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The comparative statics of tax evasion with elastic labour supply -Can we really say anything about the reactions of a tax evader?-

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** Upon completion of this version of the paper, I was informed by Frank Cowell that some of the results in section 5 have also been addressed by Claude Fluet in his paper 'Fraude fiscale et offre de travail au noir' (*L'Actualité Économique*, 63, 1987).

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ABSTRACT

In this paper, I study the effects of tax, compliance and macroeconomic parameters on the tax evading behaviour of an individual whose labour supply curve is not totally inelastic. Following Allingham & Sandmo (1972), the government is assumed to carry out a random audit policy followed by the imposition of a penalty when any evasion is detected. The decision problem under uncertainty with which the tax evader is confronted is given more structure by imposing an ordinal condition on preferences, first identified by Drèze & Modigliani (1972). If this restriction is complemented with some institutional conditions, it is sufficient to generate functional separability in the decision to evade taxes. This was the course taken by Cowell (1981, 1985).

First I show that these institutional conditions are not likely to obtain, both for positive as well as normative reasons. Second, even if they do obtain–for instance because the government restricts itself to a proportional penalty scheme–, I show that the comparative statics effects remain quite complex, despite the functional separability, and contrary to insights obtained hitherto. The reason is twofold. First because in general the risk premium a tax evader is willing to pay to avoid the risk of the audit policy is *not* independent of the amount of leisure enjoyed. Necessary and sufficient conditions both on the preference order and on the shape of the utility function are presented for such dependency to vanish. Secondly, when a consumer faces multiple budget constraints–which is the proper way to think about a random opportunity set–the non-trivial compensation schemes necessary to produce unambiguous substitution effects nullifies the predictive power of the latter in explaining the direction of the associated uncompensated effect.

Table of Contents

1. Introduction.

The theoretical literature on tax evasion was initiated, now two decades ago, with the seminal paper by Allingham & Sandmo (1972). That paper focused on the decision to evade income taxes when income stems from an inelastic labour supply. Confronted with a linear (in particular a proportional) tax scheme, the consumer could reduce his tax liability by underreporting his earned income, thus reducing the effective tax base.

The decision to evade taxes was modelled as a decision under uncertainty–the uncertainty originating from the random audit policy carried out by the government. Given the exogenous character of gross income, the results where much in the spirit of the basic portfolio problem where wealth is allocated between a safe and risky asset. The model provided useful insights. For instance, with decreasing absolute risk aversion, tax evasion is positively correlated with gross income; on the other hand, an increase in the marginal penalty rate or in the audit probability enhances tax compliance.

The framework offered by the Allingham & Sandmo article has been used in a large number of other papers, some of which also discussed the normative aspects of the audit policy. Because the income source is exogenous, these papers focused most of the time on the equity-compliance trade-off. The standard literature on redistribution has been centred around a different kind of trade-off: the one between equity and efficiency (cf Atkinson & Stiglitz, 1980, Part two). Since the late 70s and early 80s, a number of authors has introduced the efficiency aspect in the tax evasion literature by dropping the assumption of an inelastic labour supply. Important work in this area was carried out by Sandmo (1981) and Cowell (1981, 1985, 1991).

Not surprisingly, by adding labour supply effects, the responses of an individual agent did not gain in transparency. In fact, the mathematical structure of the model bears close resemblance to the two period savings *cum* portfolio model. This model was extensively studied by Drèze & Modigliani (1969, 1972), and these authors were able to identify conditions which impose some interesting and simplifying separability structure on the problem. Cowell (1981, 1985) picked up this idea, and engaged in deriving the comparative statics effects on the evasion activity of a consumer/worker. The tax evasion problem may be approached as a two-stage procedure: in a first stage the consumer decides upon the amount of leisure to deny himself; this amount is then allocated in a second stage over hours worked on the official and black labour market.

The purpose in the present paper is twofold. First, I want to make precise the institutional conditions under which the separability result will go through. I will conclude that both from a normative as well as positive point of view, such conditions are not likely to obtain. Second, I want to show that even if these conditions do hold, the results obtained by Cowell (1981, 1985) are incomplete. The reason for this incompleteness is twofold. First, as shown by Diamond & Yaari (1972) in the savings *cum* portfolio context, to generate conventional substitution effects one needs to compensate the consumer/worker in a non-trivial way. Failing to do so will produce substitution effects which cannot be signed and hence become useless in the signing of uncompensated effects. The second source of problems is related to intricate spill-over effects: any parameter which affects the decision in the first stage, will produce, next to an income effect, a substitution effect to the second stage decision variables. Unless one is willing to supplement the Drèze-Modigliani hypothesis with an extra condition on preferences which makes the risk premium independent of the amount of leisure, this second type of spill-over effect will fail to vanish and will blur the conclusions obtained otherwise.

The outline of the paper is as follows. In the next Section, I introduce the model of tax evasion when labour supply is elastic. Essentially this is a model of allocating endogenous labour supply between an official and a black labour market. Section 3 is concerned with sufficient conditions on preferences and institutions that allow for a break up of the problem in two stages (the former condition is analogous to the Drèze-Modigliani condition for separating the savings decision from the portfolio decision). In a first stage, the consumer decides on how much leisure to enjoy; in a second stage, the amount of foregone leisure is allocated between the official and black labour market. In Section 4, I take a closer look at the preferences implied by the Drèze-Modigliani condition. Section 4.1 introduces a graphical presentation of the problem. Section 4.2 is concerned with the dependency of the risk aversion function on the amount of leisure enjoyed. In Section 4.3, necessary and sufficient conditions are derived for such dependency to vanish. Finally (Section 4.4), it is checked to which extent three frequently used labour supply models satisfy both the DM and the risk independency condition. In Section 5, I derive the (uncompensated) comparative statics effects for the upper stage (5.1) and the lower stage (5.2). Both type of effects are combined in Section 5.3, and summarized in the table at the end of that section. Section 6 deals with some compensated comparative statics effects, the knowledge of which is likely to be useful in normative exercises w.r.t. income tax and compliance policy. Concluding remarks are gathered in Section 7.

2. The Model

The consumer has access to two types of labour market: the official labour market on which he can earn a gross wage rate w_L per hour of work, and the black labour market on which he is able to obtain the gross hourly wage rate w_B. Both wage rates need not be identical, but when they are, a different interpretation can be given to this decision problem, as will be explained later on. Total time available to the consumer is spent on leisure (ℓ) and labour time (H). Normalizing total time available to unity and denoting the number of hours on the official and black labour market by L and B, respectively, the time constraint of the consumer is given by

$$1-\ell = L+B =_{def} H. \tag{2.1}$$

The net disposable income of this consumer will depend on the tax treatment of income earned in either market. Income earned on the official labour market is subjected to a two-part tax scheme with a constant marginal tax rate t and a lump sum tax amounting to T. By definition, the income earned on the black labour market is not subjected to any tax treatment. If it would be prohibitively costly for the government to observe these illegal activities, the consumer will spend his foregone leisure time either on the official market, either on the black market, depending on where he can obtain the highest disposable labour income. However, usually, the government is not totally powerless against black market activity. In particular, it may announce an audit policy whereby each agent faces the probability π of being audited. If any black market activity has been going on, it is assumed it is successfully revealed by the audit, and the income earned in this way is subjected to a penalty scheme. This scheme may take the form of a two-part tariff as well: a constant marginal penalty f per illegally earned franc, supplemented with a lump sum fine F.

All disposable income (net of taxes and penalties) is spent on a single commodity (c) which serves as *numéraire*. Denoting consumption in the non-audit and in the audit state by c_1 and c_2, respectively, the random budget constraint of the consumer can be written down as follows:

$$\begin{aligned} c_1 &= (1-t)w_L L - T + w_B B && \text{with probability } (1-\pi) \\ c_2 &= (1-t)w_L L - T + (1-f)w_B B - F\cdot I(B) && \text{with probability } \pi, \end{aligned} \tag{2.2}$$

where $I(B)$ is an indicator function, being zero whenever $B=0$, and unity otherwise. Notice that by writing the budget constraint in this way, the imputation of the lump sum tax T is made unconditionally–for instance, when negative, it can be thought of as a basic income.

Two further remarks can be made about this random opportunity set. First, we have what Drèze & Modigliani (1972, p 183) coin a *temporary uncertain prospect*, because labour market decisions need to be made *before* any uncertainty is resolved. Second, it is clear that by restraining himself from any black labour market participation, the consumer is able to avoid any uncertainty about disposable income. In other words, every uncertainty is voluntary chosen, or endogenous. This property distinguishes the model form the analysis in Block & Heineke (1973) who exclude the opportunity for labour market diversification.

I assume that preferences over this opportunity set are given by the von Neumann-Morgenstern utility function $u(c,\ell)$ which is continuous and continuously differentiable at least three times[1]. Further assumptions on $u(\cdot)$ are :

(U1) Concavity: $u_{11} < 0$, $u_{22} < 0$, $u_{11}u_{22}-u_{12}^2 > 0$;

(U2) Normality of leisure: $u_{21}-\frac{u_2}{u_1}u_{11} = u_1\frac{\partial}{\partial c}(\frac{u_2}{u_1}) > 0$;

(U3) Normality of consumption: $u_{12}-\frac{u_1}{u_2}u_{22} = u_2\frac{\partial}{\partial \ell}(\frac{u_1}{u_2}) > 0$.

[1] I thus rule out state-dependent preferences. *The Economist*'s Rome correspondent would probably regard this as a wrong assumption if the consumer in my model were an Italian: "The biggest problem is that Italians think that cheating on taxes merits respect" ('Pastassessment', *The Economist*, July 14, 1984, p 81).

Assumption (U1) will of course guarantee risk averting behaviour. Assumptions (U2) and (U3) are called normality assumptions, because they guarantee a positive correlation of consumption and leisure with exogenous income in a *certain* environment.

Expected utility is given by

$$Eu(c,\ell) =_{def} (1-\pi)u(c_1,\ell)+\pi u(c_2,\ell), \tag{2.3}$$

where E is the expectations operator. The optimal choice of consumption and leisure, and the allocation of foregone leisure over both types of labour market activities will satisfy the necessary conditions for maximization of (2.3) subject to (2.2). The presence of the non-differentiable indicator function in (2.2) invalidates inference on a global maximum from the first order conditions. However, we may proceed with a marginal analysis provided that it is indeed optimal to participate in the black labour market ($B>0$). Before doing so, let me define the random variables τ and ϕ as follows:

$$\begin{aligned}(\tau,\phi) &= (0,0) \quad \text{with probability } (1-\pi) \\ &= (f,F) \qquad\qquad\qquad\quad \pi\ .\end{aligned} \tag{2.4}$$

As mentioned before, it will be assumed that it is indeed optimal to engage into some black market activity. The constrained maximization problem can now be stated as

$$Max_{L,B}\ Eu[(1-t)w_L L-T+(1-\tau)w_B B-\phi,\ 1-L-B],\quad L \geq 0. \tag{2.5}$$

The associated first order conditions for a B-interior maximum are given by

$$\begin{aligned}&L \geq 0,\\ &L[(1-t)w_L Eu_1-Eu_2] = 0,\\ &[(1-t)w_L Eu_1-Eu_2] \leq 0.\end{aligned} \tag{2.6}$$

$$Eu_1(1-\tau)w_B = Eu_2. \tag{2.7}$$

The second condition asserts that at the optimum, the expected marginal disutility of work should exactly offset the expected marginal utility of consumption generated by the random income. Conditions (2.6) are the Kuhn-Tucker conditions w.r.t. labour supply on the official labour market. Suppose first that the term in square brackets is strictly negative at the optimum. Substituting Eu_2 for the LHS of (2.7) and rearranging, this strict inequality may be rewritten as

$$\pi f < (\frac{w_B-(1-t)w_L}{w_B})[\pi+(1-\pi)\frac{u_1(c_1,\ell)}{u_1(c_2,\ell)}]. \tag{2.8}$$

The LHS represents the expected marginal penalty per unit of non-reported income. The RHS is less transparent. The term in round brackets denotes the relative net wage differential on the two markets in case no audit follows. When it is optimal for the consumer to engage into some black market activity, positivity of this term follows as a necessary condition. Since an audit is followed by a penalty, concavity of the von Neumann-Morgenstern utility function guarantees that the term in square brackets is strictly smaller than one. Inequality (2.8) is illuminating because it identifies conditions under which it is very likely for the tax and wage structure in the economy to create "ghosts"–people of which no official employment records exist at all. Thus, a very low audit probability and marginal penalty rate, a very high marginal tax rate and a very high black market wage rate all incite people to become a ghost.

When it is optimal to participate on the official labour market, the necessary FOC's reduce to

$$\begin{aligned} E(1-t)w_L u_1 &= Eu_2, \\ E(1-t)w_L u_1 &= E(1-\tau)w_B u_1. \end{aligned} \tag{2.9}$$

From now on I will only consider allocations which satisfy these conditions for in interior optimum.

By reinterpreting the different variables and parameters, the solution to the maximization programme (2.5) can serve for at least two other problems. See Table 1 below. When $w_L = w_B$, the model above becomes a genuine tax evasion model. For now, one can think of $w_B B$ as the amount of gross labour income $w_L H$ which the consumer decides not to declare in his tax report at the end of the year.

Solving these FOC's for H and B, one obtains the overall and black market labour supply equations

$$\begin{aligned} H &= H[(1-t)w_L,\ w_B,\ T,\ g(\tau,\phi)], \\ B &= B[(1-t)w_L,\ w_B,\ T,\ g(\tau,\phi)], \end{aligned} \tag{2.10}$$

where $g(\cdot)$ is the density of the two-part penalty scheme, as given by (2.4). As argued by several authors, the comparative statics of this system are cumbersome. In fact, it is easy to understand why this is the case. There are two states of nature, and associated with them, two consumption variables, c_1 and c_2; in addition, there are the two labour market decision variables H and B. This amounts to four decision variables in total which are linked by two budget constraints. And although the expected utility hypothesis places some structure on the problem, this is not sufficient to yield any *a priori* predictions on the sign of comparative statics effects. In Appendix A, I have derived these effects using a primal state preference approach.

Nevertheless, further insight might be obtained by imposing additional structure on the problem. This is the course taken by Cowell (1981, 1985), and it will be taken up in the next section.

Table 1. Alternative interpretations of the model.

	Taxation and labour supply with risky activities (Cowell, 1981)	Consumption decisions under uncertainty (Drèze & Modigliani, 1972)
c	consumption	future consumption
ℓ	leisure	present consumption
L	hours spent at riskless activity	investment in riskless asset
B	hours spent at risky activity	investment in risky asset
$(1-t)$	wage rate of the riskless activity	rate of return on investment in riskless asset
w_L	1 minus marginal tax rate on certain wage income	
$(1-\tau)$	wage rate of the risky activity	rate of return on investment in risky asset
w_B	1 minus marginal tax rate on uncertain wage income	
$-T$	lump sum income	certain future income
$-\phi$	\	uncertain future income

3. Institutional and Preference Conditions for Separation between Labour Supply and Labour Allocation Decisions.

This section is concerned with sufficient conditions on preferences and institutions that allow for breakdown of the tax evasion problem in two stages. For convenience, let me define the following net wage rates:

$$\omega_L =_{def} (1-t)w_L, \quad \omega_B =_{def} (1-\tau)w_B, \quad \text{and}$$
$$\omega_H =_{def} \frac{\omega_L L+\omega_B B}{H} = \omega_L+(\omega_B-\omega_L)\frac{B}{H}. \tag{3.1}$$

ω_H is the overall net wage rate from a given labour supply (L,B). It can then be checked from the FOC's (2.9) that

$$\begin{aligned} Eu_1\omega_H &= \omega_L Eu_1+[Eu_1\omega_B-\omega_L Eu_1]\frac{B}{H} \\ &= Eu_2. \end{aligned} \tag{3.2}$$

The expected overall net wage rate, weighted by the marginal utility in each state, is given by

$$\omega_H^* =_{def} \frac{E\omega_H u_1}{Eu_1}. \tag{3.3}$$

From (3.1) and the FOC (2.9), it follows that

$$\omega_H^* = \omega_L. \tag{3.4}$$

Likewise, the expected–but u_1-weighted–lump sum fine is defined as

$$\phi^* =_{def} \frac{Eu_1\phi}{Eu_1}. \tag{3.5}$$

We are now in a position to state the following

Theorem 1 (Drèze & Modigliani, 1972, Cowell, 1981): *When the marginal rate of substitution between consumption and leisure, $\frac{u_2}{u_1}(c,\ell)$, is linear in consumption, the optimal amount of hours worked, H, will be identical to the amount chosen by a consumer facing the certain net wage rate ω_L and the certain lump sum tax $T+\phi^*$.*

Proof

Consider the random bundle $[\omega_L H+(\omega_B-\omega_L)B-T-\phi,1-H]=(\omega_H H-T-\phi,1-H)$ where H and B are optimal number of hours satisfying the FOC (2.9). Evaluating the MRS at this bundle and performing a second order Taylor expansion around the deterministic bundle $(\omega_H^* H-T-\phi^*,1-H)$ yields:

$$\begin{aligned}\frac{u_2}{u_1}(\omega_H H-T-\phi,1-H) &= \frac{u_2}{u_1}(\omega_H^* H-T-\phi^*,1-H) \\ &+\frac{\partial(\frac{u_2}{u_1})}{\partial c}(\omega_H^* H-T-\phi^*,1-H)\ [(\omega_H-\omega_H^*)H-(\phi-\phi^*)] \\ &+\frac{1}{2}\frac{\partial^2(\frac{u_2}{u_1})}{\partial c^2}(\tilde{c},1-H)\ [(\omega_H-\omega_H^*)H-(\phi-\phi^*)]^2,\end{aligned} \tag{3.6}$$

where $\tilde{c}$ is some convex combination of $(\omega_H H-T-\phi)$ and $(\omega_H^* H-T-\phi^*)$. Since the MRS is linear in consumption, the third term on the RHS drops out.

Next, multiplying the equation above through by $u_1(\omega_H H-T-\phi,1-H)$ and taking expectations results in

$$\begin{aligned}Eu_2(\omega_H H-T-\phi,1-H) &= \frac{u_2}{u_1}(\omega_H^* H-T-\phi^*,1-H)\ Eu_1(\omega_H H-T-\phi,1-H) \\ &+ \frac{\partial(\frac{u_2}{u_1})}{\partial c}(\omega_H^* H-T-\phi^*,1-H)\ [(Eu_1\omega_H-\omega_H^* Eu_1)H-(Eu_1\phi-\phi^* Eu_1)].\end{aligned} \tag{3.7}$$

But by definition [see (3.3) and (3.5)], the term on the RHS in square brackets is zero. Using the first of the FOCs in (2.9), the expression further simplifies to

$$\omega_L = \frac{u_2}{u_1}(\omega_H^* H-T-\phi^*,1-H), \tag{3.8}$$

or, using (3.4),

$$\omega_L = \frac{u_2}{u_1}(\omega_L H-T-\phi^*,1-H). \tag{3.9}$$

This equation is precisely the FOC for the standard labour supply problem when the net real wage rate is given by ω_L and lump sum taxes by $T+\phi^*$. QED

The linearity condition, or

$$\text{(DM)}\quad \frac{\partial^2}{\partial c^2}\left(\frac{u_2}{u_1}\right) = 0,$$

is of course crucial to the theorem, and was first identified by Drèze & Modigliani (1972) in the savings decision context. The condition implies that the overall labour supply function can be written in the standard way:

$$H = H[\omega_L, -T-\phi^*]. \tag{3.10}$$

But this does not really simplify the problem since the certainty equivalent lump sum fine, ϕ^* still depends on the endogenous "portfolio" allocation of total labour H among L and B through the argument of u_1 [see (3.1) and (2.5)]. This dependency may vanish under two institutional settings.

The first setting–and probably the most unrealistic–is one of an additional market where the tax evading consumer can exchange the uncertain lump sum fine ϕ for a certain fine Φ to the extent he wishes. This insurance price Φ for getting rid of the uncertain prospect ϕ is taken as parametric by the consumer. Let us call this *perfect fine insurance*. Denoting (1-α) the proportion of the fine risk the consumer wishes to trade away on this insurance market, the maximization problem is modified into

$$Max_{H,B,\alpha}\ Eu[\omega_L H-T+(\omega_B-\omega_L)B-\alpha\phi-(1-\alpha)\Phi,\ 1-H]. \tag{3.11}$$

The FOC w.r.t. α then reads as

$$Eu_1\phi = \Phi Eu_1, \tag{3.12}$$

from which it is immediately clear that $\phi^* = \Phi$ [see (3.5)]. Since the insurance premium Φ is observable and parametric to the individual consumer, his overall labour supply H is now fully conditional on market data and tax parameters, i.e. $(1\text{-}t)w_L$, Φ, and T. Viz

$$H = H[\omega_L, -T-\Phi]. \tag{3.13}$$

The consumer's problem has now been broken down into two stages. In a *first stage*, he decides on how much leisure to enjoy by solving the standard problem

$$Max_H\ u[(1-t)w_L H-T-\Phi,\ 1-H]. \tag{3.14}$$

Conditional on the optimal amount of overall labour supply–as given by (3.13)–he decides in a *second stage* on the allocation of this amount over the official and black labour market as well as on the fine insurance policy to obtain; this is done by solving

$$Max_{B,\alpha}\ Eu[\omega_L H - T + (\omega_B - \omega_L)B - \alpha\phi - (1-\alpha)\Phi,\ 1-H]. \tag{3.15}$$

The second setting which allows for a breakdown of the original problem is where the government does not care about the use of the additional penalty instrument F–let us call this the case of a *proportional penalty scheme*. In these circumstances $\phi^* = 0$, and again the leisure decision is taken only on the basis of market and tax parameters (this time, of course, the second stage exists only of finding the optimal allocation of foregone leisure over both labour markets).

We may therefore formulate the following

Proposition: *Let the* DM-*condition hold. Then either under perfect fine insurance or under a proportional penalty scheme the consumer's decision on leisure can be separated from his labour market "portfolio" decision.*

What is the likelihood for these conditions to obtain? Let me start to point out that the example of insurance against fines is not purely academic. More than half a century ago, Picard & Besson (1938, pp 51-56) spend some time discussing the treatment by French insurance law of insurance against fines in general and fiscal penalties in particular. More recently, the topic has been discussed by Faure & Heine (1991) (FH hereafter) who provide a comparative analysis of West-European legal systems.

From the point of view of insurance law, in most West-European countries, there is no general statutory provision prohibiting the insurance of fines. Such prohibition is generally based on the principles of contract law which prohibits contracts with an illicit cause. Such illegality could for instance stem from a violation of *public order* (FH p 41). Still, it is interesting to observe that in Belgium up till 1981, the insurance company ARAG continued to provide insurance against fines as complementary to a legal aid policy.

Also from the point of view of criminal law, insurance of fines is illegal. The deterrent function of a fine shows only to full advantage through its personal character. Given the public order aspect of criminal law, it is illicit for a person to derogate himself from criminal statutes or to exclude his criminal liability by contract (FH p 43). Moreover, an insurance company insuring a fine could also be held liable as an accessory if it were aware of the intentions of her client. Although this might be very hard to prove in many cases, it is obvious that such awareness prevails in the specific case of an insurance contract against tax evasion penalties.

According to Faure & Heine (1991, p 46), considering insurance against fines as contrary to public order and therefore void, does not guarantee nonexistence of such contracts: only when the insurer would want to sue the insurance company, or vice versa, would a judge acquire the opportunity to consider the validity of the contract. The authors consider the ARAG case as evidence for the difficulty of enforcing prohibition of such contracts.

However, the main message from contract theory is that agents will engage only into contractual agreements with one another when they can be sure that the terms of agreement are enforceable *ex post*. Such enforcement derives either from the existence of a legal system or from sanctioning by the market in case of non-compliance with the terms of agreement.[2] Since the former type of enforcement is absent in the present context, only the perspective of sanctioning of non-compliance by the market could guarantee existence of such contracts. Although such a credible threat might be available for large insurance companies–if such a company would default on the payment of the fine in case of detection, it would loose its credibility and loose its future market share *both* of insurance against fines and of contracts with a legal cause–the presence of regulatory government agencies in this industry is likely to preclude the supply of illegal contracts in the first place. On the other hand, for small independent insurers, such credible threat does not seem available–indeed, why should they not simply "take the (premium-)money and run" ?

From these considerations we may conclude that the institution of an insurance market against fiscal penalties is not likely to exist. *A fortiori* this applies to a market for insurance against the lump sum part of the penalty.

This leaves us with the second institutional sufficiency condition: a proportional penalty scheme. From the government's point of view, it would be disadvantageous to

[2] For instance, in a repeated game theoretic framework reputation could guarantee enforcement under certain conditions, as has been stressed by the tacit collusion literature.

restrict itself to a proportional penalty scheme since with the help of the additional instrument it could never do worse and in many cases even do better–whatever its objectives may be. The only circumstance under which the government could not perform strictly better using the additional instrument, is precisely the existence of a perfect fine insurance market. The reason is that the insurance market would override the efforts of the government to implement a certain policy–*in casu* the imputation of a higher lump sum tax on evaders.[3] But, as I have just argued, these conditions for such an insurance market to work are not likely to go through. One is therefore led to conclude that labour market decisions are only separable under the proportional penalty scheme operated by a shortsighted government.[4] This is what I will assume in the remainder of the paper.

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[3] The argument is similar to the one used to restrict indirect taxes to be linear on commodities which are easily retradeable. See Guesnerie (1981) and Hammond (1987).

[4] The conditions discussed were sufficient but not necessary. For instance, Drèze & Modigliani (1969, p 64) specify additional restrictions that might restore the separability result when perfect insurance markets do not exist. In terms of the present model, one of these conditions would require stochastic independence between the lump sum fine and the marginal penalty rate; this does not seem likely to occur in practice.

4. Preference Structure under the Drèze-Modigliani Assumption

Before moving on to the comparative statics results under DM, it is useful to get a better understanding of the preference structure. For this purpose, I look at the behaviour of two distinct "demand price" variables: the marginal willingness to pay for leisure, and the marginal willingness to exchange risks. In particular, I will pay attention to the cross relations between these two variables: how risk taking affects the demand price for leisure; and vice versa, how leisure influences the willingness to exchange consumption opportunities between different states of nature. The parameters that govern both relations determine the extent to which each of the two stages of the tax evasion problem can be studied in isolation.

The first question has already been addressed in the previous section. It will be briefly picked up in the next subsection for a graphical interpretation. Section 4.2 deals with the behaviour of the willingness to bear risks and introduces the notions of risk complementarity/substitutability with leisure. In Section 4.3, I give necessary and sufficient conditions for risk independence with leisure, both in terms of the preference ordering and in terms of the von Neumann-Morgenstern utility function. Finally, Section 4.4 assesses frequently used labour supply models on their implicit assumption w.r.t. the behaviour of the two demand prices.

4.1 A Graphical Presentation of the Tax Evasion Problem.

Ex ante expected utility is generated by the amount of leisure enjoyed, $1\text{-}H$, and the final consumption in both states of nature, c_1 and c_2 [see (2.3)]. Let me denote this utility level by

$$v(c_1,c_2,H) =_{def} (1-\pi)u(c_1,1-H)+\pi u(c_2,1-H). \tag{4.1}$$

Consider the double diagram in Fig 1, which is taken from Cowell (1985). In the left hand panel, I have drawn a von Neumann-Morgenstern indifference curve corresponding to the utility level $\bar{u}$. The right hand panel depicts the field of indifference curves *conditionally* on a given amount of foregone leisure H. Any bundle on the 45° line corresponds to a case of complete compliance (or absolute honesty in the self-declaration version of the model). Since $v(c_1,c_2,H)=u(\bar{c},H)$ for $c_1=c_2=\bar{c}$, the indifference curve passing through the point $(\bar{c},\bar{c})$ corresponds to the utility level generated by the bundle $(\bar{c},H)$. Cowell (1985, p 23) coins the indifference curves in the left hand quadrant *honestly equivalent*.

The import of the DM assumption is that it makes these honestly equivalent indifference curves suitable for determining the optimal choice of leisure, whatever be the risk taking behaviour in the right hand panel. Since the marginal rate of substitution between consumption and leisure, $u_2/u_1\ (c,\ell)$ is linear in consumption, it is true that

$$\frac{u_2}{u_1}(c,\ell) = \frac{u_2}{u_1}(\bar{c},\ell)+\frac{\partial(\frac{u_2}{u_1})}{\partial c}(c-\bar{c}). \tag{4.2}$$

Performing now the same kind of manipulations on eq (4.2) as in the proof of Theorem 1, it is possible to show that

$$\frac{Eu_2(c,\ell)}{Eu_1(c,\ell)} = \frac{u_2}{u_1}(\bar{c},\ell)+\frac{\partial(\frac{u_2}{u_1})}{\partial c}(\bar{c},\ell)\ [\frac{Eu_1 c}{Eu_1}-\bar{c}]. \tag{4.3}$$

Under the proper institutional conditions, we know that the optimizing behaviour of the consumer will make the term in square brackets vanish. Therefore, risk taking by participation on the black labour market does not drive a wedge between the slope of the honesty equivalent indifference curve and the ratio of expected marginal utilities. Instead of equating this ratio to the net wage rate on the official market–as the first expression in (2.9) prescribes–the consumer might as well look for a tangency point of the highest possible honesty equivalent indifference with the certainty equivalent budget line [like point A in the left hand panel of Fig 1bis]. In this sense, risk taking on the black labour market does not interfere with the choice on the labour/leisure margin; this can take place *as if* there was no uncertainty at all.

The second expression in (2.9) can be rewritten as

$$\frac{1-\pi}{\pi}\frac{u_1(c_1,1-H)}{u_1(c_2,1-H)} = \frac{\omega_L-(1-f)w_B}{w_B-\omega_L}, \tag{4.4}$$

which is the tangency condition in the (c_1,c_2) space: the marginal rate of substitution between consumption in the two states of the world should coincide with the marginal rate of transforming c_2 into c_1 via the two labour markets and the tax administration. The equilibrium at this risk/no risk margin is represented in right hand panel of Fig 1bis by point B. The equation for the trading line *EE'* is given by

$$c_2 = -\frac{\omega_L-(1-f)w_B}{w_B-\omega_L}c_1+\frac{fw_B}{w_B-\omega_L}(\omega_L H-T), \quad \omega_L H-T \le c_1 \le w_B H-T. \tag{4.5}$$

From these last two expression, it becomes clear that any reallocation on the labour/leisure margin will have a double impact on risk taking behaviour. First, there is the effect on the trading possibilities curve. This is a mere income effect, and under reasonable assumptions w.r.t. risk aversion, it will be possible to sign this effect--see Section 5. But in addition, this reallocation will bear upon willingness to substitute consumption between the two states of nature. In the next subsection, I will inquire into the nature of this second effect, and look for conditions under which it will vanish.

4.2 Risk Complementarity, Risk Substitutability and Risk Independence.

Let us now focus attention on the slope of the indifference curves in the right hand panel. Define the marginal rate of substitution between c_2 and c_1, *conditional* on H as

$$MRS(c_1,c_2;H) =_{def} -\left.\frac{dc_2}{dc_1}\right|_{dv=0} = \frac{(1-\pi)}{\pi}\frac{u_1(c_1,1-H)}{u_1(c_2,1-H)}. \tag{4.6}$$

Of course, for any bundle along the 45° line this expression is simply the odds ratio $(1-\pi)/\pi$, which is independent of the amount of foregone leisure. In general, however, this marginal rate of substitution *will* depend on H, and one may calculate the relative change of the slope of an indifference curve when H is increased marginally:

$$\frac{\partial MRS}{\partial H}\frac{1}{MRS} = \frac{u_{12}}{u_1}(c_2,1-H)-\frac{u_{12}}{u_1}(c_1,1-H) = \frac{\partial}{\partial c}\left(\frac{u_{12}}{u_1}\right)(\tilde{c},1-H)(c_2-c_1), \tag{4.7}$$

where $\tilde{c}$ is come convex combination of c_1 and c_2. Since $c_2<c_1$, the sign of the relative change in the MRS will be opposite to that of $\partial(u_{12}/u_1)/\partial c$.

Expression (4.7) is an uncompensated magnitude, since the consumer is made worse of by having to spend more hours at work. One might be interested how the marginal willingness to pay for an additional unit of c_1 (in terms of units of c_2) changes by a marginal increase in hours worked, compensation taking place by a uniform additive increase in c_i, $i=1,2$, dc, say. Total differentiation of (4.1) and restricting $dc_1=dc_2=dc$, yields

$$dc = \frac{Eu_1}{Eu_2}dH. \tag{4.8}$$

The effect of a uniform change dc on MRS is given by

$$\begin{aligned}\frac{\partial MRS}{\partial c} &= \frac{1-\pi}{\pi}\frac{1}{u_1(c_2,\ell)}\left[u_{11}(c_1,\ell)-\frac{u_1(c_1,\ell)}{u_1(c_2,\ell)}u_{11}(c_2,\ell)\right]\\ &= MRS\ [R_a(c_2,\ell)-R_a(c_1,\ell)]\\ &= MRS\ R_a'(\hat{c},\ell)\ (c_2-c_1).\end{aligned} \tag{4.9}$$

Here $R_a(\cdot) =_{def} -u_{11}/u_1$. As shown by Sandmo (1969), this is the proper risk aversion function for *temporal* uncertain prospects; it will be approximately twice the risk premium per unit of infinitesimal variance. R_a' is a shorthand for $\partial R_a/\partial c$ and $\hat{c}$ is again some convex combination of c_1 and c_2.[1]

Gathering results, the compensated relative change in the marginal willingness to pay for c_1 may be written as

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[1] Alternatively, one could compensate the consumer by adjusting consumption in both states by the same *relative* amount, in which case the coefficient of *relative* risk aversion would have shown up in expression (4.9).

$$\frac{\partial MRS}{\partial H}\frac{1}{MRS}\bigg|_{dc_1=\frac{Eu_1}{Eu_2}dH} = [\frac{\partial}{\partial c}(\frac{u_{12}}{u_1})(\tilde{c},\ell)+R_a'(\hat{c},\ell)\frac{Eu_2}{Eu_1}](c_2-c_1). \tag{4.10}$$

Expression (4.10) is a *cross* Antonelli effect, and bearing in mind that the utility function has three arguments (c_1, c_2, and ℓ), it is of no surprise that no sign restriction applies.

These effects may be illustrated geometrically. Consider first the diagram in Fig 2. The risky bundle $(c_1,c_2;H)$ in the right hand quadrant corresponds to the honesty equivalent bundle $(\tilde{c},H)$ in the left hand quadrant. A small compensated increase in H to H' will correspond to a move from A to B along the honesty equivalent indifference curve. In the right hand quadrant, this corresponds to a move from C, parallel to the 45° line to point D on the indifference curve containing $(\tilde{c}',\tilde{c}')$. Whether the indifference curve at D is steeper than the one through C depends on the sign of expression (4.10). Clearly, with additive utility ($u_{12}=0$), the change in slope will have the sign of R_a'; with constant absolute risk aversion, there is no change of slope.

Consider now a small *un*compensated increase in H. Figure 3a illustrates. The effect in the left hand quadrant is a small horizontal shift to point B on a lower indifference curve. The corresponding riskless bundle to point B is $(\tilde{c},\tilde{c})$ as well. Since the slope of an indifference curve should be the odds ratio where it intersects with the 45° line, the new indifference curve in the right hand quadrant corresponding to a utility level of $\bar{u}'$ and a labour supply of H' should be tangent to the old one in point $(\tilde{c},\tilde{c})$.

It is possible to prove the following

Theorem 2: *Let* (c_1,c_2) *and* $(\tilde{c},\tilde{c})$ *both belong to the same indifference curve with utility level* $\bar{u}$ *when labour supply is H. Then*

$$\frac{\partial MRS}{\partial H} \gtrless 0 \Rightarrow v(c_1,c_2;H+dH) \gtrless v(\tilde{c},\tilde{c};H+dH). \tag{4.11}$$

Proof: See Appendix B.

Consequently, when expression (4.7) is negative, point C must lie on a lower indifference curve than point D, and therefore the new field of indifference curves (dashed line) must be more convex to the origin than the old field. Conversely (Fig 3b), when the marginal rate of substitution rises everywhere with an decrease in leisure, the new field of indifference curves must exhibit less curvature than the curves of the old field. Under a constant *MRS*, the new field of indifference curves will coincide with the old field; only a relabelling has taken place.

Expression $\partial \frac{u_{12}}{u_1} / \partial c$ thus controls the influence of the decisions on the labour/leisure margin on the attitude towards risk taking. This becomes even more clear when working out this derivative:

$$\frac{\partial (\frac{u_{12}}{u_1})}{\partial c} = \frac{\partial \log u_1}{\partial \ell \partial c} = \frac{\partial \log u_1}{\partial c \partial \ell} = \frac{\partial (\frac{u_{11}}{u_1})}{\partial \ell} = -\frac{\partial R_a}{\partial \ell} = \frac{\partial R_a}{\partial H}. \tag{4.12}$$

In other words, the expression which controls for the effect of hours worked on the marginal rate of substitution is precisely the effect of foregone leisure on the risk aversion function for temporal risks.

In his analysis of savings decisions under uncertainty, Sandmo (1969, p 592) puts forward a risk premium which is *decreasing* in the uncertain prospect (in his context, future consumption) as an intuitively reasonable assumption. In the present context, this would mean $R_a' < 0$. However, the risk premium, and thus the risk aversion function, also depends upon the amount of leisure enjoyed, a variable which is under deterministic control of the consumer. In the savings decision problem, the deterministic control variable is present consumption. Sandmo assumes "that the risk premium is increasing in [present consumption]. This means that the higher is present consumption, the higher is the consumer's risk premium for gambles on future consumption. It is tempting to call this *risk complementarity* and its opposite (risk premium decreasing in [present consumption]) *risk substitutability*" (p 592). Therefore, I shall associate risk complementarity (substitutability) w.r.t. leisure with $\partial R_a / \partial \ell > 0$ (<0). The case where the risk premium is not affected by leisure ($\partial R_a / \partial \ell = 0$) will be referred to as *risk independence*.

If we may extrapolate this behavioural assumption of risk complementarity to the tax evasion problem, expression (4.12) should bear a negative sign: spending longer hours at work enhances the willingness to bear the delayed risks involved by audit. Graphically, this would mean that what happens in Fig 3b is a better reflection of the consumer's preference ordering than the scenario in Fig 3a.

Even when the extrapolation argument is distrusted, risk complementarity with leisure may be defended as a sufficient condition for a distinct behavioural assumption. Suppose the consumer has access only to the official labour market, but experiences some *exogenous* uncertainty w.r.t. the lump sum tax he will have to pay (or w.r.t. some other non-labour income he will receive). In that case, his (random) budget constraint may be formulated as

$$c = \omega_L H - \varphi. \tag{4.13}$$

Conditional on a realization of ϕ, supplying one extra hour of labour yields ω_L units of consumption: $\partial c/\partial H = \omega_L$. Consider now the total effect of this marginal reallocation of time on the risk aversion function $R_a(c,\ell)$; this is given by

$$\frac{dR_a}{dH} = \frac{\partial R_a}{\partial c}\omega_L + \frac{\partial R_a}{\partial H}. \tag{4.14}$$

When this expression is negative, the consumer is said to exhibit *endogenously diminishing absolute risk aversion*. Let this be the case. It can then be shown that increased uncertainty about the random lump sum tax–defined as a stretching of the distribution of ϕ around a constant mean–will incite the consumer to work longer hours (see Sandmo, 1970, and Block and Heineke, 1973). The reason is that the consumer uses the labour market to hedge against the uncertainty. Although not a necessary condition, it is clear from (4.14) that risk complementarity with leisure, as defined above, is a sufficient condition for endogenously decreasing risk aversion to obtain.

The question arises whether the DM condition imposes any further restrictions on expressions (4.12). The answer is negative under reasonable assumptions. This can be seen by partially working out (DM):

$$\begin{aligned}\frac{\partial^2}{\partial c^2}\left(\frac{u_2}{u_1}\right) &= \frac{\partial}{\partial c}\left[\frac{\partial}{\partial c}\left(\frac{u_2}{u_1}\right)\right] = \frac{\partial}{\partial c}\left(\frac{u_{21}}{u_1} - \frac{u_2}{u_1}\frac{u_{11}}{u_1}\right) \\ &= \left[\frac{\partial R_a}{\partial H} + \frac{u_2}{u_1}\frac{\partial R_a}{\partial c}\right] + R_a\frac{\partial}{\partial c}\left(\frac{u_2}{u_1}\right) \\ &= \frac{dR_a}{dH} + R_a\frac{\partial}{\partial c}\left(\frac{u_2}{u_1}\right) = 0.\end{aligned} \tag{4.15}$$

Under normality of leisure (U2), condition (DM) guarantees endogenously decreasing risk aversion, *but nothing more*. It does not imply risk complementarity with leisure (except under constant absolute risk aversion, $R_a'=0$), and it certainly does not imply risk independence w.r.t. leisure.

This should not come as a surprise since DM is an *ordinal* property on the preference order, while the risk (in)dependency on leisure is a *cardinal* property. Nevertheless, it is useful of emphasizing this because it means that the separability in the decision process obtained under DM is only a "one way" separability. That is, DM allows us to isolate the choice on the labour/leisure margin from the choice on the risk/no risk margin, but *not* vice versa. Only in the case of risk independence is it legitimate to study risk taking behaviour independently from the first stage–up to the income effect of course.

4.3 Necessary and Sufficient Conditions for Risk Independence.

In view of the strong implication of risk independence–as defined above–it is useful to provide a characterization of this condition, both in terms of preference order and in terms of the von Neumann-Morgenstern utility function. To do so, consider the two simple lottery tickets

$$\begin{aligned} \mathcal{L}^* &= [(c_1^*,\ell_1^*),(c_2^*,\ell_2^*);1-\pi^*], \\ \mathcal{L}^+ &= [(c_1^+,\ell_1^+),(c_2^+,\ell_2^+);1-\pi^+]. \end{aligned} \tag{4.16}$$

For instance, lottery ticket $\mathcal{L}^*$ means that with probability $1-\pi^*$ the consumer will receive the bundle (c_1^*,ℓ_1^*) and with probability π^* he will receive the bundle (c_2^*,ℓ_2^*); and likewise for $\mathcal{L}^+$. We then call $\mathcal{L}^*$ and $\mathcal{L}^+$ a pair of *c-random lottery tickets* when $\pi^*=\pi^+=½$, and $\ell_1^*=\ell_2^*=\ell_1^+=\ell_2^+=\ell^\circ$, where ℓ° denotes the common level of leisure in all four bundles. In a similar way, we could define a ℓ-random lottery ticket.

Consider the following restriction on the preference ordering:

Weak Risk Independence Axiom : *An individual's preference between the two c-random lottery tickets in a given pair is independent of the level of ℓ°, for all pairs of c-random lottery tickets.*

This risk independence axiom refers to pairs of lotteries of which the bundles differ only in the consumption part, and it requires that the consumer's preference among any two such lotteries is independent of the common level of leisure.

The following theorem now establishes the equivalence between risk independence as defined on the basis of the risk aversion function, the weak risk independence axiom, and a particular functional form of the von Neumann-Morgenstern utility function.

Theorem 3 : (i) *A tax evader's preferences satisfy the weak risk independence axiom if and only if* (ii) *his risk aversion function is independent of leisure, if and only if* (iii) *his von Neumann-Morgenstern utility function is of the semi additive form* $u(c,\ell)=f(\ell)g(c)+h(\ell)$.

Proof:

To prove the theorem, it is sufficient to show that (i)⇒(ii), (ii)⇒(iii), and (iii)⇒(i).

- (i)⇒(ii) (based on Pollak, 1973):

Suppose that for the consumer is indifferent between the c-random lottery ticket $[\frac{1}{2},(c_1,\ell^\circ),(c_2,\ell^\circ)]$ and the sure prospect $[\frac{1}{2},(c_s,\ell^\circ),(c_s,\ell^\circ)]$. Then the weak independence axiom implies that

$$\tfrac{1}{2}u(c_1,\ell)+\tfrac{1}{2}u(c_2,\ell) = u(c_s,\ell),\ \forall\ \ell. \tag{4.17}$$

Define now c_s implicitly as follows

$$\tfrac{1}{2}u(c_1,\ell)+\tfrac{1}{2}u(c_2,\ell) = u(F(c_1,c_2,\ell),\ell),\ \forall\ \ell. \tag{4.18}$$

By the independence axiom, the function $F(\cdot)$ is independent of ℓ; thus $F(c_1,c_2,\ell)=f(c_1,c_2)$. Equation (4.17) is now an identity in c_1,c_2 and ℓ. Whence,

$$\begin{aligned} \tfrac{1}{2}u_1(c_1,\ell) &= u_1[f(c_1,c_2),\ell]f_1(c_1,c_2), \\ \tfrac{1}{2}u_1(c_2,\ell) &= u_1[f(c_1,c_2),\ell]f_2(c_1,c_2). \end{aligned} \quad \forall c_1,c_2,\ell \tag{4.19}$$

Taking the ratio of both expressions, rearranging, and taking logarithms, one obtains

$$\log u_1(c_1,\ell) = \log u_1(c_2,\ell)+\log\frac{f_1(c_1,c_2)}{f_2(c_1,c_2)}. \tag{4.20}$$

Therefore,

$$\frac{\partial R_a(c_1,\ell)}{\partial \ell} = \frac{\partial^2 \log u_1(c_1,\ell)}{\partial c_1 \partial \ell} = 0\ ,\ \ \forall\ c_1,\ell. \tag{4.21}$$

This proves the first part.

- (ii)→(iii):

Consider the level of leisure ℓ°. Since the absolute risk aversion function is independent of the amount of leisure enjoyed, the following identity must hold:

$$\frac{\partial \log u_1(c,\ell)}{\partial c} = \frac{\partial \log u_1(c,\ell^{\circ})}{\partial c}, \quad \forall\; c,\ell. \tag{4.22}$$

Integrating both sides of the equation yields

$$\begin{aligned} \int_a^c \frac{\partial \log u_1(\gamma,\ell)}{\partial \gamma} d\gamma &= \int_a^c \frac{\partial \log u_1(\gamma,\ell^{\circ})}{\partial \gamma} d\gamma, \quad \forall\; c,\ell, \\ &\Updownarrow \\ \log u_1(c,\ell) + F(\ell) &= \log u_1(c,\ell^{\circ}) + F(\ell^{\circ}), \quad \forall\; c,\ell, \\ &\Updownarrow \\ u_1(c,\ell)\exp[F(\ell)] &= u_1(c,\ell^{\circ})\exp[F(\ell^{\circ})], \quad \forall\; c,\ell, \end{aligned} \tag{4.23}$$

where $F(\ell) =_{\text{def}} -\log u_1(a,\ell)$. Integrating this last expression again w.r.t. c, one obtains

$$\begin{aligned} \int_a^c u_1(\gamma,\ell)\exp[F(\ell)]d\gamma &= \int_a^c u_1(\gamma,\ell^{\circ})\exp[F(\ell^{\circ})]d\gamma, \\ &\Updownarrow \\ u(c,\ell)\exp[F(\ell)] + G(\ell) &= u(c,\ell^{\circ})\exp[F(\ell^{\circ})] + G(\ell^{\circ}). \end{aligned} \tag{4.24}$$

Here, $G(\ell) =_{\text{def}} -u(a,\ell)\exp[F(\ell)]$.

Rearranging then results in

$$\begin{aligned} u(c,\ell) &= u(c,\ell^{\circ})\exp[(F(\ell^{\circ}) - F(\ell)] + [G(\ell^{\circ}) - G(\ell)]\exp[-F(\ell)], \quad \text{or} \\ &= f(\ell)g(c) + h(\ell), \end{aligned} \tag{4.25}$$

where

$$\begin{aligned} g(c) &=_{def} u(c,\ell^{\circ}), \\ f(\ell) &=_{def} \exp[F(\ell^{\circ}) - F(\ell)], \quad \text{and} \\ h(\ell) &=_{def} [G(\ell^{\circ}) - G(\ell)]\exp[-F(\ell)]. \end{aligned} \tag{4.26}$$

This proves the second part.

- (iii)→(i):

This is easily verified by evaluating the expected utility difference between the lottery tickets $\mathcal{L}^*$ and $\mathcal{L}^+$ by means of the just derived von Neumann-Morgenstern utility function; the sign of this difference will not be influenced by the common value chosen for ℓ. QED

Pollak (1973) originally put forward a stronger version of the axiom, and allowed for more than two dimensions. In my terminology, this stronger axiom would require in addition that the preference among any two ℓ-random lottery tickets is independent of the common level of c. Pollak then shows that with only two dimensions, his independence axiom is equivalent to the weak additivity axiom (Th 1 p 36), which was defined in an earlier paper (Pollak, 1967). There, he showed that such a restriction on the preference order is necessary and sufficient for the von Neumann-Morgenstern utility function to be either of the additive or log additive (that is, additive after taking the logarithm) form.

However, in the present setting, as well as in the variants listed in Table 1, talking of ℓ-random lottery tickets does not make much sense since by definition the problem is to choose from *temporal* uncertain prospects, which require the amount of leisure (or savings) to be decided upon *before* the uncertainty is revealed. The theorem above shows that by casting Pollak's (1973) independence axiom in a temporal setting, a larger class of von Neumann-Morgenstern utility functions is allowed for.

It is useful at this stage to contrast this class of functions with the ones satisfying the DM condition. Drèze & Modigliani (1972, Appendix B) showed that DM is a necessary and sufficient condition for the utility to take the form of

$$u(c,\ell) = U[F(\ell)c+H(\ell)], \quad U' > 0,\ F > 0,\ F',H' \geq 0. \tag{4.27}$$

In their words, "[condition DM] is the *ordinal* property of $u(c,\ell)$ that is *necessary* for risk neutrality in terms of [c], and *sufficient* for such neutrality to obtain under a monotonic transformation of $u(\cdot)$" (p 196).

A comparison of both classes learns that for every DM utility function the same monotonous transformation $U^{-1}(\cdot)$ which will bring about risk neutrality in terms of c will also establish risk independence. However, when we are interested in the second stage allocation process, such transformations are not allowed because they affect the cardinalization of the utility function. Two subclasses of DM utility functions that are also risk independent are the ones where either $U(\cdot)$ is a linear function or $H(\cdot)\equiv 0$ and $U(\cdot)$ is homogenous of degree k. In this last case, the DM utility function can be rewritten as

$$u(c,l) = U[F(\ell)]c^k, \tag{4.28}$$

which is a function satisfying risk independence [with $f(\ell)=U[F(\ell)]$, $g(c)=c^k$, and $h(\ell)=0$].

Andersen (1979), analyzing tax evasion under elastic labour supply, works with an additively separable von Neumann-Morgestern utility function. He recognizes additivity as a strong assumption, but finds comfort in its frequent use in intertemporal allocation problems. The analysis above throws light on what additivity can and cannot do. While under additivity, one can present a clear picture of the reallocations on the risk/no risk margin for *exogenous* perturbations in the amount of leisure (the income effect), this is no longer true for most endogenous variations. The reason is that risk taking itself will bear on the direction and the magnitude of these variations because generally DM will not be satisfied.

4.4 Parameters Controlling the Choice Among Temporal Uncertain Prospects in Frequently Used Labour Supply Models.

In Appendix C, I have calculated both the DM and the risk complementarity coefficients implied by some frequently used preference orderings in empirical labour supply models. In general, sign and value of the risk complementarity coefficient will hinge on the chosen cardinalization of the utility function. In order to avoid a multitude of possibilities, the calculations were carried out for the most common ordinal representation, *as if* this representation is also the proper cardinalization of the preference order over uncertain prospects. This should be kept firmly in mind when interpreting the results which are summarized in Table 2 below. For details about the functional forms, refer to Appendix C.

Table 2: Values for parameters controlling risk taking behaviour in frequently used labour supply models.

	$R_a' =_{def} \frac{\partial R_a}{\partial c}$	$\frac{\partial^2}{\partial c^2}(\frac{u_2}{u_1})$	$\frac{\partial R_a}{\partial H}$
Linear Labour Supply Model (Hausman, 1981)	0	0	$< 0 \leftrightarrow (U2)$
Linear Expenditure System	<0	0	0
CES model (σ=elasticity of substitution)	$\underset{>}{\leq} 0 \Leftrightarrow \sigma \underset{<}{\geq} \frac{\delta}{1+\delta}$ (a)	$\underset{>}{\leq} 0 \Leftrightarrow \sigma \underset{<}{\geq} 1$	$\underset{>}{\leq} 0 \leftrightarrow \sigma \underset{<}{\geq} 1$

(a): This is not a closed form expression since δ itself depends on σ; see Appendix C.

The preferences supporting Hausman's (1981) linear labour supply function are characterized by DM. Risk complementarity will obtain if and only if the normality condition (U2) is verified. This model is a nice example where endogenously decreasing absolute risk aversion is generated by risk complementarity, the genuine absolute risk aversion being constant. A consumer endowed with Hausman's preferences will only experience a one way separability.

No separability will be encountered under CES preferences (except in the Cobb-Douglas case, which is also nested in the LES). It can be shown that CES preferences will exhibit endogenously decreasing absolute risk aversion if and only if $\sigma > \delta$ [where $0 < \delta < 1$, $\lim_{\sigma \to \infty} \delta = \alpha$]. Although for values of σ in the interval $[\delta/(1+\delta), \delta]$, absolute risk aversion in the strict sense is still decreasing, risk substitutability w.r.t. leisure (which is present for $\sigma < 1$) has become too strong to generate endogenously decreasing absolute risk aversion.[2]

Of the three models listed in Table 2, the LES is the only one to exhibit both DM and risk independence with leisure. This means that a double separability will be present in the consumer's decision process. One the one hand, DM guarantees that the labour/leisure choice will not be influenced by any risk taking on the black labour

[2] From empirical evidence on labour supply, it turns out that the elasticity of substitution lies in ranges which are critical both for endogenously decreasing risk aversion, and for DM and risk complementarity/substitutability. For instance, Zabalza (1983), estimating a CES model of labour supply for married woman using the UK's 1974 General Household Survey, arrives at a value for σ of 1.25 (std error .09). Ashworth & Ulph (1981), also using British data but only on working women, obtain a much smaller value (σ=.44), while Wales & Woodland (1979), using the Michigan PSID, estimate σ at .91.

market. On the other hand, the only impact of the labour/leisure decision on the labour "portfolio" allocation will be through the position of the budget curve in the (c_1,c_2) space; this decision will *not* affect the field of indifference curves in this space. In this sense, the link between decisions on both margins is eliminated in both directions. This will have important implications for the comparative statics of the model, as will be shown in the next section.

5. Comparative Statics under DM preferences and a Proportional Penalty Scheme.

In this section I will calculate the comparative statics effects when the DM condition applies and the (shortsighted) government operates a proportional penalty scheme (i.e. $F=0$). In section 3, I already mentioned that without this institutional assumption, it is very unlikely that the certainty equivalent fine ϕ^* can treated as parametric and therefore a breakdown of the whole problem in two stages to take place. Under such circumstances, a comparative statics exercise will produce hardly any definite result; this was shown in Appendix A. What I want to show in this section is that even under a proportional penalty scheme, one is not likely to get many clearcut results which are robust at the same time.

5.1 Stage One

Let me start with the first stage problem. The solution to problem (3.14) will result in the labour supply function (3.13), both with $\Phi=0$. The FOC w.r.t. H is $\omega_L u_1 - u_2 = 0$ and the income and compensated wage effect are given by

$$\frac{\partial H}{\partial T} = -\frac{u_1}{\Delta}\frac{\partial(\frac{u_2}{u_1})}{\partial c}, \tag{5.1}$$

$$\left.\frac{\partial \check{H}}{\partial \omega_L}\right|_{dT=Hd\omega_L} = -\frac{u_1}{\Delta}, \tag{5.2}$$

where $\Delta =_{\text{def}} u_{22} - 2\omega_L u_{21} + \omega_L^2 u_{11}$ is the partial derivative of the FOC w.r.t. H, which is negative by assumption (U1). The inverted hat denotes a utility compensated effect *in the absence of uncertainty*: only then will the indicated increase in T exactly compensate the consumer for an increase in ω_L; otherwise it will make him worse off. This compensated wage effect is unambiguously positive, while the income effect (5.1) is negative under the normality assumption (U2). Of course,

$$\frac{\partial H}{\partial \omega_L} = \frac{\partial \check{H}}{\partial \omega_L} - \frac{\partial H}{\partial T} H. \tag{5.3}$$

5.2 Stage Two

Conditional on the overall labour supply H, the consumer solves the labour market portfolio problem (3.16) [with $\Phi = \phi = 0$]. The FOC is the second condition in (2.9), viz $z =_{def} Eu_1(\omega_B\text{-}\omega_L) = 0$; this may be solved for B to obtain the conditional black market labour supply function

$$B = B(\omega_L, w_B, T, f, p; H), \tag{5.4}$$

its partial derivatives which we are interested in. By concavity of the utility function, $D =_{def} z_B < 0$.

- The effect of an decrease in T.

$$\begin{aligned} \frac{\partial B}{\partial(-T)} &= \frac{z_T}{z_B} = -\frac{1}{D} Eu_{11}(\omega_B - \omega_L) \\ &= \frac{1}{D} Eu_1(\omega_B - \omega_L) R_a. \end{aligned} \tag{5.5}$$

In view of the FOC $Eu_1(\omega_B\text{-}\omega_L) = 0$, decreasing absolute risk aversion will give more weight to the outcome $\tau = f$ than to the outcome $\tau = 0$ so that the expectations term in (5.5) is negative; therefore $\frac{\partial B}{\partial(-T)} > 0$.

- The effect of a decrease in F. Although I am assuming a proportional penalty scheme ($F \equiv 0$), it is useful for later purposes to have an expression for this effect:

$$\frac{\partial B}{\partial(-F)} = \frac{z_F}{z_B} = -\frac{1}{D} \pi u_{11}(c_2, \ell)[(1-f)w_B - \omega_L]. \tag{5.6}$$

By concavity, this effect will have the opposite sign of the net wage differential in square brackets, and will thus be positive; alternatively, any increase in the lump sum fine will cause a reduction in black market activity.

- The effect of an increase in ω_L .

$$
\begin{aligned}
\frac{\partial B}{\partial \omega_L} &= -\frac{z_{\omega_L}}{z_B} = \frac{1}{D}[Eu_1-(H-B)Eu_{11}(\omega_B-\omega_L)] \\
&= \frac{1}{D}[Eu_1+(H-B)Eu_1R_a(\omega_B-\omega_L)] \\
&= \frac{1}{D}Eu_1+L\frac{\partial B}{\partial(-T)} \ .
\end{aligned}
\tag{5.7}
$$

The first RHS term is the pure substitution effect, which is negative. The second term on the RHS accounts for the income effect; under decreasing absolute risk aversion, this effect is positive, leaving the sign of the overall effect undetermined.

- The effect of an increase in w_B .

$$
\begin{aligned}
\frac{\partial B}{\partial w_B} &= -\frac{z_{w_B}}{z_B} = -\frac{1}{D}[E(1-\tau)u_1+BEu_{11}(1-\tau)(\omega_B-\omega_L)] \\
&= -\frac{1}{D}E(1-\tau)u_1+\frac{1}{D}BEu_1R_a(1-\tau)(\omega_B-\omega_L).
\end{aligned}
\tag{5.8}
$$

The increase in the lump sum tax T, necessary to keep the consumer on the same indifference curve after the increase in w_B is given by

$$
dT = \frac{Eu_1(1-\tau)B}{Eu_1}dw_B = \frac{\omega_L B}{w_B}dw_B,
\tag{5.9}
$$

where the second equality follows from the FOC. Therefore, the uncompensated effect on B of an increase in w_B may be decomposed as follows

$$\frac{\partial B}{\partial w_B} = \left.\frac{\partial \hat{B}}{\partial w_B}\right|_{dT=\frac{\omega_L}{w_B}Bdw_B} + \frac{\partial B}{\partial(-T)}\frac{\partial T}{\partial w_B}$$

$$= \left.\frac{\partial \hat{B}}{\partial w_B}\right|_{dT=\frac{\omega_L}{w_B}Bdw_B} + \frac{\partial B}{\partial(-T)}\frac{\omega_L}{w_B}B \; . \qquad (5.10)$$

Cowell (1981, Th 2, part 1) argues that the first RHS term is a conventional substitution effect which must be strictly positive; the second term being positive under decreasing absolute risk aversion, the total (uncompensated) effect of a black market wage rise will be positive. However, there is a flaw in this argument. First of all, from expression (A.13) in Appendix A, it is clear that the sign of the substitution effect obtained by carrying out this particular form of compensation, cannot be predicted from the basic axioms of utility maximization. Moreover, this conclusion continues to hold when restricting ourselves to the second stage comparative statics [cf (A.26)]. Rearranging (5.10) and making use of expressions (5.5) and (5.8), the following expression for this particular substitution effect can be established:

$$\left.\frac{\partial \hat{B}}{\partial w_B}\right|_{dT=\frac{\omega_L}{w_B}Bdw_B} = -\frac{1}{D}[E(1-\tau)u_1 + \frac{B}{w_B}Eu_{11}(\omega_B - \omega_L)^2]. \qquad (5.11)$$

By concavity, the two terms in square brackets take on opposite signs, leaving the sign of the substitution effect indeterminate.

To obtain any definite conclusion w.r.t. the sign of the substitution effect , it is necessary to adjust the consumer's welfare by the following double compensation scheme:

$$dT = Bdw_B, \text{ and } dF = -fBdw_B. \qquad (5.12)$$

The first part of the scheme makes the consumer unconditionally worse off, while the second part makes him better off conditionally on an audit taking place. The need for a lower lump sum fine is that a wage increase on the black market will increase the penalty fw_BB, making the consumer worse off when an audit takes place; this calls for conditional rather than unconditional compensation. Using expressions (5.5), (5.6) and (5.8), it can be shown that the implied substitution effect is of the form

$$\left.\frac{\partial\hat{B}}{\partial w_B}\right|_{\substack{dT=Bdw_B\\ dF=-fBdw_B}} = \frac{\partial B}{\partial w_B}-\frac{\partial B}{\partial(-T)}B+\frac{\partial B}{\partial(-F)}fB$$
$$= -\frac{1}{D}E(1-\tau)u_1. \tag{5.13}$$

From expression (A.11c) in Appendix A we know such a substitution effect must be positive for the unconditional black market labour supply equation and that this result carries over to the conditional labour supply behaviour as well [cf (A.26)]. This is indeed the case since (5.13) is positive. Can we now say something about the *un*compensated wage effect? The answer is negative. Consider the following rearrangement of (5.13):

$$\underset{}{\frac{\partial B}{\partial w_B}} = \underset{+}{\left.\frac{\partial\hat{B}}{\partial w_B}\right|_{\substack{dT=Bdw_B\\ dF=-fBdw_B}}} + \underset{\substack{+\\ R_a'<0}}{\frac{\partial B}{\partial(-T)}B} - \underset{+}{\frac{\partial B}{\partial(-F)}fB} \tag{5.14}$$

Since the last two terms pull into a different direction, the sign of the entire expression is indeterminate.

- The effect of an increase in f.

$$\frac{\partial B}{\partial f} = -\frac{z_f}{z_B} = \frac{1}{D}w_B\pi u_1(c_2,\ell)+w_BB\{\frac{1}{D}\pi u_{11}(c_2,\ell)[(1-f)w_B-\omega_L]\}. \tag{5.15}$$

If a lump sum fine would have been in place, the term in curly brackets would measure the effect on B of an increase in this fine. As shown above, this expression will be negative since the net wage differential in square brackets is–otherwise it would not be optimal to participate a single hour on the official labour market. This effect is premultiplied by the amount w_BB which gives the decrease in the lump sum fine necessary to compensate for the increase in marginal penalty rate f. Therefore, the second term represents the income effect of an increase in f, and bears a negative sign. The first term can than be identified as the substitution effect, which explains its negative sign. In short: $\frac{\partial B}{\partial f} < 0$.

- The effect of an increase in π.

$$\frac{\partial B}{\partial \pi} = -\frac{z_\pi}{z_B} \qquad (5.16)$$
$$= \frac{1}{D}\{u_1(c_1,\ell)(w_B-\omega_L)-u_1(c_2,\ell)[(1-f)w_B-\omega_L]\}.$$

This expression is unambiguously negative because $(1\text{-}f)w_B<\omega_L<w_B$. Whence $\frac{\partial B}{\partial \pi}<0$.

• The effect of a simultaneous increase in w_L and w_B. This effect is interesting in the case of the self-declaration version of the model. It is given by

$$\left.\frac{\partial B}{\partial w_B}\right|_{w_B=w_L} = \frac{\partial B}{\partial w_B}+(1-t)\frac{\partial B}{\partial \omega_L}$$
$$= -\frac{1}{D}\{Eu_1(t-\tau)+Eu_{11}(t-\tau)w_B[(1-t)L+(1-\tau)B]\} \qquad (5.17)$$
$$= -\frac{1}{D}\{0+E(t-\tau)u_{11}(c_1+T)\}$$
$$= \frac{1}{D}Eu_1(t-\tau)R_r+\frac{T}{w_B}\frac{\partial B}{\partial(-T)},$$

where $Eu_1(t\text{-}\tau)=0$ by the FOC and where $R_r =_{\text{def}} -u_{11}c_1/u_1$ denotes the coefficient of relative risk aversion. Under decreasing absolute risk aversion, the second RHS term has the same sign as T. With relative risk aversion increasing in c_1, the outcome $\tau=0$ will be given more weight than the outcome $\tau=f$; this will make the first RHS term negative. Thus, the sign of the whole expression becomes indeterminate, except when T denotes a transfer to the consumer; in this case both effects pull into the same (downward) direction.

Suppose T is indeed negative, and suppose as well that $\partial B/\partial w_B>0$; in other words that the black market labour supply curve is forward bending. Under these conditions, the same assumptions w.r.t. absolute and relative risk aversion will imply that $\partial B/\partial \omega_L$ bears a negative sign (Cowell, 1981, Th 2, part 2). But as I have tried to show above, there are no *a priori* reasons for assuming the first term to be positive, and Cowell's conclusion w.r.t. $\partial B/\partial \omega_L$ in the second part of his Theorem 2 is less general than it appears to be.

• The effect of an increase in H.

$$\frac{\partial B}{\partial H} = -\frac{\partial z_H}{\partial z_B} = \omega_L \frac{\partial B}{\partial(-T)} + \frac{1}{D} E u_{12}(\omega_B - \omega_L)$$
$$= \omega_L \frac{\partial B}{\partial(-T)} + \frac{1}{D} E u_1(\omega_B - \omega_L)(\frac{u_{12}}{u_1}) \qquad (5.18)$$

When the consumer decides to spend more hours at work, the budget curve in the (c_1,c_2) space will shift outward. Under decreasing absolute risk aversion, the consumer will want to take advantage of this opportunity to spend more time on the black labour market. This is the income effect reflected by the first RHS term.

But there is a substitution effect at play as well. This is because a fall in leisure will affect the willingness to bear risks and will cause a change in the curvature of the indifference curves in the (c_1,c_2) space as was shown in Section 4. Using the FOC the second RHS term may also be written as

$$\frac{(1-\pi)}{D}(w_B-\omega_L)u_1(c_1,\ell)[\frac{u_{21}}{u_1}(c_1,\ell)-\frac{u_{21}}{u_1}(c_2,\ell)]$$
$$= \frac{(1-\pi)}{D}(w_B-\omega_L)u_1(c_1,\ell)\ \frac{\partial}{\partial c}(\frac{u_{21}}{u_1})(\tilde{c},\ell)\ (c_1-c_2) \qquad (5.19)$$
$$= \eta\frac{\partial R_a}{\partial \ell} \propto \frac{\partial MRS}{\partial H}\bigg|_{(\tilde{c},\ell)},$$

where

$$\eta =_{def} -\frac{(1-\pi)}{D}(w_B-\omega_L)u_1(c_1,\ell)$$

is a positive number.

For any *given* trading possibilities between c_1 and c_2 a small decrease in leisure will favour a higher (resp. lower) participation in the black labour market if and only if the fall in leisure increases (resp. decreases) the substitution possibilities between consumption in the two states. In the terminology of Section 4, black labour market participation will increase (decrease) if and only if the consumer's preferences exhibit risk complementarity (risk substitutability) w.r.t. leisure. Only in the case of risk neutrality w.r.t. leisure (as for instance in the Linear Expenditure System) will this effect vanish and will the overall effect coincide with an income effect. This is in fact what

Cowell (1981, eq 34) is implicitly assuming.

5.3 Collecting Results.

I now combine the effects taking place in the two stages.

- The effect of a reduction in T.

$$\begin{aligned}\frac{dB}{d(-T)} &= \frac{\partial B}{\partial(-T)} + \frac{\partial B}{\partial H}\frac{\partial H}{\partial(-T)} \\ &= [1+\omega_L\frac{\partial H}{\partial(-T)}]\frac{\partial B}{\partial(-T)} + \eta\frac{\partial R_a}{\partial \ell}\frac{\partial H}{\partial(-T)} \\ &= \underset{\substack{+\\U3}}{\frac{\partial c}{(-T)}}\;\underset{\substack{+\\R_a'<0}}{\frac{\partial B}{\partial(-T)}} + \eta\underset{?}{\frac{\partial R_a}{\partial \ell}}\;\underset{\substack{-\\U2}}{\frac{\partial H}{\partial(-T)}}\;.\end{aligned} \tag{5.20}$$

The second equality follows from the (5.18-19) and rearranging. The term in square brackets is equal to the income effect on consumption under certainty, since $c=\omega_L H\text{-}T$. Under normality of consumption (U3), this term is positive. When there is absence of complementarity between the risk premium and leisure–when the substitution possibilities between c_1 and c_2 do not diminish when leisure is increased–the second RHS term on the last row takes on a positive sign under normality of leisure, and the entire expression becomes positive. Otherwise its sign is ambiguous.

- The effect of an increase in ω_L .

$$\begin{aligned}\frac{dB}{d\omega_L} &= \frac{\partial B}{\partial\omega_L} + \frac{\partial B}{\partial H}\frac{\partial H}{\partial\omega_L} \\ &= \frac{\partial B}{\partial\omega_L} + [\omega_L\frac{\partial B}{\partial(-T)} + \eta\frac{\partial R_a}{\partial \ell}]\frac{\partial H}{\partial\omega_L} \\ &= \underset{-}{\left.\frac{\partial \hat{B}}{\partial\omega_L}\right|_{dT=Ld\omega_L}} + [L+\omega_L\underset{+/-}{\frac{\partial H}{\partial\omega_L}}]\underset{\substack{+\\R_a'<0}}{\frac{\partial B}{\partial(-T)}} + \eta\underset{?}{\frac{\partial R_a}{\partial \ell}}\;\underset{+/-}{\frac{\partial H}{\partial\omega_L}}\;.\end{aligned} \tag{5.21}$$

The first equality follows from the decomposition (5.18-19), and the second equality follows from the Slutsky decomposition in (5.7).

Investigating the last equation, one cannot make any statements about the sign of the overall effect. The first RHS term is negative because of the substitution effect. When the labour supply curve is forward bending, the term in square brackets–and thus the entire second term under decreasing absolute risk aversion–is positive. Further ambiguity is added by the last RHS term whenever preferences do *not* exhibit risk independency with leisure. When the labour supply curve is backward bending, the sign of the square bracket term is indeterminate.

We may proceed via a different route, however. Consider again the second equality. Under $R_a' < 0$, $R_r' > 0$, $T < 0$, and a forward bending labour supply curve on the black market, the first RHS term is negative [cf the discussion above following expression (5.17)]. Risk complementarity or risk independence with leisure guarantee, together with $R_a' < 0$, a positive sign for the term in square brackets. When the overall labour supply curve is backward bending, the entire expression becomes negative. This is essentially Cowell's (1985, p 28) result. But one should be aware that his conditions are only sufficient under a negative lump sum tax, risk independency with leisure, and a forward bending black market labour supply curve. However, it seems hard to reconcile such a forward bending curve with a backward bending overall labour supply function, and whether such behaviour is empirically important is doubtful.

- The effect of a simultaneous increase in w_L and w_B .

$$\begin{aligned}
\left.\frac{dB}{dw_B}\right|_{w_B=w_L} &= \left.\frac{\partial B}{\partial w_B}\right|_{w_B=w_L} + \frac{\partial B}{\partial H}\frac{\partial H}{\partial \omega_L}(1-t) \\
&= \frac{1}{D}Eu_1(t-\tau)R_r + \frac{T}{w_B}\frac{\partial B}{\partial(-T)} + [\omega_L\frac{\partial B}{\partial(-T)} + \eta\frac{\partial R_a}{\partial \ell}]\frac{\partial H}{\partial \omega_L}(1-t) \\
&= \underset{\substack{- \\ R_r'>0}}{\frac{1}{D}Eu_1(t-\tau)R_r} + [\underset{\substack{- \\ \text{grant}}}{\frac{T}{w_B}} + \underset{+/-}{\omega_L\frac{\partial H}{\partial \omega_L}(1-t)}]\underset{\substack{+ \\ R_a'<0}}{\frac{\partial B}{\partial(-T)}} + \eta\underset{?}{\frac{\partial R_a}{\partial \ell}}\underset{+/-}{\frac{\partial H}{\partial \omega_L}(1-t)}\ .
\end{aligned} \tag{5.22}$$

A quick look at the sign restrictions informs that the overall effect on black market participation is indeterminate. Suppose however that the state indifference curves become flatter or remain unaffected when available leisure decreases–in other words that there is absence of risk substitutability with leisure. Assume further that R_a is decreasing, that R_r is increasing, that a lump sum grant ($T<0$) is provided, and that the overall labour supply schedule is backward bending. Then under all these conditions, the black market participation rate of the consumer will fall with an increase of the wage level.

As to the effects of the remaining parameters–w_B (in isolation), f, and π–, these are exactly as derived in the previous subsection since these parameters do not interfere with the first stage of the decision process; their effects therefore continue to hold for the overall problem.

Table 3 below summarizes the comparative statics results for black market activity, with an indication of the required assumptions. Observe how conflicting assumptions need to be made both w.r.t. the influence of leisure on the risk premium and w.r.t. the slope of the overall and black market labour supply schedules, in order to arrive at definite conclusions.

Table 3. Comparative Statics Effects on Black Market Activity: Uncompensated Effects[a]

Rise in	U2	U3	$R_a'<0$	$R_r'>0$	$T\leq 0$	$\frac{\partial B}{\partial w_B}\geq 0$	$\frac{\partial H}{\partial \omega_L}\leq 0$	$\frac{\partial R_a}{\partial \ell}$	Effect on B
$(-T)$	✓	✓	✓					≤ 0	>0
F									<0
ω_L			✓	✓	✓	✓	✓	≥ 0	<0
$w_B \equiv w_L$			✓	✓	✓		✓	≥ 0	<0
w_B						✓			≥ 0
f									<0
π									<0

[a] All signs are conditional on the assumptions of concavity (U1) and DM.

6. Compensated Effects.

In any normative analysis of optimal income taxation and compliance policy, the structure of the tax and penalty scheme will be governed by compensated rather than uncompensated price effects whenever the social planner can make use of a sufficient wide set of lump sum instruments that take care of the compensation. This was clearly shown by Sandmo (1981).

In this section, I want to calculate some compensated effects, by adjusting the lump sum parts of the tax and penalty scheme in very specific way. In particular, I will make use of the same compensation schemes as in Appendix A. This will enable me to see whether the ambiguities obtained there for a general utility function, can be removed in the DM framework.

For any tax and penalty scheme, the expected utility of a consumer is given by

$$\begin{aligned} v &= Eu[\omega_L H - T + (\omega_B - \omega_L)B - \phi,\ 1-H] \\ &= (1-\pi)u[\omega_L H - T + (w_B - \omega_L)B,\ 1-H] + \pi u[\omega_L H - T + ((1-f)w_B - \omega_L)B - F,\ 1-H], \end{aligned} \tag{6.1}$$

where the lump sum fine F has been reintroduced. Nevertheless, I will assume this fine is arbitrary small, such that the separation result continues to apply under DM-preferences.

- Consider first the compensated effect of an increase in the net wage rate, $d\omega_L$. This increase may either stem from an increased wage level on the official labour market, or from a small reduction in the marginal tax rate of the amount $d\omega_L/w_L$. From (6.1), it is easy to see that the increase in the lump sum tax, T, necessary to keep the consumer on the same indifference curve, is given by

$$dT = L d\omega_L \,. \tag{6.2}$$

Therefore, the overall compensated effect on black market activity is given by

$$
\begin{aligned}
\left.\frac{d\hat{B}}{d\omega_L}\right|_{dT=Ld\omega_L} &= \frac{dB}{d\omega_L}-\frac{dB}{d(-T)}L \\
&= \frac{\partial B}{\partial \omega_L}+\frac{\partial B}{\partial H}\frac{\partial H}{\partial \omega_L}-[\frac{\partial B}{\partial(-T)}+\frac{\partial B}{\partial H}\frac{\partial H}{\partial(-T)}]L \\
&= \left.\frac{\partial \hat{B}}{\partial \omega_L}\right|_{dT=Ld\omega_L}+\frac{\partial B}{\partial H}[\left.\frac{\partial \check{H}}{\partial \omega_L}\right|_{dT=Hd\omega_L}+\frac{\partial H}{\partial(-T)}B] \qquad (6.3) \\
&= \underset{-}{\left.\frac{\partial \hat{B}}{\partial \omega_L}\right|_{dT=Ld\omega_L}}+\{\omega_L\underset{\substack{+\\ R_a'<0}}{\frac{\partial B}{\partial(-T)}}+\eta\underset{?}{\frac{\partial R_a}{\partial \ell}}\}[\underset{+}{\left.\frac{\partial \check{H}}{\partial \omega_L}\right|_{dT=Hd\omega_L}}+\underset{\substack{+\\ U2}}{\frac{\partial H}{\partial(-T)}}B]\ .
\end{aligned}
$$

The first/second compensated partial derivative is unambiguously negative/positive–see (5.7) and (5.2), respectively. The normality assumption w.r.t. leisure (U2) ensures that the square brackets term is positive. Under the usual risk aversion assumption, the sign of the term in curly brackets hinges on the way leisure affects the risk premium. Under risk complementarity with leisure, this substitution effect works in the same direction as the income effect and the term in curly brackets is positive, leaving the entire expression unsigned. On the other hand, when a fall in leisure results in a decrease of substitution possibilities, both effects in curly brackets work in opposite directions, and the ambiguity remains. Therefore, we may conclude that even under reasonable risk aversion and normality assumptions, the DM hypothesis does not succeed in signing this cross substitution effect. This should not seem too surprising. Inspecting expression (A.11a) of Appendix A shows that in a general model, the sign of this effect is governed by an off-diagonal term of a (4×4) negative semi-definite matrix. On *a priori* grounds, nothing can be said on the sign of such a term.

- When a small increase in the marginal penalty rate, df, is accompanied by a reduction in the lump sum fine F to the amount $d(\text{-}F)=Bw_B\,df$, the consumer is equally well off as before the reform. Since neither of these two parameters enter the first stage problem of the consumer, the overall compensated effect coincides with the second stage effect. From expression (5.15), it may be readily established this amounts to

$$\left.\frac{d\hat{B}}{df}\right|_{dF=-Bw_B df} = \frac{1}{D}\pi w_B u_1(c_2,\ell), \tag{6.4}$$

which is negative by concavity. This result is hardly surprising since it was established at the more general level in Appendix A [cf eq (A.11b)].

- A small wage increase on the black labour market, dw_B, could be compensated by an increase in the lump sum fine, F, alone. However, this would not produce any clearcut result at all since such a wage increase makes the consumer better of in both states of the world. It could also be accompanied by an increase in the lump sum tax T. This has been studied in Section 5.2, but there [eq (5.11)], I concluded that the resulting substitution effect was ambiguous. In the same section it was shown that a definite (positive) substitution effect could be obtained by the following compensation scheme:

$$\begin{aligned} d(-T) &= -Bdw_B, \text{ and} \\ d(-F) &= fBdw_B \ . \end{aligned} \tag{6.5}$$

The entailed substitution effect was given by expression (5.13) and is unambiguously positive. However, this effect describes the behaviour of the consumer at the second stage of the decision problem, *conditional* on the predetermined level of overall labour supply, H. And although neither the parameters w_B and F influence the decision on how much leisure to enjoy, the compensation taking place via T will, of course. Working out the overall substitution effect, we arrive at

$$\begin{aligned} \left.\frac{d\hat{B}}{dw_B}\right|_{\substack{dT=Bdw_B \\ dF=-fBdw_B}} &= \frac{dB}{dw_B} - \frac{dB}{d(-T)}B + \frac{dB}{d(-F)}fB \\ &= \frac{\partial B}{\partial w_B} - [\frac{\partial B}{\partial(-T)} + \frac{\partial B}{\partial H}\frac{\partial H}{\partial(-T)}]B + \frac{\partial B}{\partial(-F)}fB \\ &= \underset{+}{\left.\frac{\partial \hat{B}}{\partial w_B}\right|_{\substack{dT=Bdw_B \\ dF=-fBdw_B}}} - [\omega_L \underset{\substack{+ \\ R_a'<0}}{\frac{\partial B}{\partial(-T)}} + \eta \underset{?}{\frac{\partial R_a}{\partial \ell}}] \underset{\substack{- \\ U2}}{\frac{\partial H}{\partial(-T)}} B \ . \end{aligned} \tag{6.6}$$

The transition from the first to the second equality is explained by the fact that only the reform in T produces any first stage effects. The first term on the third line stems from expression (5.13), while the term in square brackets makes up the effect $\partial B/\partial H$ [see (5.18-19)].

At first sight, one is inclined to conclude that even this overall substitution effect has no determinate sign. But from (A.11c) in Appendix A, we know that the basic axioms ensure this kind of effect is positive. Under normality of leisure (U2), this means that whenever $\partial B/\partial H$ is negative, the axioms impose a lower bound on the value of this spill-over effect. In particular, it means that under decreasing absolute risk aversion (i.e. when the first term in square brackets is positive), a reduction in leisure can never lower the substitution possibilities between c_1 and c_2 "too much"–alternatively, that risk substitutability with leisure can never be "too high". It is an open question to me whether the negative value for $\partial R_a/\partial \ell$ which ensures (5.20) to be positive and (6.3) to be negative is precluded by this lower bound.

• When the consumer earns the same hourly wage rate on both markets, $w_L = w_B$, real wage growth makes him better of in both states of nature. Let us tax away the higher wage income by raising T: $dT = [(1\text{-}t)L + B]dw_B$. In his capacity as a tax evader, the consumer will be made worse off because the penalty he is required to pay when caught, fw_BB, has gone up as well. To compensate for this loss in disposable income in the audit state, the government lowers the lump sum fine: $dF = \text{-}fBdw_B$. By carrying out this operation, the *ex ante* expected utility level of the consumer is at the same level before wage growth occurred. The substituting behaviour w.r.t. the black market activity may then be decomposed as follows:

$$\left.\frac{d\hat{B}}{dw}\right|_{\substack{dT=[(1-t)L+B]dw \\ dF=-fbdw}} = \frac{dB}{dw_B} + (1-t)\frac{dB}{d\omega_L} - \frac{dB}{d(-T)}[(1-t)L+B] + \frac{dB}{d(-F)}fB$$

$$= \underset{+}{\left.\frac{d\hat{B}}{dw_B}\right|_{\substack{dT=Bdw_B \\ dF=-fBdw_B}}} + (1-t)\underset{?}{\left.\frac{d\hat{B}}{d\omega_L}\right|_{dT=Ld\omega_L}} . \tag{6.7}$$

Above, it was established that the first substitution effect is unambiguously positive, but that the second term can take either sign, making it impossible to make any definite conclusions about the adjustment of B. This confirms the more general finding in Appendix A [(A.15)]. Little additional insight can be gained by decomposing the two substitution effects along the lines of (6.3) and (6.6).

7. Conclusion.

In this paper, I have tried to analyze the behaviour of a tax evader whose labour supply curve is not perfectly inelastic. Following Drèze & Modigliani (1972) and Cowell (1981, 1985) an ordinal condition was imposed on preferences. Combined with a restriction on institutions, this condition is sufficient to guarantee functional separability of the tax evasion problem: in a first stage, the agent decides on how much leisure to enjoy independently of any risk taking; in a second stage, the risk taking is decided upon by allocating the amount of forgone leisure over the official and black labour market.

It turns out, however, that the institutional restriction is not likely to obtain: a perfect insurance market against lump sum fine risk is not likely to exist, neither would a welfare maximizing government want to abstain from the additional lump sum fine instrument. The most plausible sufficiency condition to establish functional separability is a proportional penalty scheme operated by a shortsighted government. This was the stance taken in the rest of the paper.

For this particular setting, I engaged in calculating the uncompensated and compensated effects on black market activity from perturbations in tax and compliance parameters, as well as wage growth. As to uncompensated effects, it turns out that the only easily predictable effects stem from the compliance parameters: both the marginal penalty rate and the audit probability temper black market activity. As regards the other effects, their direction is blurred by the effect leisure exerts on the risk premium. But even assuming this effect away–which was shown to be equivalent to invoking the weak risk independence axiom–did not lead to a clearcut result in the case of an isolated wage rise on the black market. The reason is that the non-trivial schemes of compensation, necessary to obtain an unambiguous substitution effect, nullify the predictive power of the latter in explaining the direction of the uncompensated effect.

Turning attention to compensated effects, it appears that the functional separability does not enhance very much the insight obtained from a framework with general preferences.

Making any definite conclusions w.r.t. black market activity under elastic labour supply thus turns out to be much harder than we could have guessed from the literature on consumption decisions under uncertainty–despite the analytical equivalence between both type of models. The reason has to be sought in the difference in questions these models attempt to address. While the models of Drèze & Modigliani (1972) and Sandmo (1969, 1970) are mainly concerned with the effects of uncertainty on present consumption (and thus on total savings), the models of tax evasion go further in that they seek to predict responses in one component of the equivalent to total savings (foregone leisure).

These are rather unfortunate results, both from an incidence as well as a normative point of view. At the same time, they emphasize the need for empirical evidence on tax evading behaviour: both microeconometric research on carefully designed household surveys (of which the anonymity seems sufficiently credible), and the organization of experiments[1] may produce such evidence. This would allow us to make direct assumptions on certain effects rather than decomposing them into parts on which we believe other models might shed light. Just as in many models under certainty the normality of a particular good is regarded as a maintained hypothesis because it is supported by sufficient empirical evidence, we may for instance attempt to make direct assumptions on the income effect on black market activity and see how far this would carry us.

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1 See for instance the recent paper by Alm, McClelland & Schulze (1992) which reports on experimental results w.r.t. tax compliance.

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Appendix A

In this appendix, I will derive the comparative statics effects for a general expected utility function, not necessarily satisfying the Drèze-Modigliani condition. The optimization problem takes the form

$$\max_{c_1,c_2,L,B} \; v(c_1,c_2,L+B) =_{def} (1-\pi)u(c_1,1-L-B)+\pi u(c_2,1-L-B)$$

$$\text{s.t.} \quad c_1 = \omega_L L+w_B B-T \qquad (\lambda_1) \qquad \text{(A.1)}$$

$$c_2 = \omega_L L+(1-f)w_B B-T-F \qquad (\lambda_2).$$

where λ_i denotes the Lagrange multiplier with the budget constraint in state i, and ω_L is the net wage rate on the official labour market. In fact, formulating the problem in this way, makes it reminiscent to the coupon rationing problem studied by Diamond & Yaari (1972) who apply the framework to the savings *cum* portfolio problem.

Define the following vectors and matrices:

$$q' =_{def} (c_1,c_2,-L,-B), \quad (m_1,m_2) =_{def} (-T,-T-F)$$

$$I_2 =_{def} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad W =_{def} \begin{pmatrix} w^{1'} \\ w^{2'} \end{pmatrix} = \begin{pmatrix} \omega_L & w_B \\ \omega_L & (1-f)w_B \end{pmatrix}, \quad P =_{def} \begin{pmatrix} p^{1'} \\ p^{2'} \end{pmatrix} =_{def} (I_2,W). \qquad \text{(A.2)}$$

I.e. p^i is the 'price system' w.r.t. the 'consumption bundle' q and m_i is autonomous income when state of nature i obtains.

Problem (A.1) can then be summarised as

$$\max_q \quad v(q)$$

$$\text{s.t.} \quad Pq = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}, \qquad \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}. \qquad \text{(A.3)}$$

The First Order Condition for an interior maximum of this problem are given by

$$v_q = \Sigma_i \, \lambda_i \, p^i \, ,$$
$$p^{i'}q = m_i \, , \qquad (i=1,2), \qquad \text{(A.4)}$$

where v_q is the gradient of $v(\cdot)$. After total differentiation, we obtain the Fundamental Matrix Equation:

$$\begin{pmatrix} V & p^1 & p^2 \\ p^{1\prime} & 0 & 0 \\ p^{2\prime} & 0 & 0 \end{pmatrix}\begin{pmatrix} dq \\ -d\lambda_1 \\ -d\lambda_2 \end{pmatrix} = \begin{pmatrix} 0 \\ dm_1 \\ dm_2 \end{pmatrix} + \begin{pmatrix} \Sigma_i \lambda_i dp^i \\ -q' dp^1 \\ -q' dp^2 \end{pmatrix}. \tag{A.5}$$

Here V denotes the Hessian of the utility function $v(\cdot)$.

Since the two types of work, L and B, appear as perfect substitutes in the utility function, this Hessian will be only of rank 2. However, because the wage vectors w^1 and w^2 are linearly independent when a marginal penalty rate applies, it can be shown that the strong quasi concave nature of $v(\cdot)$ guarantees non-singularity of the bordered Hessian on the LHS. In other words, the demand and supply functions will be differentiable.

Before solving this system, let me draw attention to the following

Lemma (Barten, 1972 p 3.4): *In the* (n+h)×(n+h) *symmetric matrix*

$$Z^* = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{12}' & Z_{22} \end{pmatrix} = \begin{pmatrix} X_{11} & X_{12} \\ X_{12}' & O \end{pmatrix}^{-1} = X^{*^{-1}}, \tag{A.6}$$

where Z_{11} *and* X_{11} *are both matrices of* n×n *and* n≥h: $\text{rk}(Z_{11})=\text{n-h}$, $\text{rk}(Z_{12})=\text{rk}(X_{12})=\text{h}$, *and* $\text{rk}(Z_{22})\leq \text{h}+\text{rk}(X_{11})\text{-n}$.

Let us now denote the inverse of the bordered Hessian by

$$\begin{bmatrix} V & (p^1, p^2) \\ \begin{pmatrix} p^{1\prime} \\ p^{2\prime} \end{pmatrix} & O \end{bmatrix}^{-1} = \begin{bmatrix} Z & (q_m^1, q_m^2) \\ \begin{pmatrix} q_m^{1\prime} \\ q_m^{2\prime} \end{pmatrix} & Y \end{bmatrix}, \tag{A.7}$$

where Z and Y are 4×4 matrices, and q_m^i is a 2×1 vector. It may then be verified that this inverse shares the following properties:

(i) the matrix Z is symmetric;
(ii) $\text{rk}(Z)=4-2=2$;
(iii) $\text{rk}[(q_m^1, q_m^2)]=\text{rk}(P')=2$;
(iv) $Zp^i=0$, $i=1,2$;
(v) $p^{i\prime}q_m^j=1$ if $i=j$, $i,j=1,2$, and
$=0$ otherwise.

Symmetry of V implies (i); (ii) and (iii) follow from Barten's lemma; and (iv) and (v) are obtained by postmultiplying the inverse by the original bordered Hessian.

Solving (A.5) for the endogenous variables, the demand system can be written as

$$dq = \Sigma_i q_m^i(dm_i - q'dp^i) + \Sigma_i Z\lambda_i dp^i \ . \tag{A.8}$$

The vectors q_m^i $(i=1,2)$ are the vectors of state contingent income effects; they satisfy the adding up property (cf v). This identifies the second RHS term as the substitution effects. Since it matters via which state contingent lump sum income the consumer is compensated, the substitution matrices will not be identical. However, from (A.8) it is apparent that they will differ only in their scale factor to the matrix Z; this scale factor being the marginal utility of income in the corresponding state of nature.

These substitution matrices are symmetric [cf (i)], and of less than full rank [cf (ii)]; any proportional change in one of the two price systems will leave the Hicksian demand vectors unaffected [cf (iv)]. Finally, it is possible to show that the matrix Z and therefore any of the two substitution matrices, is negative semi definite, implying that own compensated price effects must always reduce demand (increase supply).

Let me now single out labour supply on the black market, B, from the vector q, and investigate its comparative statics effects. From (A.2) and (A.8) it follows that

$$\begin{aligned} dB = {} & \frac{\partial B}{\partial m_1}[d(-T)+Ld\omega_L+Bdw_B]+\frac{\partial B}{\partial m_2}[d(-T)+d(-F)+Ld\omega_L+B(1-f)dw_B-Bw_B df] \\ & -\lambda_1[z_{43}d\omega_L+z_{44}dw_B]-\lambda_2[z_{43}d\omega_L+z_{44}[(1-f)dw_B-w_B df], \end{aligned} \tag{A.9}$$

where $\partial B/\partial m_i = -(q_m^i)_4$. The effects of a fall in the poll tax T and the lump sum fine F are straightforward:

$$\begin{aligned} \frac{\partial B}{\partial(-F)} &= \frac{\partial B}{\partial m_2} \\ \frac{\partial B}{\partial(-T)} &= \frac{\partial B}{\partial m_1}+\frac{\partial B}{\partial m_2} \end{aligned} \tag{A.10}$$

As usual with income effects, their sign cannot be predicted from the basic axioms.

The following three equations represent compensated price effects, with the form of compensation denoted besides the vertical bar.

$$\left.\frac{\partial \hat{B}}{\partial \omega_L}\right|_{dT=Ld\omega_L} = -(\lambda_1+\lambda_2)z_{43} \ , \tag{A.11a}$$

$$\left.\frac{\partial \hat{B}}{\partial f}\right|_{dF=-Bw_B df} = \lambda_2 z_{44} w_B, \tag{A.11b}$$

$$\left.\frac{\partial \hat{B}}{\partial w_B}\right|_{\substack{dT=Bdw_B \\ dF=fBdw_B}} = -[\lambda_1+(1-f)\lambda_2]z_{44}. \tag{A.11c}$$

The first of these compensated effects is related to a net wage increase on the official labour market. It is proportional to an off-diagonal element of the negative semi definite matrix Z. Since there are four 'commodities' present in our problem, no inference can be made about the sign.

The second equation gives the effect of an increased marginal penalty rate on black market activity. Since this rise affects the consumer only in the 'audit' state, he can be kept on his original indifference curve by adjusting the lump sum fine F accordingly. The resulting effect on B will be negative since $z_{44}<0$.

The final equation pertains to an isolated rise of the wage rate obtained on the black market. This perturbation hits the consumer in both states of nature, and the only way to guarantee any predictable–positive–reaction on his behalf is by adjusting *both* the poll tax T and the lump sum fine F appropriately. Although a rise in w_B increases the return to black market labour supply in both states of nature, it would be incorrect to conclude that by increasing the universal lump sum tax T, a conventional substitution effect follows. Sure, an increase in T of the amount

$$dT = \frac{\omega_L}{w_B} B dw_B \tag{A.12}$$

will bring the consumer back on his original indifference curve. But the resulting substitution effect is given by

$$\left.\frac{\partial \hat{B}}{\partial w_B}\right|_{dT=\frac{\omega_L}{w_B}Bdw_B} = \{\underset{+}{\frac{\partial B}{\partial m_1}}(w_B-\omega_L)+\underset{-}{\frac{\partial B}{\partial m_2}}[(1-f)w_B-\omega_L]\}\frac{B}{w_B} \underset{+}{-}[\lambda_1+\lambda_2(1-f)]\underset{-}{z_{44}}. \tag{A.13}$$

Only under the peculiar assumption that black market activity is a normal good w.r.t. income in the no-audit state (m_1), but an inferior good w.r.t. income in the audit state (m_2), will this substitution effect take on a positive sign.

One might also be interested in the effect of a universal rise in the wage rate, for instance when both wage rates are linked in a one-to-one way. In this case, $d\omega_L=(1-t)dw_B$. This general pay rise will make the consumer better off in both states of nature. If the government adjusts the lump sum tax and fine as follows:

$$dT = [(1-t)L+B]dw_B \qquad dF = -fBdw_B \tag{A.14}$$

the consumer will find himself back on the original indifference curve, but he will have adjusted his labour supply on the black market in the following direction:

$$\left.\frac{\partial \hat{B}}{\partial w_B}\right|_{(A.14)} = -(1-t)(\lambda_1+\lambda_2)z_{43} - [\lambda_1+\lambda_2(1-f)]z_{44} \tag{A.15}$$

The sign of this compensated effect is indeterminate.

One may conclude from this exercise that without any further restrictions on preferences, the direction of most compensated parameter changes on black market activity remains ambiguous. *A fortiori* this holds for uncompensated changes.

In Section 5.2, attention has been devoted to comparative statics effects *conditional* on a predetermined amount of foregone leisure, H. In the remainder of this appendix, I will derive these effects for any general preference order by using the concepts of virtual prices and income. These concepts have recently been reintroduced in the literature by Neary & Roberts (1980); their approach is a dual one. However, in the present context with multiple budget constraints, a primal approach seems more desirable. I will therefore take a similar route as in an earlier paper (Schroyen, 1991). This amounts to imposing the additional restriction $L+B=H$ by adjusting some prices and income such that this constraint is indeed verified.

Consider problem (A.3) again. Let us partition the vector of decision variables, q, as well as the price system P, as follows:

$$q' = (q_a', q_b'), \quad P = \begin{pmatrix} p_a^{1'} & p_b^{1'} \\ p_a^{2'} & p_b^{2'} \end{pmatrix}. \tag{A.16}$$

Suppose now that the subvector q_b is subject to the additional linear restriction

$$r'q_b = R \qquad (\mu), \tag{A.17}$$

where the vector r has the same dimension as q_b and R is a scalar; μ denotes the corresponding Lagrange multiplier. For instance, in the model of tax evasion with predetermined total labour supply, $r'=(1,1)$ and $R=-H$.

While the FOC w.r.t. q_a is as in (A.4), the one w.r.t. q_b now needs to be modified to

$$v_{q_b} = \Sigma_i \lambda_i p_b^i + \mu r = \lambda_1 p_b^1 + \lambda_2 (p_b^2 + \frac{\mu}{\lambda_2} r). \tag{A.18}$$

The term in round brackets is the *virtual* price vector that will induce the consumer solving problem (A.3) to comply with the additional constraint (A.17). Denote this price vector as $\tilde{p}_b$, and let $\beta =_{\text{def}} \mu/\lambda_2$. In order to allow the consumer to transact at these virtual prices, it is necessary to adjust his budget set accordingly:

$$\begin{aligned} p_a^{1'}q_a + p_b^{1'}q_b &= m_1, \\ p_a^{2'}q_a + \tilde{p}_b^{2'}q_b &= m_2 + \beta R =_{def} \tilde{m}_2. \end{aligned} \tag{A.19}$$

$\tilde{m}_2$ is the virtual income that enables the consumer to transact at the virtual price vector $\tilde{p}_b$ [alternatively, if we had decided to transform p_b^1 into a virtual price, m_1 would have been adjusted accordingly].

After total differentiation of the FOC's and budget constraint, one obtains a modified version of the Fundamental Matrix Equation (A.5):

$$\begin{pmatrix} V_{aa} & V_{ab} & p_a^1 & p_a^2 \\ V_{ba} & V_{bb} & p_b^1 & \tilde{p}_b^2 \\ p_a^{1'} & p_b^{1'} & 0 & 0 \\ p_a^{2'} & \tilde{p}_b^{2'} & 0 & 0 \end{pmatrix} \begin{pmatrix} dq_a \\ dq_b \\ -d\lambda_1 \\ -d\lambda_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ dm_1 \\ dm_2 \end{pmatrix} + \begin{pmatrix} \Sigma_i\ \lambda_i dp_a^i \\ \Sigma_i\ \lambda_i dp_b^i \\ -q'dp^1 \\ -q'dp^2 \end{pmatrix} + \begin{pmatrix} 0 \\ \lambda_2 r d\beta \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \beta dR \end{pmatrix}. \tag{A.20}$$

Let the inverse of the LHS bordered Hessian be given by

$$\begin{pmatrix} Z_{aa} & Z_{ab} & a_m^1 & a_m^2 \\ Z_{ba} & Z_{bb} & b_m^1 & b_m^2 \\ a_m^{1'} & b_m^{1'} & \multicolumn{2}{c}{\multirow{2}{*}{Y}} \\ a_m^{2'} & b_m^{2'} & & \end{pmatrix}. \tag{A.21}$$

This matrix shares again the properties (i)-(v), but now w.r.t. the *virtual* prices and income.

Multiplying (A.20) through by this inverse yields the following system:

$$\begin{aligned} dq_a &= \Sigma_i\ a_m^i(dm_i - q'dp^i) + Z_{ab}\Sigma_i\ \lambda_i dp_b^i + Z_{ab}\lambda_2 r d\beta + \beta a_m^2 dR \\ dq_b &= \Sigma_i\ b_m^i(dm_i - q'dp^i) + Z_{bb}\Sigma_i\ \lambda_i dp_b^i + Z_{bb}\lambda_2 r d\beta + \beta b_m^2 dR. \end{aligned} \tag{A.22}$$

Here, I have assumed that $dp_a = 0$, which will be true in the tax evasion model where q_a' corresponds to (c_1,c_2) [see (A.2)].

Multiplying the last equation through by r', recognizing that $r'dq_b=dR$, and rearranging, produces the equilibrium change in β:

$$d\beta = (\lambda_2 r' Z_{bb} r)^{-1}[dR - \Sigma_i\ r' b_m^i (dm_i - q' dp^i) - r' Z_{bb} \Sigma_i\ \lambda_i dp^i - \beta r' b_m^2 dR]. \tag{A.23}$$

By substituting this expression for $d\beta$ in the two equation of (A.22), one obtains the equilibrium changes in demand/supply in terms of the exogenous perturbations of the parameters and the predetermined variable R. In particular, for the vector dq_b, the solution looks like:

$$dq_b = \Sigma_i\ \tilde{b}_m^i (dm_i - q' dp^i) + \tilde{Z}_{bb} \Sigma_i\ \lambda_i dp_b^i + [\tilde{Z}_{bR} + \beta \tilde{b}_m^2] dR, \tag{A.23}$$

where

$$\begin{aligned} \tilde{b}_m^i &=_{def} b_m^i - Z_{bb} r (r' Z_{bb} r)^{-1} r' b_m^i \\ \tilde{Z}_{bb} &=_{def} Z_{bb} - Z_{bb} r (r' Z_{bb} r)^{-1} r' Z_{bb} \\ \tilde{Z}_{bR} &=_{def} Z_{bb} r (r' Z_{bb} r)^{-1}\ . \end{aligned} \tag{A.25}$$

The first vector is the vector of constrained (state contingent) income effects. The elements of the matrix on the second line are proportional to the constrained substitution effects. It is not difficult to verify that the negative definite nature of the matrix Z_{bb} carries over to this matrix. Neither that every diagonal element on this matrix will, in absolute value, be smaller than the corresponding element of Z_{bb}. This last statement is of course an application of the LeChatelier principle. The final vector gives the substitution component of the spill-over effect generated by a change in the parameter $R - m_2$ being adjusted to establish compensation.

Applying now these results to the tax evasion model, we may again single out the equilibrium response in black market activity to perturbations in the environment:

$$\begin{aligned} dB = &\frac{\partial \tilde{B}}{\partial m_1}[d(-T) + Ld\omega_L + Bdw_B] + \frac{\partial \tilde{B}}{\partial m_2}[d(-T) + d(-F) + Ld\omega_L + B(1-f)dw_B - Bw_B df] \\ &- \lambda_1[\tilde{z}_{43} d\omega_L + \tilde{z}_{44} dw_B] - \lambda_2[\tilde{z}_{43} d\omega_L + \tilde{z}_{44}[(1-f)dw_B - w_B df] \\ &+ [\frac{\partial \hat{B}}{\partial H} - \beta \frac{\partial \tilde{B}}{\partial m_2}] dH \end{aligned} \tag{A.26}$$

A $\tilde{}$ denotes a constrained effect, and $\partial\hat{B}/\partial H$ is the last element of $\tilde{Z}_{bR}$. Apart from the spill-over effect, this supply equation has the same structure as when leisure is not predetermined. Consequently, similar same conclusions hold w.r.t. the compensated comparative statics effects (keeping in mind, of course, the LeChatelier property). In particular, a rise in w_B compensated only by an upward adjustment of T will generate an ambiguous substitution effect, as it did in (A.13).

Appendix B

Proof of Theorem 2:

Let (c_1,c_2) and $(\bar{c},\bar{c})$ belong to the same indifference curve yielding utility $\bar{u}$ when overall labour supply is H, and assume $c_1>\bar{c}>c_2$. Thus

$$\begin{aligned} v(c_1,c_2;H) &= (1-\pi)u(c_1,1-H)+\pi u(c_2,1-H) = \bar{u}, \\ v(\bar{c},\bar{c};H) &= (1-\pi)u(\bar{c},1-H)+\pi u(\bar{c},1-H) = \bar{u}. \end{aligned} \tag{B.1}$$

Let the function $c_b(\cdot)$ be implicitly defined as follows:

$$v[c_a,c_b(c_a);H] = \bar{u}, \quad \forall\ c_a \in R_o^+. \tag{B.2}$$

Since $v[\cdot]$ is continuous, so is $c_b(\cdot)$. Moreover, $c_b(c_1)=c_2$ and $c_b(\bar{c})=\bar{c}$. After total differentiation of (B.2) it can be established that

$$c_b' =_{def} \frac{\partial c_b}{\partial c_a} = -\frac{(1-\pi)}{\pi}\frac{u_1(c_a,1-H)}{u_1(c_b(c_a),1-H)}. \tag{B.3}$$

Next, define $c_a(s)=_{\text{def}} sc_1+(1\text{-}s)\bar{c}$, so that $c_a'(s)=c_1\text{-}\bar{c}$. The utility differential between the two consumption bundles conditional on an overall labour supply H' is then given by

$$\begin{aligned} & v(c_1,c_2,1-H')-v(\bar{c},\bar{c},1-H') \\ &= \int_0^1 [\tfrac{\partial v}{\partial c_a}+\tfrac{\partial v}{\partial c_b}c_b']c_a(s)\ ds \\ &= \int_0^1 \left((1-\pi)u_1[c_a(s),1-H']+\pi u_1[c_b(c_a(s)),1-H']\cdot\{-\tfrac{(1-\pi)}{\pi}\tfrac{u_1[c_a(s),1-H]}{u_1[c_b(c_a(s)),1-H]}\}\right)(c_1-\bar{c})\ ds \\ &= \int_0^1 \pi u_1[c_b(c_a(s)),1-H']\left(\tfrac{(1-\pi)}{\pi}\tfrac{u_1[c_a(s),1-H']}{u_1[c_b(c_a(s)),1-H']} - \tfrac{(1-\pi)}{\pi}\tfrac{u_1[c_a(s),1-H]}{u_1[c_b(c_a(s)),1-H]}\right)(c_1-\bar{c})\ ds \\ &= \int_0^1 \pi u_1[c_b(c_a(s)),1-H']\left(MRS(s,H') - MRS(s,H)\right)(c_1-\bar{c})\ ds, \end{aligned} \tag{B.4}$$

with obvious definition of $MRS(s,\cdot)$.

Since both $\pi u_1[\cdot]$ and $(c_1\text{-}\bar{c})$ are strictly positive, and because because I assume $MRS(\cdot, h)$ to be monotonous in h, it must be true that

$$MRS(s,H') \gtrless MRS(s,H) \Rightarrow v(c_1,c_2,H') \gtrless v(\bar{c},\bar{c},H), \quad \forall\ s \in[0,1]. \tag{B.5}$$

Letting $H'=H+dH$, proves the theorem. QED

Appendix C

In the paper, several parameters were identified as being of importance in controlling the willingness to bear risk and the simultaneity between decision taken on the labour/leisure and risk/no risk margins. For the von Neumann-Morgenstern utility function $u(c,\ell)=u(c,1\text{-}H)$, these parameters are:

$$\frac{\partial^2}{\partial c^2}\left(\frac{u_2}{u_1}\right),\quad R_a =_{def} -\frac{u_{11}}{u_1},\quad \frac{\partial R_a}{\partial c},\ \text{and}\ \frac{\partial R_a}{\partial H}.$$

In this Appendix, I will derive the values for these parameters for three frequently used labour supply models, to wit: the Linear Expenditure System (LES), the Constant Elasticity of Substitution model (CES), and Hausman's (1981) linear labour supply model. See Stern's (1986) thorough survey paper for a critical evaluation of these models and many others.

- **The Linear Expenditure System:**

$$u(c,\ell) = (c-\gamma)^{\alpha_1}(\ell-\lambda)^{\alpha_2},\ 0<\alpha_1,\alpha_2<1 \tag{C.1}$$

where γ and λ can be thought of as minimal consumption and leisure requirements.

The following derivations are straightforward:

$$u_1 = \frac{\alpha_1}{(c-\gamma)}u,\ u_2 = \frac{\alpha_2}{(\ell-\lambda)}u \tag{C.2}$$

$$\frac{u_2}{u_1} = \frac{\alpha_2}{\alpha_1}\frac{(c-\gamma)}{(\ell-\lambda)},\ \frac{\partial}{\partial c}\left(\frac{u_2}{u_1}\right) = \frac{\alpha_2}{\alpha_1}\frac{1}{(\ell-\lambda)},\ \frac{\partial^2}{\partial c^2}\left(\frac{u_2}{u_1}\right) = 0 \tag{C.3}$$

$$u_{11} = \alpha_1(\alpha_1-1)\frac{u}{(c-\gamma)^2},\ R_a = \frac{1-\alpha_1}{c-\gamma} > 0,\ \frac{\partial R_a}{\partial c} = \frac{\alpha_1-1}{(c-\gamma)^2} < 0,\ \frac{\partial R_a}{\partial H} = 0 \tag{C.4}$$

- **The CES model:**

$$u(c,\ell) = [\alpha(c-\gamma)^\rho+(1-\alpha)(\ell-\lambda)^\rho]^{1/\rho},\quad -\infty<\rho\le 1,\quad 0<\alpha<1. \tag{C.5}$$

Here, α is the intensity parameter with consumption, γ and λ can again be interpreted as minimal amounts of consumption and leisure, and $\sigma=_{\text{def}} 1/(1\text{-}\rho)$ is the (constant) elasticity of substitution.

It is useful to define the following alternative measure of the consumption intensity in total utility:

$$\delta = \frac{\alpha(c-\gamma)^{\rho}}{\alpha(c-\gamma)^{\rho}+(1-\alpha)(\ell-\lambda)^{\rho}} \tag{C.6}$$

It is clear that $0<\delta<1$ and $\lim_{\rho\to 0}\delta=\alpha$. With this in mind, the following expressions can be verified:

$$u_1 = \alpha[\alpha(c-\gamma)^{\rho}+(1-\alpha)(\ell-l)^{\rho}]^{\frac{1}{\rho}-1}(c-\gamma)^{\rho-1} \tag{C.7}$$

$$u_2 = (1-\alpha)[\alpha(c-\gamma)^{\rho}+(1-\alpha)(\ell-l)^{\rho}]^{\frac{1}{\rho}-1}(\ell-\lambda)^{\rho-1} \tag{C.8}$$

$$\frac{u_2}{u_1} = \frac{1-\alpha}{\alpha}\left(\frac{c-\gamma}{\ell-\lambda}\right)^{1/\sigma},\quad \frac{\partial}{\partial c}\left(\frac{u_2}{u_1}\right) = \frac{1-\alpha}{\alpha}\frac{1}{\sigma}\left(\frac{c-\gamma}{\ell-\lambda}\right)^{\frac{1}{\sigma}-1}\frac{1}{\ell-\lambda} \tag{C.9}$$

$$\frac{\partial^2}{\partial c^2}\left(\frac{u_2}{u_1}\right) = \frac{1-\alpha}{\alpha}\frac{1}{\sigma}\left(\frac{1}{\sigma}-1\right)\left(\frac{c-\gamma}{\ell-\lambda}\right)^{\frac{1}{\sigma}-2}\frac{1}{(\ell-\lambda)^2} \lesseqgtr 0 \Leftrightarrow \sigma \gtreqless 1 \tag{C.10}$$

$$\begin{aligned} u_{11} =\ & \alpha^2(1-\rho)[\alpha(c-\gamma)^{\rho}+(1-\alpha)(\ell-l)^{\rho}]^{\frac{1}{\rho}-2}(c-\gamma)^{2\rho-2} \\ & +\alpha[\alpha(c-\gamma)^{\rho}+(1-\alpha)(\ell-\lambda)^{\rho}]^{\frac{1}{\rho}-1}(\rho-1)(c-\gamma)^{\rho-2} \end{aligned} \tag{C.11}$$

$$R_a = \frac{1-\rho}{c-\gamma}\left(1-\frac{\alpha(c-\gamma)^{\rho}}{\alpha(c-\gamma)^{\rho}+(1-\alpha)\ell-\lambda)^{\rho}}\right) = \frac{1-\rho}{c-\gamma}(1-\delta) > 0 \tag{C.12}$$

$$\begin{aligned} \frac{\partial R_a}{\partial c} &= \frac{\rho-1}{(c-\gamma)^2}(1-\delta)-\frac{1-\rho}{c-\gamma}\left\{\frac{\rho\alpha(c-\gamma)^{\rho-1}}{[...]}-\frac{\rho\alpha^2(c-\gamma)^{2\rho-1}}{[...]^2}\right\} \\ &= \frac{\rho-1}{(c-\gamma)^2}\{(1-\delta)+\rho\delta(1-\delta)\} \\ &= \frac{\rho-1}{(c-\gamma)^2}(1-\delta)(1+\rho\delta) \lesseqgtr 0 \Leftrightarrow (1+\rho\delta) \gtreqless 0 \Leftrightarrow \sigma \gtreqless \frac{\delta}{1+\delta} \end{aligned} \tag{C.13}$$

where [...] is a shorthand for the denominator of δ.

$$\begin{aligned} \frac{\partial R_a}{\partial H} &= -\frac{\partial R_a}{\partial \ell} = \frac{\rho-1}{c-\gamma}\frac{\alpha(c-\gamma)^{\rho}\rho(1-\alpha)(\ell-\lambda)^{\rho-1}}{[...]^2}, \\ &= \frac{(\rho-1)\delta(1-\delta)}{(c-\gamma)(\ell-\lambda)}\rho \lesseqgtr 0 \Leftrightarrow \rho \lesseqgtr 0 \Leftrightarrow \sigma \gtreqless 1. \end{aligned} \tag{C.14}$$

- **The Linear Labour Supply Model:**

Consider the following linear labour supply function:

$$H = \alpha\omega + \beta m + \gamma \tag{C.15}$$

where ω is the net real wage rate, m is non-labour income, α is the (constant) uncompensated wage effect, β is the (constant) income effect, and γ is the intercept. It is not difficult to check that the compensated wage effect is give by α-βH, which should be strictly positive to be consistent with utility maximization. Therefore

$$b =_{def} \frac{\alpha}{\beta} \gtrless H \Leftrightarrow \beta \gtrless 0. \tag{C.16}$$

Hausman (1981) derived the direct utility function from which this supply function stems. It is given by

$$u(c,1-H) = \frac{H-b}{\beta} \exp\{-[1+\frac{\beta(c+a)}{b-H}]\}, \quad a =_{def} \frac{\gamma}{\beta} - \frac{\alpha}{\beta^2} \tag{C.17}$$

The different partial derivatives look as follows:

$$u_1 = \exp\{...\}, \quad u_2 = -\frac{1}{\beta}[1+\frac{\beta(c+a)}{b-H}]\exp\{...\} \tag{C.18}$$

where {...} is a shorthand for the argument of the exponential function;

$$\frac{u_2}{u_1} = \frac{1}{\beta}\frac{H-\beta c-\gamma}{b-H}, \quad \frac{\partial}{\partial c}(\frac{u_2}{u_1}) = -\frac{1}{b-H}, \quad \frac{\partial^2}{\partial c^2}(\frac{u_2}{u_1}) = 0 \tag{C.19}$$

$$u_{11} = -\frac{\beta}{b-H}\exp\{...\}, \quad R_a = \frac{\beta}{b-H} > 0 \tag{C.20}$$

$$\frac{\partial R_a}{\partial c} = 0, \quad \frac{\partial R_a}{\partial H} = \frac{\beta}{(b-H)^2} \tag{C.21}$$

Fig 1

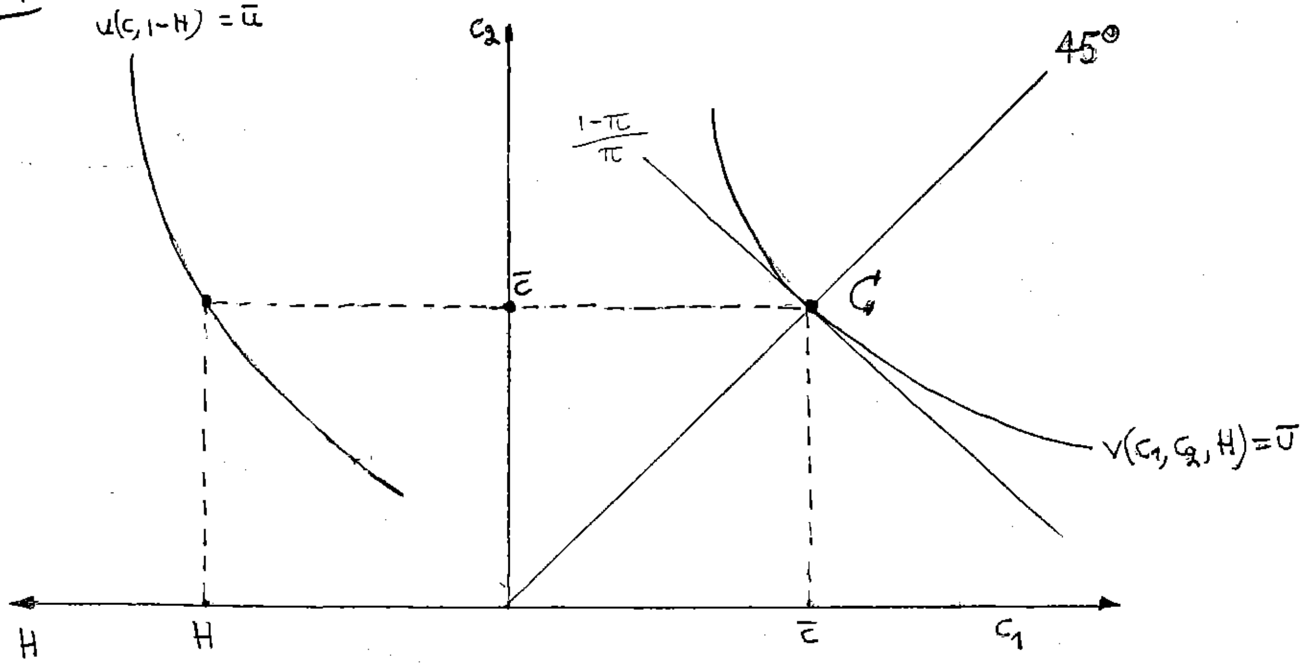


Fig 2.

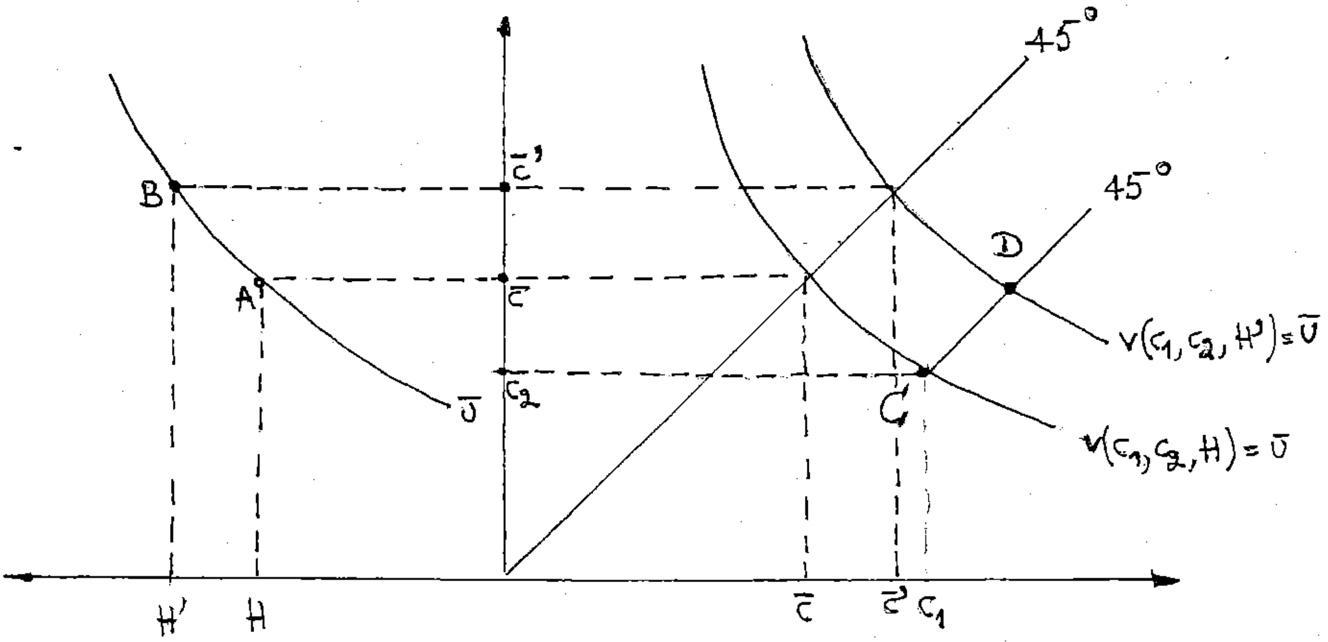


Fig 3a

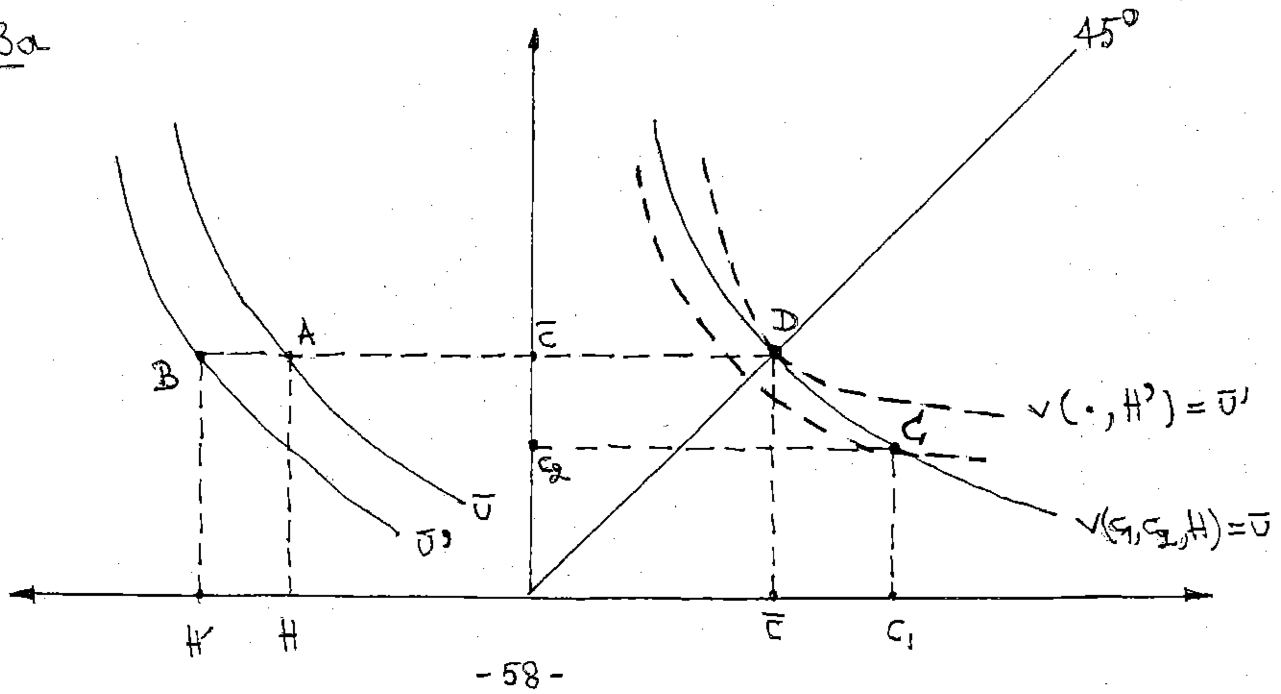


Fig 3b

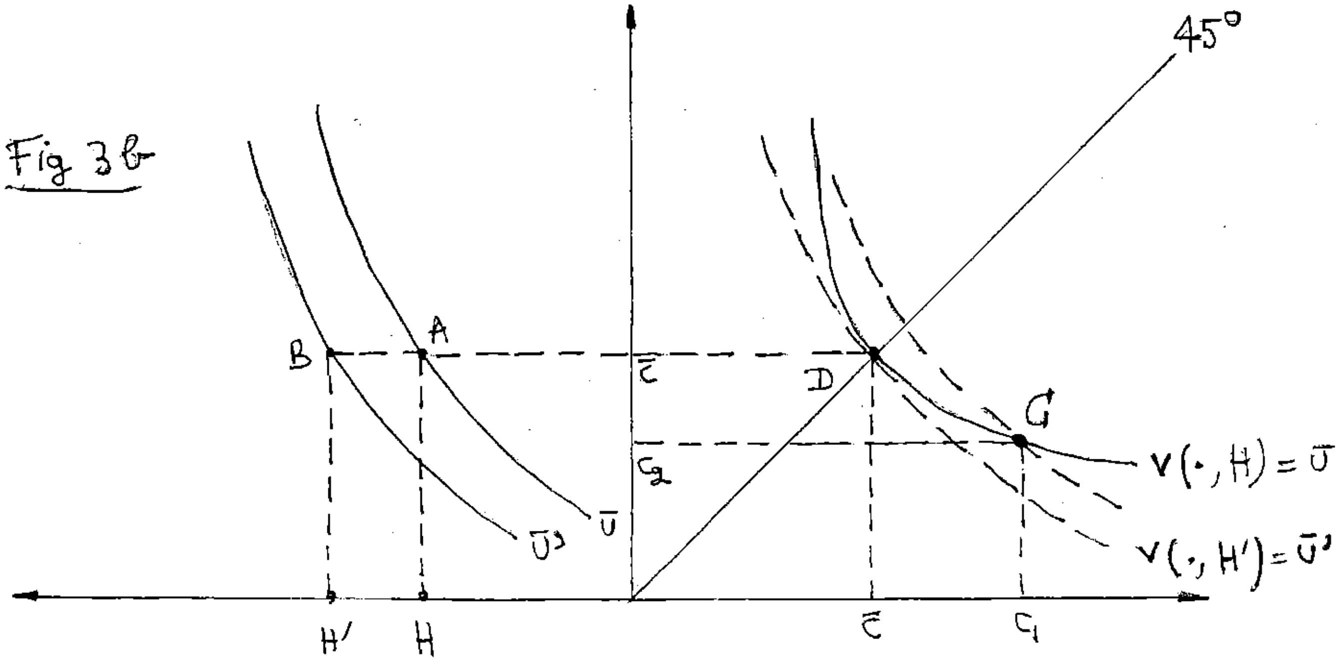
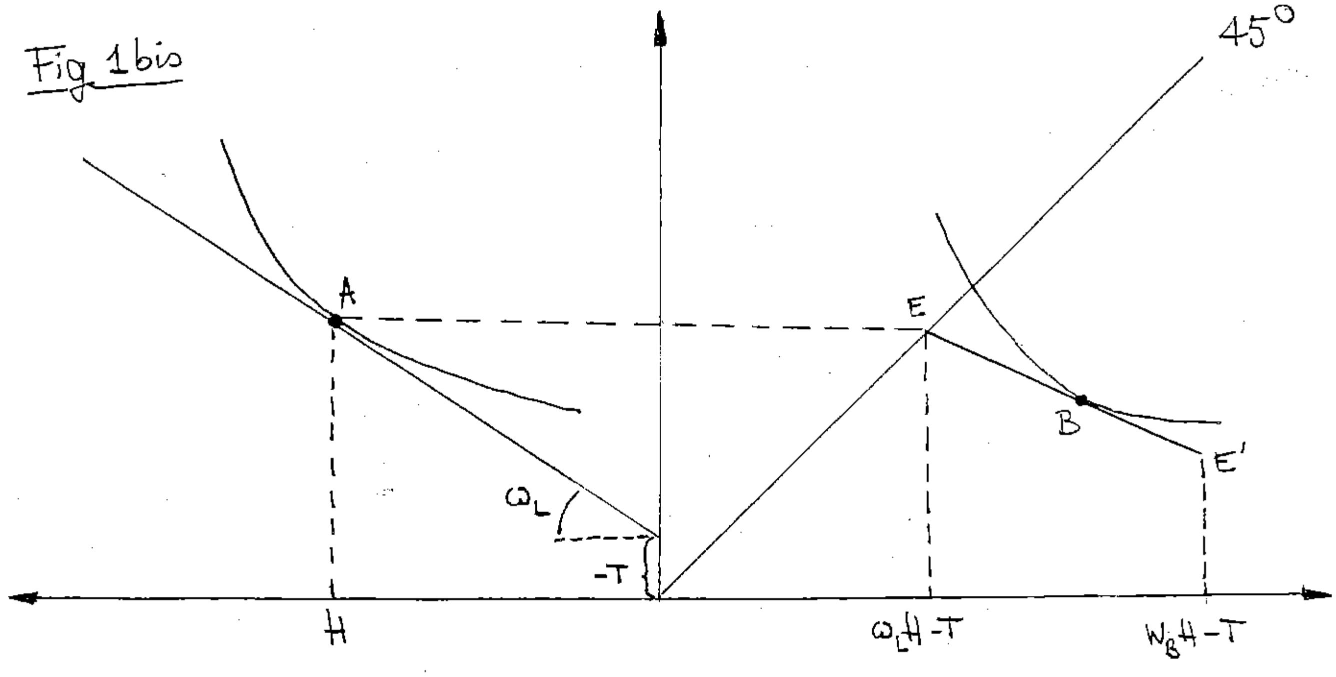


Fig 1bis



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