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VAKGROEP MACRO-ECONOMIE

**Modelling and Forecasting
Belgian Stock Market Prices**

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Abstract

In this paper a selection of seven stock market prices is analysed. We model daily (compound) returns of closing prices for a sample period, running from Eastern 1988 to the end of November 1990.

Since we are principally looking for deviations from the traditional random walk stock returns, also by using the recent concept of 'self-organized criticality' in chaos theory, we construct ARIMA- and transfer function models for the daily stock returns. Finally, forecasts are generated for a 20-day, up to a 60-day, forecasting period.

It is verified that random walk model forecasts are better (in the short run) in only one case, and that in more than half of the cases the time series analyses get better forecasts than the technical (and fundamental) analyses.

Finally, a rough test on robustness is performed by computing new estimates over a sample period from the end of June 1988 to the beginning of February 1991 and making forecasts for 20 periods ahead, and by comparing these results to the estimates and forecasts previously mentioned. Again, technical and fundamental analyses are clearly beaten.

JEL codes: C220, E440, G120

"Study the past if you want to
forecast the future"

Confucius (551 - 478 a.D.)

Introduction

As Campbell (1991) in his recent H.G. Johnson Lecture to the Royal Economic Society states, it is important to distinguish between interpreting and forecasting the movements of the stock market. "To forecast the (financial) market means to predict price changes in the near future. To interpret the (financial) market means to explain, with the benefit of hindsight, why prices have changed in the way they have. This is something which the financial press does almost every day. But the financial press does not impose on itself the discipline of consistency; one day's explanation need not cohere logically with the next day's story. The task for academics is to find an interpretation which can consistently explain stock market movements over a long period of time."

We shall analyse the (recent) development of seven Belgian closing market prices at the Brussels Stock Exchange in this paper, i.e., these of Petrofina, GB-Inno-BM (GIB), Gevaert, Solvay, Glaverbel, CBR and Electrabel (EBES). We shall try to interpret daily stock market returns (excluding weekend data) for a sample period, running from April 11, 1988 to November 26, 1990, and afterwards for a sample period between June 24, 1988 and February 8, 1991, and to forecast these stock market prices over a forecasting period of at least 20 days.

Since the (often advocated) strict 'random walk' theory of stock market prices implies that stock returns are (strictly) unforecastable, so that, under the condition that 'rational bubbles' are ruled out, all unexpected movements in stock prices are assumed to be due to 'news' about future dividends, we shall try to study deviations from random walk stock market prices.

In studying these deviations, we shall make use of ARIMA- and transfer function time series modelling. For the latter aspect, we shall consider possible explanatory variables as exchange rates, oil prices, interest rates, etc.... In this respect, we shall also use some recently developed aspects of chaos theory. Hence, extensive use will be made of time series analysis. Application of multivariate, simultaneous time series analysis will be delayed to a subsequent paper.

In an introductory section, a brief discussion about the measurement of stock market returns and some aspects of time series analysis are stated. The sample data are clarified in section 2, where some brief concepts of 'predictable chaos' are also mentioned. Section 3 contains the analysis and forecasting of the exchange rates, while section 4 discusses the statistical estimation and prediction of the above called Belgian closing prices. Finally, section 5 retrieves some conclusions.

1. Stock Market Returns and some brief Aspects of Time Series Analysis.

In general, we can distinguish three types of stock (market) returns, which can be either returns of shares (or assets) or returns of exchange rates (see, e.g., Taylor (1986)). Denoting the returns, or alternatively, the price differentials, as x_t , we may define :

i) "the absolute or first difference returns" :

$$x_{1t} := s_t + d_t - s_{t-1} \quad (1)$$

ii) "the compound returns" :

$$x_{2t} := \log(s_t + d_t) - \log s_{t-1} \quad (2)$$

iii) "the simple or relative first difference returns" :

$$x_{3t} := \frac{s_t + d_t - s_{t-1}}{s_{t-1}} \quad (3)$$

with s_t the price of, e.g., an asset at period t (month, week, day, hour or even minute) (usually closing prices)

and d_t the dividend (if any) of an asset, payable at period t .

Since the absolute returns x_{1t} depend on the individual price units, comparison among the various absolute returns is very difficult. Moreover, variances of returns are proportional to the price level in this case ('variable-heteroskedasticity').

Therefore, almost everyone uses either the compound returns or the simple returns. According to the Taylor expansion :

$$s_{t-1}^{-1}(s_t + d_t) = x_{3t} + 1 = e^{x_{2t}} = 1 + x_{2t} + \frac{1}{2!} x_{2t}^2 + \frac{1}{3!} x_{2t}^3 + \dots \quad (4)$$

we immediately observe that, if the compound returns are small (which is likely to be true if t stands for days), compound and simple returns are more or less the same. Hectic developments, however, with $|x_{2t}| > 0.10$, can disturb this statement.

Moreover, there exist two fundamental reasons why the compound returns x_{2t} are used more often than the simple returns x_{3t} :

- a) generalizations of discrete time results to continuous time results are simpler with x_{2t} (e-powers !) than with x_{3t} and
- b) compound returns of more than one period are just sums of the (compound) returns of one period, which cannot be said for the simple returns.

For example, neglecting dividends, the simple returns for 2 periods (e.g., 2-day returns) satisfy:

$$X_{3,t+1}^{(2)} := \frac{S_{t+1} - S_{t-1}}{S_{t-1}} = \frac{S_t - S_{t-1}}{S_{t-1}} + \frac{S_{t+1} - S_t}{S_t} \cdot \frac{S_t}{S_{t-1}} = X_{3t} + X_{3,t+1} + X_{3,t+1} \cdot X_{3t} \quad , (5)$$

where an embarrassing product term emerges. For 3-day returns, we get:

$$X_{3,t+1}^{(3)} := \frac{S_{t+2} - S_{t-1}}{S_{t-1}} = X_{3t} + X_{3,t+1} + X_{3,t+2} + X_{3,t+1} \cdot X_{3,t} + X_{3,t+2} \cdot X_{3t} + X_{3,t+2} \cdot X_{3,t+1} + X_{3t} \cdot X_{3,t+1} \cdot X_{3,t+2} \quad , (6)$$

Hence, we prefer the compound returns-definition (2) for our paper (too).

Now, we shall break the compound returns into a component which is a reaction to (other) measured (news) variables, and a residual (often called "noise"). Hence, we can model the compound returns x_{2t} as a time series model.

Inspecting the autocorrelation function and the partial autocorrelation function of the compound returns x_{2t} ($t=1,2,\dots,T$), up to a maximum lag length depending on the sample size (e.g., for about 700 observations, the maximum lag length should be at least 50), we may derive a univariate ARIMA (p, d, q)-model, where p is the (maximum) order of the autoregressive process, d is the order of integration (i.e., the compound returns are found to be stationary after differencing d times) and q is the (maximum) order of the moving average process of the compound returns, viewed as a time series. If no signifi-

cant (partial) autocorrelations of the residuals remain¹, we may generate our forecasts for the ARIMA-model over the forecasting period.

In order to model the compound returns x_{2t} into a 'transitory' and a more 'systematic' or 'permanent' component, we may look for omitted variables which have an impact on the compound returns investigated. Hence, by computing the cross correlation function between the compound returns on the one side and important explanatory variables or "input variables", possessing an impact on these returns, as, e.g., exchange rates, (oil) prices, interest rates, etc..., on the other side, we may identify a transfer function for x_{2t} .

In general, such a transfer function should quantify the economic variables which would determine the compound returns of assets, as, e.g. (see also Fase (1990)):

- i) the growth of the quantity of money, defined as the difference between the nominal money growth and the inflation rate, which might have a positive impact on the general index of asset returns; notice, however, that such a relationship is not very convincing, since a sound economic reasoning is lacking for it, asset returns are more and more internationally determined and the true relationship may also be the reverse: enlarged asset portfolios require more liquidity and quantity of money ('reverse causation', quantity of money demand relationship);
- ii) the external balance (exports minus imports), exchange rates, interest rates, inflation and unemployment rates as main general economic indicators;
- iii) variables underlying the efficient market hypothesis, saying that the expected asset price equals the present value of all expected future after tax dividend payments (i.e., the expected dividends, inflation rates, growth

¹ Under the null hypothesis that the (partial) autocorrelation is equal to zero, the variance of a (partial) autocorrelation coefficient may be approximated by T^{-1} , so that a rough confidence interval for the (partial) autocorrelation coefficient is $\pm 2T^{-1/2}$ (see Plasmans (1990) for further details).

- rates and interest rates are the 'fundamentals');
 iv) the 'speculative bubbles' hypothesis, assuming that speculative behaviour dominates the asset market.

The above mentioned cross correlation analysis is usually made with 'prewhitening', i.e., cross correlation coefficients between an input and the output x_{2t} are usually computed after, firstly, deriving an ARIMA-model for the input variable (and, so, obtaining a white noise residual input) and applying the same 'filter' to the output variable. Simulation studies have demonstrated (see also Plasmans (1990)), however, that it is sometimes better not to 'prewhiten' (PW), i.e., in the case that the standard deviation of the noise term of the endogenous (output) variable is much higher (e.g., 5 times) than the standard deviation of the noise term of the exogenous (input) variable. Hence, identification of transfer functions could be improved by combining cross correlation analyses with PW- and non-PW (or "unwhitened") inputs and outputs.

We shall use the SAS-package (SAS-ETS), both for PC and Main-frame, to perform the statistical estimation. In this paper, only the conditional least squares (CLS) method, setting the pre-sample starting values at zero, is used. Empirical evidence and simulation studies have shown (see Plasmans (1988)) that this method performs well, compared with the method of unconditional least squares (ULS), with back-forecasting of the pre-sample values, and with the exact ML-method, which maximizes the log-likelihood-function of the observed output variable with back-forecasting of its pre-sample values.

2. Our Sample Data and some Elements from Chaos Theory.

In this paper, we investigate compound returns of daily closing rates for seven (large) Belgian firms: Petrofina, GB Inno BM (GIB), Solvay, Glaverbel, CBR and Electrabel (formerly EBES) for a sample period running from April 11, 1988 to November 26, 1990. If we exclude weekends and interpolate linearly for remaining missing values (as, e.g., holidays, days without any closing prices at the Brussels stock exchange), we get 686 daily observations in this way. Afterwards, we shall generate forecasts, in general for a 20 - day forecasting period ($t = 687$ (27/11/90), ..., 706 (24/12/1990)), but also for 30 days ($t = 716$ on January 7, 1991), 40 days ($t = 726$ on January 21, 1991), 50 days ($t = 736$ on February 2, 1991) and 60 days (February 18, 1991). Finally, the sample period is changed to 24/6/88 - 8/2/91 with a 20 period forecasting period.

Since we are studying deviations from random walk modelling, we may search for ARIMA- and transfer functions. Candidate variables having a possible impact on asset (compound) returns are (the returns of) exchange rates, inflation rates, interest rates, growth rates, unemployment rates, quantity of money changes, expected future dividends, external balance variables etc.... Since we have to stick to daily observations, we have chosen the spot rate of Brent-oil in US Dollars per barrel on the London International Oil Market as a measure for prices and the interest rate on 3 months - Eurobonds as a measure for interest rates (the interest rates on 1 month - Eurobonds have a similar pattern). Other variables, except exchange rates, are not available on a daily basis. One can find a list of variables selected and their corresponding symbols in appendix A.

Several authors have advocated the unpredictability of (compound) returns (see, e.g. Fase (1990)). Moreover, when catastrophe strikes, analysts typically blame some rare set of circumstances or some combination of powerful mechanisms. When the (New York) stock market crashed on Black Monday in

1987, economists pointed to the destabilizing effect of computer trading. Therefore, some economists utilized elements of **chaos theory** to explain exchange rate movements and/or asset returns (see, e.g., De Grauwe and Vansanten (1990)). Nevertheless, it was observed that large interactive systems on stock markets organize themselves to a critical state in which a minor event starts a chain reaction that can lead to a catastrophe (see, e.g., Bak and Chen (1991)).

Bak a.o. (1988, 1991) developed the theory of self-organized criticality: many composite systems naturally evolve to a critical state in which a minor event starts a chain reaction that can affect any number of elements in the system.

"According to the theory, the mechanism that leads to minor events is the same one that leads to major events. Furthermore, composite systems never reach equilibrium but instead evolve from one metastable state to the next." (Bak and Chen (1991), p. 26).

These findings originate from the study of earthquakes. According to the **Gutenberg - Richter law** (1956), e.g., the number of earthquakes each year that release a certain amount of energy (intensity), E , is proportional to one divided by E to the power b , where the exponent b is about $3/2$.

Hence, large earthquakes are much more rare than small ones. Because the number of small earthquakes is systematically related to the number of large earthquakes, Bak a.o. suggested that small and large events arise from the same (mechanical) process. Under the Gutenberg - Richter empirical law, the system evolves on the border of chaos ('weak chaos'). Weak chaos differs significantly from fully chaotic behaviour. Fully chaotic systems are characterized by a time scale beyond which it is impossible to make predictions. Weakly chaotic systems lack such a time scale and, so, allow (even) long - term predictions, using, e.g., transfer functions (being similar to 'flicker noise' in physics; see Bak and Chen (1991), p. 28).

In order to test the Gutenberg-Richter law, and, hence, the above theory, for our daily data, we have established two

pictures, i.e., the GIB - returns/US Dollar cross correlations and the Gevaert - returns/DM cross correlations. Considering classes of cross - correlations (being multiplied by 100) as measures for intensity on the x-axis and the number of (absolute) cross correlations falling in each class on the ordinate, we get an approximate (negative) exponential empirical law with exponent at about $-3/2$ indeed. This can be directly seen from the charts below.

This is not true for each cross correlogram, but a γ - type of distribution is always obtained. According to the theory of self - criticality, there is room for forecastability in this case.

Modelling and forecasts will first be made for the exchange rates, since they seem to possess (more or less) important impacts on closing price returns.

$\Delta \ln \text{GIB} / \Delta \ln \text{US\$}$

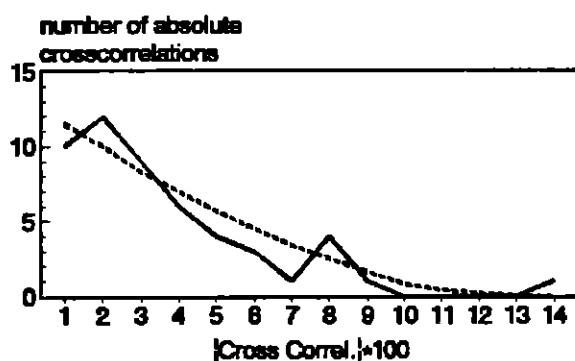


Chart 1

$\Delta \ln \text{Gevaert} / \Delta \ln \text{DM}$

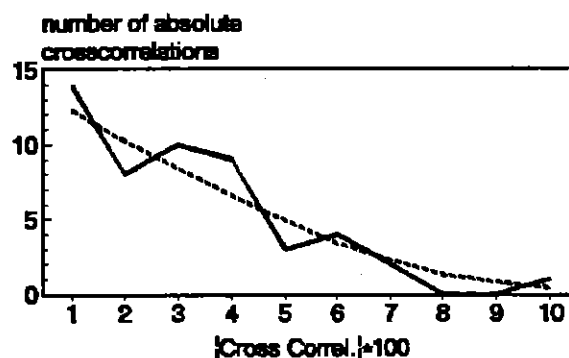


Chart 2

3. Modelling and Forecasting the Exchange Rates

As to Koedijk and Schotman (1990), the low explanatory power of models which attempt to explain exchange rate changes need not be in conflict with the theory : "The overshooting model of Dornbusch (1976), and the asset market approach in general, stresses that exchange rates will be highly sensitive to news, and that the variance of the error term in an exchange rate equation can be large compared with the variance of the explanatory variables" (p. 311).

Symptoms of misspecified exchange rate equations are then : autocorrelated residuals, time varying parameters, structural breaks, heteroskedasticity and omitted variables. Aiming at forecasting the exchange rates of the Belgian Franc (BF) vis à vis the US Dollar (USD), the Deutsche Mark (DM) and the Japanese Yen (JY) respectively, we shall focus on the first and last mentioned sources of misspecification in this paragraph.

In a recent survey, however, Takagi (1988) concluded that empirical exchange rates of major currencies followed a time series process that is closely approximated by a random walk. Monthly data generally showed greater serial dependence than daily data, possibly suggesting the presence of systematic information in low frequency data corresponding to macroeconomic variables. This feature was confirmed by the above cited study of Koedijk and Schotman (1990), who observed that the mutual monthly exchange rates between the USD, the DM, the JY and the BP (British Pound) do not follow a random walk at all, but are strongly influenced by price differentials between wholesale and consumption prices and by relating interest rates during the sample period February 1977 - July 1987.

Although Koedijk and Schotman (1990) observed that individual interest rates had a larger impact than interest rate differentials, we decided to test the Uncovered Interest Rate Parity (UIP-) hypothesis, also for high frequency data as daily exchange rate returns. This UIP-hypothesis follows from the Covered Interest Rate Parity (CIP-) hypothesis, if the expected spot rate and the forward rate of the exchange rate coincide (risk premium equal to zero), so that the nominal

interest rate differential between two countries will be equal to the expected relative change of the exchange rate under (strict) UIP. Hence, for the validity of this (strict) UIP, one has to make some rather strong assumptions (see, e.g., Kirchgässner and Wolters (1989)) : capital has to be perfectly mobile and domestic and foreign bonds are perfect substitutes, which implies that there are to be no transaction costs, no differences in national tax systems regarding capital markets and no risk premia in forward markets, which are in addition regarded as efficient.²

Notice also that departures from random walk can also point to heteroskedasticity. Therefore, we performed statistical tests on the existence of ARMA-models with generalized autoregressive conditional heteroskedasticity (GARCH-effect) for the compound returns of exchange rates Δne_t in a model as (see Hsieh (1989)) :

$$\Delta ne_t = \alpha_0 + \alpha_M D_{Mt} + \alpha_T D_{Tt} + \alpha_W D_{Wt} + \alpha_R D_{Rt} + \alpha_H Hol_t + \sum_{i=1}^p \alpha_i \Delta ne_{t-i} + \eta_t \quad (7)$$

with the error term η_t distributed as $N(0, h_t)$, with the conditional variance h_t satisfying :

$$h_t = \beta_0 + \beta_M D_{Mt} + \beta_T D_{Tt} + \beta_W D_{Wt} + \beta_R D_{Rt} + \beta_H Hol_t + \sum_{i=1}^q \beta_i \eta_{t-i}^2 + \beta h_{t-1} \quad (8)$$

where D_M , D_T , D_W , D_R and Hol are dummies for Monday, Tuesday, Wednesday, Thursday and Holidays (excl. Weekends) respectively.

² In contrast to UIP, CIP states that interest rate differentials equal the difference between the forward and the spot exchange rate. Hence, if a risk premium is not negligible in the foreign exchange market, the forward rate is no longer an unbiased predictor of the next period's spot rate.

Utilizing the method of Maximum Likelihood (ML) for relationships (7-8), maximizing the log likelihood function of the sample data :

$$L(\theta) = -\frac{1}{2T} \sum_{t=1}^T \ln h_t - \frac{1}{T} \sum_{t=1}^T \left(f\left(\frac{\eta_t}{h_t^{1/2}}\right) \right) \quad (9)$$

with respect to the $(m + q + 13)$ -dimensional parameter vector

$$\theta := (\alpha_0, \alpha_M, \alpha_T, \alpha_W, \alpha_R, \alpha_B, \alpha_1, \dots, \alpha_m, \beta_0, \beta_M, \beta_T, \beta_W, \beta_R, \beta_B, \beta_1, \dots, \beta_q, \beta)' , \text{ where } f(.) \text{ is the standard}$$

normal density function, yielded a joint estimation of the parameters in the mean- and variance equations (7-8), also imposing the restrictions $\beta_1 \geq 0$ and $h_t > 0$. Application of ML on the daily returns of the BF vis à vis the USD, the DM and the JY respectively for the sample period $t = 1$ (19/4/1988), ..., 686 (26/11/1990) did not yield any day-effect at all.

It should be noticed that a significant GARCH-effect, $(\beta_1, \beta_2, \dots, \beta_q, \beta)' \neq 0$, was sometimes found. This could already be checked by computing the coefficient of kurtosis, $a_4 = m_4/\sigma^4$, with m_4 the fourth moment about the mean and σ the standard deviation of the compound returns of the relating exchange rates; usually a_4 was found to be larger than 3, pointing to a leptokurtic distribution. Moreover, ML-estimates of a fat-tailed Student t-distributed GARCH(1,1)-model as:

$$\begin{aligned} \Delta \ln e_t &= \mu + \varepsilon_t, \quad \varepsilon_t \sim t(h_t, v) \quad \text{with} \\ h_t &= \beta_0 + \beta_M D_{Mt} + \beta_T D_{Tt} + \beta_W D_{Wt} + \beta_R D_{Rt} + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \end{aligned} \quad (10)$$

and v degrees of freedom, for the BF/DM - [and the BF/USD] exchange rates (685 time periods), yielded no significant day-effect either (t-ratios for $\hat{\beta}_M$, $\hat{\beta}_T$, $\hat{\beta}_W$ and $\hat{\beta}_R$ were 1.36 [0.96], 0.58 [0.84], 1.0 [-0.02] and -1.15 [1.38] respectively). GARCH(1,1)-effects, nevertheless, were very significant:

$$\begin{array}{llll} \hat{\alpha} = 0.088 & [0.087] & \hat{\beta} = 0.905 & [0.851] & \hat{v} = 4.38 & [5.11] \\ (t\text{-ratio}) & (8.72) & [(7.48)] & (79.85) & [(45.22)] & (38.35) & [(34.17)] \end{array}$$

The mean equation is not disturbed too strongly by this GARCH(1,1)-effect, however, so that ARIMA- and transfer functions for the logarithmic exchange rates may be modelled,

although the error terms are in fact not identically distributed with constant variance.

Utilizing the symbols of appendix A, the following departures from a random walk for the three daily exchange rates returns considered were observed for the above mentioned sampling period of 686 time points (where $(|\hat{\epsilon}_i|)$ are the absolute values of the t-ratios):

BF / USD

i) Random walk: $\Delta \ln z_{1t} = \eta_{1t}$; $\hat{\sigma}_{\eta_1} = 0.00664$ (11)

ii) ARIMA: $\Delta \ln \hat{z}_{1t} = (1 - 0.114L + 0.12058L^{15}) \hat{\eta}_{1t}$
 $(|\hat{\epsilon}_i|) \quad (3.02) \quad (3.18) \quad (12)$
 $\hat{\sigma}_{\hat{\eta}_1} = 0.00657197$

iii) Transfer function:

$$\begin{aligned} \Delta \ln \hat{z}_{1t} = & (-2.79213 - 1.09939L^{37}) \Delta \ln z_{2t} + (0.33283 + 0.11606L^{16}) \Delta \ln z_{3t} \\ & (|\hat{\epsilon}_i|) \quad (5.62) \quad (2.29) \quad (7.62) \quad (2.65) \\ & - 0.00532(z_4 - z_5)_{t-28} + 0.02942 \Delta \ln z_{8,t-49} + (1 - 0.1199L) \cdot \\ & (1.89) \quad (3.11) \quad (3.02) \\ & (1 + 0.08376L^{13})(1 + 0.14901L^{15}) \hat{\eta}_{1t} \\ & (2.09) \quad (3.73) \\ & \hat{\sigma}_{\hat{\eta}_1} = 0.00614985 \end{aligned} \quad (13)$$

BF/DM

i) Random walk : $\Delta \ln z_{2t} = \eta_{2t}$; $\hat{\sigma}_{\eta_2} = 0.000491$ (14)

ii) ARIMA: $(1 + 0.13643L^{11})(1 + 0.08937L^{17} - 0.07484L^{18})(1 - 0.09503L^{40}) \cdot$
 $(|\hat{\epsilon}_i|) \quad (3.53) \quad (2.30) \quad (1.93) \quad (2.43)$
 $(1 - 0.10778L^{44}) \Delta \ln \hat{z}_{2t} = (1 - 0.07625L^7)(1 + 0.11458L^{37}) \hat{\eta}_{2t}$
 $(2.76) \quad (1.97) \quad (2.94)$
 $\hat{\sigma}_{\hat{\eta}_2} = 0.00047577 \quad (15)$

iii) Transfer function :

$$\begin{aligned} \Delta \ln \hat{z}_{2t} = & -0.01653 \Delta \ln z_{1t} + (0.0119 - 0.00567L^{12} + 0.01084L^{28} \\ & (|\hat{\epsilon}_i|) \quad (6.60) \quad (3.88) \quad (1.84) \quad (3.38) \\ & + 0.01116L^{34}) \Delta \ln z_{3,t-16} + 0.00058(z_4 - z_6)_{t-10} \\ & (3.43) \quad (2.55) \\ & + [(1 + 0.17052L^{11})(1 + 0.14667L^{17})(1 - 0.08671L^{37} \\ & (4.26) \quad (3.64) \quad (2.14) \\ & - 0.15586L^{44})]^{-1} (1 - 0.10369L^7)(1 + 0.09230L^{24}) \hat{\eta}_{2t} \\ & (3.82) \quad (2.59) \quad (2.28) \end{aligned} \quad (16)$$

$$\hat{\sigma}_{\hat{\eta}_2} = 0.00044562$$

BF / JY

i) Random walk : $\Delta \ln z_{3t} = \eta_{3t} ; \quad \hat{\sigma}_{\eta_3} = 0.005355 \quad (17)$

ii) ARIMA :

$$\begin{aligned} & (1-0.09100L^2-0.09839L^8)(1+0.08019L^{20})(1+0.10502L^{47}) \Delta \ln z_{3t} = \\ & (|\hat{\epsilon}_1|) \quad (2.38) \quad (2.57) \quad (2.07) \quad (2.60) \\ & (1-0.07881L^{13})(1+0.10169L^{33}) \hat{\eta}_{3t} \\ & \quad (2.04) \quad (2.57) \\ & \hat{\sigma}_{\eta_3} = 0.0052561 \quad (18) \end{aligned}$$

iii) Transfer function :

$$\begin{aligned} \Delta \ln z_{3t} = & (0.22476-0.07170L^{15}) \Delta \ln z_{1t} + (1.27295-0.69917L^6 \\ & (|\hat{\epsilon}_1|) \quad (7.99) \quad (2.56) \quad (3.26) \quad (1.84) \\ & + 1.53902L^7 + 1.40758L^{23}) \Delta \ln z_{2,t-22} - 0.00576(z_4 - z_7)_{t-11} \\ & \quad (4.00) \quad (3.72) \quad (2.16) \\ & + (-0.02094 + 0.01865L^2 - 0.01849L^3 + 0.02815L^6) \Delta \ln z_{8,t-37} \\ & \quad (2.82) \quad (2.45) \quad (2.47) \quad (3.77) \\ & + [(1-0.10880L^2 - 0.08456L^7)(1-0.09026L^{15})]^{-1} \\ & \quad (2.70) \quad (2.09) \quad (2.20) \\ & (1 - 0.08335L^{20})(1 + 0.07105L^{30} + 0.10277L^{33}) \hat{\eta}_{3t} \\ & \quad (2.02) \quad (1.67) \quad (2.43) \\ & \hat{\sigma}_{\eta_3} = 0.00486897 \quad (19) \end{aligned}$$

Since the sample standard error difference between the estimated ARIMA-model and the estimated transfer function model is relatively smallest for the BF/USD exchange rate compound returns, and since we do not use simultaneous time series analysis in this paper, the transfer functions (16) and (19) and the ARIMA-function (12) were chosen to generate (best possible) forecasts of the BF/DM-, BF/JY- and BF/USD exchange rates respectively (see Tables 1 and 2; 20-day forecasts for the BF/DM and BF/JY).

**Table 1: Forecasts of the BF/DM exchange rates: $t=687$
(27/11/90) - 706 (24/12/90)**

t	Observed DEM exchange rate	Random walk (14)	ARIMA- model (15)	Transfer function model (16)
686	20.6008	20.6008	20.6008	20.6008
687	20.598	20.6008	20.5975	20.6005
688	20.6040	20.6008	20.5963	20.5981
689	20.5975	20.6008	20.5934	20.5987
690	20.6080	20.6008	20.5941	20.5967
691	20.6205	20.6008	20.5917	20.5988
692	20.6335	20.6008	20.5901	20.5930
693	20.6460	20.6008	20.5927	20.5945
694	20.664	20.6008	20.5910	20.5913
695	20.666	20.6008	20.5886	20.5887
696	20.6775	20.6008	20.5908	20.5870
697	20.6755	20.6008	20.5930	20.5919
698	20.6705	20.6008	20.5924	20.5953
699	20.6740	20.6008	20.5951	20.6013
700	20.6755	20.6008	20.5929	20.6078
701	20.6665	20.6008	20.5941	20.6113
702	20.6445	20.6008	20.5928	20.6134
703	20.6505	20.6008	20.5936	20.6152
704	20.6425	20.6008	20.5936	20.6129
705	20.6250	20.6008	20.5934	20.6173
706	20.5950	20.6008	20.5937	20.6180
Exchange rates U1		0.00243	0.00277	0.00251
Exchange rates U2		0.00121	0.00138	0.00126
Compound returns U1		1.00000	1.00458	1.12605
Compound returns U2		1.00000	0.87138	0.88354

Table 2: Forecasts of BF/JY exchange rates: t=687
(27/11/90) - 706 (24/12/90)

t	Observed YEN exchange rate	Random walk (17)	ARIMA- model (18)	Transfer function model (19)
686	23.8350	23.8350	23.8350	23.8350
687	23.8000	23.8350	23.8688	23.7681
688	23.5725	23.8350	23.8632	23.9307
689	23.4550	23.8350	23.9174	23.8528
690	23.2700	23.8350	23.9028	23.7585
691	23.1050	23.8350	23.9651	23.8223
692	23.2075	23.8350	23.9972	23.8134
693	23.0225	23.8350	23.9700	23.7542
694	23.1625	23.8350	23.9654	23.8124
695	23.3100	23.8350	23.9586	23.7768
696	23.1475	23.8350	23.9122	23.8269
697	23.0850	23.8350	23.8914	23.9043
698	23.2325	23.8350	23.8494	23.9411
699	23.1825	23.8350	23.8451	23.9540
700	23.1625	23.8350	23.8352	23.8840
701	23.0775	23.8350	23.8113	23.8775
702	23.0725	23.8350	23.7931	23.7917
703	22.8375	23.8350	23.7611	23.7656
704	22.9000	23.8350	23.7745	23.6473
705	23.1050	23.8350	23.7138	23.6713
706	23.3150	23.8350	23.7132	23.6815
Exchange rates U1		0.02884	0.03008	0.02788
Exchange rates U2		0.01422	0.01483	0.01376
Compound returns U1		1.00000	1.03640	1.11326
Compound returns U2		1.00000	0.85418	0.76009

These ex ante forecasts are compared with the ex post observations utilizing as forecast performance indices the absolute and relative Theil's inequality indices:

$$U_1 := \frac{\sqrt{\sum_{t \in T_f} (x_t - \hat{x}_t)^2}}{\sqrt{\sum_{t \in T_f} x_t^2}} \quad \text{and} \quad U_2 := \frac{\sqrt{\sum_{t \in T_f} (x_t - \hat{x}_t)^2}}{\sqrt{\sum_{t \in T_f} x_t^2} + \sqrt{\sum_{t \in T_f} \hat{x}_t^2}} \quad (20)$$

respectively, where T_f is the forecasting period.

U_1 is (assumed to be) positive and is considered to be good if it is smaller than 0.4, while U_2 is always between zero and one.

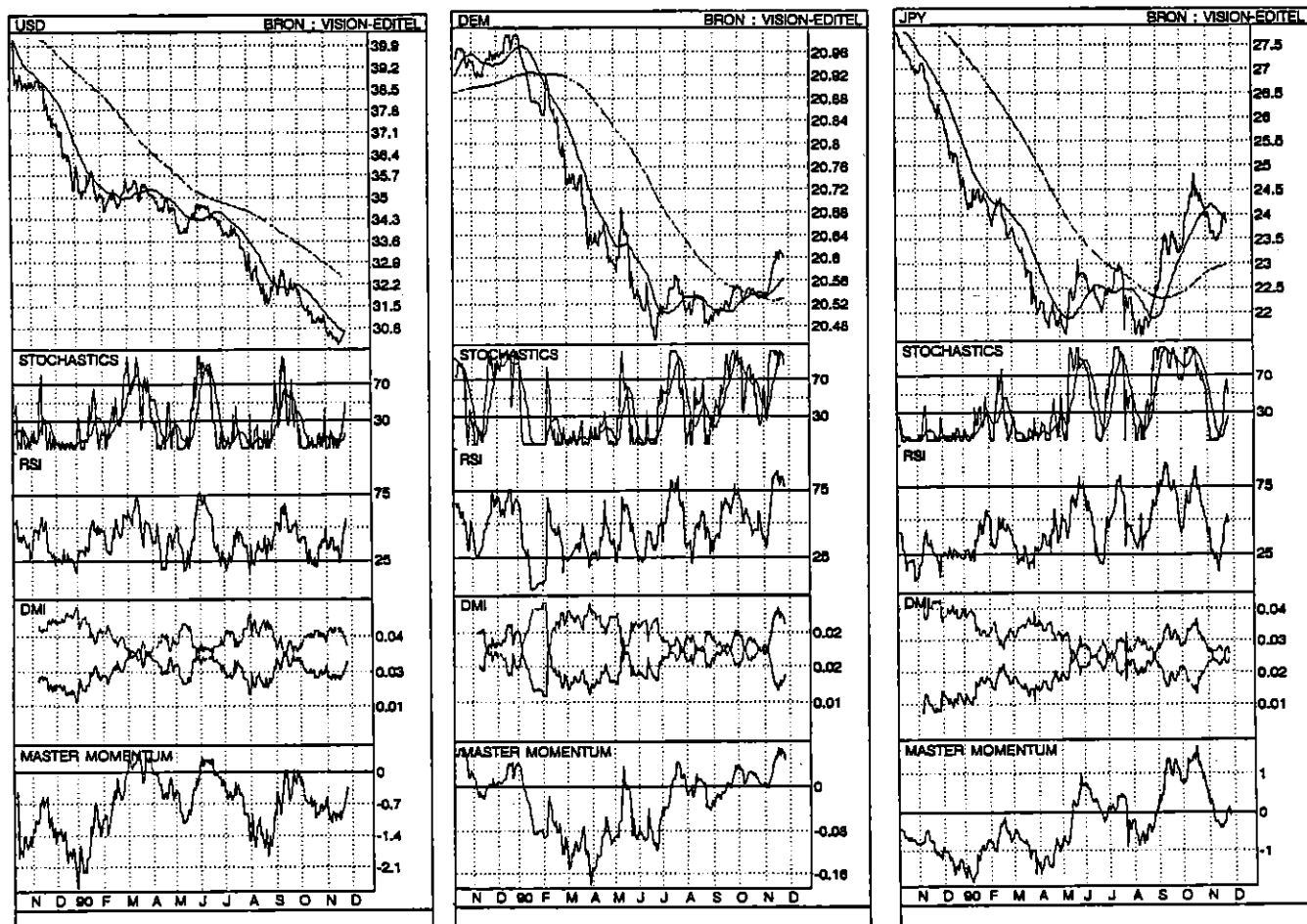
For the BF/DM exchange rate a random walk predicts the best within the 20-day period; for the BF/JY exchange rate the transfer function performs better.

On a sixty-day basis (± 3 months), the BF/DM-exchange rate is predicted to be more or less constant at about 20.6 BF per DM, while the BF/JY-exchange rate is predicted to vary between 23.6 and 23.9 BF per JY.

According to the technical analysis (see Table 3), performed on the 27th of November 1990, the BF/USD exchange rate was predicted to (further) decrease and the BF/DM- and BF/JY-exchange rates were predicted to increase. We may observe that reality developed differently, and that our exchange rate forecasts were not that bad.

Recomputing univariate time-series models for the various exchange rates for a sampling period running from June 24, 1988 to February 8, 1991 (observation 55 to observation 740) and comparing for best forecasts over 20 days, we found that the random walk models for the three exchange rates perform relatively best.

Table 3: Technical analysis for the BF/USD, BF/DM and BF/JY exchange rates.



	45.9399
	44.5888
	43.2775
	42.0047
	40.7893
	39.5703
	38.4098
	37.277
	36.1807
	35.1186
	34.0839
	33.0615
	32.1085
	31.1642
	30.2477
	30.7075

	31.0684
	30.1547
	29.2878
	28.407
	27.5716
	26.7807
	25.9737
	25.2088
	24.4884
	23.7488
	23.0503
	22.3724
	21.7146
	21.0758
	20.456
	20.8008

	32.8882
	31.7288
	30.7837
	29.8881
	29.0081
	28.1556
	27.3279
	26.5242
	25.7441
	24.987
	24.2521
	23.5386
	22.8486
	22.1747
	21.5225
	23.835

4. Estimating and Forecasting (the Returns of) the Closing Prices.

Tables 1 to 7 in appendix B of this paper contain the specification for the selected random walk-, ARIMA- and transfer function models for the Petrofina, GIB, Gevaert, Solvay, Glaverbel, CBR and Electrabel (EBES) shares respectively. The transfer function models differ in that respect only that the identification phase occurred differently: either the inputs and the relating output were all prewhitened (PW) or a combination of PW and non-PW ("unwhitened") inputs and relating output is considered in order to compute the crosscorrelations. In general, the statistical estimations were improved by also considering unwhitened inputs and output. Furthermore, the ex ante forecasts of the exchange rates could be improved by also considering the other exchange rates and other exogenous variables. Hence, ex ante forecasts for those exchange rates, occurring in the transfer functions for the asset returns, can be constructed, either by simple ARIMA-structures (see (12), (15) and (18)) or by the transfer functions (16) and (19). Since the BF/USD was explained relatively best of all 3 exchange rates by an ARIMA-model and since we do not model simultaneously (yet), we preferred to utilize equations (12), (16) and (19) for the exchange rates of the BF vis à vis the USD, the DM and the JY respectively. In the sequel we shall briefly discuss the forecasting capability of each asset price model.

Notice, however, that constant error variances of the ARIMA- and transfer function models for the 7 asset returns considered are assumed for estimation, and even normality of the corresponding errors is adopted for (most) testing.

In fact, daily data of stock returns are not (always) normally distributed with constant variance, so that ARIMA-modelling may be (very) doubtful. Time varying conditional variances and fat-tailed error distributions are often observed and should be treated appropriately. In the early seventies it was reported that many stock return distributions are fat-tailed (see, e.g., Blattberg and Gonedes (1974) and, more recently, the GARCH-model has been frequently used in studies on stock return behaviour (e.g. in Chou (1988), Akgiray (1989)

and Connolly (1989)). We shall study this eventually occurring GARCH-behaviour of the 7 asset returns considered in a subsequent paper. Since emphasis is concentrated on forecasting of asset prices in this paper, eventually occurring GARCH-characteristics of these 7 asset returns may be considered as 'not too harmful' for our sake³.

4.1 Petrofina

Inspecting the predictions of the Petrofina closing prices in table 1B we may remark that we overestimated the ex post development of the closing prices (Kuwait war!), but that for the first 50 forecasting periods the random walk model overestimated even more. Looking at the inequality coefficients below we remark that the best closing price forecasts are obtained with the help of transfer function (1.3.2) with improved exchange rate equations (16) and (19). Compound returns of the Petrofina closing prices, however, are predicted best by equation (1.3.1). From the technical analysis charts in appendix C, a further increase of the Petrofina closing prices is predicted (buy signal) while reality showed a decrease (from the end of November 90 until half of January 1991). Notice, that our forecasts do not indicate an increase either, but rather a very moderate decrease (even in the long run up to 60 periods; while on February 4 1991 equation (1.3.2) predicted 10174 for the closing price and in reality it was only 9920, 10 working days later (February 18, 1991) a too low prediction emerged (10188 compared to 10950 in reality)).

Petrofina	Random walk (1.1)	ARIMA- model (1.2)	Transfer function models		
			(1.3.1) ARIMA exch. rates	(1.3.2) improved exch. rates	(1.3.2) improved exch. rates
Clos.prices U1	0.02975	0.03162	0.03091	0.03060	0.02711
Clos.prices U2	0.01469	0.01560	0.01525	0.01510	0.01339
Comp>Returns U1	1.00000	0.99390	0.96625	0.97032	1.06053
Comp>Returns U2	1.00000	0.95855	0.63278	0.62951	0.66386

³ See Baillie and Bollerslev (1990) for predictions in dynamic models with time dependent conditional variances.

Changing the sample period from 24/6/88 to 8/2/91, we find ARIMA-model (1.2)' and transfer function (1.3.1)', with corresponding inequality coefficients:

Petrofina	Random walk (1.1)'	ARIMA- model (1.2)'	Transfer function models (1.3.1)' (1.3.2)'	
Clos.prices U1	0.06346	0.06211	0.04727	0.04810
Clos.prices U2	0.03267	0.03196	0.02415	0.02458
Comp>Returns U1	1.00000	0.98969	1.07973	1.08795
Comp>Returns U2	1.00000	0.95430	0.77421	0.78630

Inequality coefficients are somewhat larger, for an important part due to the Gulf War. Notice that the best forecasts are obtained now from the transfer function, being identified with PW inputs only.

4.2 GIB

As becomes clear from the inequality coefficients below, it is directly verified that all coefficients in the estimated relationships for the first sampling period are very low, but that the best 20-day prediction for the GIB closing price is obtained by the ARIMA-model (2.2). At the end of November 1990, the technical analysis charts indicated a further decrease, which did not come out in our transfer function forecasts. Our ARIMA-predictions decreased very moderately. Concluding, we may state that our models generate good GIB closing price forecasts.

GIB	Random walk (2.1)	ARIMA- model (2.2)	Transfer function models (2.3.1) (2.3.2) ARIMA improved improved exch. exch. exch. rates rates rates		
Clos.prices U1	0.01730	0.01057	0.01932	0.01923	0.01751
Clos.prices U2	0.00859	0.00528	0.00959	0.00954	0.00870
Comp.returns U1	1.00000	1.02283	1.03736	1.03522	1.06144
Comp.returns U2	1.00000	0.85821	0.81575	0.80990	0.78023

For the second sampling period, the random walk predicts best. The increase with $\pm 11\%$ in the GIB closing prices can not be predicted very well with the ARIMA model nor with the transfer function model.

GIB	Random walk (2.1)'	ARIMA- model (2.2)'	Transfer function model (2.3.1)'
Clos.prices U1	0.10312	0.10545	0.12114
Clos.prices U2	0.05426	0.05555	0.06435
Comp>Returns U1	1.00000	1.01072	1.00176
Comp>Returns U2	1.00000	0.95027	0.79533

4.3 Gevaert

A striking result from the inspection of the inequality coefficients for a 20-day forecasting experiment, given below, is that the simple random walk model (3.1) predicts the Gevaert closing prices best vis à vis ARIMA-model (3.2) and transfer function models (3.3.1) and (3.3.2), although the corresponding residual sample standard errors are lower. This originates from the fact that the ARIMA-model and the transfer function models overestimate too much the realized development of the Gevaert asset closing price during the 20-day forecasting period. In the long run (60 days), however, the transfer function performs relatively best (6566 on February 18 1992 by (3.3.1) and 6520 by (3.1), compared with 6960 as realized value).

Gevaert	Random walk (3.1)	ARIMA- model (3.2)	Transfer function models		
			(3.3.1) ARIMA exch. rates	(3.3.2) improved exch. rates	(3.3.2) improved exch. rates
Clos.prices U1	0.02265	0.02742	0.02653	0.02591	0.03206
Clos.prices U2	0.01125	0.01357	0.01313	0.01283	0.01583
Comp.returns U1	1.00000	1.02156	1.01714	1.02991	1.02330
Comp.returns U2	1.00000	0.91690	0.83575	0.85388	0.84514

In the second forecasting period transfer function (3.3.1)' performs best, while the random walk forecasting value remains at a too low price level.

Gevaert	Random walk (3.1)'	ARIMA- model (3.2)'	Transfer function models	
			(3.3.1)'	(3.3.2)'
Clos.prices U1	0.11700	0.12623	0.11521	0.11617
Clos.prices U2	0.06172	0.06697	0.06080	0.06135
Comp>Returns U1	1.00000	1.02061	1.03537	1.10107
Comp>Returns U2	1.00000	0.88574	0.86284	0.83760

4.4 Solvay

According to the inequality coefficients below the best predictions occur with the help of transfer function (4.3.2), although the ARIMA-model (4.2) also generates reasonable forecasts on a 20-day basis. Notice that, in contrary to the other variables' forecasts, our forecast on a 20-day basis was too low. This underestimation also remains in the long run. From the detailed inspection of the technical analysis in appendix C a buy signal could emerge and a price increase can be expected. Why? Three reasons can be mentioned. Firstly, the Solvay closing price line crosses the moving average line from downstairs to upstairs at the end of October '90; secondly, we can observe a triangle with upwards breakthrough and finally, the (long term) Solvay asset price comes in the neighbourhood of the support line. Predictions should be improved.

Solvay	Random walk (4.1)	ARIMA- model (4.2)	Transfer function models		
			(4.3.1) ARIMA exch. rates	(4.3.2) improved exch. rates	(4.3.2) improved exch. rates
Clos.prices U1	0.05607	0.05595	0.06325	0.06327	0.05574
Clos.prices U2	0.02871	0.02869	0.03253	0.03254	0.02856
Comp.returns U1	1.00000	0.92959	1.09125	1.09143	1.06520
Comp.returns U2	1.00000	0.77875	0.84051	0.84062	0.79620

Transfer function (4.3.2)' performs also best in the forecasting period, using the sampling estimates for the second period. Notice also that the Solvay closing price predictions are (much) better for this second forecasting period than for the first one, although the Gulf War came to its end in this period.

Solvay	Random walk (4.1)'	ARIMA- model (4.2)'	Transfer function models	
			(4.3.1)'	(4.3.2)'
Clos.prices U1	0.05572	0.04273	0.05011	0.03394
Clos.prices U2	0.02849	0.02167	0.02554	0.01710
Comp.Returns U1	1.00000	1.01529	1.07885	1.11503
Comp.Returns U2	1.00000	0.79950	0.83124	0.80279

4.5 Glaverbel

The random walk (5.1) performs very badly and, according to the inequality coefficients presented below, transfer function (5.3.2) is preferred. Nevertheless, the Glaverbel closing price is generally overestimated (in the long run). Notice, however, that the sell-signal (predicted decrease) from the technical analysis is neither realized nor predicted; on the contrary, equation (5.3.2) forecasts an increase and the Glaverbel asset price increased in practice too.

Glaverbel	Random walk (5.1)	ARIMA- model (5.2)	Transfer function models (5.3.1) (5.3.2)		
			ARIMA exch. rates	improved exch. rates	improved exch. rates
Clos.prices U1	0.05824	0.03935	0.05436	0.05482	0.05509
Clos.prices U2	0.02980	0.01981	0.02663	0.02686	0.02693
Comp.returns U1	1.00000	1.14190	1.29156	1.31767	1.25953
Comp.returns U2	1.00000	0.76520	0.73610	0.73592	0.69547

The random walk model, however, yields the best predictions in the second experiment when changing the sample period.

Glaverbel	Random walk (5.1)'	ARIMA- model (5.2)'	Transfer function models (5.3.1)' (5.3.2)'	
Clos.prices U1	0.12176	0.12670	0.13288	0.12618
Clos.prices U2	0.06426	0.06703	0.07066	0.06691
Comp>Returns U1	1.00000	1.04093	1.05060	1.13311
Comp>Returns U2	1.00000	0.83348	0.75379	0.76446

4.6 CBR

The random walk model seriously underestimates the future development of the CBR closing prices. According to the inequality coefficients, the best forecasts emerge by far from the transfer function (6.3.2), which is also true for the compound returns of this asset. Good forecasts are obtained, which point to an increase and not to a 'steady state' as suggested from the technical analysis.

CBR	Random walk (6.1)	ARIMA- model (6.2)	Transfer function models		
			(6.3.1) ARIMA exch. rates	(6.3.2) improved exch. rates	(6.3.2) improved exch. rates
Clos.prices U1	0.03871	0.04052	0.02693	0.02644	0.02638
Clos.prices U2	0.01966	0.02058	0.01354	0.01330	0.01327
Comp.returns U1	1.00000	1.10692	1.18006	1.14468	0.95971
Comp.returns U2	1.00000	0.88153	0.80044	0.77602	0.65790

A similar picture is obtained when translating the sampling and forecasting periods to more recent dates.

CBR	Random walk (6.1)'	ARIMA- model (6.2)'	Transfer function models	
			(6.3.1)'	(6.3.2)'
Clos.prices U1	0.16033	0.15941	0.13185	0.12247
Clos.prices U2	0.08655	0.08601	0.07017	0.06489
Comp>Returns U1	1.00000	1.00282	0.96156	0.94826
Comp>Returns U2	1.00000	0.95832	0.79491	0.73982

4.7 Electrabel (EBES)

Transfer function (7.3.2) with underlying identification based on a combination of prewhitened and unwhitened inputs and output yields the best forecasts, being considerably better than the random walk forecasts.

Electrabel	Random walk (7.1)	ARIMA- model (7.2)	Transfer function models		
			(7.3.1) ARIMA exch. rates	(7.3.2) improved exch. rates	(7.3.2) improved exch. rates
Clos.prices U1	0.01736	0.01325	0.01225	0.01204	0.01132
Clos.prices U2	0.00861	0.00659	0.00610	0.00599	0.00564
Comp.returns U1	1.00000	1.00412	1.03040	1.03078	1.07968
Comp.returns U2	1.00000	0.94166	0.84537	0.82882	0.87134

For the second sampling period, however, the random walk predicts best.

Electrabel	Random walk (7.1)'	ARIMA- model (7.2)'	Transfer function models	
			(7.3.1)'	(7.3.2)'
Clos.prices U1	0.02190	0.02208	0.02840	0.02747
Clos.prices U2	0.01106	0.01116	0.01439	0.01391
Comp>Returns U1	1.00000	1.00667	1.15013	1.14798
Comp>Returns U2	1.00000	0.99102	0.80551	0.79774

5. Some concluding Remarks

In this paper deviations from random walk processes for (compound) returns of some major Belgian closing prices at the Brussels Stock Exchange have been investigated. Although utilizing high frequency data, the often supported random walk hypothesis yielded better (short term) predictions only once, i.e., for the Gevaert closing prices.

Furthermore, the technical analyses showed in more than half of the cases a wrong strategy direction (e.g. Petrofina, Glaverbel, CBR, Electrabel and to a lesser extent GIB).

By changing the sampling period from 11/4/1988 - 26/11/1990 to 24/6/1988 - 8/2/1991, both with sample size 686 observations, and corresponding 20-day forecasting periods, we could observe that, although the point estimates changed, the corresponding forecasts were comparable. Due to the Gulf War, most of the predictions worsened, except for GIB and Solvay. In this second period, the technical analyses give only a correct picture for Solvay. In the other cases a decrease of the closing prices is predicted instead of an increase.

All in all, this (very) rough test of robustness was rather successful, except for the prediction of the exchange rates, where we had to work with naive random walk predictions for the second forecasting period.

The theoretical and empirical apparatus has still to be improved in the near future, as, e.g., the use of multivariate, simultaneous models where compound asset returns can explain each other (better), intervention models (as, e.g. the Kuwait-war), structural break models implying models with time-varying coefficients. The analysis should be broadened to foreign assets and foreign stock exchanges as well. Comparison with weekly/monthly time series could also be very informative.

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Appendix AList of SymbolsA. Endogenous Variables

- $y_{1t} :=$ the (compound) return of one Petrofina share at period t , defined as the difference between the natural logarithm of the sum of the closing price of one Petrofina share and its dividend paid at period (day) t (which is zero most of the time) and the natural logarithm of the closing price of one Petrofina share at period (day) $t-1$;
- $y_{2t} :=$ the (compound) return of one GIB share at period t ;
- $y_{3t} :=$ the (compound) return of one Gevaert share at period t ;
- $y_{4t} :=$ the (compound) return of one Solvay share at period t ;
- $y_{5t} :=$ the (compound) return of one Glaverbel share at period t ;
- $y_{6t} :=$ the (compound) return of one CBR share at period t ;
- $y_{7t} :=$ the (compound) return of one Electrabel (Ebes) share at period t ;

B. Exogenous Variables

- $z_{1t} :=$ the nominal exchange rate of Belgian Franc w.r.t. one US Dollar at period t ;
- $z_{2t} :=$ the nominal exchange rate of Belgian Franc w.r.t. one German Mark at period t ;
- $z_{3t} :=$ the nominal exchange rate of Belgian Franc w.r.t. one Japanese Yen at period t ;
- $z_{4t} :=$ the nominal interest rate for Eurobonds on 3 months in Belgian Francs at period t ;
- $z_{5t} :=$ the nominal interest rate for Eurobonds on 3 months in US Dollars at period t ;
- $z_{6t} :=$ the nominal interest rate for Eurobonds on 3 months in German Marks at period t ;
- $z_{7t} :=$ the nominal interest rate for Eurobonds on 3 months in Japanese Yens at period t ;
- $z_{8t} :=$ the spot price of Brent-oil in US Dollar per barrel at the London International Oil Market.

Appendix BTable 1Petrofina closing pricesA Sampling Estimates of Petrofina returns

$$\underline{y_{1t} := \ln (s_{1t} + d_{1t}) - \ln s_{1,t-1}}$$

$$(t = 1 \text{ (11/4/88)}, \dots, 686 \text{ (24/11/90)}) \\ [(t = 55 \text{ (24/6/88)}, \dots, 740 \text{ (8/2/91)})]$$

(1.1) Random walk model

$$y_{1t} = \varepsilon_{1t}$$

$$\hat{\theta}_{\varepsilon_1} = 0.011639$$

(significant (partial) autocorrelations at peaks 1 and 11)

$$[\hat{\theta}_{\varepsilon_1} = 0.011915] \quad (1.1)'$$

(1.2) ARIMA model (CLS)

$$(1 + 0.08481L)(1 - 0.08109L^{11})\hat{y}_{1t} = \hat{\varepsilon}_{1t}$$

$$(|\hat{\varepsilon}_i|) \quad (2.22) \quad (2.12)$$

$$\hat{\theta}_{\varepsilon_1} = 0.01157993$$

(no significant (partial) autocorrelation of the residuals)

$$[\hat{y}_{1t} = (1 + 0.008293L^{10} - 0.07773L^{11})\hat{\varepsilon}_{1t}] \quad (1.2)'$$

$$(2.17) \quad (2.03)$$

$$[\hat{\theta}_{\varepsilon_1} = 0.01186449]$$

(1.3) Transfer function models (CLS)(1.3.1) Identification with prewhitened (PW) inputs and output

$$\hat{y}_{1t} = 0.16252\Delta \ln z_{1t} + (-1.51940 - 2.79942L^{25})\Delta \ln z_{2,t-23} +$$

$$(|\hat{\varepsilon}_i|) \quad (2.51) \quad (1.67) \quad (3.09)$$

$$+ (0.18517 + 0.18612L^9)\Delta \ln z_{3,t-14} - 0.20096\Delta \ln z_{4,t-11} +$$

$$(2.29) \quad (2.29) \quad (2.97)$$

$$+ (-0.06261 + 0.04444L^{13})\Delta \ln z_{8,t-35} + [(1 - 0.08671L)$$

$$(3.59) \quad (2.52) \quad (2.16)$$

$$(1 - 0.08095L^{10})(1 + 0.12512L^{11})(1 + 0.06947L^{42})]^{-1}\hat{\varepsilon}_{1t}$$

$$(2.01) \quad (3.13) \quad (1.70)$$

$$\hat{\theta}_{\epsilon_1} = 0.01124505$$

(no significant (partial) residual-autocorrelations)

$$\begin{aligned} [\hat{y}_{1t} = & (-0.01909 + 0.02686L) \Delta \ln z_{2,t-1} + (0.15606 + 0.21666L^{10} + \\ & + 0.16954L^{19} + 0.23399L^{26}) \Delta \ln z_{3,t-4} + (-0.23513 + 0.19461L^{36}) \Delta \ln z_{4,t-11} \\ & + (-0.04427 - 0.02708L^{13} - 0.05264L^{35}) \Delta \ln z_{8,t} + \\ & + [(1 - 0.08165L^{10})(1 + 0.11157L^{11})]^{-1} \hat{\epsilon}_{1t}] \end{aligned}$$

(1.3.1)'

$$[\hat{\theta}_{\epsilon_1} = 0.0111115]$$

(1.3.2) Identification with a combination of PW and unwhitened inputs and output

$$\hat{y}_{1t} = 0.16813 \Delta \ln z_{1t} + (-1.60080 - 3.1639L^{25}) \Delta \ln z_{2,t-23} +$$

$$(|\hat{\epsilon}_t|) \quad (2.61) \quad (1.79) \quad (3.53)$$

$$+ (0.17409 + 0.15225L^9 + 0.23108L^{16}) \Delta \ln z_{3,t-14} + (-0.20546 +$$

$$(2.15) \quad (1.88) \quad (2.82) \quad (3.14)$$

$$+ 0.19028L^{35} + 0.14729L^{36}) \Delta \ln z_{4,t-11} + (-0.06020 + 0.03496L^{13}) \Delta \ln z_{8,t-35}$$

$$(2.86) \quad (2.22) \quad (3.52) \quad (2.02)$$

$$+ [(1 + 0.08191L^9)(1 + 0.12763L^{11})]^{-1} \hat{\epsilon}_{1t}$$

$$(2.02) \quad (3.19)$$

$$\hat{\theta}_{\epsilon_1} = 0.01111375$$

(no significant (partial) residual autocorrelations)

$$\begin{aligned} [\hat{y}_{1t} = & 0.13836 \Delta \ln z_{1,t-8} + 0.03506 \Delta \ln z_{2,t-2} + (0.15200 + 0.22255L^{10} + \\ & + 0.16778L^{16} - 0.15116L^{17} + 0.17443L^{19} + 0.23295L^{26}) \Delta \ln z_{3,t-4} + \\ & + (-0.24077 + 0.15508L^{35} + 0.18024L^{36}) \Delta \ln z_{4,t-11} + (-0.04531 - \\ & - 0.03171L^{13} - 0.05100L^{35}) \Delta \ln z_{8,t} + [(1 - 0.07381L^{10}) \\ & (1 + 0.12640L^{11})]^{-1} (1 - 0.08978L^9) \hat{\epsilon}_{1t}] \end{aligned}$$

(1.3.2)'

$$[\hat{\theta}_{\epsilon_1} = 0.01097246]$$

B. Forecasts of Petrofina closing prices:t=687 (27/11/90) - 706 (24/12/90)t=741 (09/02/91) - 760 (08/03/91)

t	Observed Petrofina closing prices	Random walk (1.1)	ARIMA- model (1.2)	Transfer function models		
				(1.3.1) ARIMA exch. rates	(1.3.2) improved exch. rates	(1.3.2) improved exch. rates
686	10300	10300	10300	10300	10300	10300
687	10225	10300	10300	10295	10295	10277
688	10025	10300	10299	10287	10287	10316
689	10030	10300	10304	10284	10284	10321
690	10095	10300	10311	10262	10262	10254
691	10140	10300	10316	10276	10276	10277
692	10185	10300	10312	10260	10260	10257
693	10230	10300	10313	10259	10259	10253
694	10275	10300	10320	10333	10333	10344
695	10200	10300	10321	10330	10330	10333
696	10250	10300	10323	10428	10428	10418
697	10175	10300	10324	10430	10430	10411
698	10100	10300	10325	10414	10414	10387
699	10050	10300	10326	10401	10401	10353
700	9960	10300	10326	10369	10369	10294
701	9860	10300	10326	10324	10316	10265
702	9840	10300	10326	10330	10336	10271
703	9940	10300	10327	10306	10302	10233
704	9820	10300	10328	10235	10224	10153
705	9795	10300	10328	10213	10202	10134
706	9770	10300	10329	10315	10300	10227

t	Observed Petrofina closing prices	Random walk (1.1)'	ARIMA- model (1.2)'	Transfer function models	
				(1.3.1)'	(1.3.2)'
740	10425	10425	10425	10425	10425
741	10575	10425	10424	10468	10471
742	10550	10425	10425	10488	10461
743	10625	10425	10437	10454	10441
744	10625	10425	10424	10542	10524
745	11100	10425	10437	10532	10501
746	10950	10425	10439	10479	10443
747	11075	10425	10442	10510	10482
748	10850	10425	10443	10521	10514
749	10950	10425	10446	10627	10611
750	10950	10425	10451	10663	10649
751	11125	10425	10439	10664	10653
752	11150	10425	10439	10662	10684
753	11275	10425	10440	10735	10747
754	11350	10425	10441	10665	10665
755	11100	10425	10441	10672	10675
756	11175	10425	10442	10616	10606
757	11275	10425	10443	10585	10577
758	11525	10425	10444	10548	10543
759	11475	10425	10444	10737	10708
760	11525	10425	10445	10764	10761

Table 2**GIB closing prices****A Sampling Estimates of GIB-returns**

$$\underline{y_{2t} := \ln(s_{2t} + d_{2t}) - \ln s_{2,t-1}}$$

$$(t = 1(11/4/88), \dots, 686 (24/11/90)) \\ [(t = 55 (24/6/88), \dots, 740 (8/2/91))]$$

(2.1) Random walk model

$$y_{2t} = \varepsilon_{2t}$$

$$\hat{\theta}_{\varepsilon_2} = 0.010218 \quad (\text{significant autocorrelation at peaks } 1, 2 \text{ and } 35 \text{ and significant partial autocorrelation at peaks } 1 \text{ and } 35)$$

$$[\hat{\theta}_{\varepsilon_2} = 0.010608] \quad (2.1)'$$

(2.2) ARIMA model (CLS)

$$(1 - 0.11349L^{35}) \hat{y}_{2t} = (1 + 0.11706L + 0.08319L^2) (1 - 0.08592L^{21})$$

$$(|\hat{\varepsilon}_i|) \quad (2.88) \quad (3.06) \quad (2.17) \quad (2.22)$$

$$(1 + 0.08844L^{26}) \hat{\varepsilon}_{2t}$$

$$(2.26)$$

$$\hat{\theta}_{\varepsilon_2} = 0.01004442 \quad (\text{no (partial) autocorrelation of the residuals})$$

$$[(1 - 0.08446L^{35}) \hat{y}_{2t} = \hat{\varepsilon}_{2t}] \quad (2.2)' \\ (2.09)$$

$$[\hat{\theta}_{\varepsilon_2} = 0.01058394]$$

(2.3) Transfer function models (CLS)**(2.3.1) Identification with prewhitened (PW) inputs and output**

$$\hat{y}_{2t} = (0.18777 + 0.11187L^6 - 0.10722L^9) \Delta \ln z_{1t} + (-1.49053 - 1.73030L^{14}) \Delta \ln z_{2,t-17}$$

$$(|\hat{\varepsilon}_i|) \quad (3.39) \quad (2.01) \quad (1.91) \quad (1.94) \quad (2.28)$$

$$\begin{aligned}
& +(-0.12773-0.16814L+0.27147L^6)\Delta\ln z_{3,t-24}-0.14957\Delta\ln z_{4,t-11}+ \\
(1.81) \quad (2.39) \quad (3.81) \quad & (2.54) \\
& +(-0.03504-0.03663L^{12})\Delta\ln z_{8t}+(1+0.07427L)(1-0.08737L^{21})\hat{\varepsilon}_{2t} \\
& (2.58) \quad (2.65) \quad (1.87) \quad (2.17)
\end{aligned}$$

$\hat{\theta}_{\varepsilon_2}=0.00966336$ (no significant (partial) residual autocorrelations)

$$\begin{aligned}
[\hat{y}_{2t} = & (0.16823-0.14116L^9+0.12117L^{23})\Delta\ln z_{1t}+(0.03375+ \\
& +0.02340L^6)\Delta\ln z_{2,t-2}+(-0.13108-0.13976L+0.16143L^6- \\
& -0.14019L^{26})\Delta\ln z_{3,t-24}-0.14816\Delta\ln z_{4,t-11}+(-0.03850- \\
& -0.02355L^{12}-0.03171L^{15}+0.03762L^{18})\Delta\ln z_{8t}+ \\
& +[(1-0.08881L^{26})]^{-1}\hat{\varepsilon}_{2t}]
\end{aligned}$$

(2.3.1)'

$$[\hat{\theta}_{\varepsilon_2}=0.01002855]$$

(2.3.2) Identification with a combination of PW and unwhitened inputs and output

$$\hat{y}_{2t}=(0.17085+0.09692L^6-0.08937L^9+0.10092L^{23})\Delta\ln z_{1t}+$$

$$(|\hat{\varepsilon}_i|) \quad (3.09) \quad (1.74) \quad (1.60) \quad (1.82)$$

$$+(-1.66307-1.51062L+1.76027L^{16})\Delta\ln z_{2,t-17}+$$

$$(2.18) \quad (1.96) \quad (2.33)$$

$$+(-0.11821-0.16895L+0.23961L^6)\Delta\ln z_{3,t-24}$$

$$(1.69) \quad (2.40) \quad (3.37)$$

$$-0.16987\Delta\ln z_{4,t-11}+(-0.03039-0.02214L-0.03119L^{12}+0.03028L^{18}-$$

$$(2.89) \quad (2.26) \quad (1.64) \quad (2.27) \quad (2.14)$$

$$-(0.03183L^{29}-0.02914L^{37})\Delta\ln z_{8t}+(1+0.06068L)(1-0.09569L^{21})\hat{\varepsilon}_{2t}$$

$$(2.18) \quad (1.97) \quad (1.51) \quad (2.35)$$

$\hat{\theta}_{\varepsilon_2}=0.00956198$ (slightly significant residual autocorrelation at peak 40:-0.0793; no significant partial residual autocorrelation)

B. Forecasts of GIB closing prices:t=687 (27/11/90) - 706 (24/12/90)t=741 (09/02/91) - 760 (08/03/91)

t	Observed GIB closing prices	Random walk (2.1)	ARIMA- model (2.2)	Transfer function models		
				(2.3.1) ARIMA exch. rates	(2.3.2) improved exch. rates	(2.3.2) improved exch. rates
686	1108	1108	1108	1108	1108	1108
687	1104	1108	1101	1104	1104	1102
688	1104	1108	1097	1114	1114	1117
689	1082	1108	1098	1115	1115	1114
690	1085	1108	1097	1110	1110	1115
691	1089	1108	1094	1112	1112	1116
692	1092	1108	1095	1115	1115	1117
693	1096	1108	1096	1115	1115	1115
694	1100	1108	1097	1118	1118	1111
695	1106	1108	1098	1116	1116	1107
696	1114	1108	1096	1113	1113	1107
697	1108	1108	1096	1112	1112	1108
698	1102	1108	1094	1107	1107	1103
699	1106	1108	1093	1109	1109	1101
700	1096	1108	1090	1108	1108	1104
701	1060	1108	1092	1110	1110	1104
702	1082	1108	1094	1111	1111	1102
703	1092	1108	1092	1109	1109	1099
704	1080	1108	1092	1106	1105	1098
705	1085	1108	1092	1107	1107	1104
706	1090	1108	1093	1107	1106	1104

t	Observed GIB closing prices	Random walk (2.1)'	ARIMA- model (2.2)'	Transfer function model (2.3.1)'
740	1098	1098	1098	1098
741	1152	1098	1099	1102
742	1146	1098	1098	1077
743	1162	1098	1098	1074
744	1172	1098	1098	1075
745	1230	1098	1097	1073
746	1270	1098	1097	1077
747	1256	1098	1097	1073
748	1228	1098	1098	1073
749	1232	1098	1096	1071
750	1210	1098	1097	1074
751	1234	1098	1095	1077
752	1200	1098	1095	1077
753	1204	1098	1095	1078
754	1230	1098	1094	1077
755	1222	1098	1094	1076
756	1240	1098	1092	1073
757	1252	1098	1091	1073
758	1244	1098	1091	1074
759	1252	1098	1095	1075
760	1240	1098	1092	1074

Table 3**Gevaert closing prices****A Sampling Estimates of the Gevaert returns**

$$y_{3t} := \ln(s_{3t} + d_{3t}) - \ln s_{3,t-1}$$

$$(t = 1(11/4/88), \dots, 686(26/11/90)) \\ [(t = 55(24/6/88), \dots, 740(8/2/91))]$$

(3.1) Random walk model

$$y_{3t} = \varepsilon_{3t}$$

$$\hat{\theta}_{\varepsilon_3} = 0.011731 \quad (\text{significant (partial) autocorrelation at peaks 1 and 9})$$

$$[\hat{\theta}_{\varepsilon_3} = 0.01248] \quad (3.1)'$$

(3.2) ARIMA-model (CLS)

$$(1+0.13575L)(1+0.07799L^2)\hat{y}_{3t} = (1-0.09246L^9)\hat{\varepsilon}_{3t}$$

$$(|\hat{\varepsilon}_t|) \quad (3.54) \quad (2.02) \quad (2.55)$$

$$\hat{\theta}_{\varepsilon_3} = 0.01158276 \quad (\text{no significant (partial) autocorrelation of the residuals})$$

$$[(1+0.11644L)(1+0.08228L^2)(1+0.08207L^7)(1+0.09741L^9)\hat{y}_{3t} =$$

$$(3.02) \quad (2.13) \quad (2.12) \quad (2.53)$$

$$(1+0.0826L^{14})(1-0.08238L^{17})\hat{\varepsilon}_{3t}] \quad (3.2)'$$

$$(2.11) \quad (2.08)$$

$$[\hat{\theta}_{\varepsilon_3} = 0.012296]$$

(3.3) Transfer function models (CLS)**(3.3.1) Identification with prewhitened (PW) inputs and output**

$$\hat{y}_{3t} = -2.02785\Delta \ln z_{2,t-1} + (0.24046 - 0.17395L^3 + 0.31683L^5$$

$$(|\hat{\varepsilon}_t|) \quad (2.37) \quad (3.10) \quad (2.21) \quad (4.05)$$

$$+ 0.14449L^{11})\Delta \ln z_{3t} + (-0.11731 + 0.16097L^4)\Delta \ln z_{4,t-12} +$$

$$(1.83) \quad (1.85) \quad (2.50)$$

$$+ (-0.04602 - 0.04223L^{10})\Delta \ln z_{8,t-1} + (1 - 0.16036L - 0.08932L^9)$$

$$(3.03) \quad (2.74) \quad (4.12) \quad (2.29)$$

$$(1+0.07743L^{14})\hat{\varepsilon}_{3t}$$

$$(1.94)$$

$\hat{\theta}_{\epsilon_3}=0.01109422$ (no significant (partial) residual autocorrelation)

$$[\hat{Y}_{3t} = (-0.15529 + 0.14020L^{13})\Delta \ln z_{1,t-26} + (0.02860 - 0.02117L^4 - 0.01732L^{15})\Delta \ln z_{2,t-2} + (0.28437 + 0.28938L^5 - 0.21326L^{28} + 0.17393L^{45})\Delta \ln z_{3t} + (0.20002 - 0.13423L^{12} + 0.10904L^{27})\Delta \ln z_{4,t-16} + (-0.03229 - 0.04279L - 0.03243L^{11})\Delta \ln z_{8t} + (1 - 0.14547L)(1 - 0.09737L^9)(1 + 0.10416L^{14})\hat{\epsilon}_{3t}] \quad (3.3.1)'$$

$[\hat{\theta}_{\epsilon_3}=0.0115593]$

(3.3.2) Identification with a combination of PW and unwhitened inputs and output

$$\hat{Y}_{3t} = (0.13321 - 0.15886L^{11})\Delta \ln z_{1,t-39} - 2.30679\Delta \ln z_{2,t-1} +$$

$$(|\hat{\epsilon}_i|) \quad (2.03) \quad (2.41) \quad (2.59)$$

$$+ (0.24580 - 0.15521L^3 + 0.29061L^5 + 0.14027L^{11} - 0.22139L^{28})\Delta \ln z_{3,t}$$

$$(3.11) \quad (1.94) \quad (3.66) \quad (1.76) \quad (2.75)$$

$$+ (-0.11928 + 0.14620L^4)\Delta \ln z_{4,t-12} +$$

$$(1.85) \quad (2.22)$$

$$+ (-0.04631 - 0.04249L^{10})\Delta \ln z_{8,t-1} +$$

$$(3.04) \quad (2.73)$$

$$+ (1 - 0.16041L - 0.08754L^9)\hat{\epsilon}_{3t}$$

$$(3.99) \quad (2.18)$$

$\hat{\theta}_{\epsilon_3}=0.01115835$ (no significant (partial) residual autocorrelation)

$$[\hat{Y}_{3t} = (-0.14373 + 0.13291L^{13})\Delta \ln z_{1,t-26} + (0.03003 - 0.01916L^4 - 0.01841L^{15})\Delta \ln z_{2,t-2} + (0.29338 + 0.29347L^5 - 0.22586L^{28} + 0.16667L^{45})\Delta \ln z_{3t} + (-0.16970 + 0.21249L^7 - 0.12073L^{19} + 0.09706L^{34})\Delta \ln z_{4,t-9} + (-0.02531 - 0.04299L + 0.02398L^3 - 0.03133L^{11} - 0.04140L^{35})\Delta \ln z_{8t} + (1 - 0.15230L)(1 - 0.11611L^9)(1 + 0.09864L^{14})\hat{\epsilon}_{3t}] \quad (3.3.2)'$$

$[\hat{\theta}_{\epsilon_3}=0.01144363]$

B. Forecasts of Gevaert closing prices:t=687 (27/11/90) - 706 (24/12/90)t=741 (09/02/91) - 760 (08/03/91)

t	Observed Gevaert closing prices	Random walk (3.1)	ARIMA- model (3.2)	Transfer function models		
				(3.3.1) ARIMA exch. rates	(3.3.2) improved exch. rates	(3.3.2) improved exch. rates
686	6520	6520	6520	6520	6520	6520
687	6480	6520	6544	6557	6551	6547
688	6420	6520	6559	6550	6552	6576
689	6300	6520	6568	6578	6574	6572
690	6340	6520	6564	6578	6570	6570
691	6380	6520	6548	6556	6543	6569
692	6420	6520	6561	6588	6570	6588
693	6460	6520	6556	6569	6569	6587
694	6500	6520	6557	6575	6566	6593
695	6620	6520	6571	6597	6584	6616
696	6680	6520	6570	6603	6601	6640
697	6610	6520	6569	6574	6575	6608
698	6650	6520	6570	6570	6569	6608
699	6480	6520	6570	6571	6575	6614
700	6380	6520	6571	6566	6557	6613
701	6400	6520	6571	6557	6547	6612
702	6300	6520	6572	6553	6546	6629
703	6250	6520	6572	6550	6555	6633
704	6360	6520	6573	6549	6541	6616
705	6355	6520	6573	6548	6544	6631
706	6350	6520	6574	6553	6553	6651

t	Observed Gevaert closing prices	Random walk (3.1)'	ARIMA- model (3.2)'	Transfer function models	
				(3.3.1)'	(3.3.2)'
740	6450	6450	6450	6450	6450
741	6500	6450	6376	6412	6408
742	6600	6450	6348	6357	6353
743	6530	6450	6357	6365	6350
744	6580	6450	6370	6396	6375
745	6900	6450	6386	6441	6423
746	6960	6450	6379	6410	6381
747	7100	6450	6367	6404	6375
748	7110	6450	6386	6403	6379
749	7200	6450	6367	6405	6408
750	7320	6450	6379	6415	6416
751	7400	6450	6380	6456	6457
752	7450	6450	6375	6454	6448
753	7300	6450	6366	6468	6492
754	7400	6450	6376	6459	6438
755	7210	6450	6378	6516	6498
756	7340	6450	6384	6506	6460
757	7560	6450	6369	6490	6441
758	7700	6450	6374	6499	6446
759	7640	6450	6376	6514	6563
760	8000	6450	6377	6518	6569

Table 4**Solvay closing prices****A Sampling Estimates of the Solvay returns**

$$\underline{y_{4t} := \ln (s_{4t} + d_{4t}) - \ln s_{4,t-1}}$$

$$(t = 1 \text{ (11/4/88)}, \dots, 686 \text{ (26/11/90)}) \\ [(t = 55 \text{ (24/6/88)}, \dots, 740 \text{ (8/2/91)})]$$

(4.1) Random walk model

$$y_{4t} = e_{4t}$$

$$\hat{\sigma}_{\epsilon_4} = 0.011187$$

(significant autocorrelation at peaks
1, 2, 13, 17, 26, 27, 34 and 41 and
significant partial autocorrelation at
peaks 1, 2, 13 and 27)

$$[\hat{\sigma}_{\epsilon_4} = 0.011952] \quad (4.1)'$$

(4.2) ARIMA model (CLS)

$$(1 + 0.09380L^{27})(1 - 0.10476L^{34})\hat{y}_{4t} = (1 + 0.11116L^2 + 0.11672L^{13})$$

$$(|\hat{\epsilon}_i|) \quad (2.39) \quad (2.66) \quad (2.92) \quad (3.06)$$

$$(1 - 0.08716L^{17})(1 + 0.09221L^{30})(1 - 0.11836L^{41})\hat{\epsilon}_{4t}$$

$$(2.25) \quad (2.35) \quad (2.99)$$

$$\hat{\sigma}_{\epsilon_4} = 0.01087716 \quad (\text{no significant (partial) autocorrelation of the residuals})$$

$$[(1 - 0.12652L^2)(1 - 0.13733L^{13})\hat{y}_{4t} = (1 - 0.09028L^{17}) \\ (3.29) \quad (3.55) \quad (2.24)]$$

$$(1 + 0.12193L^{30} + 0.10647L^{34} - 0.14545L^{40})\hat{\epsilon}_{4t}] \quad (4.2)'$$

$$(3.01) \quad (2.62) \quad (3.55)$$

$$[\hat{\sigma}_{\epsilon_4} = 0.01159773]$$

(4.3) Transfer function models (CLS)(4.3.1) Identification with prewhitened (PW) inputs and output

$$\hat{y}_{4t} = (0.15491 - 0.11648L^{46}) \Delta \ln z_{1,t-4} + 2.22546 \Delta \ln z_{2,t-3} +$$

$$(|\hat{\epsilon}_t|) \quad (2.59) \quad (1.93) \quad (2.67)$$

$$+ (-0.17873 + 0.25208L^9) \Delta \ln z_{3,t-21} +$$

$$(2.39) \quad (3.32)$$

$$+ (-0.19293 + 0.14326L^6 + 0.143302L^{10}) \Delta \ln z_{4,t-39} +$$

$$(3.04) \quad (2.29) \quad (2.24)$$

$$+ (-0.02690 - 0.04271L - 0.04022L^4 - 0.06111L^{47} + 0.05348L^{48}) \Delta \ln z_{8,t} +$$

$$(1.85) \quad (2.89) \quad (2.74) \quad (3.72) \quad (3.28)$$

$$+ [(1 + 0.11559L^{17})(1 - 0.06669L^{21})]^{-1} (1 + 0.08502L^2 + 0.10418L^{13})$$

$$(2.84) \quad (1.63) \quad (2.10) \quad (2.57)$$

$$(1 - 0.08468L^{40} - 0.08162L^{44}) \hat{\epsilon}_{4t}$$

$$(2.00) \quad (1.93)$$

$$\hat{\sigma}_{\epsilon_4} = 0.01036086$$

(no significant (partial) residual autocorrelations)

$$[\hat{y}_{4t} = 0.13087 \Delta \ln z_{1,t-4} + 0.02885 \Delta \ln z_{2,t-2} + (-0.17710 + 0.20946L^9 - 0.21119L^{29}) \Delta \ln z_{3,t-21} + 0.17776 \Delta \ln z_{4,t-45} + (-0.07210 - 0.02540L^{15} - 0.04211L^{35} + 0.04086L^{48}) \Delta \ln z_{8,t} + [(1 - 0.11373L^2 + 0.07631L^{15})]^{-1} (1 + 0.09108L^{13} + 0.09640L^{26} - 0.07974L^{40}) \hat{\epsilon}_{4t}]$$

$$[\hat{\sigma}_{\epsilon_4} = 0.01086778]$$

(4.3.1)'

(4.3.2) Identification with a combination of PW and unwhitened inputs and output

$$\hat{y}_{4t} = (0.14265 - 0.12231L^{46}) \Delta \ln z_{1,t-4} +$$

$$(|\hat{\epsilon}_1|) \quad (2.45) \quad (2.08)$$

$$+ (2.17823 + 1.68428L^{30} - 1.66045L^{45}) \Delta \ln z_{2,t-3} +$$

$$(2.73) \quad (2.17) \quad (2.08)$$

$$+ (0.15388 - 0.18757L^{12} + 0.21744L^{21} + 0.18496L^{29}) \Delta \ln z_{3,t-9} +$$

$$(2.08) \quad (2.60) \quad (2.95) \quad (2.46)$$

$$+ (-0.18279 + 0.15074L^6 + 0.15618L^7 + 0.15824L^{10}) \Delta \ln z_{4,t-39} +$$

$$(2.95) \quad (2.46) \quad (2.53) \quad (2.54)$$

$$+ (-0.0269 - 0.04353L - 0.04227L^3 - 0.04019L^4 - 0.05970L^{47} +$$

$$(1.90) \quad (3.05) \quad (2.95) \quad (2.11) \quad (3.77)$$

$$+ 0.05482L^{48}) \Delta \ln z_{8,t} + [(1 + 0.11265L^{17} + 0.08593L^{18})$$

$$(3.46) \quad (2.75) \quad (2.09)$$

$$(1 + 0.11290L^{39})]^{-1} (1 + 0.07868L^2 + 0.10740L^{13})$$

$$(2.67) \quad (1.93) \quad (2.61)$$

$$(1 - 0.09305L^{40} - 0.07988L^{44}) \hat{\epsilon}_{4t}$$

$$(2.17) \quad (1.87)$$

$$\hat{\sigma}_{\epsilon_4} = 0.01010727$$

(no significant (partial) residual autocorrelations)

$$\begin{aligned} [\hat{y}_{4t} = & 0.02580 \Delta \ln z_{2,t-2} + (0.23170 - 0.15545L^{17} + 0.18930L^{26} - \\ & + 0.19283L^{34} - 0.16017L^{42} - 0.22616L^{46}) \Delta \ln z_{3,t-4} + \\ & + (-0.13802 + 0.18135L^{36} + 0.12174L^{38}) \Delta \ln z_{4,t-9} + (-0.06859 - \\ & - 0.02495L^{15} - 0.02526L^{30} - 0.04351L^{35} - 0.05306L^{47} + \\ & + 0.04393L^{48}) \Delta \ln z_{8,t} + [(1 - 0.11897L^2)]^{-1} \\ & (1 + 0.11714L^{13} + 0.11735L^{26} - 0.08579L^{40}) \hat{\epsilon}_{4t}] \end{aligned}$$

(4.3.2)'

$$[\hat{\sigma}_{\epsilon_4} = 0.01059377]$$

B. Forecasts of Solvay closing prices:t=687 (27/11/90) - 706 (24/12/90)t=741 (09/02/91) - 760 (08/03/91)

t	Observed Solvay closing prices	Random walk (4.1)	ARIMA- model (4.2)	Transfer function models		
				(4.3.1) ARIMA exch. rates	(4.3.2) improved exch. rates	(4.3.2) improved exch. rates
686	10075	10075	10075	10075	10075	10075
687	10075	10075	10016	10006	10006	10034
688	10125	10075	9958	10021	10021	10056
689	10125	10075	9962	10040	10040	10055
690	10250	10075	9946	9956	9956	9965
691	10375	10075	9949	9908	9907	9897
692	10500	10075	9974	9909	9909	9939
693	10625	10075	10000	9958	9957	10014
694	10750	10075	10064	10022	10021	10098
695	10725	10075	10065	10036	10035	10102
696	11125	10075	10107	9970	9970	10087
697	10975	10075	10122	10016	10016	10169
698	11225	10075	10149	10008	10008	10128
699	11050	10075	10124	9984	9984	10118
700	10800	10075	10119	10009	10010	10137
701	10525	10075	10106	10014	10014	10115
702	10600	10075	10085	9997	9997	10093
703	10475	10075	10072	10007	10007	10083
704	10300	10075	10047	9918	9918	10012
705	10325	10075	10030	9902	9902	9965
706	10350	10075	10044	9965	9966	10050

t	Observed Solvay closing prices	Random walk (4.1)'	ARIMA- model (4.2)'	Transfer function models	
				(4.3.1)'	(4.3.2)'
740	11700	11700	11700	11700	11700
741	11800	11700	11702	11673	11755
742	11800	11700	11807	11600	11715
743	11850	11700	11810	11581	11699
744	11825	11700	11896	11706	11909
745	12375	11700	11906	11640	11849
746	12475	11700	11878	11674	11920
747	12600	11700	11866	11717	11910
748	12250	11700	11937	11737	12071
749	12200	11700	11881	11789	12108
750	12150	11700	11960	11849	12131
751	12175	11700	11858	11929	12150
752	11950	11700	11956	11899	12150
753	11950	11700	11936	11933	12188
754	12200	11700	12032	11844	12236
755	11925	11700	11949	11816	12227
756	11950	11700	11925	11751	12120
757	12175	11700	11876	11802	12174
758	12575	11700	11987	11744	12128
759	12925	11700	11922	11941	12400
760	13550	11700	11958	11859	12278

Table 5**Glaverbel closing prices****A Sampling Estimates of the Glaverbel returns**

$$\underline{y_{5t} := \ln(s_{5t} + d_{5t}) - \ln s_{5,t-1}} \\ (t = 1 \text{ (11/4/88)}, \dots, 686 \text{ (26/11/90)}) \\ [(t = 55 \text{ (24/6/88)}, \dots, 740 \text{ (8/2/91)})]$$

(5.1) Random walk model:

$$y_{5t} = \epsilon_{5t}$$

$$\hat{\theta}_{\epsilon_5} = 0.016784$$

(significant autocorrelation at peaks, 26, 40 and 48; significant partial autocorrelation at peaks 26 and 40).

$$[\hat{\theta}_{\epsilon_5} = 0.017505] \quad (5.1)'$$

(5.2) ARIMA - model (CLS)

$$\hat{y}_{5t} = (1 + 0.20851L^{26}) (1 + 0.12500L^{40}) (1 + 0.10405L^{33} - 0.13695L^{48}) \hat{\epsilon}_{5t}$$

$$(|\hat{\epsilon}_i|) \quad (5.37) \quad (3.05) \quad (2.63) \quad (3.21)$$

$$\hat{\theta}_{\epsilon_5} = 0.01628827$$

(no significant (partial) autocorrelation of the residuals).

$$[(1 - 0.07011L^2) (1 + 0.07291L^{21}) (1 - 0.16470L^{26}) \hat{y}_{5t} = (1 + 0.09081L^{40}) \hat{\epsilon}_{5t}] \\ (1.83) \quad (1.83) \quad (4.15) \quad (2.24)$$

$$[\hat{\theta}_{\epsilon_5} = 0.01719853]$$

(5.3) Transfer function models (CLS)**(5.3.1) Identification with prewhitened (PW) inputs and output**

$$\hat{y}_{5t} = 0.26936 \Delta \ln z_{1,t} + 3.17390 \Delta \ln z_{2,t-33} + (0.29633 - 0.37407L^{20}$$

$$(|\hat{\epsilon}_i|) \quad (3.15) \quad (2.71) \quad (2.71) \quad (3.47)$$

$$- 0.25859L^{21} + 0.23182L^{26} + 0.40288L^{34}) \Delta \ln z_{3,t-4} +$$

$$(2.41) \quad (2.08) \quad (3.62)$$

$$+ 0.24427 \Delta \ln z_{4,t-32} + (-0.05352 - 0.06087L^{29}) \Delta \ln z_{8,t} +$$

$$(2.80) \quad (2.63) \quad (2.77)$$

$$+(1-0.11489L^{33})^{-1}(1-0.10240L^7)(1-0.08630L^{18})(1+0.21236L^{26})$$

$$(2.72) \quad (2.54) \quad (2.09) \quad (5.15)$$

$$(1-0.07956L^{39}+0.11185L^{40}-0.19012L^{48})\hat{\epsilon}_{5t}$$

$$(1.87) \quad (2.64) \quad (4.32)$$

$$\hat{\theta}_{\epsilon_5}=0.01550203 \quad (\text{no significant (partial) residual autocorrelations})$$

$$\begin{aligned} [\hat{y}_{5t} = & (0.19737+0.23919L^4-0.22774L^{15})\Delta \ln z_{1t} + (0.02403- \\ & -0.05174L^4)\Delta \ln z_{2,t-2} + (0.28909-0.32320L^{24}-0.23097L^{25}+ \\ & +0.34723L^{38}-0.34795L^{46}+0.25858L^{49}-0.29608L^{50})\Delta \ln z_{3t} + \\ & +(-0.25565+0.27115L^5)\Delta \ln z_{4,t-5} + (-0.07061-0.03560L^{11}- \\ & -0.03721L^{12}+0.05484L^{18})\Delta \ln z_{5t} + [(1+0.10597L^{21})]^{-1} \\ & (1+0.15465L^{26})(1+0.8866L^{33}-0.09169L^{38})\hat{\epsilon}_{5t}] \end{aligned}$$

(5.3.1)'

$$[\hat{\theta}_{\epsilon_5}=0.01609417]$$

(5.3.2) Identification with a combination of PW and unwhitened inputs and output

$$\hat{y}_{5t}=0.28741\Delta \ln z_{1t}+3.03164\Delta \ln z_{2,t-33}+(0.24214-0.37781L^{20}$$

$$(|\hat{\epsilon}_i|) \quad (3.46) \quad (2.64) \quad (2.28) \quad (3.59)$$

$$-0.19478L^{21}+0.26410L^{26}+0.39678L^{34})\Delta \ln z_{3,t-4}+$$

$$(1.85) \quad (2.44) \quad (3.65)$$

$$+(0.17037+0.22993L^6+0.23636L^{20})\Delta \ln z_{4,t-26}+$$

$$(2.00) \quad (2.73) \quad (2.82)$$

$$+(-0.05983-0.05818L^{12}-0.04843L^{29}-0.04906L^{36}$$

$$(2.95) \quad (2.85) \quad (2.23) \quad (2.20)$$

$$-0.04350L^{37})\Delta \ln z_{5,t}+[(1+0.10229L^{10})(1-0.12097L^{33})]^{-1}$$

$$(2.01) \quad (2.46) \quad (2.82)$$

$$(1-0.10775L^7)(1-0.07995L^{18})(1+0.19283L^{26})$$

$$(2.64) \quad (1.91) \quad (4.58)$$

$$(1-0.06807L^{39}+0.11864L^{40}-0.24098L^{48})\hat{\epsilon}_{5t}$$

$$(1.59) \quad (2.79) \quad (5.45)$$

$$\hat{\theta}_{\epsilon_5}=0.01526799 \quad (\text{no significant (partial) residual-autocorrelations})$$

$$\begin{aligned}
[\hat{y}_{st} = & (0.19931 + 0.19019L^4 - 0.19635L^{15}) \Delta \ln z_{1t} + (0.33454 + \\
& + 0.25693L^4 - 0.25059L^{24} - 0.24380L^{25} + 0.39648L^{38} - \\
& - 0.32526L^{46} + 0.18857L^{49} - 0.28517L^{50}) \Delta \ln z_{3t} + (-0.23919 + \\
& + 0.25671L^5) \Delta \ln z_{4,t-5} + (-0.09208 - 0.04907L^{11} + 0.05420L^{18} - \\
& - 0.05413L^{35} - 0.05705L^{36}) \Delta \ln z_{8t} + [(1 + 0.09691L) \\
& (1 + 0.08073L^{21})]^{-1} (1 + 0.16152L^{26}) (1 - 0.09817L^{38}) \hat{\varepsilon}_{st}] \\
& \quad \quad \quad (5.3.2)' \\
[\hat{\sigma}_{\varepsilon_s} = & 0.01607318]
\end{aligned}$$

(5.3.3) Identification with PW inputs (incl. GIB and Petrofina returns) and output

$$\hat{y}_{5t} = 0.23623 \Delta \ln z_{1,t} + 3.67374 \Delta \ln z_{2,t-33} + (0.21733 - 0.26558L^{20} -$$

$$(|\hat{\varepsilon}_i|) \quad (2.84) \quad (3.26) \quad (2.11) \quad (2.53)$$

$$-0.24687L^{21} + 0.36042L^{34}) \Delta \ln z_{3,t-4} + 0.22508 \Delta \ln z_{4,t-32} +$$

$$(2.38) \quad (3.37) \quad (2.67)$$

$$+ (-0.05218 - 0.05709L^{29}) \Delta \ln z_{8,t} + (0.18467 + 0.06392L^{10} -$$

$$(2.60) \quad (2.62) \quad (4.96) \quad (1.78)$$

$$-0.11984L^{35} - 0.08496L^{39}) y_{1t} + (0.09617 - 0.11752L^6 + 0.08742L^{12}) y_{2t} +$$

$$(3.35) \quad (2.39) \quad (2.24) \quad (2.85) \quad (2.15)$$

$$+ (1 - 0.09846L^{33})^{-1} (1 - 0.09130L^7) (1 - 0.09340L^{18} - 0.10389L^{19})$$

$$(2.31) \quad (2.22) \quad (2.27) \quad (2.51)$$

$$(1 + 0.20154L^{26}) (1 + 0.09361L^{40} - 0.16344L^{48}) \hat{\varepsilon}_{5t}$$

$$(4.84) \quad (2.17) \quad (3.66)$$

$$\hat{\sigma}_{\varepsilon_s} = 0.01484509$$

(no significant (partial) residual autocorrelation)

B. Forecasts of Glaverbel closing prices:t=687 (27/11/90) - 706 (24/12/90)t=741 (09/02/91) - 760 (08/03/91)

t	Observed Glaverbel closing prices	Random walk (5.1)	ARIMA- model (5.2)	Transfer function models		
				(5.3.1) ARIMA exch. rates	(5.3.2) improved exch. rates	(5.3.2) improved exch. rates
686	2850	2850	2850	2850	2850	2850
687	2850	2850	2915	2954	2969	2969
688	2850	2850	2920	3010	3010	3006
689	2810	2850	2948	3075	3076	3056
690	2868	2850	2953	3095	3096	3080
691	2926	2850	2926	3093	3090	3071
692	2984	2850	2967	3165	3168	3171
693	3042	2850	2926	3104	3102	3115
694	3100	2850	2957	3130	3125	3133
695	3195	2850	2927	3084	3079	3110
696	3220	2850	2963	3140	3133	3178
697	3125	2850	2952	3137	3129	3198
698	3100	2850	2929	3104	3098	3170
699	2950	2850	2918	3103	3096	3136
700	2975	2850	2918	3120	3117	3124
701	2905	2850	2959	3156	3157	3165
702	2920	2850	2956	3138	3142	3146
703	2945	2850	2975	3170	3174	3183
704	2970	2850	2971	3145	3147	3149
705	2970	2850	2969	3134	3137	3140
706	2970	2850	2961	3116	3116	3120

t	Observed Glaverbel closing prices	Random walk (5.1)'	ARIMA- model (5.2)'	Transfer function models	
				(5.3.1)'	(5.3.2)'
740	2930	2930	2930	2930	2930
741	3030	2930	2911	2930	2910
742	3020	2930	2931	2874	2860
743	2900	2930	2917	2873	2860
744	2975	2930	2939	2877	2867
745	3000	2930	2899	2850	2846
746	3090	2930	2901	2848	2848
747	3250	2930	2895	2856	2851
748	3160	2930	2901	2885	2897
749	3180	2930	2915	2886	2914
750	3260	2930	2947	2880	2924
751	3300	2930	2948	2891	2919
752	3280	2930	2944	2863	2902
753	3330	2930	2938	2884	2944
754	3460	2930	2903	2876	2915
755	3450	2930	2909	2875	2900
756	3460	2930	2910	2832	2854
757	3460	2930	2896	2873	2864
758	3590	2930	2907	2915	2900
759	3570	2930	2893	2963	2998
760	3580	2930	2893	2927	3029

Table 6

CBR closing pricesA Sampling Estimates of the CBR returns

$$\underline{y_{6t} := \ln(s_{6t} + d_{6t}) - \ln s_{6,t-1}} \quad \begin{array}{l} (t = 1 \text{ (11/4/88)}, \dots, 686 \text{ (26/11/90)}) \\ [(t = 55 \text{ (24/6/88)}, \dots, 740 \text{ (8/2/91)})] \end{array}$$

(6.1) Random walk model

$$y_{6t} = \varepsilon_{6t}$$

$$\hat{\theta}_{\varepsilon} = 0.013492$$

(significant autocorrelations at peaks 1, 11 and 41; significant partial autocorrelations at peaks 1, 22, 24, 25, 40 and 41)

$$[\hat{\theta}_{\varepsilon} = 0.014155] \quad (6.1)'$$

(6.2) ARIMA model (CLS)

$$(1 - 0.09105L)(1 + 0.08029L^{22} - 0.07538L^{25})(1 - 0.08467L^{41})\hat{y}_{6t}$$

$$(|\hat{\varepsilon}_i|) \quad (2.36) \quad (2.03) \quad (1.88) \quad (2.10)$$

$$= (1 + 0.08362L^{11})\hat{\varepsilon}_{6t}$$

$$(2.13)$$

$$\hat{\theta}_{\varepsilon} = 0.01329350$$

(no significant (partial) autocorrelations of the residuals)

$$[(1 - 0.07784L)(1 - 0.07729L^{41})\hat{y}_{6t} = \hat{\varepsilon}_{6t}] \quad (6.2)'$$

$$(2.09) \quad (1.89)$$

$$[\hat{\theta}_{\varepsilon} = 0.01408985]$$

(6.3) Transfer function model (CLS)(6.3.1) Identification with prewhitened (PW) inputs and output

$$\hat{y}_{6t} = 0.15244\Delta \ln z_{1,t-39} + (-2.05449 - 2.11757L^{25})\Delta \ln z_{2,t-23} +$$

$$(|\hat{\varepsilon}_i|) \quad (2.04) \quad (2.06) \quad (2.10)$$

$$+ (-0.21024 + 0.27758L^4 - 0.27878L^{17})\Delta \ln z_{3,t-7} +$$

$$(2.26) \quad (2.95) \quad (2.94)$$

$$\begin{aligned}
& +0.19954\Delta\ln z_{4,t-47} + (-0.05933-0.05646L^{11}-0.05822L^{28})\Delta\ln z_{8,t-1} + \\
& \quad (2.57) \quad (3.37) \quad (3.14) \quad (3.08) \\
& + [(1+0.11524L^{20})(1-0.10008L^{25})(1-0.08694L^{41}+0.09559L^{46}+ \\
& \quad (2.77) \quad (2.36) \quad (2.06) \quad (2.25) \\
& +0.12072L^{47})]^{-1}\hat{\epsilon}_{6t} \\
& (2.83)
\end{aligned}$$

$\hat{\sigma}_{\epsilon} = 0.0128282$ (residual autocorrelation at peak 17: -0.08052, residual partial autocorrelation at peak 17: -0.07689; hence, MA(17) could be added)

$$\begin{aligned}
[\hat{y}_{6t} = & 0.13406\Delta\ln z_{1,t-39} + (0.04141-0.02283L^{12})\Delta\ln z_{2,t-2} + \\
& + (0.20496-0.24924L^{13}+0.30203L^{16}-0.25660L^{17}- \\
& -0.19524L^{20}-0.20999L^{24})\Delta\ln z_{3,t-4} + (0.16373- \\
& -0.23721L^3+0.23316L^{31})\Delta\ln z_{4,t-16} + (-0.05583+0.04358L^9- \\
& -0.03307L^{12}-0.04583L^{13})\Delta\ln z_{6t} + [(1-0.07345L- \\
& -0.10096L^5)(1+0.11560L^{22})(1+0.08470L^{37}-0.06022L^{41})]^{-1}\hat{\epsilon}_{6t}] \\
& \quad (6.3.1)' \\
[\hat{\sigma}_{\epsilon} = & 0.0129756]
\end{aligned}$$

(6.3.2) Identification with combination of PW and unwhitened inputs and output

$$\hat{y}_{6t} = 0.13787\Delta\ln z_{1,t-39} + (-2.34646-1.83575L^{25})\Delta\ln z_{2,t-23} +$$

$$(|\hat{\epsilon}_i|) \quad (1.87) \quad (2.44) \quad (1.86)$$

$$\begin{aligned}
& + (-0.17323+0.23607L^4-0.18601L^{10}+0.16334L^{13}-0.26261L^{17}- \\
& \quad (1.88) \quad (2.52) \quad (1.99) \quad (1.74) \quad (2.78) \\
& -0.16905L^{21})\Delta\ln z_{3,t-7} + (0.19529+0.16865L^{31})\Delta\ln z_{4,t-16} + \\
& \quad (1.78) \quad (2.53) \quad (2.20) \\
& + (-0.05089-0.03385L-0.04743L^{11}-0.04103L^{21}-0.05525L^{28}- \\
& \quad (2.90) \quad (1.90) \quad (2.65) \quad (2.25) \quad (2.95) \\
& -0.04954L^{34})\Delta\ln z_{8,t-1} + [(1+0.13014L^{20})(1+0.10348L^{22}) \\
& \quad (2.57) \quad (3.11) \quad (2.46) \\
& (1+0.09814L^{46}+0.13384L^{47})]^{-1}\hat{\epsilon}_{6t} \\
& (2.29) \quad (3.12)
\end{aligned}$$

$$\hat{\theta}_{\epsilon_t} = 0.01261798$$

(no significant (partial) residual autocorrelations)

$$\begin{aligned} [\hat{Y}_{\epsilon_t} = & 0.20446 \Delta \ln z_{1,t-8} + (0.04168 - 0.02474L^{12}) \Delta \ln z_{2,t-2} + \\ & + (0.22821 - 0.23970L^{13} + 0.30099L^{16} - 0.17943L^{17} - \\ & - 0.16575L^{24} + 0.10604L^{37}) \Delta \ln z_{3,t-4} + (0.20117 - \\ & - 0.21690L^3 + 0.21041L^{31}) \Delta \ln z_{4,t-16} + (-0.05594 + 0.04480L^9 - \\ & - 0.02760L^{12} - 0.04638L^{13} - 0.04547L^{35} - 0.04826L^{49}) \Delta \ln z_{\epsilon_t} + \\ & + [(1 - 0.08300L - 0.11170L^5)(1 + 0.10033L^{22})(1 + 0.07609L^{37})]^{-1} \\ & (1 - 0.10105L^{47}) \hat{\epsilon}_{\epsilon_t}] \end{aligned} \quad (6.3.2)'$$

$$[\hat{\theta}_{\epsilon_t} = 0.0128402]$$

B. Forecasts of CBR closing prices:t=687 (27/11/90) - 706 (24/12/90)t=741 (09/02/91) - 760 (08/03/91)

t	Observed CBR closing prices	Random walk (6.1)	ARIMA- model (6.2)	Transfer function models		
				(6.3.1) ARIMA exch. rates	(6.3.2) improved exch. rates	(6.3.2) improved exch. rates
686	5550	5550	5550	5550	5550	5550
687	5600	5550	5567	5588	5588	5554
688	5570	5550	5591	5666	5666	5588
689	5550	5550	5585	5661	5661	5563
690	5620	5550	5552	5688	5688	5612
691	5690	5550	5537	5667	5667	5622
692	5760	5550	5548	5691	5691	5624
693	5830	5550	5548	5662	5662	5607
694	5900	5550	5538	5652	5657	5648
695	5930	5550	5521	5656	5653	5662
696	5980	5550	5537	5670	5674	5715
697	5940	5550	5525	5639	5646	5678
698	5820	5550	5536	5640	5640	5657
699	5860	5550	5515	5641	5654	5673
700	5690	5550	5528	5705	5712	5698
701	5650	5550	5555	5707	5705	5722
702	5650	5550	5567	5680	5680	5736
703	5650	5550	5579	5654	5647	5708
704	5580	5550	5561	5632	5618	5674
705	5605	5550	5557	5655	5640	5693
706	5630	5550	5557	5680	5663	5742

t	Observed CBR closing prices	Random walk (6.1)'	ARIMA- model (6.2)'	Transfer function models	
				(6.3.1)'	(6.3.2)'
740	6100	6100	6100	6100	6100
741	6280	6100	6103	6136	6128
742	6490	6100	6102	6122	6133
743	6470	6100	6102	6114	6157
744	6500	6100	6103	6212	6221
745	6530	6100	6098	6237	6257
746	6700	6100	6100	6224	6238
747	7070	6100	6103	6257	6283
748	6900	6100	6106	6232	6289
749	7070	6100	6108	6297	6363
750	7100	6100	6111	6312	6399
751	7540	6100	6111	6333	6463
752	7300	6100	6110	6359	6459
753	7330	6100	6108	6388	6521
754	7590	6100	6107	6352	6437
755	7470	6100	6121	6349	6407
756	7530	6100	6116	6361	6386
757	7560	6100	6098	6340	6388
758	7730	6100	6097	6371	6403
759	7770	6100	6115	6371	6509
760	7840	6100	6103	6385	6519

Table 7

Electrabel (EBES) closing pricesA Sampling Estimates of the Electrabel (EBES) returns

$$y_{7t} := \ln(s_{7t} + d_{7t}) - \ln s_{7,t-1}$$

(t = 1 (11/4/88), ..., 686 (26/11/90))
 [(t = 55 (24/6/88), ..., 740 (8/2/91))]

(7.1) Random walk model

$$y_{7t} = \varepsilon_{7t}$$

$$\hat{\sigma}_{\varepsilon_7} = 0.008402$$

(significant autocorrelations at peaks 1, 2, 8, 27 and significant partial autocorrelations at peaks 1, 2 and 27)

$$[\hat{\sigma}_{\varepsilon_7} = 0.008644] \quad (7.1)'$$

(7.2) ARIMA - model (CLS)

$$\hat{y}_{7t} = (1 + 0.07625L) (1 + 0.08980L^2) (1 - 0.09466L^{27}) \varepsilon_{7t}$$

(| $\hat{\varepsilon}_i$ |) (1.99) (2.35) (2.45)

$$\hat{\sigma}_{\varepsilon_7} = 0.0083323$$

$$[(1 - 0.008644) \hat{y}_{7t} = \varepsilon_{7t}] \quad (7.2)'$$

(2.68)

$$[\hat{\sigma}_{\varepsilon_7} = 0.00863019]$$

(7.3) Transfer function models (CLS)(7.3.1) Identification with prewhitened (PW) inputs and output

$$\hat{y}_{7t} = (0.09154 + 0.08378L^2) \Delta \ln z_{1,t-37} - 1.80151 \Delta \ln z_{2,t-48} + (0.15790 + 0.16786L^{11} - 0.13207L^{15} + 0.13783L^{23}) \Delta \ln z_{3t} + 0.11373 \Delta \ln z_{4,t-14} + (-0.02605 - 0.02526L^{37} + 0.03269L^{47}) \Delta \ln z_{8,t-1} + (1 + 0.06073L)$$

(| $\hat{\varepsilon}_i$ |) (1.93) (1.77) (2.80) (2.69) (2.86) (2.25) (2.33) (2.28) (2.26) (2.03) (2.57) (1.51)

$$(1 + 0.06909L^2) (1 - 0.08872L^{27}) \varepsilon_{7t}$$

(1.72) (2.18)

$$\hat{\sigma}_{\varepsilon_7} = 0.00808113$$

$$[\hat{y}_{7t} = (0.12194 + 0.08443L^7) \Delta \ln z_{1,t-30} - 0.02168 \Delta \ln z_{2,t-1} + \\ + 0.14192 \Delta \ln z_{3t} + (0.02890 - 0.02949L^{18} + 0.03278L^{28}) \Delta \ln z_{8,t-20} + \\ + (1 - 0.13238L)^{-1} (1 + 0.08007L^2) (1 - 0.10069L^{27}) \hat{\varepsilon}_{7t}]$$

$$[\hat{\theta}_{\varepsilon_7} = 0.0083909]$$

(7.3.2) Identification with combination of PW and unwhitened inputs and output

$$\begin{aligned} \hat{y}_{7t} = & (0.09931 + 0.08883L^2) \Delta \ln z_{1,t-37} + (-1.53559 - 1.91172L^{38}) \Delta \ln z_{2,t-10} + \\ & (|\hat{\varepsilon}_i|) \quad (2.08) \quad (1.87) \quad (2.35) \quad (2.97) \\ & + (0.15679 + 0.17463L^{11} - 0.13031L^{15} + 0.13339L^{23} + 0.11578L^{33}) \Delta \ln z_{3t} + \\ & \quad (2.67) \quad (2.97) \quad (2.22) \quad (2.26) \quad (1.91) \\ & + 0.11276 \Delta \ln z_{4,t-14} + (-0.02442 - 0.02568L^{37} + 0.02976L^{47}) \Delta \ln z_{8,t-1} + \\ & \quad (2.28) \quad (2.14) \quad (2.08) \quad (2.36) \\ & + (1 + 0.07512L^2) (1 - 0.08581L^{27}) \hat{\varepsilon}_{7t} \\ & \quad (1.87) \quad (2.11) \end{aligned}$$

$$\hat{\theta}_{\varepsilon_7} = 0.00804393$$

$$[\hat{y}_{7t} = (0.12998 + 0.08039L^7) \Delta \ln z_{1,t-30} - 0.02187 \Delta \ln z_{2,t-1} + \\ + (0.13309 + 0.14209L^{23}) \Delta \ln z_{3t} + (0.02703 - 0.03138L^{18} + \\ + 0.03365L^{28}) \Delta \ln z_{8,t-20} + (1 - 0.13434L)^{-1} (1 + 0.08844L^2) \\ (1 - 0.09742L^{27}) \hat{\varepsilon}_{7t}]$$

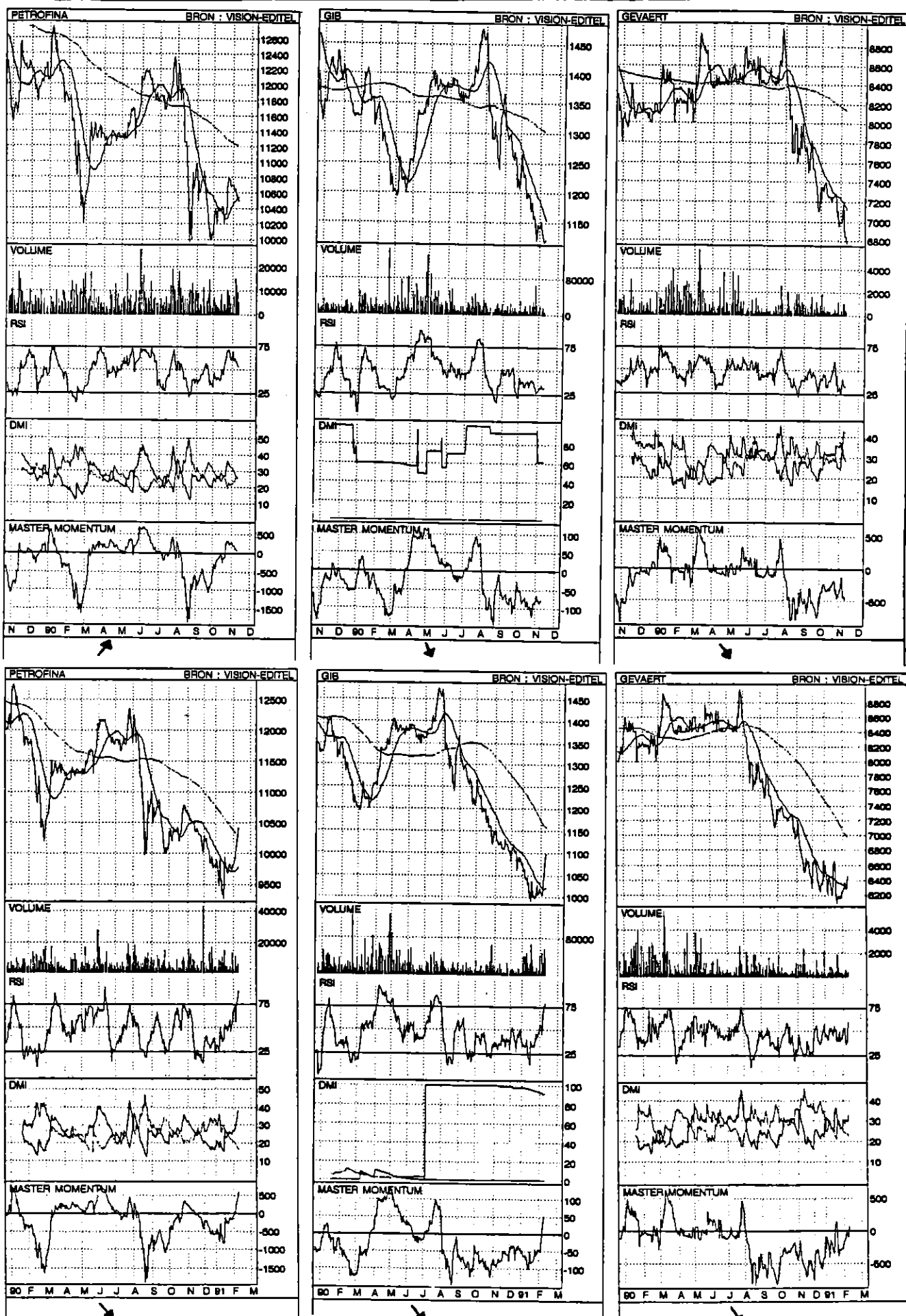
$$[\hat{\theta}_{\varepsilon_7} = 0.00835708]$$

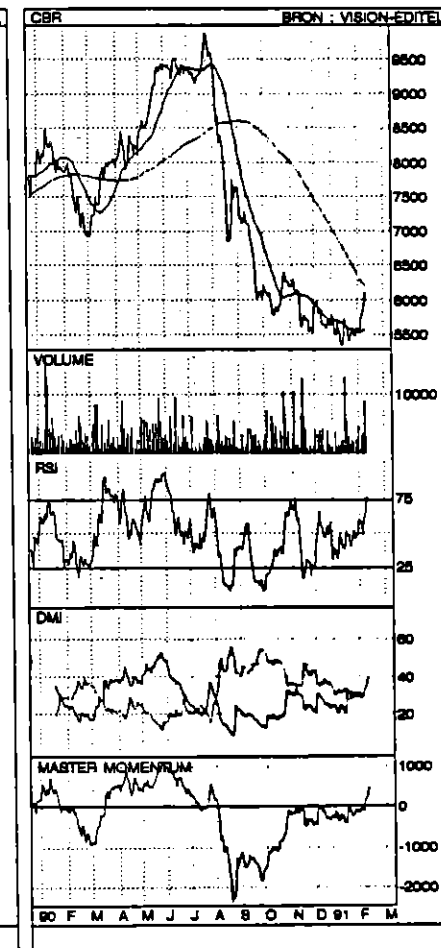
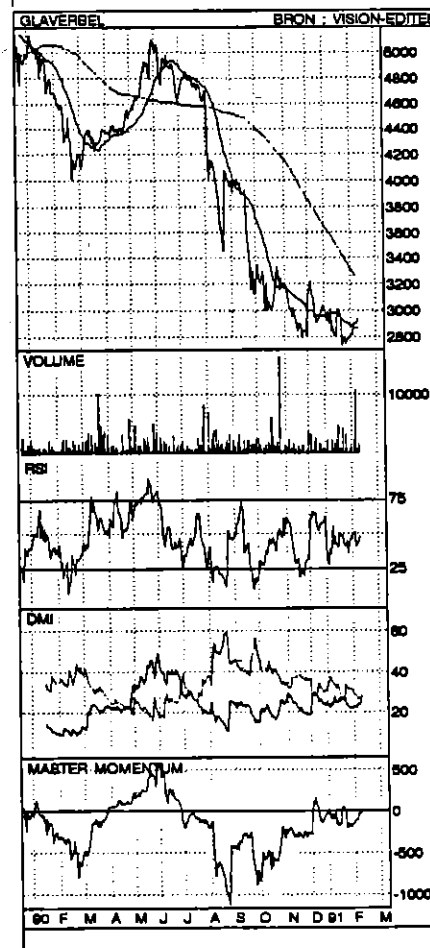
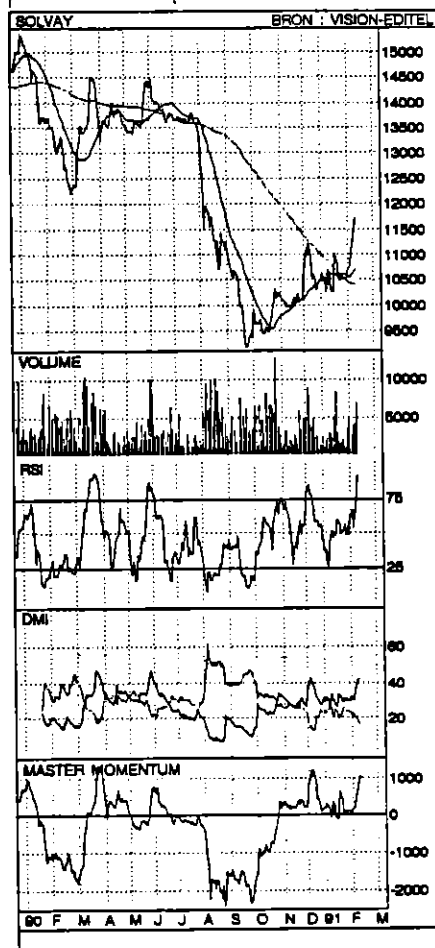
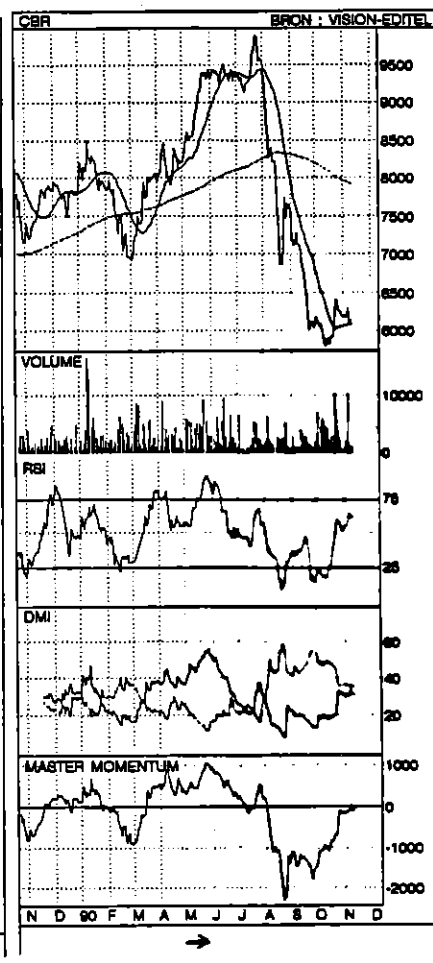
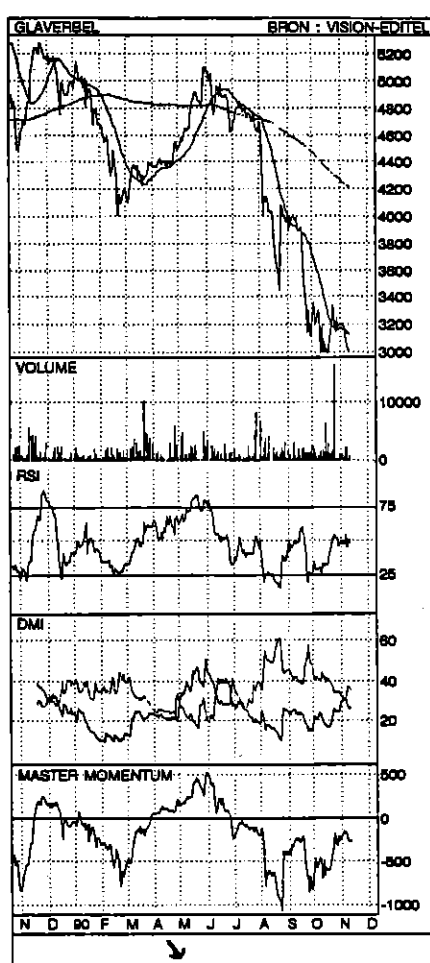
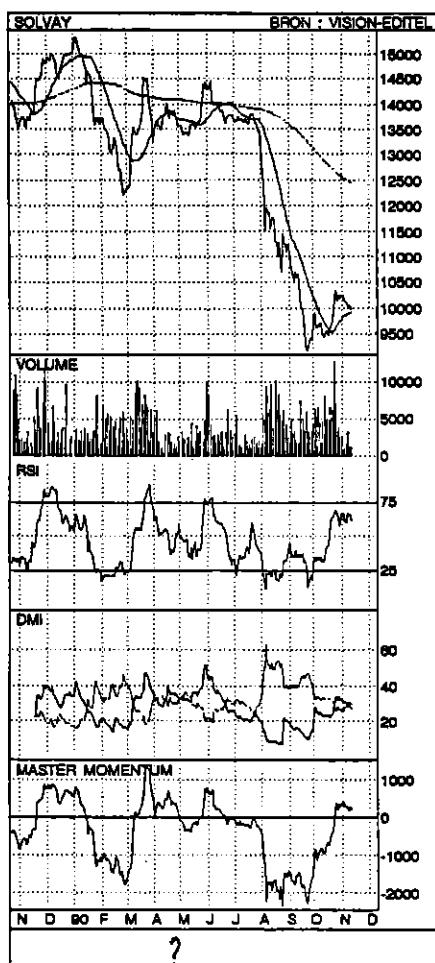
B. Forecasts of Elecrabel closing prices:t=687 (27/11/90) - 706 (24/12/90)t=741 (09/02/91) - 760 (08/03/91)

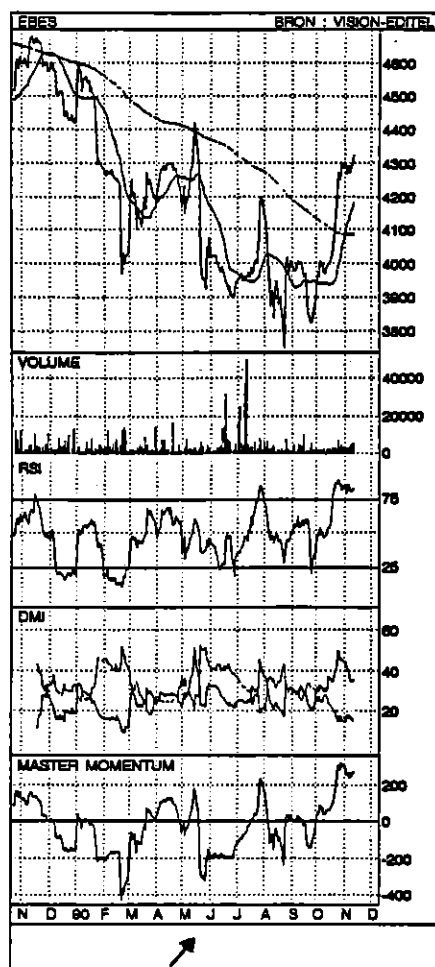
t	Observed Elecrabel closing prices	Random walk (7.1)	ARIMA- model (7.2)	Transfer function models		
				(7.3.1) ARIMA exch. rates	(7.3.2) improved exch. rates	(7.3.2) improved exch. rates
686	4375	4375	4375	4375	4375	4375
687	4290	4375	4372	4370	4367	4365
688	4285	4375	4366	4359	4361	4364
689	4280	4375	4361	4361	4359	4367
690	4400	4375	4358	4356	4352	4348
691	4350	4375	4352	4336	4332	4339
692	4280	4375	4354	4337	4331	4342
693	4300	4375	4350	4350	4344	4335
694	4340	4375	4352	4351	4347	4345
695	4320	4375	4351	4345	4340	4336
696	4307	4375	4352	4340	4337	4331
697	4295	4375	4353	4341	4341	4337
698	4290	4375	4354	4343	4343	4336
699	4285	4375	4352	4361	4367	4355
700	4280	4375	4354	4366	4366	4354
701	4325	4375	4352	4354	4352	4341
702	4320	4375	4351	4348	4346	4332
703	4315	4375	4348	4330	4323	4307
704	4310	4375	4350	4329	4319	4305
705	4320	4375	4348	4322	4319	4304
706	4250	4375	4349	4335	4332	4316

t	Observed Elecrabel closing prices	Random walk (7.1)'	ARIMA- model (7.2)'	Transfer function models	
				(7.3.1)'	(7.3.2)'
740	4675	4675	4675	4675	4675
741	4800	4675	4673	4675	4676
742	4825	4675	4673	4674	4677
743	4801	4675	4673	4698	4708
744	4785	4675	4673	4650	4662
745	4775	4675	4673	4637	4650
746	4775	4675	4673	4647	4653
747	4760	4675	4673	4645	4655
748	4745	4675	4674	4659	4666
749	4735	4675	4674	4643	4649
750	4750	4675	4674	4642	4651
751	4780	4675	4674	4641	4647
752	4740	4675	4674	4646	4650
753	4725	4675	4674	4654	4658
754	4730	4675	4675	4648	4650
755	4725	4675	4675	4642	4648
756	4750	4675	4675	4650	4658
757	4760	4675	4675	4623	4626
758	4770	4675	4675	4628	4627
759	4815	4675	4675	4611	4608
760	4890	4675	4676	4596	4598

Appendix C: Technical analysis of the closing prices







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