STUDIECENTRUM VOOR ECONOMISCH EN SOCIAAL ONDERZOEK

VAKGROEP PUBLIEKE ECONOMIE

A NON-PARAMETRIC NON-CONVEX APPROACH TO TECHNICAL EFFICIENCY: An illustration of radial efficiency measures and some sensitivity results (*)

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Report 92/276
June 1992

(*) The authors thank J. Couder for his superb programming assistance and I. Janssens and D. De Graeve for their comments on an earlier version. The authors are responsible for any remaining errors.

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D/1992/1169/11

ABSTRACT

The purpose of this paper is to investigate the sensitivity of a recently proposed non-parametric approach to technical efficiency measurement. Using a data set of Belgian municipalities we analyze the sensitivity of the Free Disposal Hull (FDH) approach with respect to the number of input and output dimensions and with respect to sample size, and we consider the impact of outliers on efficiency scores. We finally investigate the effects of using a variety of alternative (radial) efficiency measures.

0. INTRODUCTION

A variety of different approaches to the measurement of technical efficiency coexist in the literature[1]. Methodologically, they can be a categorized according to at least two criteria. First, one can distinguish between stochastic and deterministic methods. Whereas the former make explicit assumptions with respect to the stochastic nature of the data, the latter do not. A second classification distinguishes between parametric and non-parametric methods. In the parametric approach it is assumed that the boundary of the production set can be represented by a particular functional form with constant parameters. The non-parametric approach on the contrary concentrates on the regularity assumptions of the production set itself. Imposing some plausible restrictions on the production process these methods directly construct a piecewise linear reference technology or best practice frontier on the basis of observed input-output combinations.

Recently, Deprins, Simar and Tulkens (1984) and Tulkens (1986a,1986b) suggested the Free Disposal Hull (FDH) reference technology as a new deterministic and non-parametric basis for the evaluation of productive efficiency. Compared to other existing methods the FDH approach requires minimal assumptions with respect to the production technology. For example, it does not require convexity. As there is no generally accepted model of governmental behavior, the minimal technical and behavioral assumptions underlying the FDH make it a particularly useful tool for analyzing public sector efficiency questions. Not surprisingly, since its introduction a number of empirical studies have appeared in which the approach is applied to evaluate the technical efficiency of a number of public service providers as well as a few private enterprises (for a review, see

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[1] For surveys of the various approaches see, e.g., Bauer (1990), Førsund, Lovell and Schmidt (1980), Schmidt (1986), and Seiford and Thrall (1990).

Pestieau and Tulkens (1990) and Tulkens (1990)). Both the theoretical and empirical work have clarified the main advantages and disadvantages of the FDH reference technology.

This paper elaborates upon De Borger et al. (1992) and serves three purposes. First, using information on 589 local authorities we apply the FDH method to evaluate the technical efficiency of the production of municipal services in Belgium. Second, we use this data set to illustrate in a systematic way the strengths and weaknesses of the FDH approach. Specifically, we assess the sensitivity of the results with respect to the number of input and output dimensions and with respect to sample size, and we consider the impact of the existence of outliers on efficiency scores. Third, contrary to common practice in non-parametric efficiency analyses we argue in favour of global or graph efficiency measurement. Almost all existing empirical studies confine the attention to measuring either input or output efficiency (see, e.g., Tulkens (1990), Färe, Grosskopf and Logan (1985)). In this paper, however, we do not restrict the analysis to separate input and output indices but also calculate two global efficiency measures that take account of all dimensions simultaneously. This procedure has the advantage that all available information is used in the construction of the ranking of observations in terms of their productive efficiency.

The paper unfolds as follows. The first section deals with methodological issues. We review the FDH reference technology for measuring technical efficiency, and we systematically discuss its advantages and shortcomings. We then continue to define the different input, output and graph efficiency measures that will be used in the empirical analysis. We apply the suggested methodology in Section 2 to study the efficiency of local public service provision by Belgian municipalities. The sensitivity of the results with respect to sample size, the existence of outliers and the number of dimensions is illustrated in Section 3. Some further reflections and a conclusion are provided in Section 4.

1. THE FREE DISPOSAL HULL APPROACH TO PRODUCTIVE EFFICIENCY

A production unit is technically efficient if it produces the maximum output which is technically feasible for given inputs, or uses minimal inputs for the production of a given level of output. In other words, technical or productive efficiency of a production unit is to be defined in terms of the ability of the unit to produce on the boundary of its production set[2]. Consequently, any methodology for evaluating technical efficiency requires the complete specification of the production possibilities set as well as some concept of distance to relate the observed input-output combinations to the boundary of the specified set. We therefore first characterize the FDH approach by specifying its assumptions regarding the production set, and then present various efficiency measures. We conclude this section with a review of the advantages and shortcomings of the FDH approach.

1.1 The FDH reference technology

Let $y = y(y_1, y_2, \ldots, y_n)$ be the n non-negative outputs produced by using various combinations of the m non-negative inputs $x = x(x_1, x_2, \ldots, x_m)$. The production possibilities set Y is the set of all input and output combinations which are technically feasible :

$$Y = \{ (x,y) \mid x \in \mathbb{R}_+^m, y \in \mathbb{R}_+^n, (x,y) \ \textit{is feasible} \} \qquad \textbf{(1)}$$

It is convenient to model the production technology by an input correspondence[3] $y \to L(y) \subseteq \mathbb{R}^n_+$. For any y, the level set L(y) denotes the subset of all input vectors x which yield at least the output vector y.

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[2] A complete characterization of types of efficiency can be found in Färe,Grosskopf and Lovell (1985).

[3] See Färe, Grosskopf, and Lovell (1985) and Varian (1984).

Different production technologies can be defined by imposing various restrictions on L(y). The non-parametric approaches typically impose very weak assumptions. Although they vary widely, they are generally less restrictive than those used in the parametric approaches[4]. Moreover, it is fair to say that the FDH reference technology imposes the mildest assumptions among the non-parametric alternatives. Specifically, the following axioms define the Free Disposal Hull[5] :

$$0 \notin \mathrm{L}(y) \textit{ for } y \geq 0, \textit{and } \mathrm{L}(0) = \mathbb{R}_+^n \tag{2.1}$$

$$\textit{If } \|y^l\| \to +\infty \textit{ as } l \to +\infty, \textit{ then } \bigcap_{l=1}^{+\infty} \mathrm{L}(y^l) \textit{ is empty} \tag{2.2}$$

$$\textit{If } x \in \mathrm{L}(y) \textit{ and } x' \geq x, \textit{ then } x' \in \mathrm{L}(y) \tag{2.3}$$

$$\mathrm{L}(y) \textit{ is a closed correspondence} \tag{2.4}$$

$$\textit{If } y' \geq y, \textit{ then } \mathrm{L}(y') \subseteq \mathrm{L}(y) \tag{2.5}$$

The intuition behind each of these axioms is straightforward. Axiom 1 states that a semipositive output cannot be obtained from a null input vector - thus excluding free production - and that any nonnegative input results at least in a zero output. The second axiom implies that finite inputs cannot produce infinite outputs. Axiom 3 is known as strong free disposability of inputs or positive monotonicity. An increase in inputs can not result in a decrease in outputs. In axiom 4 it is stated that if a sequence of input vectors can each produce y and converges to x^* then x^* can also produce y. Closedness is an axiom postulated for mathematical convenience. It cannot be

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[4] Grosskopf (1986) and Seiford and Thrall (1990) review the reference technologies used in the non-parametric approach.

[5] See, e.g., Deprins, Simar and Tulkens (1984). Note that the notion of a free disposal hull originally referred to the property of strong free disposal and not to any particular reference technology (See McFadden (1978)).

contradicted by any empirical observation[6]. Axiom 5 is known as strong free disposability of outputs. It implies that any reduction in outputs remains producible with the same amount of inputs. Note that this assumption allows for variable returns to scale.

The FDH reference technology is now easily defined. It is a piecewise linear technology, constructed on the basis of observed input-output combinations, that satisfies the above axioms. The FDH input correspondence can be specified as:

$$L(y)^{FDH} = \{ x \mid x \in \mathbb{R}_+^m,\ z'N \geq y,\ z'M \leq x,\ I_k' z = 1,\ z_i \in \{0,1\} \} \qquad (3)$$

where N is the k x n matrix of observed outputs, M is the k x m matrix of observed inputs, z is a k x 1 vector of intensity parameters, and I_k is a k x 1 vector of ones. Consistent with the idea of variable returns to scale the intensity vector is restricted to sum to one. Since the intensity vector contains either zeros or ones linear combinations of several observations are excluded. Note that the axioms did not impose convexity on the technology.

We have focused so far on the FDH-input correspondence L(y). Obviously, the technology can equivalently be characterized using the output correspondence or the graph correspondence. The output correspondence is the subset of all output vectors y which is obtained from the input vector x. Based on analogous axioms the FDH output correspondence is given by:

$$P(x)^{FDH} = \{ y \mid y \in \mathbb{R}_+^n,\ z'N \geq y,\ z'M \leq x,\ I_k' z = 1,\ z_i \in \{0,1\} \} \qquad (4)$$

Finally, the FDH graph correspondence can easily be defined with respect to either the input or the output correspondence :

$$\begin{aligned} GR(x,y)^{FDH} &= \{(x,y) \mid x \in L(y)^{FDH},\ x \in \mathbb{R}_+^m,\ y \in \mathbb{R}_+^n\} \\ &= \{(x,y) \mid y \in P(x)^{FDH},\ x \in \mathbb{R}_+^m,\ y \in \mathbb{R}_+^n\} \end{aligned} \qquad (5)$$

[6] For further interpretation, see Färe, Grosskopf and Lovell (1985), p. 25.

To develop some intuition for the graphical representation of the FDH reference technology, note that, reflecting free disposal, each observed input-output combination adds one orthant, positive in the inputs and negative in the outputs, to the production set. The FDH reference technology is then the boundary to the union of all such orthants. Its graph as well as its isoquants typically have a staircase form. They are illustrated in Figures 1 and 2, respectively. These Figures also clarify the above definitions of the correspondences. Note that for an observation on the frontier the corresponding component of z equals one and the inequalities in (3) and (4) are binding. For an inefficient observation in an orthant spanned by a boundary observation j, the j-th component of z equals one and the inequalities hold, as the dominated observation uses more inputs to produce less outputs than observation j.

To construct the reference technology and to separate efficient from inefficient observations on the basis of a sample of input-output combinations, a data classification algorithm based on simple vector dominance reasoning can be used (see, e.g., Tulkens, Thiry and Palm (1988))[7]. The procedure operates basically as follows. Each observation in the sample is sequentially compared to all others. An observation is declared inefficient if it is possible to find another observation which contains the same or more outputs but strictly less of at least one input, or which uses the same or less inputs to produce strictly more of at least one output. In this sense they are dominated by at least one other observation. Input-output combinations which are undominated are declared efficient. However, efficient observations which never dominate another

[7] Note that a linear programming approach similar to the ones used in other non-parametric methods is also available. For details we refer to Tulkens (1990). We selected the data classification algorithm because of its ease of programming and the fact that it allows one to generate some useful additional information along the way. Our program was developed in Turbo-pascal.

observation have been aptly called 'efficient by default'. Due to the partial ordering implied in the dominance reasoning the method is unable to make precise statements concerning their technical efficiency. They have to be distinguished from efficient observations that do dominate others.

The above concepts are illustrated on Figure 2. Observation a is dominated by observations 3, 4, 5 and 6 but is less inefficient than observation b. Observations 1,5,6 and 7 are efficient by default. Finally note the effect of not imposing convexity. Observation 4 is efficient although, had convexity been imposed, it would have been dominated by a linear combination of observations 5 and 6.

1.2. Defining alternative measures of technical efficiency

Once the reference technology has been determined technical efficiency is measured as the distance between an observed production unit and the postulated boundary. In the non-parametric approach attention is often restricted to the measurement of either input efficiency or output efficiency, depending on whether the inputs or the outputs are the decision variables under the control of the production unit[8]. For example, if it can be assumed that for the public sector cost minimization is a more likely behavioral postulate than output maximization or any other objective, then restricting attention to input efficiency is considered legitimate[9]. Furthermore, it is common to restrict the attention to radial or Farrell measures. Note, however, that due to the non-convex nature of the FDH reference technology one could easily argue in favour of non-radial efficiency measures[10].

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[8] See Färe, Grosskopf and Lovell (1985), p.16.

[9] Tulkens (1986b), p. 311.

[10] See Russell (1988) for an overview.

For the ease of comparison we stick to the tradition of radial measurement in this paper. However, we do not limit the analysis to input and output efficiency. We also calculate graph or global technical efficiency measures in the empirical work reported below. This can be justified on several accounts. First, the reference technology is constructed using all dimensions of input and output vectors. Restricting the measurement of technical efficiency to either the input or the output dimensions implies an information loss as it allows remaining slack in the dimensions not captured by the efficiency measure. Second, if the purpose of the analysis is to rank the production units according to technical efficiency, then a priori some overall measure may prove more useful than a detailed two part analysis.

The various measures used in this paper are easily defined. The Farrell input measure of technical efficiency is given by:

$$F_i(x,y) = min\{\lambda \mid \lambda \geq 0,\ \lambda x \in L(y)\} \tag{7}$$

In standard textbook language it measures the distance from the observed production unit to the isoquant along a ray through the origin. It varies between zero and one, with unity representing efficient production. This measure of input efficiency determines by what percentage inputs could be proportionately reduced while still producing the same output. Analogously, the Farrell output measure is defined as:

$$F_o(x,y) = max\{\mu \mid \mu \geq 0,\ \mu y \in P(x)\} \tag{8}$$

It determines the maximal proportional expansion in all outputs while still using the same input.

Various graph measures have been proposed in the literature[11]. In this paper we used two measures of the Farrell type. First, the Farrell graph measure of technical efficiency is defined as:

[11] For a detailed discussion, see Färe, Grosskopf and Lovell (1985).

$$F_g(x,y) = \min\{\lambda \mid \lambda \geq 0,\ (\lambda x, \lambda^{-1} y) \in GR(x,y)\} \qquad \textbf{(9)}$$

It looks for the maximal equiproportionate reduction of all inputs and increase of all outputs. Finally, the Generalized Farrell graph measure allows the proportional reduction of all inputs to be different from the proportional increase of all outputs and simply takes the average. It is given by:

$$F_g^G(x,y) = \min\{\frac{\lambda+\mu}{2} \mid \lambda \geq 0,\ \mu \geq 0,\ (\lambda x, \mu^{-1} y) \in GR(x,y)\} \qquad \textbf{(10)}$$

It has been shown that $F_i(x,y) = F_o(x,y)$ if and only if the technology satisfies constant returns to scale. As the FDH reference technology allows for variable returns to scale the Farrell input and output measures will generally differ. Also note that $F_g(x,y) \geq \max(F_i(x,y), F_o(x,y))$ and that $F_g(x,y) = 1$ if and only if either $F_i(x,y) = 1$ or $F_o(x,y) = 1$. Finally, observe that $F_g^G(x,y) < F_g(x,y)$ for $\lambda \neq \mu$[12].

Conceptually, the efficiency measures are easily calculated. First, for each inefficient observation the set of dominating observations is searched for. Then the various efficiency indices are obtained by directly applying the above definitions to the elements of this set. The element of the set which optimizes an efficiency measure is called the most dominating observation. Note that the most dominating observation may be different depending on the efficiency measure being used. Identifying the most dominating observations provides useful information concerning the opportunities available for improving efficiency.

[12] More details on the relations between these measures can be found in Chapter 6 of Färe, Grosskopf, and Lovell (1985).

1.3. FDH and efficiency measurement : advantages and shortcomings

The advantages and disadvantages of the FDH-approach[13] are summarized from two perspectives. First, we evaluate the method from the theoretical and empirical point of view. Then we discuss its merits and inconveniences from the managerial viewpoint.

From a theoretical and an empirical point of view, the FDH-based methodology makes very weak assumptions regarding the modelling of the production technology. The least restrictive technology used so far in the non-parametric approach assumes weak disposability instead of strong disposability. But these technologies always assumed convexity[14]. Furthermore, it can be argued that the assumptions of strong free disposal in inputs and outputs have a strong intuitive appeal since they are closest to the concept of technical efficiency itself. A dominated observation is inefficient due to its excessive usage of resources or due to its lack of outputs compared to another observation, irrespective of formal convexity or functional form considerations[15].

In the FDH approach the problem of assessing technical efficiency of a production set is separated from the issue of representing its boundary. This reference technology is less useful in answering other questions. The evaluation of productive efficiency indeed is an altogether different exercise than e.g the determination of factor productivity, of economies of scale

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[13] See e.g. Bös (1988), Färe, Grosskopf and Lovell (1985), and Thiry and Tulkens (1989).

[14] See Grosskopf (1986), p. 504. But see the recent contribution of Petersen (1990) which relaxes the assumption of convexity.

[15] Note that in the case of undesirable outputs the assumption of strong free disposal of outputs is disputable : see Färe, Grosskopf, Lovell and Pasurka (1989) for details.

and of scope, etc[16]. These problems require focussing on the boundary of the production set and are difficult to solve without resort to production or transformation functions. Here the more restrictive technologies considered in the parametric approach may well be indispensable[17].

A second advantage of the FDH-approach is its non-parametric nature. It is a general methododological requirement that the results of theoretical economic analysis should not depend on specific parametric forms chosen. However, in empirical work specific parametrizations are often crucial. It is then implicitly postulated that the parametric forms chosen are good approximations of the true functional relationships. This maintained hypothesis is however not directly testable. Therefore, it has been argued that both theoretical and empirical work should attempt to stay as close as possible to the raw data[18]. Furthermore, the non-parametric reference technologies and the resulting efficiency measures can be related to the results of the parametric approach. It has recently been argued that the former provide upper bounds to the latter[19]. Both with respect to the parametric approaches and the non-parametric methods that impose convexity, the FDH approach may therefore be considered conservative.

As any methodology the FDH-approach has some drawbacks. The most obvious problem is due to the partial ordering based on the vector dominance reasoning. It implies that the approach may be sensitive both to the number and distribution of the observations in the data set, and to the number of input and

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[16] A point developed in Tulkens (1990).

[17] The non-parametric approach can still be useful, viz. as a first step in the estimation of parametric frontiers. For applications of this method, see Thiry and Tulkens (1988) and Simar (1989).

[18] See ,e.g., Varian (1984).

[19] See Banker and Maindiratta (1988).

output dimensions considered. Increasing the sample size increases the possibility of dominance for any given observation, and therefore the probability of being denoted inefficient. Also a rather uniform distribution of the observations over the dimensions in the data set increases the possibility of dominance. On the other hand, an increase in the dimensions considered decreases this possibility. Therefore, one expects that incorporating more inputs or outputs into the analysis will increase the probability of efficiency. Moreover, all non-parametric approaches, which envelop the data as closely as possible, may be sensitive to outliers. Notice however that the FDH-approach is least sensitive to this defect. Each observation potentially only adds a small subset, i.e. an orthant, to the reconstruction of the production set. Hence outliers only affect a small subset of observations.

From a managerial viewpoint, the major advantage of the FDH-approach is that the resulting efficiency measures relate to an observed production unit. In most other methods the point of reference is a hypothetical construct. For example, an observation may be inefficient with respect to some convex combination of observations in the non-parametric DEA or with respect to some fitted value on a postulated frontier in the stochastic frontier approach. It may be difficult to convince managers that they are outperformed by such a hypothetical unit. They can always object that these convex combinations of observed activities are not feasible, or that they can not learn how to improve from an unobservable standard of comparison[20]. A final advantage is that additional information is readily available. For example, the set of dominating observations can provide useful information in designing stepwise improvements in the direction of the production unit on the frontier. The

[20] See the remarks in Epstein and Henderson (1989).

possibilities of the FDH-approach to improve productivity, to reward production units, etc., are clear[21].

2. AN APPLICATION TO BELGIAN MUNICIPALITIES

In this section we apply the FDH-approach to determine technical efficiency of all 589 Belgian municipalities[22]. The choice of input and output indicators has been motivated both by the desire to account for the most important local public services provided, and by the availability of data. Our basic data set has one input indicator, defined as total municipal staff, and five output indicators. The latter capture important aspects of local production in the field of education, transportation, and social and recreational services. The following outputs were used:

(i) the length of municipal roads
(ii) the number of beneficiaries of minimal subsistence grants
(iii) the number of students enrolled in local primary schools
(iv) the surface of public recreational facilities
(v) a 'residual' output defined as total municipal outlays minus the identifiable outlays on outputs (i),(ii), (iii) and (iv).

Several remarks are in order. First, some justification

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[21] Its use in public sector management has been developed in Pestieau and Tulkens (1990).

[22] Vanden Eeckaut and Tulkens (1988) and Vanden Eeckaut, Tulkens and Jamar (1991) have reported results for the Belgian local authorities using FDH. This paper differs from their studies on four accounts. First, their sample is restricted to the Walloon region. Second, they use somewhat different input and output indicators. Third, their analysis does not consider graph efficiency measures. Finally, they do not engage in the kind of sensitivity testing reported in Section 3 below.

on the inclusion of the residual output may be warranted. From municipal accounts we verified that the first four output indicators capture between 30% and 75% of municipal outlays. Therefore, the fifth output attempts to correct for other unobserved outputs. If it were not included then municipalities that spend a large fraction of their budget for the production of outputs not captured by our first four indicators would be incorrectly assigned very low efficiency scores. The residual output should largely eliminate the possible bias in the efficiency ranking on this account. Second, note that the available outputs only very crudely proxy for the underlying services being provided by local authorities, and that no information on capital inputs was available. As a consequence our study may have a limited scope and the results should be interpreted with care.

Application of the FDH-approach on our main data set yields the summary results reported in Table 1[23]. The results indicate that about 50% of the observations are inefficient. Among the efficient observations some 60 observations are efficient by default. The inefficient observations are most dominated by a subset of 11% to 14% of the observations depending on the efficiency index used.

Mean efficiency varies between 0.86 and 0.93. It is the lowest for the Farrell input measure and the highest for the Farrell graph measure. The Farrell input and output measures have the largest standard deviations and the lowest minimum. Note that the distribution of the efficiency measures covers a range from 0.23 to 1. The distribution of the efficiency measures is obviously rather skewed. A histogram of the frequencies is drawn in Figure 3. Since the non-parametric approach provides upper bounds for the estimation of efficiency, the mode at 1 can be interpreted as the discrete part of a censored distribution.

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[23] Note that in the empirical results we report the reciprocal of the output Farrell to facilitate comparison with the other measures.

Although the use of different efficiency measures does not lead to dramatically different mean efficiency levels one may wonder to what extent they imply different rankings of individual observations. Correlation coefficients, once for all observations and once for the inefficient observations only, are reported in Table 2. Note that the ranking implied by the Farrell input and output measures correlate least. If one considers the inefficient observations only, the correlation coefficient amounts to 0.59. The Farrell generalized graph measure clearly correlates best with the other measures. This is not entirely surprising. It is the only measure that takes account of differences in all inputs and outputs while at the same time allowing different proportional changes in each of these two major dimensions. Given that it is no more difficult to calculate than the other measures it probably deserves more attention in empirical applications.

3. SOME SENSITIVITY RESULTS

In this section we discuss the results of a sensitivity analysis on our data set of Belgian municipalities. In Section 1.3 we indicated the major strengths and weaknesses of the FDH approach. It was suggested that the method could be sensitive to the number of inputs and outputs taken into account, to the sample size and to the existence of outliers. The degree to which these claims hold true for our data set is investigated in some detail below.

First, we test the effect of the sample size by taking random samples of increasing size from 50 up to 550. For each size we considered five random samples[24]. In each case we report

[24] A more satisfactory procedure is to use bootstrapping techniques to approximate the sampling distribution of the efficiency measures. This is an obvious direction for future work.

the average results over the five samples in Table 3. The results indicate clearly that increasing the sample size increases not only the absolute but also the relative number of inefficient observations. The process is apparently highly nonlinear. Although the differences in the proportion of inefficient observations seem to level off for sample sizes above 400, there is no strong indication that further changes somehow converge to zero. Interestingly, larger sample sizes seem to have a much less pronounced impact on the absolute number of observations that are efficient by default, except for the smallest samples. Differences between samples of 200 observations and up are almost negligible.

As expected, it follows from the table that increasing the sample size decreases the average efficiency measures and increases their standard deviation. However, except for the smallest samples, the differences are trivial. Also note that the average Farrell input and output measures vary most and have larger standard deviations than the Farrell graph measure and the generalized Farrell measure. Finally observe that for all four measures larger sample sizes increase the number of most dominating observations.

Second, to determine the impact of outliers we first eliminated outliers from our main data set of 589 municipalities using one of the procedures outlined in Belsley, Kuh and Welsch (1980). The method employed constructs a test statistic based on the so-called leverage value $h_i = x_i(X'X)^{-1}x_i'$ of each observation, i.e. the diagonal elements of the matrix $X(X'X)^{-1}X'$. In our case X is the k x (m+n) datamatrix with k observations and (m+n) {input and output} dimensions. The leverage value determines the importance of the observations in the space spanned by all dimensions in the data set. Use of the appropriate test statistic resulted in the detection of 35 outliers, including the 5 largest

Belgian cities[25]. From these outliers 31 were efficient in the original analysis, and 13 among these were efficient by default.

We repeated the calculation of the four efficiency measures based on the data set obtained after deleting the 35 outliers. The result of this exercise is also reported in Table 3. Despite the fact that most of the outliers were efficient, their impact both on the number of inefficient observations and on the distribution of the efficiency scores is very small. Dropping the outliers results in a decrease in both the relative number of efficient observations and most dominating observations. Furthermore, we observe a marginal decrease in the average efficiency measure. These findings obviously do not necessarily imply the unimportance of correcting for the existence of outliers, as the effect on some individual observations may still be substantial.

Third, we tried to illustrate the effects of disaggregation and aggregation, i.e., the impact of variations in the number of dimensions. Because of data limitations there was unfortunately no scope at all for increasing the number of inputs and outputs taken into account in the analysis. Therefore, this part of the sensitivity analysis must necessarily remain somewhat unsatisfactory. We proceeded as follows. The main data set has 5 output dimensions and 1 input dimension, a total of 6 dimensions. Aggregation was achieved by dropping output(s) while in each case recalculating the 'residual' output. We calculated efficiency measures for the four combinations to drop one output and for the six combinations to drop two outputs, while in each of these cases the additional output was recalculated. To keep the results tractable we only report the average results per level of aggregation. The results of these exercises are in Table 4. They suggest, consistent with a priori expectations, that increasing the number of dimensions decreases the number of inefficient observations and increases the number of observations

[25] The complete list of outliers is available upon request.

efficient by default. Mean efficiency scores increase while their standard deviations decrease. It is somewhat reassuring that the variability in mean efficiency is quite small, despite the large impact on the fraction of efficient municipalities. This implies that, if the analysts' main interest is in calculating average efficiency levels, aggregation may not be too harmful[26]. Of course, if one is interested in the precise distribution of efficiency scores over the sample, this statement will probably be incorrect, as the impact of aggregation on individual observations may be nontrivial. Finally note that the number of observations on which inefficiency measurement depends, i.e., the set of most dominating observations, does not seem to vary systematically with the number of dimensions.

Finally, we attempted to detect the sensitivity to variable selection. Including or excluding critical variables may be interpreted as providing information on the importance of possible misspecification. A variation of the exercise to test for the effect of aggregation was used to investigate the impact of critical variables. Whereas in the case of testing for aggregation the residual output was systematically recalculated, in the present exercise the residual output was completely ignored. We simply varied the number of outputs taken into account in the analysis. The base case for this exercise has five dimensions : one input and four outputs. We calculated efficiency measures for the four combinations to drop one output and for the six combinations to drop two outputs. The results are also in Table 4. The results of this exercise show similar directions than the aggregation exercise. Although it is difficult to compare both exercises it seems that omitting critical variables leads to somewhat more variability in the efficiency measures, as is clear from the increased standard deviation. This is as expected : misspecification can have a significant effect on any

[26] Tulkens, Thiry and Palm (1988) found similar indications for the FDH reference technology. This is also analoguous to the results in Data Envelopment Analysis reported in Seiford and Thrall (1990).

estimation procedure. It is however comforting to know that the FDH reference technology is not particularly vulnerable to this problem[27].

4. SUMMARY AND CONCLUSIONS

The Free Disposal Hull approach is an alternative deterministic and non-parametric method for the evaluation of productive efficiency. The purpose of this paper was threefold. First, we calculated various measures of technical efficiency for a data set of 589 Belgian local governments using the FDH approach. Second, based on a priori reasoning as well as on the basis of the empirical results obtained we argued in favour of global efficiency measurement instead of limiting the analysis to either input or output efficiency. Finally, we attempted to illustrate - insofar as possible - the strengths and weaknesses of the FDH method using our municipal data set.

In the first section we presented the methodology for measuring productive efficiency based on the FDH reference technology. Apart from input and output efficiency measures, two global or graph efficiency indices were defined. The advantages and drawbacks of the FDH approach were systematically discussed. In Section 2 the method was applied to study the efficiency of local public service provision by Belgian municipalities. The main conclusions reached were that the FDH-approach has considerable advantages with respect to alternative methods from the theoretical, empirical and managerial viewpoint. These have to be traded off against some obvious disadvantages such as the sensitivity with respect to sample size and the number of inputs and outputs taken into account into the analysis. This sensitivity was illustrated in a third section.

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[27] Analoguous results in Data Envelopment Analysis are reported in Seiford and Thrall (1990).

Two final conclusions emerge from this paper. First, it is warranted to state that the FDH reference technology offers a simple but powerful approach to the evaluation of technical efficiency. It will work best when all aspects of the production process can be captured in a limited number of inputs and outputs, and when a relatively large sample is available for analysis. Moreover, it generates a wealth of additional information which can be easily made available for managerial purposes. For example, for each inefficient municipality the set of dominating observations and the identification of the most dominating municipality is particularly useful. Second, the empirical results provide evidence in favor of the use of global efficiency measures. Especially the Generalized Farrell Graph measure seems to be a promising efficiency index that deserves more attention in future work.

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Figure 1 : FDH input section

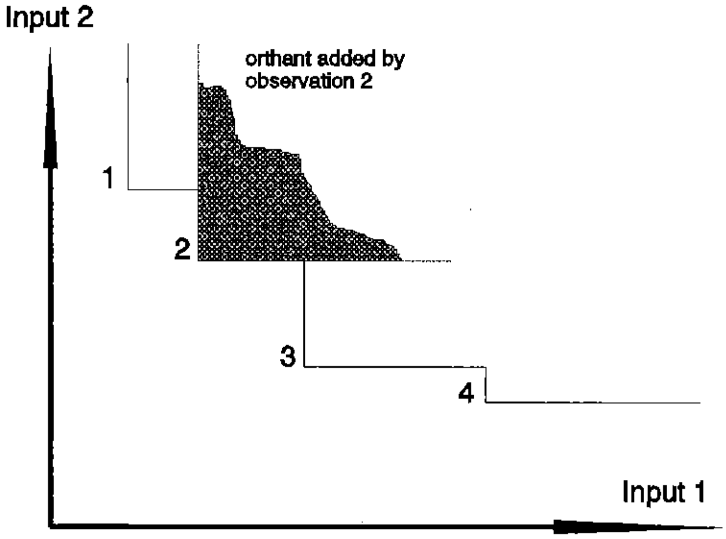

Figure 2 : FDH graph section

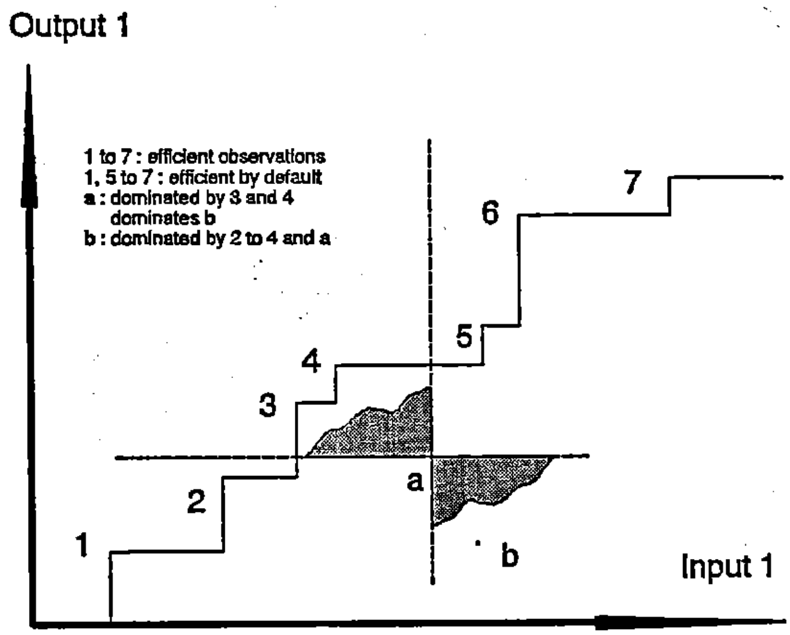

Table 1 : Descriptive statistics : Farrell technical efficiency in the main data set

	Farrell Graph	Farrell Input	Farrell Output	Farrell Gen.Graph
Mean	.927[a]	.861	.878	.893
	.855[b]	**.724**	**.757**	**.788**
Median	1.000	1.000	.999	.995
Minimum	.490	.235	.268	.466
Std Dev	.105	.189	.167	.130
	.107	**.181**	**.160**	**.107**
Kurtosis	4.732	3.245	3.582	2.701
Skewness	-1.492	-1.162	-1.152	-0.901
# Most dom. observations	74(13%)	72(12%)	80(14%)	65(11%)
# Ineffic		297(50%)		
# Efficient by default		59(10%)		

a All observations
b Inefficient obs. only

Table 2 : Correlations between Farrell efficiency measures

	All observations				Inefficient obs. only			
	Farrell Graph	Farrell Inp	Farrell Outp	Farrell G.Graph	Farrell Graph	Farrell Inp	Farrell Outp	Farrell G.Graph
Farrell Graph	1.00				1.00			
Farrell Input	.89	1.00			.78	1.00		
Farrell Output	.89	.81	1.00		.78	.59	1.00	
Farrell Gen. Graph	.94	.94	.93	1.00	.89	.87	.84	1.00

Figure 3 : Histogram frequency : Farrell technical efficiency (inefficient observations only)

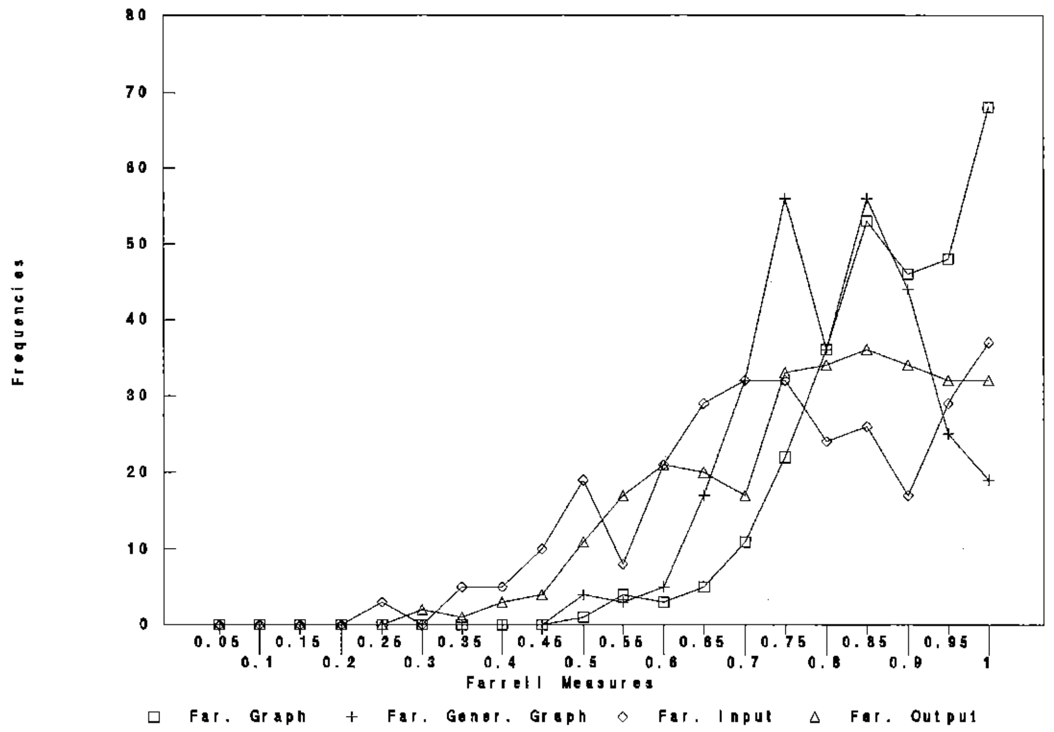

Table 3 : Sensitivity results for samples of different sizes

Size	#Inef Obs.(%)	# Effic Obs. by def(%)	Farrell Graph Mean	Farrell Graph StDev	Farrell Graph #Most domin. Obs.	Farrell Input Mean	Farrell Input StDev	Farrell Input #Most domin. Obs.	Farrell Output Mean	Farrell Output StDev	Farrell Output #Most domin. Obs.	Gen. Farrell Graph Mean	Gen. Farrell Graph StDev	Gen. Farrell Graph #Most domin. Obs.
Main Data Set :														
589	297(50.4)	59(10.0)	.927	.105	74	.861	.189	72	.878	.166	80	.893	.130	65
Size :														
50	8(15.6)	33(6.6)	.980	.054	5	.968	.087	5	.965	.087	5	.970	.072	5
100	24(23.8)	48(4.8)	.973	.064	16	.946	.124	13	.956	.102	13	.957	.089	12
150	51(34.0)	53(35.1)	.959	.079	22	.917	.150	23	.929	.128	23	.935	.107	21
200	72(36.2)	55(27.7)	.955	.082	32	.911	.153	30	.923	.133	29	.929	.111	26
250	92(37.0)	57(23.0)	.949	.090	33	.905	.160	33	.914	.147	35	.924	.118	29
300	124(41.5)	54(18.1)	.945	.093	43	.897	.166	42	.907	.148	46	.918	.120	38
350	148(42.3)	59(17.0)	.943	.095	49	.892	.170	47	.902	.154	48	.914	.122	40
400	183(45.8)	59(14.6)	.937	.099	53	.881	.177	52	.892	.158	56	.906	.125	46
450	218(48.5)	56(12.5)	.931	.102	61	.869	.185	57	.886	.160	65	.898	.129	52
500	237(47.4)	59(11.9)	.934	.099	67	.872	.182	64	.887	.161	69	.901	.126	56
550	269(49.0)	61(11.2)	.930	.103	72	.865	.186	68	.882	.164	77	.896	.129	62
Outliers :														
554	287(51.8)	49(8.8)	.924	.107	68	.856	.190	66	.874	.167	73	.888	.132	60

Table 4 : Sensitivity results for the number of dimensions

#Dim	#Inef Obs.(%)	# Effic Obs. by def(%)	Farrell Graph Mean	Farrell Graph StDev	Farrell Graph #Most domin. Obs.	Farrell Input Mean	Farrell Input StDev	Farrell Input #Most domin. Obs.	Farrell Output Mean	Farrell Output StDev	Farrell Output #Most domin. Obs.	Gen. Farrell Graph Mean	Gen. Farrell Graph StDev	Gen. Farrell Graph #Most domin. Obs.
Aggregation :														
4	440(74.7)	16(2.8)	.878	.125	78	.764	.204	72	.793	.182	76	.827	.134	58
5	371(63.1)	29(4.9)	.912	.102	81	.825	.189	73	.849	.165	84	.870	.125	63
Misspecification:														
3	519(88.2)	4(0.7)	.703	.188	51	.514	.268	44	.558	.251	49	.621	.192	35
4	447(75.8)	9(1.6)	.813	.167	72	.664	.269	65	.710	.240	75	.742	.190	56
5	358(60.8)	20(3.4)	.876	.147	78	.773	.247	88	.808	.216	87	.823	.179	77

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