SESO UNIVERSITAIRE FACULTEITEN ST.-IGNATIUS TE ANTWERPEN

STUDIECENTRUM VOOR ECONOMISCH EN SOCIAAL ONDERZOEK

VAKGROEP PUBLIEKE ECONOMIE

Some Properties of the Hicks and Morishima Elasticities of Substitution

Wilfried PAUWELS

Report 91/261

September 1991

Universitaire Faculteiten St.-Ignatius
Prinsstraat 13 - B 2000 Antwerpen
D/1991/1169/16

Abstract

It is shown that the Hicks elasticity of substitution can be calculated only on the basis of the conditional demand functions for inputs. Also, necessary and sufficient conditions are given for the Morishima elasticity of substitution to be symmetric, and for the Hicks and Morishima elasticity to be equal to each other.

The purpose of this paper is to present some properties of the Hicks Elasticity of Substitution (HES) and of the Morishima Elasticity of Substitution (MES). In particular, we show that the HES can be derived only on the basis of the conditional demand functions for the inputs. We also give necessary and sufficient conditions for the MES to be symmetric, and for the HES and the MES to be equal to each other.

Our results concerning the symmetry of the MES, and concerning the equality of the HES and the MES, can be compared with those obtained by C. Blackorby and R.R. Russell [1]. Our results are of a more "local" nature. E.g. C. Blackorby and R.R. Russell investigate the question of when the MES is symmetric for <u>all</u> pairs of inputs, and for <u>all</u> vectors of input prices and output levels. Our results allow for the possibility that the MES is symmetric for some pair of inputs, but not for another pair. Also, symmetry may hold for some, but not all, vectors of input prices and output levels. Our results concerning the equality of the HES and the MES are of a similar "local" nature.

The paper is organized as follows. In section 1 we show how the HES can be calculated on the basis of the conditional demand functions for the inputs. In section 2 we investigate the symmetry of the MES. The HES and the MES are compared in section 3. A final section contains two simple examples. Some mathematical proofs are given in the appendix.

1. The Hicks Elasticity of Substitution

Let y = f(x) be a production function which associates with every vector of inputs $x \in R_+^n$ the maximum possible output level $y \in R_+$. Let $w \in R_{++}^n$ be the vector of input prices. The conditional demand functions for the inputs are denoted by

$h_i\ (w,y)$, $i:1,\ldots,n$.

The elasticity of substitution of inputs i and j, both in the sense of Hicks and of Morishima, refers to the percentage change in input proportions $h_j(w,y)/h_i(w,y)$ resulting from a one percent change in the relative prices w_i/w_j . Let us denote this elasticity by $\sigma_{ij}(w,y)$. Without loss of generality we take i=1 and j=2. We then have

$$\sigma_{12}(w,y) = \frac{d\left[\dfrac{h_2(w,y)}{h_1(w,y)}\right] \Big/ \left[\dfrac{h_2(w,y)}{h_1(w,y)}\right]}{d\left[\dfrac{w_1}{w_2}\right] \Big/ \left[\dfrac{w_1}{w_2}\right]} = \frac{\dfrac{dh_2(w,y)}{h_2(w,y)} - \dfrac{dh_1(w,y)}{h_1(w,y)}}{\dfrac{dw_1}{w_1} - \dfrac{dw_2}{w_2}} \qquad (1)$$

This value of $\sigma_{12}(w,y)$ also gives us information on the change in relative factor shares $w_2h_2(w,y)/w_1h_1(w,y)$ resulting from a change in the relative price w_1/w_2 . Indeed, we have

$$\frac{d\left[\dfrac{w_2h_2(w,y)}{w_1h_1(w,y)}\right] \Big/ \left[\dfrac{w_2h_2(w,y)}{w_1h_1(w,y)}\right]}{d\left[\dfrac{w_1}{w_2}\right] \Big/ \left[\dfrac{w_1}{w_2}\right]} = \sigma_{12}(w,y) - 1 \qquad (2)$$

When taking the total differentials in the numerator of (1), one always assumes that dy = 0, so that movements of input proportions have to take place on the same isoquant. However, when n > 2, the resulting expression does not define a unique value of $\sigma_{12}(w,y)$. To see this, consider the total differentials

$$\begin{bmatrix} \frac{dh_1(w,y)}{h_1(w,y)} \\ \frac{dh_2(w,y)}{h_2(w,y)} \end{bmatrix} = A_{11} \begin{bmatrix} \frac{dw_1}{w_1} \\ \frac{dw_2}{w_2} \end{bmatrix} + A_{12} \begin{bmatrix} \frac{dw_3}{w_3} \\ \cdots \\ \frac{dw_n}{w_n} \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} \frac{dh_3(w,y)}{h_3(w,y)} \\ \cdots \\ \frac{dh_n(w,y)}{h_n(w,y)} \end{bmatrix} = A_{21} \begin{bmatrix} \frac{dw_1}{w_1} \\ \frac{dw_2}{w_2} \end{bmatrix} + A_{22} \begin{bmatrix} \frac{dw_3}{w_3} \\ \cdots \\ \frac{dw_n}{w_n} \end{bmatrix} \tag{4}$$

The elements of the matrices A_{11}, A_{12}, A_{21} and A_{22} contain the price elasticities of the conditional demand functions.

In order to define a unique value of $\sigma_{12}(w,y)$, one has to make specific assumptions with respect to $dw_3/w_3, \ldots, dw_n/w_n$ and/or with respect to $dh_3(w,y)/h_3(w,y), \ldots, dh_n(w,y)/h_n(w,y)$. In an important publication [2] C. Blackorby and R.R. Russell show that no such specific assumptions can be made such that $\sigma_{12}(w,y)$

reduces to the Allen partial elasticity of substitution. They therefore conclude that this concept does not measure the ease of substitution between the two inputs, and that it provides no information about the comparative statics of income shares. Hence, this notion of elasticity of substitution can better be dropped.

Let us now turn to the HES. Here it is assumed that $dh_3(w,y)/h_3(w,y)=\ldots=dh_n(w,y)/dh_n(w,y)=0$, so that the input quantities different from $h_1(w,y)$ and $h_2(w,y)$ are required to remain constant. For given changes dw_1/w_1 and dw_2/w_2, this requires that $dw_3/w_3,\ldots,dw_n/w_n$ are chosen such that the left hand side of (4) equals zero.
This gives

$$\begin{bmatrix} \frac{dw_3}{w_3} \\ \cdots \\ \frac{dw_n}{w_n} \end{bmatrix} = - A_{22}^{-1} A_{21} \begin{bmatrix} \frac{dw_1}{w_1} \\ \frac{dw_2}{w_2} \end{bmatrix} \tag{5}$$

In the appendix it is shown that the inverse A_{22}^{-1} exists.

If we then use (5) in (3), we obtain

$$\begin{bmatrix} \dfrac{dh_1(w,y)}{h_1(w,y)} \\ \dfrac{dh_2(w,y)}{h_2(w,y)} \end{bmatrix} = \left[A_{11} - A_{12}\, A_{22}^{-1}\, A_{21}\right] \begin{bmatrix} \dfrac{dw_1}{w_1} \\ \dfrac{dw_2}{w_2} \end{bmatrix} \tag{6}$$

Equation (6) clearly shows that the total effect of $dw_1/w_1, dw_2/w_2$ on $dh_1(w,y)/h_1(w,y), dh_2(w,y)/h_2(w,y)$ contains a direct effect, given by the matrix A_{11}, and an indirect effect, given by the matrix $-A_{12}\, A_{22}^{-1}\, A_{21}$.

Let us denote

$$A_{11} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{bmatrix}$$

$$A_{12}\, A_{22}^{-1}\, A_{21} = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix}$$

We can then calculate from (6) that

$$\frac{dh_2(w,y)}{h_2(w,y)} - \frac{dh_1(w,y)}{h_1(w,y)} = [\varepsilon_{21} - \varepsilon_{11} - e_{21} + e_{11}]\ \frac{dw_1}{w_1} - [\varepsilon_{12} - \varepsilon_{22} - e_{12} + e_{22}]\ \frac{dw_2}{w_2} \tag{7}$$

In the appendix it is show that

$$\varepsilon_{21} - \varepsilon_{11} - e_{21} + e_{11} = \varepsilon_{12} - \varepsilon_{22} - e_{12} + e_{22} \tag{8}$$

Equation (7) can then be written as

$$\frac{dh_2(w,y)}{h_2(w,y)} - \frac{dh_1(w,y)}{h_1(w,y)} = [\varepsilon_{21} - \varepsilon_{11} - e_{21} + e_{11}]\left[\frac{dw_1}{w_1} - \frac{dw_2}{w_2}\right] =$$

$$[\varepsilon_{12} - \varepsilon_{22} - e_{12} + e_{22}]\left[\frac{dw_1}{w_1} - \frac{dw_2}{w_2}\right] \tag{9}$$

If we then denote the HES by $\sigma_{12}^{H}(w,y)$, we obtain from (1) and (9)

$$\begin{aligned} \sigma_{12}^{H}(w,y) &= \varepsilon_{12} - \varepsilon_{11} - e_{21} + e_{11} \\ &= \varepsilon_{12} - \varepsilon_{22} - e_{12} + e_{22} \end{aligned} \tag{10}$$

From (10) it is also clear that

$$\sigma_{12}^{H}(w,y) = \sigma_{21}^{H}(w,y) \tag{11}$$

Note that $\sigma_{12}^{H}(w,y)$ is calculated only on the basis of the conditional demand functions for the inputs.

If the number of inputs, n, equals 2, there are no indirect effects of dw_1/w_1 and dw_2/w_2 in (6), so that all the elements e_{11}, e_{12}, e_{21} and e_{22} are identically zero. (10) then reduces to

$$\sigma_{12}^{H}(w,y) = \varepsilon_{21} - \varepsilon_{11} = \varepsilon_{12} - \varepsilon_{22} \tag{12}$$

Finally, it is clear that the HES can also be defined on the basis of the production function f(x). The definition is then

$$\sigma_{12}^{H}(x) = \left. \frac{d\left[\dfrac{x_2}{x_1}\right] \Big/ \left[\dfrac{x_2}{x_1}\right]}{d\left[\dfrac{f_1(x)}{f_2(x)}\right] \Big/ \left[\dfrac{f_1(x)}{f_2(x)}\right]} \right|_{\substack{dy = 0 \\ dx_3 = \ldots = dx_n = 0}} =$$

$$\frac{f_1(x)\ f_2(x)}{x_1\ x_2} \cdot \frac{x_1\ f_1(x) + x_2\ f_2(x)}{\left[2f_1(x)\ f_2(x)\ f_{12}(x) - f_1^2(x)\ f_{11}(x) - f_1^2(x)\ f_{22}(x)\right]} \tag{13}$$

See e.g. R. G. Chambers [3, pp. 27-36]. Of course, if x=h(w,y) definitions (10) and (13) coincide.

2. The Morishima Elasticity of Substitution

As with the HES, let us start from equation (1). Here the percentage change

$$\frac{d\left[\frac{w_1}{w_2}\right]}{\left[\frac{w_1}{w_2}\right]} = \frac{dw_1}{w_1} - \frac{dw_2}{w_2}$$

is arbitrary. In the case of the MES, it is required that all other relative prices do <u>not</u> change, i.e. that

$$\frac{d\left[\frac{w_k}{w_2}\right]}{\left[\frac{w_k}{w_2}\right]} = \frac{dw_k}{w_k} - \frac{dw_2}{w_2} = 0 \quad , \quad k{:}3, \; \ldots, \; n$$

We must, therefore, have that

$$\frac{dw_2}{w_2} = \frac{dw_3}{w_3} = \ldots = \frac{dw_n}{w_n} \tag{14}$$

From (4) it is clear that (14) does allow changes in $h_3(w,y)$, ..., $h_n(w,y)$.

Consider then

$$\frac{dh_1(w,y)}{h_1(w,y)} = \varepsilon_{11} \frac{dw_1}{w_1} + \sum_{j=2}^{n} \varepsilon_{1j} \frac{dw_j}{w_j}$$

Applying (14) gives

$$\frac{dh_1(w,y)}{h_1(w,y)} = \varepsilon_{11} \frac{dw_1}{w_1} + \left[\sum_{j=2}^{n} \varepsilon_{1j} \right] \frac{dw_2}{w_2}$$

As the demand function $h_1(w,y)$ is homogeneous of degree zero in w, we must have that

$$\sum_{j=2}^{n} \varepsilon_{1j} = - \varepsilon_{11}$$

We then finally obtain

$$\frac{dh_1(w,y)}{h_1(w,y)} = \varepsilon_{11} \left[\frac{dw_1}{w_1} - \frac{dw_2}{w_2} \right] \tag{15}$$

In a similar way we obtain

$$\frac{dh_2(w,y)}{h_2(w,y)} = \varepsilon_{21} \left[\frac{dw_1}{w_1} - \frac{dw_2}{w_2} \right] \tag{16}$$

Denoting the MES by $\sigma_{12}^{M}(w,y)$, we obtain from (1), (15) and (16)

$$\sigma_{12}^{M}(w,y) = \varepsilon_{21} - \varepsilon_{11} \tag{17}$$

This is, of course, a well-known result. See e.g. [1], [2], [3]. In a similar way we obtain

$$\sigma_{21}^{M}(w,y) = \varepsilon_{12} - \varepsilon_{22} \tag{18}$$

There is no reason why $\sigma_{12}^{M}(w,y)$ should equal $\sigma_{21}^{M}(w,y)$. In fact from (8), (17) and (19), we see that

$$\sigma_{12}^{M}(w,y) - e_{21} + e_{11} = \sigma_{21}^{M}(w,y) - e_{12} + e_{22} \tag{19}$$

It follows from (19) that

$$\sigma_{12}^{M}(w,y) = \sigma_{21}^{M}(w,y) \leftrightarrow e_{11} + e_{12} = e_{21} + e_{22} \tag{20}$$

This means that the sum of the indirect effects of $dw_1/w_1, dw_2/w_2$ on $dh_1(w,y)/h_1(w,y)$ and on $dh_2(w,y)/h_2(w,y)$ must be the same. Of course, if the number of inputs equals two, (20) is automatically satisfied.

3. Comparing the HES and the MES

From (10, (11), (17) and (18) we can derive that

$$\sigma_{12}^{H}(w,y) = \sigma_{21}^{H}(w,y) = \sigma_{12}^{M}(w,y) - e_{21} + e_{11} \qquad (21)$$
$$= \sigma_{21}^{M}(w,y) - e_{12} + e_{22}$$

It follows that

$$\sigma_{12}^{H}(w,y) = \sigma_{21}^{H}(w,y) = \sigma_{12}^{M}(w,y) = \sigma_{21}^{M}(w,y) \leftrightarrow e_{11} = e_{21} \text{ and } e_{12} = e_{22} \qquad (22)$$

Of course, (22) is trivially satisfied if the number of inputs, n, equals 2.

More explicit results can be obtained in the case where n=3. One then easily calculates

$$A_{12}\, A_{22}^{-1}\, A_{21} = \frac{1}{\varepsilon_{33}} \begin{bmatrix} \varepsilon_{12}\,\varepsilon_{31} & \varepsilon_{13}\,\varepsilon_{32} \\ \varepsilon_{23}\,\varepsilon_{31} & \varepsilon_{23}\,\varepsilon_{32} \end{bmatrix}$$

From (20) and (22) we then have

$$
\begin{aligned}
\sigma_{12}^{M}(w,y) &= \sigma_{21}^{M}(w,y) \\
&\Leftrightarrow e_{11} + e_{12} = e_{21} + e_{22} \\
&\Leftrightarrow \varepsilon_{13}\,\varepsilon_{31} + \varepsilon_{13}\,\varepsilon_{32} = \varepsilon_{23}\,\varepsilon_{31} + \varepsilon_{23}\,\varepsilon_{32} \\
&\Leftrightarrow \varepsilon_{13} = \varepsilon_{23} \\
&\Leftrightarrow e_{11} = e_{21} \text{ and } e_{12} = e_{22} \\
&\Leftrightarrow \sigma_{12}^{M}(w,y) = \sigma_{21}^{M}(w,y) = \sigma_{12}^{H}(w,y) = \sigma_{21}^{H}(w,y)
\end{aligned}
$$

It follows that, for n=3,

$$
\begin{aligned}
\sigma_{12}^{M}(w,y) &= \sigma_{21}^{M}(w,y) \\
&\Leftrightarrow \sigma_{12}^{M}(w,y) = \sigma_{21}^{M}(w,y) = \sigma_{12}^{H}(w,y) = \sigma_{21}^{N}(w,y) \\
&\Leftrightarrow \varepsilon_{13} = \varepsilon_{23}
\end{aligned}
\tag{23}
$$

This property will be illustrated in the following examples.

4. Two Simple Examples

Let us first take the production function

$$f(x_1,\ x_2,\ x_3) = \min\ \{x_1,\ (x_1)^{1/2}\ (x_2)^{1/2}\} \tag{24}$$

examined in C. Blackorby and R.R. Russell [2, p. 883]. The corresponding cost function is

$$C(w_1,\ w_2,\ w_3,\ y) = \left[w_1 + 2\,(w_2)^{1/2}\ (w_3)^{1/2}\right] y \tag{25}$$

The matrix of price elasticities of the conditional demand functions is given by

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1/2 & 1/2 \\ 0 & 1/2 & -1/2 \end{bmatrix}$$

Hence, all these elasticities are constant, i.e. independent of w_1, w_2, w_3 and y . One then easily calculates

$$\begin{cases} \sigma_{12}^M = 0 \\ \sigma_{21}^M = 1/2 \end{cases} \qquad \sigma_{12}^H = \sigma_{21}^H = 0 \qquad (25.a)$$

$$\begin{cases} \sigma_{13}^M = 0 \\ \sigma_{31}^M = 1/2 \end{cases} \qquad \sigma_{13}^H = \sigma_{31}^H = 0 \qquad (25.b)$$

$$\begin{cases} \sigma_{23}^M = 1 \\ \sigma_{32}^M = 1 \end{cases} \qquad \sigma_{23}^H = \sigma_{32}^H = 1 \qquad (25.c)$$

As $\varepsilon_{13} = 0 \neq \varepsilon_{23} = 1/2$, one expects, on the basis of (23), that $\sigma_{12}^M \neq \sigma_{21}^M$. This is confirmed by (25.a). As $\varepsilon_{12} = 0 \neq \varepsilon_{32} = 1/2$, the same applies to (25.b). However, as $\varepsilon_{21} = \varepsilon_{31} = 0$, one must have $\sigma^M_{23} = \sigma^M_{32} = \sigma^H_{23} = \sigma^H_{32}$ as is implied by (25.c).

The second example uses a production function of the form

$$f(x_1, x_2, x_3) = 2(x_1)^{1/2}\left[(x_2)^{1/2} + (x_3)^{1/2}\right] \qquad (26)$$

This is taken from P.B. Dixon, S. Bowless and D. Kendrick [4, p. 291]. The implied cost function is

$$C(w_1, w_2, w_3, y) = \left[\frac{w_1 \; w_2 \; w_3}{(w_2 + w_3)}\right]^{1/2} y \qquad (27)$$

The matrix of price elasticities of the conditional demand functions is now given by

$$A = \begin{bmatrix} -1/2 & \dfrac{w_3}{2(w_2 + w_3)} & \dfrac{w_2}{2(w_2 + w_3)} \\ 1/2 & \dfrac{-(4w_2 + w_3)}{2(w_2 + w_3)} & \dfrac{3w_2}{2(w_2 + w_3)} \\ 1/2 & \dfrac{3w_3}{2(w_2 + w_3)} & \dfrac{-(w_2 + 4w_3)}{2(w_2 + w_3)} \end{bmatrix} \qquad (28)$$

One can then calculate

$$\begin{cases} \sigma_{12}^{M} = 1 \\ \sigma_{21}^{M} = \dfrac{2w_2 + w_3}{w_2 + w_3} \end{cases} \qquad \sigma_{12}^{H} = \sigma_{21}^{H} = \frac{2(w_2 + 2w_3)}{w_2 + 4w_3} \tag{29.a}$$

$$\begin{cases} \sigma_{13}^{M} = 1 \\ \sigma_{31}^{M} = \dfrac{w_2 + 3w_3}{w_2 + w_3} \end{cases} \qquad \sigma_{13}^{H} = \sigma_{31}^{H} = \frac{2(2w_2 + w_3)}{4w_2 + w_3} \tag{29.b}$$

$$\begin{cases} \sigma_{23}^{M} = 2 \\ \sigma_{32}^{M} = 2 \end{cases} \qquad \sigma_{23}^{H} = \sigma_{32}^{H} = 2 \tag{29.c}$$

As $\varepsilon_{13} \neq \varepsilon_{23}$, it follows that $\sigma_{12}^{M} \neq \sigma_{21}^{M}$. Similarly, $\sigma_{13}^{M} \neq \sigma_{31}^{M}$ as $\varepsilon_{12} \neq \varepsilon_{32}$. As $\varepsilon_{21} = \varepsilon_{31} = 1/2$, we must have that $\sigma_{23}^{M} = \sigma_{32}^{M}$.

Appendix

It is well known that the cost function C(w,y) is concave in w, so that the n x n Hessian matrix

$$\left[\frac{\partial^2 C(w,y)}{\partial w_i \, \partial w_j} \right]$$

is negative semi definite. By Shephard's lemma, we have that

$$\left[\frac{\partial^2 C(w,y)}{\partial w_i \, \partial w_j} \right] = \left[\frac{\partial h_i(w,y)}{\partial w_j} \right]$$

so that

$$\forall\, z \in R^n,\ z^t \left[\frac{\partial h_i(w,y)}{\partial w_j} \right] z \leq 0$$

(z^t is the transposed of the column vector z). It can also be shown that

$$z^t \left[\frac{\partial h_i(w,y)}{\partial w_j} \right] z = 0$$

if and only if z is proportional to w, i.e. $z = \lambda w$ with $\lambda \in R$. See e.g. W. Pauwels [5].

The matrix A_{22} , defined in (4) can be written as

$$A_{22} = \begin{bmatrix} \frac{1}{h_3} & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & \frac{1}{h_n} \end{bmatrix} \begin{bmatrix} \frac{\partial h_3}{\partial w_3} & \cdots & \frac{\partial h_3}{\partial w_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial h_n}{\partial w_3} & \cdots & \frac{\partial h_n}{\partial w_n} \end{bmatrix} \begin{bmatrix} w_3 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & w_n \end{bmatrix}$$

We first show that A_{22} must be nonsingular. If A_{22} would be singular, the system of equations

$$\begin{bmatrix} \frac{\partial h_3}{\partial w_n} & \cdots & \frac{\partial h_3}{\partial w_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial h_n}{\partial w_3} & \cdots & \frac{\partial h_n}{\partial w_n} \end{bmatrix} v = 0$$

would have a non zero solution $v \in R^{n-2}$. It would then follow that

$$[0,\ 0,\ v^t] \begin{bmatrix} \frac{\partial h_1}{\partial w_1} & \cdots & \frac{\partial h_1}{\partial w_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial h_n}{\partial w_1} & \cdots & \frac{\partial h_n}{\partial w_n} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ v \end{bmatrix} = 0$$

This is impossible as this quadratic form can only be zero for vectors proportional to w.

As the conditional demand functions are homogeneous of degree zero in w, we must have that

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ - \\ 1 \\ . \\ . \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ - \\ 0 \\ . \\ . \\ 0 \end{bmatrix}$$

It follows that

$$\begin{bmatrix} I_2 & -A_{12} \ A_{22}^{-1} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ - \\ 1 \\ . \\ . \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ - \\ 0 \\ . \\ . \\ 0 \end{bmatrix}$$

or

$$\begin{bmatrix} A_{11} - A_{12} \ A_{22}^{-1} \ A_{21} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ - \\ 1 \\ . \\ . \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ - \\ 0 \\ . \\ . \\ 0 \end{bmatrix}$$

so that

$$A_{11} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = A_{12}\; A_{22}^{-1}\; A_{21} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

(I_2 is the 2 x 2 identity matrix). This last equality proves that

$$\varepsilon_{11} + \varepsilon_{12} = e_{11} + e_{12}$$

$$\varepsilon_{21} + \varepsilon_{22} = e_{21} + e_{22}$$

If we add the left hand sides and the right hand sides of these equations, we obtain (8).

References

[1] BLACKORBY C. and RUSSELL R.R., The Morishima Elasticity of Substitution; Symmetry, Constancy, Separability, and its Relationship to the Hicks and Allen Elasticities, Review of Economic Studies, 1981, XLVIII, pp. 147-158.

[2] BLACKORBY C. and RUSSELL R.R., Will the Real Elasticity of Substitution Please Stand Up ? The American Economic Review, September 1989, vol. 79, number 4, pp. 882-888.

[3] CHAMBERS R.G., Applied Production Analysis, A Dual Approach, Cambridge University Press, Cambridge, 1988.

[4] DIXON P.B., BOWLESS S., and KENDRICK D., Notes and Problems in Mircroeconomic Theory, North Holland Publishing Company, Amsterdam, 1980.

[5] PAUWELS W., On Some Results in Comparative Statics Analysis, Journal of Economic Theory, vol. 21, nr. 3, December 1989, pp. 483-490.

LIJST VAN RECENTE SESO-RAPPORTEN

DE BORGER B., KERSTENS K., MOESEN W. en J. VANNESTE, Efficiency and equity in block grant design : simulating some alternatives for Flemish municipalities, August 1990, 35 blz. **(90/246)**

DE BRABANDER G. en E. GIJSBRECHTS, City marketing, van promotie tot plan ? een verkennend overzicht van een nieuw gebied, augustus 1990, 38 blz. **(90/247)**

VAN POECK A. en J. VAN GOMPEL, Unemployment and wage formation in small industrial countries (1973-1989), September 1990, 44 blz. **(90/248)**

DE BORGER B., The economic environment and public enterprise behavior : Belgian railroads 1950-1986, October 1990, 44 blz. **(90/249)**

KESENNE S., A guaranteed basic income as a cultural policy, October 1990, 12 blz. **(90/250)**

VANNESTE J. en W. MOESEN, De gemeentefinanciën in Vlaanderen : verkenning en verklaringen, oktober 1990, 47 blz. **(90/251)**

HENDRICKX K., Sectoriële bronnen voor produktiviteitsmeting : Definities, classificaties en methodologie, december 1990, 122 blz. **(90/252)**

PLASMANS J. en J. VANNESTE, The incidence of corporate taxation in Belgium on employment and investment, December 1990, 30 blz. **(90/253)**

DE BRABANDER G., m.m.v. BLOMMAERT K., GILLE A., LEZY L., MAES T. en F. Witlox, Aspecten van het Antwerpse economische en financiële draagvlak, maart 1991, 60 blz. **(91/254)**

SCHROYEN F., Demand system under rationing : an introduction with special reference to the implications of separability assumptions, March 1991, 26 blz. **(91/255)**

TORFS K. en D. DE GRAEVE, A cost-effectiveness analysis of AOTAL, a drug used to prevent relapse in weaned alcoholics, April 1991, 43 blz. **(91/256)**

VAN POECK A. en J. VAN GOMPEL, The decline in unemployment (1984-90) and the wage formation hypothesis, May 1991, 31 blz. **(91/257)**

HEYLEN F., Long-term unemployment in the OECD-countries. The relevance of structural labour market and labour market policy characteristics, May 1991, 40 blz. **(91/258)**

KONINGS J., Experimentele economie als onderzoeksmethodologie : basisprincipes en resultaten van een duopolie-experiment, juni 1991, 28 blz. **(91/259)**

VAN TRIER W., "State Bonus" or basic income in the age of reconstruction, September 1991, 145 blz. **(91/260)**