sum_{i=1}^4 \frac{\delta C^i}{\delta p^j} + \sum_{i=1}^4 p^i X^i_j + X^j] - 0 \quad j=1, \dots, 4 \end{aligned} \quad (20)$$

where X^i_j represents the aggregate price effect of price j on traffic level i .

In this formulation use is made of the relative marginal social utility of income σ_h . For one reference individual ($h=1$), this welfare weight is put equal to one. This convention is necessary to have a fully defined marginal cost of public funds $(1+\lambda)$.

⁴ In this model the presence of fixed costs and externalities implies non-convexities. It is well known that in this case the first order conditions may be insufficient and that moreover corner solutions could be optimal. We restrict ourselves to a discussion of non-corner solutions (Guesnerie, 1980 and Bös, 1985)

It is useful to define the marginal social costs S^i associated with an additional unit of X^i as follows (for $i=1, \dots, 4$):

$$S^i = \sum_h \sigma_h \text{mec}_h^i + \sum_j \frac{\delta C^j}{\delta X^i} \quad (21)$$

The social marginal cost of an extra unit of traffic of type i equals the sum of the marginal external costs caused by an extra unit of X^i as defined in (15) and (16) and the direct effect on resource costs.

Using (21), (20) can be rewritten as:

$$\begin{aligned} X^j \left(\sum_h \sigma_h \frac{x_h^j}{X^j} - (1+\lambda) \right) + \sum_{i=1}^4 [S^i - p^i] X_j^i \\ + \lambda \left(\sum_{i=1}^4 \left(\sum_{k=1}^4 \frac{\delta C^k}{\delta X^i} - p^i \right) X_j^i \right) = 0 \quad j=1, \dots, 4 \end{aligned} \quad (22)$$

The interpretation of this optimum condition is easiest if the government can use first best instruments so that ($\lambda=0$) and all welfare weights are equal to 1. In that case the first and the last term disappear and social marginal cost pricing is a solution to the optimality conditions:

$$S^i = p^i \quad \forall i \quad (23)$$

The first term will not disappear if there is a systematically larger welfare weight for the consumers of passenger transport (the σ 's are on average greater than 1) or if the welfare weights and the shares in the consumption of transport goods differ over the population. To facilitate the interpretation of the results in this case, it is useful to add a simplifying hypothesis, viz. that all cross-price elasticities of demand are zero. Optimal prices are then given by (λ is still assumed to be zero):

$$p^i = \frac{S^i \eta_i^i}{\eta_i^i - (r^i \bar{\sigma} - 1)} \quad (24)$$

where η_i is the own price elasticity of good i . r^i is the distributional characteristic and is defined as:

$$r^i = \sum_h \frac{\sigma_h}{\bar{\sigma}} \frac{x_h^i}{X^i} \quad (25)$$

In this expression $\bar{\sigma}$ stands for the average value of the marginal welfare weights. r^i will take higher values for goods which are consumed proportionately more by people with a high marginal welfare weight. For $(r^i \bar{\sigma} - 1) > \eta_i$, a higher value of r^i will, ceteris paribus, lead to a lower price for the good concerned.

When the marginal cost of public funds becomes higher than one, distortions between price and marginal costs are also the rule. As long as one has not reached a point where increasing prices have negative revenue effects, the third term in equation (22) tends to increase profit or tax margins on transportation services.

D. Pricing restrictions

Up to now we have considered the case in which there are no restrictions on pricing. In reality, however, it is often infeasible, due to practical reasons, to charge a different price for peak and off-peak travel. Therefore, it is interesting to investigate what the optimal prices are if additional restrictions are imposed on pricing.

We can consider the case where price differentiation between peak and off-peak travel is impossible for all modes. As a consequence, peak prices must equal off-peak prices both for private and public transport. We introduce p^p and p^p as the price charged to resp. private car and public transport users. Our problem (19) can be reformulated by using only two price variables as controls.

Assuming again that all cross-price elasticities are zero and that first best redistributive taxation instruments are available we obtain an analog of (24)

$$p^c = \frac{\eta^1_c S^1 X^1 + \eta^2_c S^2 X^2}{\eta^1_c X^1 + \eta^2_c X^2} \quad (26)$$

Prices do no longer equal marginal social costs. They are a weighed average of marginal social costs in both periods where the period that is most price responsive will get the highest weight. The same kind of result holds for public transportation.

Of course, one can also imagine the mixed case in which public transport prices can be varied between peak and off-peak, but private transport prices cannot. The results in this case will be a combination of the results of the two previous cases.

III. IMPLEMENTING A SIMPLIFIED VERSION OF THE MODEL

In the previous section we extended the Glaister-Lewis model to account for various external effects other than congestion (noise, pollution, accident risks) and to include distributional considerations. Different routes can be taken to apply pricing models of this kind. The choice will have to be dictated by the available empirical information⁵. Unfortunately, strict application of the theoretical model as presented before is not feasible given current limitations on data availability in Belgium. Therefore, in what follows a simplified version of the model is applied to analyze optimal transport prices. The model was simplified in two respects. First, it follows Glaister and Lewis (1978) in that it uses aggregate demand data. Demand for transport is represented for the total population and in a reduced form. This means that the congestion variable (X^i in (12)) has disappeared from the demand equation. The drawback of this formulation is that while market equilibrium computations are still possible (reduced demand equations are available), a welfare comparison of different equilibria becomes impossible because the necessary information about the real demand equations and the implicit evaluation of congestion is missing. Second, the marginal external cost functions used in the empirical analysis are less general than those presented in the

⁵ For example, Small (1983) uses individual demand functions which are in structural form with the congestion levels explicitly as arguments of the demand functions. The main advantage of this type of information is that it allows to easily make welfare evaluations using the structural demand functions.

theoretical section of the paper. However, the model does capture all the above-mentioned external effects generated by urban transport activities.

A. Specification of the demand functions

The model has to be interpreted as reflecting an 'aggregate' Belgian urban area, consisting of all Belgian cities offering public urban transport. These include Antwerp, Brussels, Ghent, Charleroi, Liège and Verviers. Implementation of the model is based on two transport modes, viz. the private car and an aggregate public transport mode, consisting of both bus and tram⁶. For the two modes considered, a distinction is made between peak and off-peak periods. Based on information in STRATEC (1992) and NIS (1985), the peak period is assumed to cover five hours a day, viz. from 7h till 9h and from 16h till 19h. The off-peak period covers seventeen hours: 4h-7h, 9h-16h and 19h-2h. Traffic between 2h and 4h is negligible.

In the simulation exercises both constant-elasticity and variable-elasticity demand functions are considered. In the former case the aggregate demand functions for the different transport services (X^i) are taken to be simple loglinear functions of the relevant prices

$$X^i = \alpha^i \exp \left(\sum_{j=1}^4 \eta_j^i \ln (p^j) \right)$$

where the η_j^i are the constant price elasticities. Using elasticity values derived from the empirical literature together with observed price-quantity combinations for each mode and period allows us to calibrate the parameters α . The data refer to the year 1989. Relevant information is summarized in Tables 1 and 2.

As an alternative to constant-elasticity demand functions we also experimented with the following specifications (see, e.g., UK Department of Transport (1982)):

⁶ In Brussels metro transport is also provided. However, we left this transport mode out of the analysis for reasons of inadequate data on marginal social costs. We have not assumed the marginal social costs of the metro to be the same as the marginal social costs of the other public transport modes, because of the totally different impact these transport modes have on environment, accidents, ... Passenger-kilometres travelled by metro are therefore ignored.

$$X^i = X_B^i \exp \left(\frac{\sum_{j=1}^4 \eta_{B_j}^i}{P_B^j} (p^j - p_B^j) \right)$$

where X_B = the initial quantity demanded
 η_B = the initial price elasticities
 p_B = the initial prices
 p = the optimal prices.

This formulation implies elasticities that vary with prices. Indeed, the price elasticities are easily shown to be given by

$$\eta_j^i = \frac{\eta_{B_j}^i}{P_B^j} p^j$$

For the base simulation the values of X_B , η_B and p_B are again those reported in Tables 1 and 2.

TABLE 1 : Price elasticities⁷

$$\begin{bmatrix} -0.3 & 0.049 & 0.708 & 0 \\ 0.05 & -0.6 & 0 & 0.578 \\ 0.03 & 0 & -0.35 & 0.036 \\ 0 & 0.02 & 0.03 & -0.87 \end{bmatrix}$$
TABLE 2 : Daily demand and prices (1989)⁸

VARIABLE	VALUE	DIMENSION	DESCRIPTION
X ¹	47 312 533	passenger-km	demand for peak car
X ²	48 695 041	passenger-km	demand for off-peak car
X ³	1 544 494	passenger-km	demand for peak bus & tram
X ⁴	1 297 262	passenger-km	demand for off-peak bus & tram
p ¹	2.665	BF per passenger-km	price for peak car
p ²	2.665	BF per passenger-km	price for off-peak car
p ³	3.46	BF per passenger-km	price for peak bus & tram
p ⁴	3.46	BF per passenger-km	price for off-peak bus & tram

B. Marginal social costs of the different modes

The marginal social costs caused by an additional vehicle-kilometre consist of external costs (congestion costs, air pollution costs, noise costs, and accident costs) and private money costs.

⁷ A careful discussion of the elasticity values used and their sources is provided in Appendix 1.

⁸ The sources and methods of calculation of these data are discussed in Appendix 2.

We first consider calculation of the various external costs, and then turn to private money costs.

1. Marginal congestion costs

The marginal external congestion costs vary with the total number of vehicle-kilometres travelled. The time loss suffered by road users when an additional vehicle travels one kilometre gives an indication of marginal congestion. Let L^i be the change in travel time for one individual car traveller due to an additional vehicle kilometre. Assuming that average speed of public transport vehicles amounts to 77% of average car speed (UK Department of Transport (1982)) the total time loss of an additional vehicle-kilometre driven in the peak period can be expressed as

$$L^i \cdot X^1 + \frac{L^i}{0.77} \cdot X^3 \quad i=1, 3$$

where as before X^1 is the number of passenger-kilometre travelled by car and X^3 is the number of passenger-kilometre travelled by public transport.

To determine marginal congestion costs in the peak period, we multiply the time loss suffered by car and public transport users by their respective values of in-vehicle time. Using values of 230 BF per hour and 124 BF per hour for car and public transport users, respectively, the following relation expresses the monetary value of the total marginal time loss⁹

$$L^i \cdot [X^1 \cdot 230] + \frac{L^i}{0.77} \cdot [X^3 \cdot 124] \quad i=1, 3$$

We finally have to determine L^i , the change in total travel time due to an additional vehicle-kilometre, in order to operationalise marginal congestion costs. To do so, it is necessary to introduce a relation describing how average speed is influenced by the number of passenger

⁹ Based on an extensive survey, Hague Consulting Group (1990) report 11.99 guilder per hour for car travel and 6.48 guilder for public transport in 1988. Converting these values into 1989 BF, we find 230 BF and 124 BF respectively.

car equivalent unit (PCU)¹⁰ kilometres¹¹. This 'capacity-speed' relation is based on several relatively crude 'observations' on average speed and traffic levels. It is assumed that travel speed of freely flowing traffic is 50 kilometres per hour, that average speed at the current peak traffic level¹² is approximately 30 km/h, and that average speed drops to 10 kilometre per hour with a traffic level of approximately 7 000 000 PCU per hour. Based on this information, and denoting the number of vehicle equivalent kilometre per hour travelled in the aggregate city by W and average speed by S the following parabolic relation was derived

$$W = 6\,133\,949 + 138\,926.12 \cdot S - 5\,232.102 \cdot S^2$$

where $W = (PCkm + 2 \cdot PTVkm)/5$ for the peak period and $W = (PCkm + 2 \cdot PTVkm)/17$ for the off-peak period. In these expressions PCkm stands for the number of passenger car-kilometre, and PTVkm is the number of public transport vehicle-kilometre. Note that account has been taken of the respective lengths of peak and off-peak periods. The average time needed to drive one kilometre as a function of the PCU kilometres travelled is obtained by inverting the flow speed relation shown above. The congestion costs of the peak period are obtained by combining this information with the composition of traffic and their value of time. A similar analysis was performed for the off-peak period.

2. Marginal external air pollution costs

Due to the limited data available, all external marginal costs other than congestion costs are assumed to be independent of traffic levels. Marginal external air pollution costs are based on Mayeres (1992). For private car use these costs are estimated to be 0.6018 BF per kilometre. The corresponding figure for public transport is 2.346 BF per vehicle-kilometre.

¹⁰ 1 passenger car (PC) = 1 passenger car equivalent unit (PCU), and 1 public transport vehicle (PTV) = 2 passenger car equivalent units (PCU)

¹¹ A more common procedure in the transport literature is to use a formal speed-flow relation. However, as we are looking at an aggregate city, the concept of traffic flows does not seem to be very useful. Instead we introduced a relation between speed and vehicle-kilometers travelled in the city.

¹² For the computation of the current peak traffic level in the Belgian urban areas (expressed in PCU), see Appendix 3.

3. Marginal external noise costs

The external noise costs of an additional public transport vehicle-kilometre for the peak and the off-peak period were taken from Boniver (1993). She reports 3.45 BF and 9.34 BF, respectively. Marginal external noise costs for passenger cars were found to be negligible (Mayeres (1993)).

4. Marginal external accident costs

The marginal external accident costs for car use are estimated at 1.6145 BF per kilometre (Mayeres (1993)). The additional accident cost incurred by an additional public transport vehicle-kilometre is calculated to be 7.5286 BF¹³.

5. Marginal private money costs

We now turn to an evaluation of marginal private money costs. For car use these include expenses on fuel, tyres, oil, reparation and insurance associated with an extra vehicle-kilometre. The average variable private money costs, exclusive of taxes¹⁴, are used as an approximation of the relevant marginal costs. The average variable private money costs for

¹³ For the method of computation of the marginal accident costs for public transport, see Appendix 4. It should be noted that these marginal external accident costs contain an allowance for the pain, grief and suffering experienced by relatives and friends. However, one might argue that road users, when deciding to make a trip, already take into account the psychological effects on relatives and friends of a possible accident. Therefore we also recalculated marginal accident costs without this allowance and found a cost of 1.1032 BF per kilometer for car use, and a cost of 2.5188 BF for public transport use. Due to the substantial differences and the controversial nature of marginal accident cost figures, the latter results were also used in a sensitivity exercise (see De Borger et al. (1993))

¹⁴ The price consumers have to pay for fuel, tyres, oil, reparation and insurance can be interpreted as the producer price plus taxes. The difference between the optimal price (to be determined) and the corresponding producer price equals the optimal tax to be levied on that good.

car use are based on Cuijpers (1992), Zierock et al. (1989), and NIS (1990)¹⁵. They were estimated at 2.82005 BF per kilometre. Note that these costs were assumed not to depend on traffic levels. In other words, the nonzero but empirically small effect of traffic levels on energy consumption was ignored.

Contrary to the case of car traffic, we were able, using the 'capacity-speed' relationship, to distinguish between peak and off-peak costs for public transport¹⁶. Again, the marginal private cost was approximated by the average variable cost, consisting of expenditures on drivers, on energy (insofar as related to rolling stock operations), on materials, and on reparations and deliveries with respect to rolling stock¹⁷. We found average variable private money costs to be 37.7 BF per kilometre for off-peak public transport. For the peak period the corresponding costs amounted to 48.7 BF per kilometre. However, consistent with the usual approach in the literature, to approximate peak-period long-run marginal costs the cost of rolling stock of public transport was totally assigned to the peak period¹⁸. This implies peak costs of 185.2 BF per kilometre.

6. Marginal social costs: summary of numerical results

Table 3 recapitulates all marginal social costs of an additional vehicle-kilometre produced by public and private transport. Summarizing the previous discussion suggests that marginal social costs, exclusive of marginal external congestion costs, amount to 5.03635 BF per kilometre for private passenger cars, 198.5246 BF per vehicle-kilometre for public transport in the peak period and 56.9146 BF per vehicle-kilometre for public transport in the off-peak period. Marginal congestion costs positively depend on traffic levels.

¹⁵ The variable private money costs are weighted by the proportion of total vehicle-kilometres travelled with gasoline, diesel and LPG cars.

¹⁶ More information can be found in Appendix 5.

¹⁷ The method of computation and the sources of the data needed are discussed in Appendix 5.

¹⁸ The procedure for calculating costs of rolling stock can also be found in Appendix 5.

**TABLE 3 : Marginal social costs of different transport modes : monetary valuations
(BF/vehicle-km)**

	PASSENGER CAR		BUS & TRAM	
	PEAK	OFF-PEAK	PEAK	OFF-PEAK
MARGINAL EXTERNAL AIR POLLUTION COSTS	0.6018	0.6018	2.346	2.346
MARGINAL EXTERNAL NOISE COSTS	NEGLIGIBLE		3.45	9.34
MARGINAL EXTERNAL ACCIDENT COSTS	1.6145	1.6145	7.5286	7.5286
AVERAGE VARIABLE PRIVATE MONEY COSTS	2.82005	2.82005	185.2	37.7
MARGINAL EXTERNAL CONGESTION COSTS	PEAK	$L^i \cdot [X^1 \cdot 230] + \frac{L^i}{0.77} \cdot [X^3 \cdot 124]$ $i=1, 3$		
	OFF-PEAK	$L^i \cdot [X^2 \cdot 230] + \frac{L^i}{0.77} \cdot [X^4 \cdot 124]$ $i=2, 4$		

Finally note that for public transport we transform all calculated costs due to an additional vehicle-kilometre to marginal costs per passenger-kilometre by dividing the marginal costs of an additional vehicle-kilometre by the average occupancy-rate during the period under consideration. The average occupancy-rate of a peak bus or tram is assumed to be 50, the average rate for an off-peak public transport vehicle is assumed to be 30. For example, the monetary value of the marginal social cost due to an extra public transport passenger-kilometre is then :

in the peak : $((\text{marginal external congestion costs at the peak traffic level}) / 50) + 3.97$ BF per kilometre

and

in the off-peak : $((\text{marginal external congestion costs at the off-peak traffic level}) / 30) + 1.8966667$ BF per kilometre

Table 4 reports all marginal social costs of an additional passenger-kilometre, both for public and private transport. Congestion costs have been evaluated using observed traffic for 1989. Interestingly, the results suggest that during the peak period marginal social costs due to congestion by far exceed those due to pollution, noise and accident risks.

TABLE 4 : Marginal social costs of an additional passenger-kilometre in 1989 for the different transport modes : monetary valuations (BF/passenger-km)

	PASSENGER CAR		BUS & TRAM	
	PEAK	OFF-PEAK	PEAK	OFF-PEAK
MARGINAL EXTERNAL CONGESTION COSTS	8.2292	0.5739	0.559	0.0639
MARGINAL EXTERNAL AIR POLLUTION COSTS	0.354	0.354	0.0469	0.0782
MARGINAL EXTERNAL NOISE COSTS	NEGLIGIBLE		0.069	0.3113
MARGINAL EXTERNAL ACCIDENT COSTS	0.9497	0.9497	0.1506	0.2509
AVERAGE VARIABLE PRIVATE MONEY COSTS	1.65885	1.65885	3.704	1.257

C. The budget constraint

In some of the simulation exercises reported below the public transport sector is budget-constrained. The constraint simply states that the sum of all variable private money costs plus total fixed costs should be covered by total revenues. Fixed costs of the public transport companies for the Belgian urban areas are calculated as explained in Appendix 5. We found a value of 16 765 086 BF per operating day.

IV. SOME SIMULATION RESULTS FOR BELGIUM

In this section, we report a variety of simulation results obtained using the models outlined before. The structure of this section is as follows. First, we carefully discuss the basic model with constant-elasticity demand functions, in which no budgetary or other restrictions are imposed. We look at optimal prices and corresponding traffic levels, consider marginal social costs and average speeds at the optimum, and we compare the optimal values with the observed values of the current situation. Second, we analyze optimal prices for cases which explicitly impose a formal restriction on the model. Three restrictions are considered, a budget constraint for the public transport sector, a restriction on the number of passengers travelling, and a restriction on the level of the public transport fares. Finally, we analyze models with variable elasticities, both with and without additional restrictions.

An overview of the different simulation exercises is provided in Table 5. Note that our selection is to some extent arbitrary and can easily be extended.

A. Basic model : no budget restriction, constant elasticities

Ignoring distributional and budgetary considerations, and assuming constant price elasticities highly simplifies the theoretical model presented in the first section. Under these circumstances optimal transport prices p^i were shown to equal marginal social costs S^i for all modes and periods (see equation system 23), where it is important to emphasize that the S^i are a function of the corresponding traffic levels (see equations 15, 16 and 21).

TABLE 5 : The simulation exercises reported

	Constant elasticities	Variable elasticities
No budgetary or other restrictions	X	X
Budget constraint on public transport sector	X	
Restriction on the number of passenger-kilometre travelled	X	
Restriction on the level of the public transport fares	X	
Restriction : prices in peak equal prices in off-peak		X
Restriction : car price in peak equals car price in off-peak		X

Using the elasticity values given before the solution of the equation system (22) leads to the results reported in Table 6. The first column presents current prices, marginal social costs and average speed of the different transport services in the 'aggregate' Belgian urban area¹⁹. The second column gives the results of the basic model. The third column reports the percentage deviation of the optimal values as compared to the current situation.

First consider the optimal prices. For private transport in both periods and for public transport in the peak period they turn out to be substantially higher than current price levels, while optimal prices for public transport in the off-peak period are below current prices. Internalisation of external costs obviously implies that optimal prices are higher in the peak period than in the off-peak period. Note that in the peak period the percentage rise is larger for the car mode than for public transport. In the off-peak period car prices rise, while public transport prices are reduced. Specifically, car prices in the peak rise by 141% as compared to a 30% increase in the off-peak period. Public transport prices decline by more than 40%

¹⁹ Please note that the traffic levels reported in Table 6 are the values per day, not per hour.

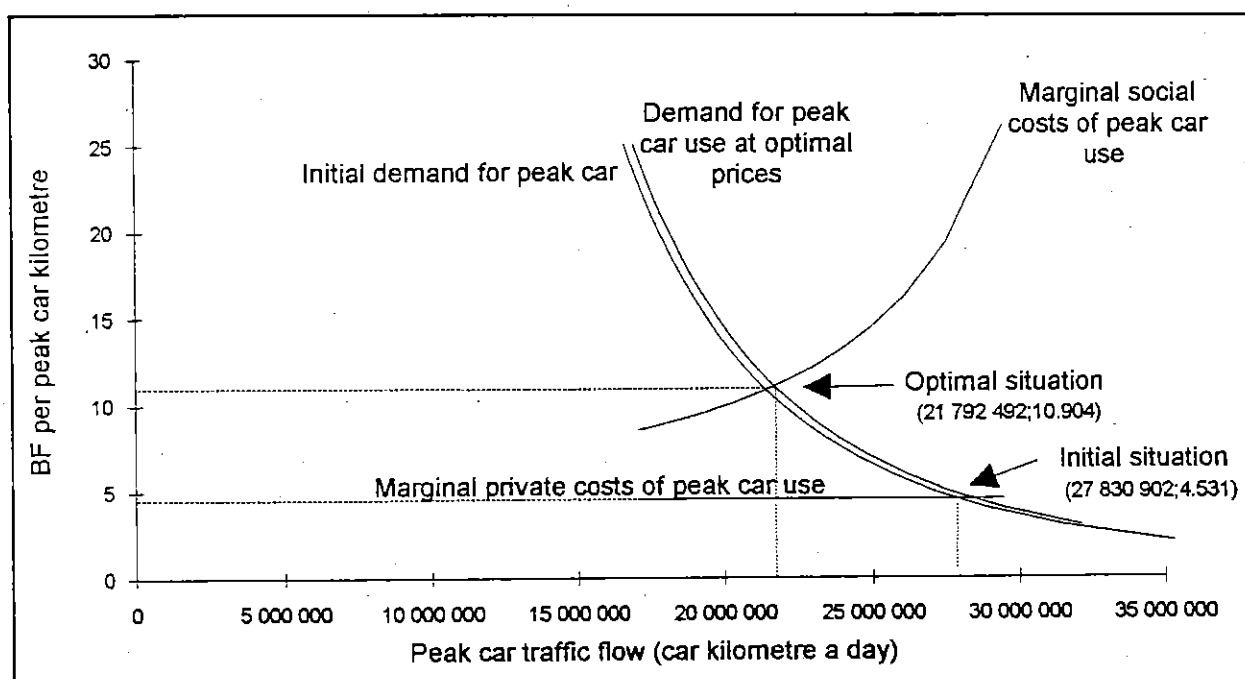
TABLE 6: Optimal pricing results (no budget restriction, constant price elasticities)

		INITIAL SITUATION	BASIC OPTIMUM Constant elasticities	
			Optimal values	% change w.r.t. initial situation
PRICES (BF per passenger km)				
Car	Peak	2.67	6.41	140.68%
	Off-peak	2.67	3.46	29.79%
Bus & tram	Peak	3.46	4.21	21.53%
	Off-peak	3.46	1.95	-43.55%
MARGINAL SOCIAL COSTS (BF per passenger km)				
Car	Peak	11.19	6.41	-42.69%
	Off-peak	3.54	3.46	-2.20%
Bus & tram	Peak	4.53	4.21	-7.17%
	Off-peak	1.96	1.95	-0.46%
TRAFFIC FLOW (mio passenger km a day)				
Car	Peak	47.31	37.05	-21.70%
	Off-peak	48.70	42.98	-11.73%
	Total	96.01	80.03	-16.64%
Bus & tram	Peak	1.54	2.64	70.98%
	Off-peak	1.30	2.50	92.56%
	Total	2.84	5.14	80.83%
Total		98.85	85.17	-13.84%
AVERAGE SPEED (km/h)				
Car	Peak	30.08	35.89	19.32%
	Off-peak	45.30	45.87	1.26%
Bus & tram	Peak	23.16	27.64	19.32%
	Off-peak	34.88	35.32	1.26%

in the off-peak, and they rise by 22% in the peak period. Finally note that optimal prices per passenger-kilometre are higher for private transport than for public transport, unlike current prices.

The optimal fare structure causes a 22% decrease in peak car traffic. The drastic increase in the price of peak car use causes a large reduction in demand along the demand schedule which is only slightly compensated by the upward shift in the demand function resulting from the price increases of the relevant substitutes, viz. off-peak car use and peak public transport. The combined effect is a substantial decrease in car traffic in the peak period. Figure 2 illustrates the optimum relative to the current situation.

FIGURE 1 : Comparison of the current situation with the basic optimum



We further find a 12% decrease in off-peak car traffic. This results from the fact that the combination of the own price effect and the effect of a decrease in off-peak public transport fares is larger than the compensating impact of the increase in peak car prices. We further note a substantial increase in peak bus and tram traffic (71%). This is mainly the result of the price increase of peak private transport. This shifts the demand curve for public transportation to the right. Despite the simultaneous demand reduction due to the increase in the price of public

transport and the decrease in the off-peak public transport fares, the combined effect of all price changes is towards more public transport use in the peak period. The 93% increase in off-peak public transport can be explained in a similar fashion.

The ultimate result is a reduction of the total traffic level by 14%. Total car traffic decreases by 17%, while public transport use increases by 81%. Total peak traffic and total off-peak traffic both decrease.

As should be the case, all prices equal marginal social costs in the optimum. Note, however, that in comparison with the initial situation, marginal social costs of all modes and in all periods have been reduced. This finding allows us to emphasize a simple but important insight: it is not the current level of marginal social costs which should guide optimal price determination, but the level at optimal traffic levels. At the optimum, marginal social costs are well below their values at current traffic volumes.

The marginal social costs associated with peak traffic are reduced more than those associated with off-peak traffic. In both periods, the reduction in marginal social costs is larger for the private transport mode than for the public transport mode. Note that the decline is actually quite substantial (43%) for peak car traffic. The reduction in car use is also responsible for the lower marginal social cost of public transport use: despite higher public transport use its social marginal cost declines due to lower congestion associated with lower car traffic levels. A final remark relates to average speeds: reduced congestion increases average speed for both transport modes relative to the initial situation, especially in the peak period.

Summarizing the discussion of the results we may conclude that, in the optimum, peak car traffic will be considerably reduced, whereas the number of peak public transport users will be increased by 71%. Moreover, the marginal social costs of all transport services will be lower than in the initial situation, and the average speed of all transport modes will increase.

Since uncertainty is associated with the elasticity values used, we performed a sensitivity analysis in order to investigate the robustness of the optimal prices resulting from the model. These results are presented and discussed in Appendix 6. The major conclusion was that only peak prices are sensitive to elasticity estimates and the optimal pricing results for cars are quite

sensitive to the own price elasticities of the car mode and less to the other (cross) price elasticity assumptions. This is due to the dominant share of the car mode. Of course all optimum traffic levels are dramatically affected by variations in all price elasticities.

B. Models with constant elasticities and restrictions on pricing

In this subsection we introduce restrictions into the model. First we consider the impact of imposing an explicit institutional budget constraint on the public transport firm. Next, we briefly analyze the effects of restricting overall traffic. Finally, we look at the consequences of imposing a maximum for the public transport fares.

1. Introducing a formal budget restriction

To put the budgetary situation of urban public transport in perspective it is useful to start from the 1989 situation. Total fixed and variable costs of the Belgian urban public transport sector were calculated to be 24 116 118 BF per day. Revenues amounted to 9 832 476 BF per day. The resulting daily deficit was 14 283 642 BF per day.

To analyze the implications of a formal budget constraint we impose the restriction

$$FC + \sum_{i=3}^4 C^i - \sum_{i=3}^4 p^i \cdot X^i$$

on the public transport sector. The marginal cost of public funds still equals 1 so that this constraint must be motivated by managerial incentive motivations. Imposing this restriction, we implicitly assume that the fixed costs in the optimum remain at the 1989 level²⁰. The resulting first-order conditions describing optimal pricing can conveniently be rewritten as

²⁰ The model assumes a fixed occupancy rate for urban transport vehicles. In other words, it is assumed that rolling stock is adjusted according to demand variations. As a consequence, fixed costs at optimal traffic levels will typically not remain at their observed 1989 level. The budget restriction imposed is used as one simple example among a large number of alternatives to illustrate the impact of budgetary constraints.

$$\begin{bmatrix} \eta^1_1 & \eta^2_1 & \eta^3_1 & \eta^4_1 \\ \eta^1_2 & \eta^2_2 & \eta^3_2 & \eta^4_2 \\ \eta^1_3 & \eta^2_3 & \eta^3_3 & \eta^4_3 \\ \eta^1_4 & \eta^2_4 & \eta^3_4 & \eta^4_4 \end{bmatrix} \cdot \begin{bmatrix} (S^1 - \mu \sum_{j=1}^4 \frac{\partial C^j}{\partial X^1} - p^1)X^1 \\ (S^2 - \mu \sum_{j=1}^4 \frac{\partial C^j}{\partial X^2} - p^2)X^2 \\ (S^3 - \mu \sum_{j=1}^4 \frac{\partial C^j}{\partial X^3} - (1-\mu)p^3)X^3 \\ (S^4 - \mu \sum_{j=1}^4 \frac{\partial C^j}{\partial X^4} - (1-\mu)p^4)X^4 \end{bmatrix} = -\mu \begin{bmatrix} 0 \\ 0 \\ p^3 X^3 \\ p^4 X^4 \end{bmatrix}$$

with $\mu \geq 0$, $X^i \geq 0$, $p^i \geq 0$ and η_j^i constant.

In solving the above system of equations we returned to the use of the base values for the price and cross-price elasticities. The results are reported in the left part of Table 7. We observe that in order to satisfy the budget restriction all optimal prices rise with respect to the base case. Interestingly, note that the budget restriction on the public transport sector also makes car use more expensive. The budget restriction forces the public firm to raise its prices. With nonzero cross-price effects, this in turn implies increasing congestion. To counteract this negative congestion effect optimal private transport prices rise as well.

Note that, as expected, prices are no longer equal to marginal social costs. For the peak and off-peak periods, car and public transport prices exceed marginal social costs.

2. Introducing quantity restrictions

In some of the solutions considered so far there were substantial effects on total traffic as compared to the current situation. One could argue that extremely large changes in traffic are somewhat unrealistic because of the captive nature of a substantial fraction of all work trips. To see the implications of an extreme form of quantity restriction we therefore simulated optimal prices under the restriction that total traffic has to remain at the currently observed level²¹. In other words, the appropriate restriction is

²¹ Several other variants would have been possible. For example, one could assume that peak period traffic remains constant; alternatively, one could impose an upper and/or lower limit to the change in total traffic.

TABLE 7: Comparison of the results of the basic situation with the results of some extended models

	BASIC OPTIMUM	Model with budget restriction on the public transport sector Optimal values % change w.r.t. basic optimum	Model with a restriction on the total number of passengers Optimal values % change w.r.t. basic optimum
PRICES (BF per passenger km)			
Car	6.41	7.31	5.32
Peak		13.97%	-17.10%
Off-peak	3.46	3.57	2.30
Peak	4.21	10.26	2.88
Off-peak	1.95	2.59	0.63
MARGINAL SOCIAL COSTS (BF per passenger km)			
Car	6.41	6.28	6.66
Peak		-2.16%	3.81%
Off-peak	3.46	3.45	3.65
Peak	4.21	4.20	4.22
Off-peak	1.95	1.95	1.97
TRAFFIC FLOW (mio passenger km a day)			
Car	37.05	36.65	37.97
Peak		-1.08%	2.49%
Off-peak	42.98	42.71	53.13
Total	80.03	79.36	91.10
Peak	2.64	2.14	2.55
Off-peak	2.50	2.05	5.19
Total	5.14	4.19	7.75
Total	85.17	83.55	98.85
AVERAGE SPEED (km/h)			
Car	35.89	36.11	35.43
Peak		0.60%	-1.28%
Off-peak	45.87	45.91	44.79
Peak	27.64	27.80	27.28
Off-peak	35.32	35.35	34.49
MULTIPLIER		-0.20	1.34

$$\sum_{i=1}^4 X^i = 98\,849\,330$$

Note that in this exercise we returned to the case of no budget restriction. The results are also contained in (the right-hand part of) Table 7. They correspond with intuition. To accommodate more travellers than in the unconstrained optimum optimal prices have to be lowered. The corresponding increase in public transport traffic in the off-peak period is most pronounced. For public transport services in the peak, traffic is slightly lower than in the basic optimum. Total car traffic increases by 14%. Note that marginal social costs exceed optimal prices for all transport services.

3. Introducing a maximum for the public transport fares

As can be seen in Table 6 on page 24, the optimal public transport fare in the peak (4.21 BF) is higher than the 1989 fare (3.46 BF). It has been argued, for distributional reasons, that it is politically infeasible to increase public transport prices above their current levels. Therefore, we investigated optimal prices under the restriction that public transport prices (in peak and off-peak) can not exceed 3.46 BF per passenger-kilometre:

$$\begin{aligned} p^3 &\leq 3.46 \\ p^4 &\leq 3.46 \end{aligned}$$

Note that in this exercise we imposed neither a budget constraint, nor a quantity restriction. The results are reported in Table 8 and correspond with intuition. As the maximum fare per passenger kilometre equals 3.46 BF for public transport, the optimal price of peak public transport has to be lowered. So, for peak public transport the price is set below its marginal social cost. This brings along changes in traffic flows and in marginal social costs of all transport services, which results in lower optimal prices for all transport services. These changes in optimal prices are small and most pronounced for peak public transport and its substitutes (peak car and off-peak public transport).

TABLE 8: Comparison of the results of the basic situation with the results of some extended models

		BASIC OPTIMUM	Model with constant elasticities maximum public transport fare fare = 3.46	
			Optimal values	% change w.r.t. basic optimum
PRICES (BF per passenger km)				
Car	Peak	6.41	6.29	-1.95%
	Off-peak	3.46	3.45	-0.32%
Bus & tram	Peak	4.21	3.46	-17.72%
	Off-peak	1.95	1.92	-1.69%
MARGINAL SOCIAL COSTS (BF per passenger km)				
Car	Peak	6.41	6.42	0.15%
	Off-peak	3.46	3.46	-0.02%
Bus & tram	Peak	4.21	4.21	0.00%
	Off-peak	1.95	1.95	0.00%
TRAFFIC FLOW (mio passenger km a day)				
Car	Peak	37.05	37.04	-0.01%
	Off-peak	42.98	43.01	0.06%
	Total	80.03	80.05	0.03%
Bus & tram	Peak	2.64	2.79	5.53%
	Off-peak	2.50	2.51	0.60%
	Total	5.14	5.30	3.13%
Total		85.17	85.35	0.21%
AVERAGE SPEED (km/h)				
Car	Peak	35.89	35.89	-0.01%
	Off-peak	45.87	45.87	-0.01%
Bus & tram	Peak	27.64	27.64	-0.01%
	Off-peak	35.32	35.32	-0.01%

C. Models with variable elasticities

Finally, we consider some models based on the variable-elasticity demand functions previously defined. In a first application, no budget constraint was imposed, nor were any other restrictions. Turning to the results reported in Table 9 (see left part of the Table), we see that with respect to the basic model with constant elasticities all optimal prices decrease. This is not surprising. Taking into account all marginal social costs for the determination of the optimal prices, we again find that prices rise with respect to the initial situation. Since the demand specification implies that price elasticities rise with price, see before, lower optimal prices are sufficient to attain equality with marginal social costs.

In a second application we impose some direct restrictions on prices to analyze their implications. In reality it is often physically or politically infeasible to charge a different price for peak and off-peak travel. Therefore, we investigated optimal prices under the restriction that no difference between peak and off-peak prices is allowed. Under these conditions optimal prices can be obtained by solution of the following equation system:

$$\begin{bmatrix} \eta^1_c & \eta^2_c & \eta^3_c & \eta^4_c \\ \eta^1_p & \eta^2_p & \eta^3_p & \eta^4_p \end{bmatrix} \cdot \begin{bmatrix} (S^1-p^a)X^1 \\ (S^2-p^a)X^2 \\ (S^3-p^b)X^3 \\ (S^4-p^b)X^4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The results of this exercise are contained in the right-hand part of Table 9. Not surprisingly, we see that optimal prices are between the optimal peak and off-peak period prices of the basic model. Total traffic decreases by 19%. This is the result of a 24% decrease in car traffic and a 63% increase in public transport traffic. All marginal social costs at the optimum are lower than in the basic model, but the effects are relatively small. In other words, the restriction on pricing does not substantially affect the marginal social cost of transport.

TABLE 9: Comparison of the results of the basic optimum with the results of some extended models

	BASIC OPTIMUM	Model with variable elasticities no restriction on pricing Optimal values	Model with variable elasticities peak prices = off-peak prices for all modes Optimal values	% change w.r.t. basic optimum	% change w.r.t. basic optimum
PRICES (BF per passenger km)					
Car	6.41	5.77	4.15	-10.04%	-35.36%
Peak					
Off-peak	3.46	3.45	4.15	-0.14%	19.86%
Bus & tram	4.21	4.16	2.22	-1.05%	-47.25%
Peak					
Off-peak	1.95	1.95	2.22	-0.05%	13.57%
MARGINAL SOCIAL COSTS (BF per passenger km)					
Car	6.41	5.77	6.02	-10.04%	-6.15%
Peak					
Off-peak	3.46	3.45	3.25	-0.14%	-6.09%
Bus & tram	4.21	4.16	4.18	-1.05%	-0.64%
Peak					
Off-peak	1.95	1.95	1.93	-0.05%	-1.23%
TRAFFIC FLOW (mio passenger km a day)					
Car	37.05	34.06	35.07	-8.06%	-5.32%
Peak					
Off-peak	42.98	42.79	25.87	-0.44%	-39.82%
Total	80.03	76.86	60.94	-3.97%	-23.85%
Bus & tram	2.64	3.24	3.79	22.67%	43.34%
Peak					
Off-peak	2.50	2.27	4.59	-9.32%	83.56%
Total	5.14	5.50	8.37	7.12%	62.89%
Total	85.17	82.36	69.31	-3.30%	-18.62%
AVERAGE SPEED (km/h)					
Car	35.89	37.31	36.82	3.95%	2.57%
Peak					
Off-peak	45.87	45.89	47.54	0.05%	3.64%
Bus & tram	27.64	28.73	28.35	3.95%	2.57%
Peak					
Off-peak	35.32	35.34	36.61	0.05%	3.64%

As a final exercise, we imposed the restriction of equal peak and off-peak prices for the car mode only²². Results are in Table 10. Looking at the optimal traffic levels, we observe some interesting results. There is a substantial decrease (24%) in car use and a substantial increase (35%) in public transport use as compared to the base case. The assumed impossibility of peak-load pricing for car transport seems to have a large impact on the traffic levels of both modes, and on modal choices. The implication of this is important: applying peak-load prices in public transportation while applying uniform rates for car use would substantially enhance public transport.

²² The relevant equation system can easily be derived from the theoretical model given before.

TABLE 10: Comparison of the results of the basic optimum with the results of some extended models

		BASIC OPTIMUM	Model with variable elasticities peak prices = off-peak prices for car mode	
			Optimal values	% change w.r.t. basic optimum
PRICES (BF per passenger km)				
Car	Peak	6.41	4.18	-34.86%
	Off-peak	3.46	4.18	20.79%
Bus & tram	Peak	4.21	2.49	-40.74%
	Off-peak	1.95	2.03	3.84%
MARGINAL SOCIAL COSTS (BF per passenger km)				
Car	Peak	6.41	6.04	-5.76%
	Off-peak	3.46	3.24	-6.29%
Bus & tram	Peak	4.21	4.18	-0.59%
	Off-peak	1.95	1.93	-1.28%
TRAFFIC FLOW (mio passenger km a day)				
Car	Peak	37.05	35.32	-4.66%
	Off-peak	42.98	25.86	-39.83%
	Total	80.03	61.19	-23.55%
Bus & tram	Peak	2.64	3.33	25.91%
	Off-peak	2.50	3.64	45.64%
	Total	5.14	6.96	35.50%
Total		85.17	68.15	-19.99%
AVERAGE SPEED (km/h)				
Car	Peak	35.89	36.71	2.28%
	Off-peak	45.87	47.55	3.66%
Bus & tram	Peak	27.64	28.27	2.28%
	Off-peak	35.32	36.62	3.66%

V. CONCLUSION

In this paper we analyzed the introduction of social cost considerations in the pricing of urban transport. We developed a theoretical extension of the Glaister-Lewis model to incorporate external environmental, accident and congestion effects and distributional considerations. We concentrated on the computation of optimal prices for automobile use and public transportation. The level of the marginal social cost of congestion and other externalities is itself a function of the intensity of car and bus use so that an equilibrium optimum price had to be computed, taking into account demand and supply responses.

Unfortunately, strict application of the theoretical model was not feasible given current limitations on data availability in Belgium. Therefore, a much simplified version of the model was applied to analyze optimal prices for urban transport. The application uses aggregate data and therefore ignores distributional issues. It captures the main external effects generated by urban transport activities.

Not surprisingly, ignoring distributional and budgetary considerations, optimal prices were found to equal marginal social costs in the optimum. A model without restrictions yields optimal prices that are substantially higher than observed prices for private transport (+141% in the peak period, +30% in the off-peak period) and for public transport in the peak period (+22%). For off-peak public transport prices decrease by more than 40% compared with the current situation. Optimal prices are higher in the peak than in the off-peak, and in both periods optimal prices are higher for private transport than for public transport. The optimal transport prices cause a substantial increase in public transport use (+81%) and a 17% decrease in total car traffic.

The results clearly illustrate a simple but important observation. Optimal pricing has to be guided by marginal social costs at optimal traffic levels, which may substantially deviate from social costs at current traffic levels. It is simply incorrect to equal prices to observed marginal social costs, because the level of the marginal social cost of congestion and other externalities is itself a function of the intensity of car and bus use.

Introduction of a budget restriction for the public transport authority yields higher optimal prices for all modes and periods. Specifically, public transport prices rise by more than 140% in the

peak and by more than 50% in the off-peak. Percentage rises in car prices are more moderate. These price rises cause all traffic levels to decrease with respect to the optimal situation without restrictions. Car traffic decreases by 1%, while public transport traffic decreases by 24%.

We finally note that the results of an elaborate sensitivity analysis suggest that optimal prices are not generally sensitive to changes in price elasticities. Only the own price elasticity of peak car demand seems to have an appreciable effect on optimal prices. On the contrary, optimal traffic levels are dramatically affected by variations in all price elasticities.

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APPENDICES

Appendix 1. Determination of the price elasticities

The price elasticity values used in the initial optimal pricing exercise were derived from Glaister and Lewis (1978) and the careful recent survey by Oum, Waters and Young (1992). They suggest a price elasticity of automobile usage of about -0.3. We take this value to be the own price elasticity of private car transport in the peak period (η^1). Both Glaister and Lewis (1978) and Bös (1986) report that off-peak own price elasticities are typically 2 to 3 times the corresponding peak elasticities. Therefore, the off-peak own price elasticity for private car (η^2) was assumed to be -0.6.

Since we were unable to obtain reliable empirical estimates of all relevant cross-price effects in the literature, we used values that we believe to be reasonable and relatively close to some of the figures used by Glaister and Lewis (1978). The elasticities used in the simulations of the basic model are summarized in the following matrix

$$\begin{bmatrix} -0.3 & 0.049 & 0.708 & 0 \\ 0.05 & -0.6 & 0 & 0.578 \\ 0.03 & 0 & -0.35 & 0.036 \\ 0 & 0.02 & 0.03 & -0.87 \end{bmatrix}$$

Because of uncertainty with respect to the values used, we carried out a substantial sensitivity analysis to determine whether or not the results were robust (see Appendix 6).

Appendix 2. Current use and prices of the different transport services

a. Current demand for private car trips and for public transport

To calculate the total number of passenger-kilometres travelled by private car we proceeded as follows. First, Cuijpers (1992, based on De Borger (1987b)) calculated the average distance travelled by cars using gasoline, diesel and LPG for 1989. We multiplied these values by the number of gasoline, diesel and LPG cars respectively (see NIS (1991)) to obtain the total number of kilometres travelled by car. Based on CBS (1989), we assume that 30% of the total number of vehicle-kilometres is travelled in cities. Dividing the number of vehicle-kilometres travelled in urban areas by 365 leads us to the average daily number of vehicle-kilometres travelled by private car. From STRATEC (1992), we know that 50.72% of the daily private car traffic travels during the off-peak time of the day, whereas 49.28% are driven in the peak period. Using an average car occupancy-rate of 1.7 (De Borger and De Borger (1987)) this leads to the values for X^1 and X^2 in Table 2 in the text.

The number of passenger-kilometres travelled by public transport in 1989 is calculated as follows. The number of public transport passengers for the different cities under consideration was taken from "Verkeer en vervoer in België" (Belgium, Ministerie van verkeer en infrastructuur (1991)). Boniver (1992) reports that, for urban areas, the average distance travelled by a passenger is approximately 4 kilometre. Dividing the annual number of passenger-kilometres by 365, we obtain the daily number. Using the proportions of daily public transport trips travelled during peak (45.65%) and off-peak (54.35%) periods proposed by STRATEC (1992) leads to the data reported in Table 2 in the text.

b. Prices

The current 'price' of car use is simply assumed to be the average variable private cost. This captures the costs of fuel, oil, tyres, reparations and insurance, all inclusive of taxes. The 1989 weighted average fuel price per vehicle-kilometre (inclusive of TVA) is given in the study by

Cuijpers (1992)²³. An indication of the price per vehicle-kilometre for oil, tyres, reparation and insurance is based on the NIS statistics (1990). Dividing the price per vehicle-kilometre by the average car occupancy-rate of 1.7 (De Borger and De Borger (1987)) yields the price per passenger-kilometre.

The price of public transport is based on the study by Boniver (1992). She reports a careful analysis (for 1991) of the average public transport price per passenger-kilometre for four cities, viz. Antwerp, Brussels, Ghent and Liège. Converting the values for 1991 into 1989 prices and using the proportion of total passengers travelling with the different public transport companies (Belgium, Ministerie van verkeer en infrastructuur (1991)), we computed a weighted average public transport price per passenger-kilometre for 1989.

Appendix 3. The current peak and off-peak traffic level in Belgian urban areas

In "Verkeer en vervoer in België" (Belgium, Ministerie van verkeer en infrastructuur (1991)), it is reported that the daily number of vehicle-kilometre travelled by public transport in the urban areas considered in this paper is 196 033. On the basis of Dorssemont (1984) and MIVB (1990), we assume that 34% of total public transport is supplied during the peak. The number of vehicle-kilometre travelled in the peak is then 66 651 kilometre a day. In the off-peak, this number is 129 382 kilometre a day.

From Table 2, we know that the number of private car kilometre travelled in the peak equals 27 830 902 kilometre a day. In the off-peak period, 28 644 142 kilometre are travelled by private car. With these data, we can calculate the number of PCU²⁴ kilometre per hour travelled in peak and in off-peak²⁵. We find 5 592 841 PCU km/h in the peak and 1 700 171 PCU km/h in the off-peak.

²³ Cuijpers weighted the gasoline, diesel and LPG prices by the proportion of total vehicle-kilometre travelled with gasoline, diesel and LPG cars.

²⁴ 1 passenger car (PC) = 1 passenger car equivalent unit (PCU), and 1 public transport vehicle (PTV) = 2 passenger car equivalent units (PCU)

²⁵ Remember that the peak period covers five hours a day, while the off-peak period covers seventeen hours.

Appendix 4. Calculation of marginal accident costs

Marginal accident costs for an additional car vehicle-kilometre were carefully analyzed by Mayeres (1993). By analogy, the marginal accident costs associated with an additional public transport vehicle-kilometre can be separated in three parts. First, marginal accident costs are associated with the risk of death or injury to the occupants of the additional public transport vehicle. Secondly, we have to consider the marginal costs associated with the increased risk of death, injury or material damage to the other motorized road users. Third, the marginal accident costs associated with the increased risk of death or injury to pedestrians and cyclists have to be taken in account. Mayeres (1993) gives a detailed discussion of each of these marginal accident costs for private car use.

The first step to calculate the marginal accident costs for public transport consists of determining the probabilities for occupants, other motorized road users and passengers or cyclists to be killed or injured. These probabilities are derived on the basis of NIS (1989). In a second step, we multiply the probabilities by the relevant accident costs (Mayeres (1993)).

Appendix 5. The money costs of public transport

The variable private money costs of public transport are assumed to consist of expenditures on drivers, on energy (insofar as related to rolling stock) and on materials, reparations and deliveries (again insofar as related to rolling stock). Fixed costs, which are only relevant for the construction of the budget restriction, were assumed to consist of the costs of non-drivers, of energy for infrastructure, of materials, reparations and deliveries with respect to infrastructure, of insurance and of depreciation expenses other than those for rolling stock.

Most relevant figures for the MIVB were found in the careful study by Evrard (1992). With respect to the MIVA and the MIVG we extracted much information directly from the corresponding annual reports. Other information was constructed as indicated in Table A1.

TABLE A1 : The costs of the Flemish public transport companies

	MIVA	MIVG
Expenditures on personnel	* Variable private money cost = expenditures on drivers * Fixed cost = expenditures on non-drivers	
Expenditures on energy	* Variable private money cost = fuel cost + 52% of electricity cost * Fixed cost = 48% of electricity cost	
Expenditures on materials, reparations and deliveries	* Variable private money cost = cost per kilometre for Brussels multiplied by the number of kilometres travelled in Antwerp and Ghent * Fixed cost = cost found in the annual report - variable private money cost	
Expenditures on insurance	Presented in the annual reports	
Depreciations other than those for rolling stock	Presented in the annual reports	

Note that with respect to the expenditures on personnel, we assigned the expenditures on drivers to the variable private money costs, while the expenditures on non-drivers were taken to be fixed.

With respect to energy we assigned all fuel costs and 52% of the electricity costs to the variable private money costs (Evrard (1992)). The other part of the electricity costs is taken to be the electricity cost for infrastructure.

Data on expenditures related to materials, reparations and deliveries with respect to rolling stock were unavailable in the annual reports. Therefore this information was indirectly based on the available data for the MIVB after correcting for the number of vehicle-kilometres travelled in the respective cities. The expenditures on materials, reparations and deliveries with respect to infrastructure were taken to be equal to the costs found in the annual reports minus the costs with respect to rolling stock.

The cost of capital of rolling stock was calculated as follows :

$$A_t (r_t + \delta) \cdot TRAMS$$

$$A_b (r_b + \delta) \cdot BUSES$$

where :

- A_t = purchasing cost of one tram = 40 000 000 BF
- A_b = purchasing cost of one bus = 5 000 000 BF
- r_t = depreciation rate trams = 2.85% (life span : 35 years)
- r_b = depreciation rate buses = 7.7% (life span : 13 years)
- δ = short-term interest rate = 8.8% (3 month treasury certificates)
- TRAMS = the number of trams in the Belgian urban areas
- BUSES = the number of buses in the Belgian urban areas

Note that we were able to distinguish between peak and off-peak variable private money costs as follows. As previously indicated the number of PCU kilometre per hour equals 5 592 841 PCU km/h in the peak and 1 700 171 PCU km/h in the off-peak. With the help of the 'speed-capacity' relationship, we determined the time needed to drive 1 kilometre by car for the peak and for the off-peak. Dividing these numbers by 0.77, we have the time needed to drive 1 kilometre by bus or tram. Multiplying these numbers by the expenditure per driver and per hour, yields the average variable private personnel costs in peak and off-peak.

Due to limited data, expenditures per kilometre on energy for rolling stock and on materials, reparations and deliveries with respect to rolling stock, are taken to be the same for peak and off-peak periods.

As we had no annual reports of the public transport companies of the Walloon region, we approximated the relevant costs for STIL, STIC and STIV as follows.

Computing a weighted average fuel cost per kilometre for the MIVA, MIVB and MIVG, and multiplying this value by the number of vehicle-kilometres travelled by STIL, STIC and STIV (Belgium, Ministerie van verkeer en infrastructuur (1991)), results in the variable energy costs for these companies²⁶. The expenditures on materials, reparations and deliveries with respect

²⁶ There are no trams in Liège, Charleroi and Verviers.

to rolling stock were approximated by multiplication of the cost per kilometre for the MIVB by the number of vehicle-kilometres travelled in the various cities.

Adding up the different variable private money costs of all the cities, and dividing these costs by the number of vehicle-kilometres travelled, we find an average variable private money cost of 48.7 BF per kilometre for the peak period and an average variable private money cost of 37.7 BF per kilometre for the off-peak times of the day.

On the basis of the data for MIVA, MIVB and MIVG, we calculated an average cost per non-driver. Multiplying this cost by the number of non-drivers for the different companies (Trends Top 25 000 (1991) + assumption : 60% of the employees are drivers), gives an indication of the unknown expenditures on personnel.

The fixed costs exclusive of personnel costs of the MIVA, MIVB and MIVG are 9% of the total costs of these companies. We assume this percentage is the same for the public transport companies of the Walloon region. The sum of the fixed costs of all the urban public transport companies together, has to be divided by 365 to get the fixed cost per day.

Appendix 6. Results of the sensitivity analysis on the price elasticities

The procedure we followed is easily summarized. Based on the available estimates in the literature and some common sense we determined for each own and cross-price elasticity a 'low' and a 'high' value. For all elasticities zero was used as the low value. The high values are summarized in the following matrix

	car peak	car off-peak	bus peak	bus off-peak
car peak	-3	0.97	2.36	2.8
car off-peak	1	-6	2.4	2.9
bus peak	0.1	0.1	-3	3.57
bus off-peak	0.1	0.1	3	-7.5

The sensitivity analysis consecutively set some elasticity values at their low, respectively high values, keeping all remaining elasticities at their base values. For example, in the first exercise the own price elasticities of the private car (η^1_1 and η^2_2) were set to their low values, whereas

all other elasticity values remain the same as in the basic model²⁷. In other words, the elasticity matrix used in the first sensitivity exercise then is

	car peak	car off-peak	bus peak	bus off-peak
car peak	0	0.049	0.708	0
car off-peak	0.05	0	0	0.578
bus peak	0.03	0	-0.35	0.036
bus off-peak	0	0.02	0.03	-0.87

We begin with a summary of the results of the simulations in Table A2. The initial prices and the optimal prices resulting from application of the base model are reproduced in the first rows of the table for purposes of comparison. For a variety of alternative elasticity estimates we further report the corresponding optimal prices.

The most striking feature of these results is that optimal prices appear to be quite sensitive only to changes in the own price elasticities of the car mode, and, to a much lesser extent, to changes in the cross-price effect between peak and off-peak car use. Moreover, only peak prices are drastically affected; both off-peak car price and public transport prices are hardly affected by varying the price sensitivities. This is a remarkable finding which undoubtedly reflects the much larger share of car traffic as compared to public transport.

²⁷ The elasticities of the base case were reported in Section III.

TABLE A2 : Summary of optimal prices for various elasticity values

		p ¹	p ²	p ³	p ⁴
	Initial	4.531	4.531	3.46	3.46
	Basic	10.904	5.880	4.205	1.953
Own elasticities private car	Low	21.723	6.123	4.637	1.969
	High	6.376	5.349	4.024	1.918
Own elasticities public transport	Low	10.919	5.867	4.205	1.952
	High	10.916	7.058	4.205	2.031
Cross elasticities peak and off-peak car	Low	10.757	5.839	4.199	1.953
	High	13.381	6.561	4.304	1.998
Cross elasticities peak car and public transport	Low	10.733	5.879	4.198	1.953
	High	12.116	5.885	4.253	1.953
Cross elasticities peak car, off-peak bus/tram	Low	10.904	5.880	4.205	1.953
	High	10.325	6.170	4.182	1.972
Cross elasticities off-peak car, peak bus/tram	Low	10.904	5.880	4.205	1.953
	High	11.115	5.900	4.213	1.954
Cross elasticities off-peak car and bus/tram	Low	10.905	5.886	4.205	1.953
	High	10.902	5.862	4.205	1.952
Cross elasticities peak and off-peak bus/tram	Low	10.908	5.879	4.205	1.953
	High	10.720	5.910	4.197	1.955

As variations in the own price of car traffic seem by far to have the most important effects we here only consider this case in more detail. Full results for this case are reported in Table A3.

We focus in our discussion of the results on the differences they imply relative to the base case presented before. Optimal prices for the car mode are clearly negatively affected by the price elasticity of car traffic. The effect is most pronounced in the peak period. Inelastic demand almost doubles prices as compared with the basic model. Highly sensitive demand for car use on the contrary substantially reduces peak car prices.

TABLE A3: Sensitivity analysis: own price elasticities private car

	BASIC OPTIMUM	Own price elasticities private car low values	Own price elasticities private car high values
		Optimal values	Optimal values
		% change w.r.t. basic optimum	% change w.r.t. basic optimum
PRICES (BF per passenger km)			
Car	6.41	12.78	3.75
Peak	3.46	3.60	3.15
Off-peak	4.21	4.64	4.02
Bus & tram	1.95	1.97	1.92
Peak			
Off-peak			
MARGINAL SOCIAL COSTS (BF per passenger km)			
Car	6.41	12.78	3.75
Peak	3.46	3.60	3.15
Off-peak	4.21	4.64	4.02
Bus & tram	1.95	1.97	1.92
Peak			
Off-peak			
TRAFFIC FLOW (mio passenger km a day)			
Car	37.05	48.45	17.20
Peak	42.98	51.99	18.08
Off-peak	80.03	100.45	35.28
Total	2.64	4.16	1.83
Bus & tram	2.50	2.55	2.40
Peak	5.14	6.71	4.23
Off-peak	85.17	107.15	39.51
Total			
AVERAGE SPEED (km/h)			
Car	35.89	29.17	44.25
Peak	45.87	44.94	48.31
Off-peak	27.64	22.46	34.07
Bus & tram	35.32	34.61	37.20
Peak			
Off-peak			

The percentage changes in the optimal off-peak car price are much less important. Public transport prices are also negatively related to the price sensitivity of car use demand, although the numerical effects are rather small.

Since, in the case of inelastic demand, all optimal prices rise with respect to the basic model, this will have nonnegligible effects on traffic levels. Total traffic increases by 26%. This is the result of an increase in private car use of 26% and an increase in public transport traffic of 30%. The increase in private car use during the peak period may seem surprising in view of the substantial rise in its price. The reason is that the demand effect due to the price increases in the substitute goods dominates the own price effect, since the latter is zero by our choice of the low elasticity values.

In the case of highly price sensitive demand for car use, we observe the opposite effects. Prices are all lower than in the base case, marginal social costs are lower, and traffic levels all substantially decline. Car travel would go down by some 56%, public transport use by approximately 18%.

To avoid potential confusion an important remark is in order. Our results suggest that at the optimum prices and traffic levels are positively correlated. For inelastic private transport demand we found higher optimal prices for all modes and periods, yet all optimal traffic levels increased as compared to the results of the base case model. Similarly, for highly elastic demand we noticed a substantial reduction in traffic levels, despite lower optimal prices than in the base case. The reason is that marginal social costs increase with traffic levels due to congestion and that optimal prices equal marginal social costs. It follows that when optimal prices rise due to more inelastic demand, marginal social cost at the optimum should correspondingly increase. This can only be achieved through an increase in traffic levels.

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