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Estimating a multiple output generalized Box-Cox cost function : cost structure and productivity growth in Belgian railroad operations, 1950-1986.

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Abstract

The purpose of this paper is to generalize Berndt and Khaled's Box-Cox specification so as to allow the estimation of multiple-output cost models. The proposed functional form is applicable to total as well as variable cost functions. In case of a variable cost function, the suggested specification allows the straightforward introduction of multiple fixed factors.

The model is applied to analyse the structure of cost and productivity growth of railroad operations in Belgium over the period 1950-1986.

Introduction

The literature has devoted considerable attention to the specification of increasingly general, flexible functional forms to be used in the empirical analysis of cost and production functions. For example, Berndt and Khaled (1979) proposed a generalised Box-Cox specification that generates both the widely used Translog (Christensen, Jorgenson and Lau (1971)) and generalised Leontief (Diewert (1971)) as special cases. The CES-translog cost function proposed by Pollak, Sickles and Wales (1984) includes both the CES and the translog as special cases. More recently, Diewert and Wales (1987) proposed generalizations of forms suggested by Mc Fadden (1978) and Barnett (1983), and evaluated their empirical performance[1].

The emphasis in these studies is typically on the specification of single-output technologies. Although a generalized Leontief specification for a technology with multiple outputs was introduced by Hall (1973), almost all empirical multi-output models have been based on the translog approximation[2] (see, for example, the railroad studies by Caves, Christensen and Swanson (1980, 1981)). The purpose of this paper is to generalize Berndt and Khaled's Box-Cox specification so as to allow the estimation of multiple-output cost models. The proposed functional form is applicable to total as well as variable cost functions. In case of a variable cost function, the suggested specification allows the straightforward introduction of multiple fixed factors.

The model is applied to analyse the structure of cost and productivity growth of railroad operations in Belgium over the period 1950-1986. Although there exists a large number of empirical papers dealing with the cost structure of the railroad industry in the US (see, e.g., Borts (1960), Keeler (1974), Breautigam et al. (1984)) there is no comparable flow of research on the economic characteristics of European railroad companies. By investigating the properties of railroad technology in Belgium and calculating indices of productivity growth over time we hope to fill a small part of this apparent gap in the applied literature.

The paper is organized as follows. In Section 1 we present the generalized Box-Cox cost function that allows the introduction of multiple outputs and, in the case of a short-run restricted cost model, multiple fixed factors. Formulas describing the economic properties of the production process are derived. In Section 2 we discuss application of the proposed cost specification to estimate the structure of railroad costs and productivity in Belgium. We discuss estimation issues and present empirical results in Section 3. The paper concludes with Section 4 in which we summarize the findings of the analysis.

1. A generalised Box-Cox cost function for multiple-output technologies

In this section we present a generalized Box-Cox multiple-output cost function (GBC) obtained by extending the specification suggested by Berndt and Khaled (1979). It encompasses the generalized Leontief (GL) as a special case and the multiple output translog (TL) as a limiting case, the adequacy of which can be empirically evaluated. This is a convenient property of the model, as comparing the relative performance of alternative cost models and consumer demand systems has generated quite some intrest in the recent literature[3]. The proposed specification directly applies to both unrestricted and restricted cost functions due to the symmetric treatment of outputs and fixed factors. Although this symmetry is also implied by several translog applications (see, for example, Caves et al. (1981)), fixed factors have been introduced in GL restricted cost or profit functions in a variety of different ways. (see Morrison (1988) and the references given therein). In many cases, introduction of multiple fixed factors in these specifications is not straightforward. The GBC model suggested here does not have this drawback.

Consider the cost function specification

$$C = [1+\Lambda G(P)]^{1/\Lambda} \; [\prod_{k=1}^{K} Z_k^{\beta_k(Z,P)}] \tag{1}$$

where

$$G(P) = \alpha_0 + \sum_{i=1}^{N} \alpha_i \; p_i(\Lambda) + \sum_{i=1}^{N}\sum_{j=1}^{N} \gamma_{ij} \; p_i(\Lambda) \; p_j(\Lambda) \tag{2}$$

$$\beta_k(Z,P) = \beta_k + \sum_{l=1}^{K} \frac{\theta_{lk}}{2} \ln Z_l + \sum_{i=1}^{N} \phi_{ki} \ln p_i \tag{3}$$

$$p_i(\Lambda) = (p_i^{\Lambda/2} - 1)/(\Lambda/2) \tag{4}$$

The vector P consists of the prices of N variable inputs, i.e., $P = (p_1, \ldots, p_N)$. The vector Z is assumed to consist of R outputs and (K-R) fixed factors, where $K \geq R$. For K = R equations (1), (2), (3) and (4) describe an unrestricted total cost model. Is $K > R$ then we are dealing with a short-run restricted cost function[4]. By assumption $\gamma_{ij} = \gamma_{ji}$ and $\theta_{lk} = \theta_{kl}$. Moreover, linear homogeneity in input prices can be shown to require the following restrictions on the parameters[5]

$$\sum_{i=1}^{N} \alpha_i = 1 + \Lambda \alpha_0 \tag{5}$$

$$\sum_{j=1}^{N} \gamma_{ij} = \frac{\Lambda}{2} \alpha_i \qquad \forall i \tag{6}$$

$$\sum_{i=1}^{N} \phi_{ki} = 0 \qquad \forall k \tag{7}$$

Imposing these restrictions on (1) yields the GBC cost model (8)

$$C = \left[\frac{2}{\Lambda} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij}\, p_i^{\Lambda/2}\, p_j^{\Lambda/2}\right]^{1/\Lambda} \left[\prod_{k=1}^{K} Z_k^{\beta_k(P,Z)}\right] \tag{8}$$

Two special cases of (8) deserve to be mentioned at this point. If we set $\Lambda = 1$ we obtain a generalized Leontief multiple-output cost function

$$C = \left[2 \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \sqrt{p_i\, p_j}\right] \left[\prod_{k=1}^{K} Z_k^{\beta_k(P,Z)}\right] \tag{9}$$

Moreover, the translog can be derived as a limiting case for Λ approaching 0. Rewrite (1) as

$$G(P) = \frac{1}{\Lambda} \left\{ \left[\frac{C}{\prod_{k=1}^{K} Z_k^{\beta_k(Z,P)}}\right]^{\Lambda} - 1 \right\}.$$

The right-hand side of this expression is the Box-Cox transformation of

$$\left[C / \prod_{k=1}^{K} Z_k^{\beta_k(Z,P)}\right]$$

It follows that for $\Lambda \to 0$ [6]

$$G(P) = \ln C - \sum_{k=1}^{K} \beta_k(Z,P) \ln Z_k.$$

Using (2), (3) and the fact that $\lim_{\Lambda \to 0} p_i(\Lambda) = \ln p_i$ we finally obtain that for $\Lambda \to 0$ expression (1) converges to

$$\ln C = \alpha_0 + \sum_{i=1}^{N} \alpha_i \ln p_i + \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \ln p_i \ln p_j + \sum_{k=1}^{K} \beta_k \ln z_k$$

$$+ \sum_{k=1}^{K} \sum_{l=1}^{K} \frac{\theta_{lk}}{2} \ln z_l \ln z_k + \sum_{k=1}^{K} \sum_{i=1}^{N} \phi_{ki} \ln p_i \ln z_k \qquad (10)$$

which is the multiple-output translog cost model.

Since we are interested in productivity measurement we adjust the GBC cost function (8) to allow for technical change. Although there are other ways of doing this we simply specify

$$C = [\frac{2}{\Lambda} \sum_i \sum_j \gamma_{ij} p_i^{\Lambda/2} p_j^{\Lambda/2}]^{1/\Lambda} [\prod_k z_k^{\beta_k(Z,P)}] [e^{T(t,P)}] \qquad (11)$$

where

$$T(t,P) = t(\tau + \sum_{i=1}^{N} \tau_i \ln p_i).$$

Linear homogeneity of the cost function in input prices requires, in addition to (5), (6) and (7), the restriction

$$\sum_{i=1}^{N} \tau_i = 0.$$

Technical change is said to be input i saving, i neutral or i using depending on whether τ_i is less than, equal to, or greater than zero. It is Hicks neutral at rate τ if all $\tau_i = 0$.

Note that our specification of technical change is quite restrictive. It implies a rate of cost diminution given by

$$\frac{\partial \ln C}{\partial t} = \tau + \sum_{i=1}^{N} \tau_i \ln p_i, \qquad (12)$$

which is independent of the elements of the vector Z. This observation suggests that we explicitly impose a lot of structure on the evolution of technical change. Given the use of pure time series data in our empirical application we did not consider this to be undesirable, however.

The factor demand system corresponding to cost function (11) is obtained by application of Shephard's lemma and some algebraic manipulation. We find

$$X_i = [\frac{2}{\Lambda} \sum_{j=1}^{N} \gamma_{ij} (\frac{p_j}{p_i})^{\frac{\Lambda}{2}}] \; [\prod_{k=1}^{K} Z_k^{\Lambda\beta_k(Z,P)}] \; [e^{\Lambda T(t,P)}] \; (\frac{C}{p_i})^{1-\Lambda}$$

$$+ \sum_{k=1}^{K} \frac{C}{p_i} (\phi_{ki} \ln Z_k + \tau_i t) \qquad (13)$$

Other properties of the cost model represented by (11) and (13) are now briefly reviewed. The cost elasticities with respect to the elements of the vector Z can be shown to be given by

$$\varepsilon_k = \frac{\partial \ln C}{\partial \ln Z_k} = \beta_k + \sum_{l=1}^{K} \theta_{lk} \ln Z_l + \sum_{i=1}^{N} \phi_{ki} \ln p_i$$

Based on these elasticities we can derive formulas related to returns to scale. If the cost model describes a restricted cost function (i.e., K > R) Caves et al (1981) have shown that the degree of returns to scale is

$$RTS = \frac{1 - \sum_{k=R+1}^{K} \varepsilon_k}{\sum_{k=1}^{R} \varepsilon_k} \qquad (14)$$

We have increasing, constant or decreasing returns to scale if RTS is greater than, equal to, or smaller than one. If (11) and (13) refer to an unrestricted total cost function, i.e. $K = R$, (14) reduces to the inverse of the sum of output cost elasticities, viz.[7]

$$\text{RTS} = \frac{1}{\sum_{k=1}^{K} \varepsilon_k} \,. \tag{15}$$

The Allen partial elasticities of substitution corresponding to an arbitrary cost function C are given by

$$\sigma_{ij} = \frac{C \quad C_{ij}}{C_i \quad C_j}$$

where

$$C_i = \frac{\partial\ C}{\partial\ p_i} \text{ and } C_{ij} = \frac{\partial^2\ C}{\partial\ p_i\ \partial\ p_j} \,.$$

For our model (11) the σ_{ij} can be calculated to be

$$\sigma_{ij} = 1 - \Lambda + \gamma_{ij} \frac{(p_i \cdot p_j)^{\Lambda/2}}{s_i\ s_j} M + \Lambda \frac{F_j(Z,t)}{s_j} + \Lambda\ [1 - \frac{F_j(z,t)}{s_j}]\ \frac{F_i(Z,t)}{s_i} \qquad i \neq j \tag{16}$$

and

$$\sigma_{ii} = 1 - \Lambda + \gamma_{ii} \frac{p_i^{\Lambda}}{s_i^2} M + \Lambda \frac{F_i(Z,t)}{s_i} + \Lambda[1 - \frac{F_i(Z,t)}{s_i}]\ \frac{F_i(Z,t)}{s_i} + \frac{\Lambda}{2}\ [1 - \frac{F_i(Z,t)}{s_i}]\ \frac{1}{s_i} - \frac{1}{s_i} \tag{17}$$

where

$$M = \left[\frac{2}{\Lambda} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \, p_i^{\Lambda/2} \, p_j^{\Lambda/2}\right]^{-1}$$

$$F_i(Z,t) = \sum_{k=1}^{K} \phi_{ki} \ln Z_k + \tau_i t \qquad i = 1, \ldots, N$$

$$s_i = \frac{p_i X_i}{C} . \qquad i = 1, \ldots, N$$

The associated input price elasticities are

$$\eta_{ij} = s_j \sigma_{ij} \qquad i, j = 1, \ldots, N \qquad (18)$$

We finally derive productivity indices based on the GBC cost function (11). We follow Caves et al (1981) and define two productivity measures. The first one, P1, is defined as the common rate at which outputs can grow over time with all inputs held constant. Following the procedure outlined in De Borger (1984, p. 39-41) one easily shows that

$$P1 = -\frac{\varepsilon_t}{\sum_{k=1}^{R} \varepsilon_k} \qquad (19)$$

where

$$\varepsilon_t = \frac{\partial \ln C}{\partial t}$$

is the rate of cost diminution, see (12). A second index, P2, is the common rate at which inputs can be reduced over time with all outputs held at a fixed level. One finds

$$P2 = - \frac{\varepsilon_t}{1 - \sum_{k=R+1}^{K} \varepsilon_k} \tag{20}$$

Note that these two productivity indices will differ unless there are constant returns to scale. Indeed, RTS = 1 implies P1 = P2, see equation (14)[8].

2. Towards an empirical application : Belgian railroads

In the US the availability of detailed information on a large cross-section of individual railroads has generated a substantial number of empirical cost models (see, e.g., Borts (1960), Griliches (1972), Keeler (1974), Hasenkamp (1976), Viton (1980)). Several authors have used estimated cost functions to carefully study productivity growth in the railroad industry (Caves et al. (1980, 1981)). Much less is known about the cost structure and the evolution of productivity changes over time for European railroads, however. In most countries railroad services are provided by a single company, which is often nationalized. This implies that differences in the institutional environment and variations in data availability between countries force the potential investigator to rely on time series data for a single company. Moreover, although studies of railroad costs based on time series have appeared in the literature (see Breautigam et al (1984)), in several countries the available data are not suited for this kind of statistical cost analysis. Fortunately, the Belgian railroad company NMBS published a relatively rich data set on its annual operations. These data are used to study cost and productivity growth over the period 1950-1986[9].

Strictly speaking, the Belgian railroad company is a public enterprise. The official statutes explicitly emphasize that one of the firm's major objectives is to 'serve the public intrest'. Output and pricing decisions are only weakly related to the profit motive, and many other social and economic factors may play a crucial role (e.g., to support declining industries, continue to serve municipalities on unprofitable lines). A second characteristic feature of the company is that the government heavily subsidizes, but in some cases also directly rations, investment in both fixed structures and rolling stock. The implied subsidy rates are not observable in the firm's annual reports and may vary substantially over time.

The previous observations eliminate profit maximization and total cost minimization as behavioral hypotheses to be used in the empirical application. We therefore assume in what follows that the firm's objective is to minimize short-run variable costs, given exogenously determined output levels and a fixed capital stock[10]. An important remark is in order, however. Given the public enterprise nature of the railroad company even variable cost minimization may not be entirely realistic. The theoretical literature has generated both normative (see, e.g., Marchand et al (1984)) and positive (see, e.g., Rees (1984)) models that suggest that under plausible conditions public enterprises may be induced to use implicit shadow prices for factors in their input choice decisions that deviate from observed market prices. More specifically, it has been argued that under conditions of high unemployment the enterprises apply shadow wages below the prevailing market wage. Introducing shadow input prices substantially complicates the already quite complex empirical model presented in the previous section, however. Given this observation and, more importantly, given our experiences with some 'shadow price' cost models based on the same data set we decided to accept short-run variable cost minimization as the behavioral hypothesis for the purpose of this paper[11].

We assume the railroad company produces freight (F) and passenger (R) services using the variable inputs labor (L) and energy (E), and the fixed input capital (K). Note that our approach differs from the usual specification in the railroad cost literature in that we do not include the variable input 'materials and equipment'. There are several reasons for our decision. First, given the Belgian institutional setting and noting that the majority of expenditures on equipment relate to modernization and replacement of the rolling stock it seemed more reasonable to treat equipment as part of the capital stock, assumed to be fixed in the short-run. Indeed, it is well known that the very long lags involved in investment in rolling stock and a number of bureaucratic procedures imply that the rolling stock cannot be greatly varied within short periods of time (De Borger and Deloddere (1982))[12]. Second, our data did not contain sufficient reliable information to construct a consistent time series for the price of non-energy materials. Fortunately, expenditures on materials typically account for less than 4 % of short-run variable costs. Therefore, we assumed, following Pindyck (1979), that in the underlying transformation function capital, labor and energy are as a group weakly separable from the input materials. Adding the assumption of weak separability in the major categories of capital, labor and energy allows us to analyze a short-run variable cost function

$$C_V(p_L, p_E, K, F, R, t)$$

where p_i is the price of input i and t is time[13]. This cost function gives the minimum expenditures on labor and energy in period t to produce outputs F and R, given input prices p_i and capital stock K.

Ignoring the materials input in our empirical model implies that the estimated substitution elasticities and input price elasticities should be interpreted as gross effects in the sense of Berndt and Wood (1979). Given the relative unimportance of non-energy materials in the production process we suspect that the bias due to ignoring the additional substitution possibilities between (K, L, E) and materials is small.

Unless otherwise noted all data were taken from the company's annual reports. The price of labor was obtained by dividing total labor expenditures by the aggregate quantity of labor input, measured in number of employees[14]. With respect to energy, a Divisia index for energy input was constructed based on three energy sources (electricity, fuel, coal) and total energy costs were divided by this aggregate input to obtain unit price p_E. A capital stock index was constructed by combining information on the initial value of five categories of capital (tracks, land, buildings, other structures, rolling stock and other equipment) with appropriate depreciation rates and gross investment figures for each category. Unlike several previous studies (Keeler (1974), Braeutigam et al (1984)) we preferred not to use 'miles of track' as a measure of the fixed factor. Variations in this variable were relatively small and investment in tracks was of limited importance over the sample period.

With respect to railroad output several authors have pointed at the multidimensionality of output and the impact of operating characteristics on costs (see, e.g. Caves et al (1981), Harmatuck (1979)). However, including more than a single output indicator for each of the two output categories, passengers and freight, would, given the short time series we have available, easily lead to a model containing too many parameters to be empirically tractable. We therefore used two hedonic output aggregates, one for passengers and one for freight, constructed in previous work (see De Borger (1989)) along the lines suggested by Wang Chiang and Friedlander (1984). Construction of the aggregates is described in detail in De Borger (1989). Here we limit ourselves to a brief overview of the procedure. We specified the output measures as

$$F = f(y_F, Z_F)$$

$$R = r(y_R, Z_R)$$

where y_F and y_R are the physical outputs measured in tonkilometers and passengerkilometers, respectively; z_F and z_R are vectors of operating characteristics; and f(.) and r(.) are aggregator functions assumed to be homogeneous of degree one in physical output[15]. The following simple Cobb-Douglas specification was used for the aggregator functions

$$\ln F = \ln y_F + \sum_{v=1}^{V} a_v \ln Z_{Fv} \tag{21}$$

$$\ln R = \ln y_R + \sum_{w=1}^{W} b_w \ln Z_{Rw} \tag{22}$$

These expressions were inserted in a translog cost model and the parameters a_v and b_w were estimated simultaneously with the cost parameters. The output aggregates used in this study were obtained by inserting the estimated a_v and b_w in (21) and (22) respectively[16].

3. An application to Belgian railroads : estimation and empirical results

In order to estimate the parameters of the model the theoretical relations (11) and (13) have to be embedded in a stochastic framework. Two issues have to be considered when choosing an appropriate stochastic specification. One concerns the relationship between the error term of the cost function and the errors of the factor demand system. A second problem relates to the appearance of cost C on the right-hand side of the factor demand equations (13).

In the empirical literature systems of cost and factor demand (or share) equations are typically estimated by adding i.i.d. error terms and applying a seemingly unrelated regression technique to capture possible correlation between error terms of different equations. This approach implicitly assumes that the economic theory of the cost minimizing firm has no implications for the specification of the stochastic part of the behavioral equations. An alternative approach is to explicitly recognize the relation between error terms of different equations implied by economic theory. For example, Berndt and Khaled (1979) assume that errors in the factor demand system are due to producers' random mistakes in adjusting to the cost minimizing input quantities and show that this implies a unique nonlinear relation between the errors in the demand system and the error in the cost function. More recently, following earlier work by Fuss, Mc Fadden and Mundlak (1978), Mc Ellroy (1987) proposes a stochastic framework, the additive general error model (AGEM), that reflects Stigler's (1976) view that observed deviations from optimizing behavior may be due to the investigator's ignorance of the true underlying optimization problem. This model assumes that for each input i there exists a parameter ε_i which is known to the firm but not to the econometrician. The distribution of the vector ε is assumed to have zero mean vector and a positive definite covariance matrix Σ, and to be uncorrelated with the explanatory variables in the factor demand system. The AGEM model implies that observed cost can be written as the sum of 'deterministic' cost (that is, the minimum cost of producing given output levels at given factor prices when all $\varepsilon_i = 0$) and a price weighted sum of the ε_i's.

The previous analyses both imply, albeit based on quite different behavioral assumptions, an exact relationship between the error terms in cost and factor demand equations. Both also imply that the cost function is redundant because all parameters and error terms in the cost function already appear in the factor demand system. Maximum likelihood estimates can in principle be obtained by applying iterated nonlinear SUR techniques (Gallant (1986)) on the factor demand system alone.

The model presented in Section 1 is complicated by the appearance of cost C on the right hand side of (13). Within the framework of Berndt-Khaled it is 'equilibrium' or 'minimum' cost, not observed cost, which appears in (13). To circumvent the unobservability of equilibrium cost Berndt-Khaled substitute observed cost in the factor demand system[17]. They note that this introduces a specification problem since observed cost is affected by errors in the factor demand system and therefore has a nonzero covariance with these error terms; consequently, SUR will produce inconsistent estimates. The authors therefore propose a maximum likelihood technique that explicitly takes account of the relation between error terms to produce consistent parameter estimates.

The suggested approach was not used in this paper for several reasons. First, the procedure used to circumvent the unobservability of equilibrium cost is quite arbitrary and based on the vague assumption that 'entrepreneurs choose input levels taking account of possible errors in cost minimization' (p. 1226). Second, since the underlying stochastic framework assumes that errors are due to errors in cost-minimizing behavior, one should, strictly speaking, impose the restriction that observed cost must be greater than or equal to equilibrium cost in the estimation procedure. This is 'analytically intractable' (p. 1227). Third, even without this complication, the suggested estimation procedure is computationally quite burdensome.

We estimated the model within the AGEM framework, which allows a variety of interpretations for the error term, including classical measurement error, and for obvious reasons does not require the constraint that observed cost should be at least as great as deterministic cost (Mc Ellroy (1987, p. 742 and 746). Within this framework it is still an unobservable, viz. deterministic cost, which appears in (13). To handle this complication, we used a proxy variable for deterministic cost that is uncorrelated with the errors of the factor demand equations. This proxy variable was constructed by purging observed cost of its correlation with the error terms through the use of a set of instrumental variables, assumed to be correlated with deterministic cost but not with the error terms. The set of instruments included lagged factor prices, costs, outputs and capital stocks. Equation system (13) was then estimated using iterative nonlinear SUR[18].

Estimation results for the generalized Box-Cox (GBC) and generalized Leontief (GL) models are presented in Table 1. The estimates are satisfactory in several respects. In both specifications the majority of coefficients is significantly different from zero. The theoretical curvature restrictions implied by economic theory were found to be satisfied for both specifications and for every sample year. Moreover, both models fitted the data very well. Since the usual single equation R^2 measures are inapplicable because the models do not contain an intercept, we calculated Berndt's generalized R^2 and obtained values of 0.9934 and 0.9917 for the GBC and GL, respectively.

Note that the parameter vectors β, θ, ϕ and τ are quite similar in the two specifications. On the other hand, the large differences in the estimated γ_{ij} are not surprising, since imposing $\Lambda = 1$ in the GL case directly affects the order of magnitude of the vector γ via the restrictions (5) and (6). To test the appropriateness of the GL model a standard likelihood ratio test was carried out. The test statistic is $-2 \ln(L_{GL}/L_{GBC})$, where L_{GL} and L_{GBC} are the values of the sample likelihood in the GL and GBC cases, respectively. It has a χ^2 distribution with 1 degree of freedom. The test statistic was estimated to be 4.39. This implies that the GL can be rejected at the 1 % significance level (critical value 6.63) but not at the 5 % level (critical value 3.84).

Several other restricted versions of the GBC model were estimated. Homotheticity, implying all elements of the vector ϕ are equal to zero, was decisively rejected. This automatically implies rejection of homogeneity in inputs and constant returns to scale since these are nested within the homothetic version. We also tested for neutrality of technical change, which would imply $\tau_L = 0$. It was rejected in favor of energy-saving technical progress, as suggested by the positive sign of τ_L. Apparently, technical progress has been driven by the massive electrification program rather than by improvements in labor productivity.

Strictly speaking the translog model (TL) cannot be obtained by imposing the restriction $\Lambda \to 0$ directly on the estimation procedure. Therefore, we did not formally attempt to test the appropriateness of the Tl specification. For purposes of comparison we did estimate a TL model, however. In order to be consistent with the AGEM error structure we did not estimate the cost function simultaneously with the associated system of factor shares, as is common in the literature. Mc Ellroy (1987, p. 748) notes that the AGEM structure implies that the system of share equations has a hightly inconvenient, nonlinear error structure. She suggests to estimate the translog factor demand equations instead. Under the AGEM assumptions this system is nonlinear in the parameters but linear in the error terms. It can be estimated by iterated nonlinear SUR.

After introducing technical change into the translog cost function (10) in the same way as it was done for the GBC specification, the derived demand system can be written as

$$X_i = \frac{1}{p_i} [e^{h(Z,P,t)}] [\alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln p_j + \sum_{k=1}^{K} \phi_{ki} \ln Z_k + \tau_i t]$$

$$i = 1, \ldots, N$$

where

$$h(Z,P,t) = \alpha_o + \sum_{i=1}^{N} \alpha_i \ln p_i + \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \ln p_i \ln p_j + \sum_{k=1}^{K} \beta_k \ln Z_k$$

$$+ \sum_{k=1}^{K} \sum_{l=1}^{K} \frac{\theta_{kl}}{2} \ln Z_k \ln Z_l + \sum_{i=1}^{N} \sum_{k=1}^{K} \theta_{ki} \ln p_i \ln Z_k$$

$$+ \tau t + \sum_{i=1}^{N} \tau_i t \ln p_i$$

Estimation results for this translog system are in Table 2. To ease the comparison with the GBC and GL models note that, given our two-variable-factor model, the restrictions (5) and (6) reduce in the translog case to $\alpha_L + \alpha_E = 1$ and $\gamma_{LL} = \gamma_{EE} = - \gamma_{LE}, = - \gamma_{EL}$, respectively. Therefore, the free GBC parameters Λ, γ_{LL}, γ_{LE}, γ_{EE} are in the TL model replaced by the free parameters α_L, γ_{LL} and α_O. The results in Table 2 indicate substantial differences between the GBC and TL estimates of the vector θ. Estimates of β, ϕ and τ are quite similar.

The estimates in Tables 1 and 2 capture information about the production process and the evolution of technical change over time. Using the formulas derived in Section 1, we calculated estimates of some important economic characteristics of the railroad company. Not surprisingly, most of these estimates showed substantial variability over time. With the exception of the final 10 years of the sample period we did not find any strong systematic trend in our estimates, however.

Rather than reporting the results for each year in the sample we present estimates of the technological characteristics evaluated at the average values of all variables (factor shares, outputs, etc.) in three successive subperiods of approximately equal length. The first subperiod, 1950-1962, can be described as a period of steady economic growth. Despite this observation, neither passenger nor freight output showed a systematic increase due to increased competition from other transport modes. The output aggregates F and R did show some variability but no trendwise evolution was observed. The subperiod can further be characterized by a slow but steady increase of the share of labor in variable costs. The second subperiod, 1963-1974, coincided with a generally booming economy. The modal shift in favor of the private car implied some reduction in total passengerkilometers early in the period. However, slight improvements in the operating characteristics implied first a relatively stable and towards the end of the period an even slightly increasing value of the output aggregate R. Freight output even substantially increased at the end of the subperiod, both in terms of tonkilometers and in terms of the aggregator F. The share of labor continued to rise, be it at a lower rate than in the first subperiod. The final subperiod, 1975-1986, obviously includes the energy crises and related slowdown in economic growth. As a consequence, both passengerkilometers and tonkilometers, especially early in the period, declined. However, for passenger transport important changes in the firm's operations counteracted this evolution. The restructuring of the existing network in this subperiod implied better traffic scheduling and substantially improved load factors. Moreover, the financial austerity program imposed upon the company in the second half of the period further encouraged efficient use of the rolling stock via optimal scheduling operations. Despite the decline in passengerkilometers, the hedonic output aggregator R actually increased over the period 1975-1986 due to improving operating characteristics. The freight aggregate F slightly declined, however. Finally, the energy crises implied a slight reduction in the share of labor in variable costs.

In Table 3 we present estimates of the elasticity of substitution between labor and energy and the associated input price elasticities for each of the three specifications (GBC, GL, TL) and for each of the three subperiods. For the GBC and GL the results are based on equations (16), (17) and (18). In the TL case it is well known (see, e.g., Pindyck (1979, p. 171)) that the elasticities of substitution are given by

$$\sigma_{ij} = \frac{\gamma_{ij} + s_i\, s_j}{s_i\, s_j} \qquad i \neq j$$

and

$$\sigma_{ii} = \frac{\gamma_{ii} + s_i(s_i - 1)}{s_i^2} .$$

Several observations are in order. First, consider the differences between specifications. The GBC and GL yield very similar estimates except for σ_{LE} in the final subperiod. The TL specification shows less variability in σ_{LE} over time and consistently higher price elaticities. Second, it is clear that despite these minor differences the economic implications of the three models are very similar. Labor demand is very inelastic throughout the sample period. Estimated elasticities range between -0.05 and -0.15. Energy price elasticities also point at inelastic demand; they vary from -0.5 to -0.76 depending on the subperiod and the specification. These values are all within the range of estimates obtained in other countries (see Caves et al (1981), Braeutigam et al (1984)). Moreover, they seem to be typical for the Belgian public transport sector as a whole. Very similar values were estimated in regional bus transportation (De Borger (1984)). There it was argued that apart from purely technological considerations, the nature of labor and energy contracts may help to explain the value of the estimated elasticities.

Estimates of the cost elasticities with respect to outputs and capital stock are reported in Table 4. Again note that all three cost function specifications yield qualitatively similar results. The evolution over time should be interpreted in light of the variations in outputs that were previously discussed, since the expressions for the cost elasticities derived in Section 1 directly depend on the values of the output aggregators. Note that as a consequence of the substantial increase in the passenger aggregate R the cost elasticities with respect to output increase in the final subperiod.

Next consider Table 5. There we present estimates of the returns to scale indicator RTS (see equation (14)) as well as the two productivity indices P1 and P2 (see equations (19) and (20)). With respect to scale economies the differences between specifications are again reasonably small. The reported point estimates suggest slight economies of scale in the first subperiod, becoming somewhat stronger in the second period. The final subperiod indicates slight diseconomies of scale, however. If one were to take these results at face value, one interpretation of this latter finding is that the thorough restructuring of the firm and the financial pressure imposed by the government in the final subperiod may have induced the company to exhaust whatever scale economies may have been present. It should be noted, however, that the estimated scale economies early in the sample period and the diseconomies in the final decade are mild and that, more importantly, none of the reported estimates for RTS was significantly different from one. In other words, the hypothesis of constant returns to scale throughout the sample period cannot be rejected. The absence of strongly increasing returns to scale is not surprising given the extremely high density of the existing railroad network.

Finally consider the estimated indices of technical change. They are presented as average annual percentage increases in productivity over the different subperiods. First note that the indices P1 and P2 lead to somewhat different results. This is not surprising since they would have yielded the same result if and only if RTS=1 would have prevailed throughout the sample period. Perhaps the most unexpected result is that there are no strong variations over time in the estimates of technical progress. Depending on the specification and the subperiod, average annual increases in productivity range between 1.3 % and 2.4 %. There is no conclusive evidence that technical progress has slowed down over time. Although P1 suggests a slowdown over the last subperiod, this conclusion is not supported by the evolution in P2. It should be noted that the absence of more variation in the productivity changes over time may be due to the particular method used to incorporate technical progress into the empirical cost models. As indicated in Section 1 we imposed a lot of structure on the evolution of technical change by making it independent of outputs and capital stock. The advantage of this procedure is that it reduces the technical problems (e.g. multicollinearity) associated with more general specifications of productivity growth. The cost of the procedure is that by imposing (too) much structure on the specification, some interesting aspects related to differences in technical progress over time may not be identifiable.

To conclude this empirical section we may note that, contrary to the findings of Diewert and Wales (1987), we did not find important qualitative differences in estimated economic characteristics of the firm between different specifications of the cost function. Based on the results of the empirical application one could argue, as does Lewbel (1989) in another context, that the controversy over the relative merits of different functional forms is quite unnecessary, as they have approximately the same explanatory power and yield similar elasticity estimates. Obviously, there is no guarantee that this will also be the case in other applications. Therefore, this paper should be viewed as either providing an appealing alternative specification in a multi-output environment, or as providing a tool for assessing the adequacy of less general models.

4. Summary and Conclusion

In this paper we have extended Berndt and Khaled's (1979) single output cost model to multiple-output production processes. The result is a multiple-output generalized Box-Cox specification that generates the generalized Leontief as a special case and the translog as a limiting case. The proposed specification captures both total and restricted cost models. If it is used to describe a restricted cost function the introduction of multiple fixed factors is straightforward.

We applied the model to study the cost structure and the evolution of productivity growth in Belgian railroad operations over the period 1950-1986. The proposed cost model was estimated using iterative nonlinear seemingly unrelated regression techniques. The parameter estimates were then used to calculate a number of economic properties of the production process. We found input price and substitution elasticities well within the range of estimates reported in the literature. The results further indicated some variation in our point estimate of returns to scale over time; however, the hypothesis of constant returns to scale throughout the sample period could not be rejected. Average annual rates of productivity growth obtained on the basis of the generalized Box-Cox model ranged between 1.65 % and 2.4 % depending on the subperiod and the productivity index used.

To compare the performance of the proposed specification with the translog and Leontief models we also estimated the parameters of these other cost functions. Contrary to the findings of Diewert and Wales (1987) we did not find important qualitative differences in the estimates of the technological characteristics of the firm between the three specifications considered in this paper. As there is obviously no guarantee that this will be the case in other applications we view this paper as either providing a very general alternative cost model in a multi-output environment, or as providing a tool for evaluating the relative performance of different empirical models.

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Parameter	GBC Estimate	GBC Standard Error	GL Estimate	GL Standard Error
γ_{LL}	0.2543	(0.0105)*	0.4064	(0.0107)*
γ_{LE}	0.0381	(0.0112)*	0.0842	(0.0116)*
γ_{EE}	0.0025	(0.0121)	-0.0152	(0.0129)
β_R	0.4984	(0.1389)*	0.51046	(0.1182)*
β_F	0.3501	(0.0623)*	0.3574	(0.0616)*
β_K	-0.1024	(0.0627)	-0.0984	(0.0511)
θ_{RR}	0.2206	(0.0941)*	0.2237	(0.0938)*
θ_{RF}	0.2705	(0.1121)*	0.2879	(0.1092)*
θ_{RK}	-0.0976	(0.0576)	-0.1168	(0.0718)
θ_{FF}	-1.5179	(0.5191)*	-1.3438	(0.5128)*
θ_{FK}	0.0235	(0.0326)	0.0784	(0.0527)
θ_{KK}	0.3086	(0.0551)*	0.3448	(0.0541)*
ϕ_{RL}	-0.1079	(0.0093)*	-0.1088	(0.0093)*
ϕ_{FL}	-0.0049	(0.0193)	-0.0068	(0.0127)
ϕ_{KL}	0.0442	(0.0107)*	0.0409	(0.0110)*
τ_t	-0.0207	(0.0005)*	-0.0211	(0.0005)*
τ_{tL}	0.0049	(0.0004)*	0.0050	(0.0005)*
λ	0.6214	(0.1239)*	1	-----

Note : * indicates significant at the 5 percent level.

Table 1 : Estimated generalized Box-Cox (GBC) and generalized Leontief (GL) short-run variable cost functions (asymptotic standard errors in parenthesis)

Parameter	Estimate	Standard Error
α_O	0.1016	(0.0123)*
α_L	0.8851	(0.0045)*
γ_{LL}	0.0134	(0.0098)
β_R	0.5002	(0.0360)*
β_F	0.2898	(0.1083)*
β_K	-0.0936	(0.0475)
θ_{RR}	0.1781	(0.1564)
θ_{RF}	0.4545	(0.1756)*
θ_{RK}	-0.1374	(0.1193)
θ_{FF}	-2.3618	(1.1262)*
θ_{FK}	0.1067	(0.3123)
θ_{KK}	0.5823	(0.1686)*
ϕ_{RL}	-0.1095	(0.0101)*
ϕ_{FL}	-0.0108	(0.0199)
ϕ_{KL}	0.0588	(0.0123)*
τ_t	-0.0191	(0.0022)*
τ_{tL}	0.0040	(0.0007)*

Note : * indicates significant at the 5 percent level.

Table 2 : Estimation result translog short-run variable cost function (asymptotic standard errors in parentheses)

	Elasticity of substitution between labor and energy σ_{LE}			Price elasticity of demand for labor η_{LL}			Price elasticity of demand for energy η_{EE}		
Period	GBC	GL	TL	GBC	GL	TL	GBC	GL	TL
1950-62	1.1058	1.0624	0.9057	-0.1027	-0.125	-0.1559	-0.6543	-0.6236	-0.7497
1963-74	0.8248	0.9361	0.8014	-0.0576	-0.0517	-0.0644	-0.5533	-0.5101	-0.7508
1975-86	0.5636	0.7428	0.8469	-0.0548	-0.0685	-0.0824	-0.5629	-0.5798	-0.7645

Notes : 1. GBC = generalized Box-Cox; GL = generalized Leontief; TL = Translog.

2. The formulas used to calculate the elasticities (see equations (16), (17) and (18)) are extremely complicated nonlinear functions of the estimated cost function parameters. We did not try to approximate the asymptotic standard errors.

Table 3 : Estimated price and substitution elasticities

Period	Cost elasticity passenger output ε_R			Cost elasticity freight output ε_F			Cost elasticity capital stock ε_K		
	GBC	GL	TL	GBC	GL	TL	GBC	GL	TL
1950-62	0.4374	0.4764	0.5026	0.4851	0.5005	0.4712	-0.1232	-0.1286	-0.1303
	(0.0831)	(0.0886)	(0.0934)	(0.1365)	(0.1508)	(0.1587)	(.0523)	(0.0594)	(0.0643)
1963-74	0.3681	0.4164	0.3274	0.4727	0.4926	0.4567	-0.1249	-0.1214	-0.1322
	(0.0781)	(0.0811)	(0.0792)	(0.1341)	(0.1622)	(0.1512)	(0.0603)	(0.0581)	(0.0681)
1975-86	0.6042	0.5699	0.5542	0.5922	0.6375	0.6720	-0.0985	-0.0631	-0.0370
	(0.1138)	(0.1307)	(0.1289)	(0.1719)	(0.2084)	(0.2211)	(0.0497)	(0.0421)	(0.0341)

NOTE : See Note 1 in Table 3.

Table 4 : Estimated cost elasticities (Asymptotic standard errors in parenthesis)

	Economics of scale Indicator RTS			Productivity growth index P1 (%)			Productivity growth index P2 (%)		
Period	GBC	GL	TL	GBC	GL	TL	GBC	GL	TL
1950-62	1.2176	1.1553	1.1607	2.0161	1.9625	1.8771	1.6558	1.6987	1.6172
	(0.3814)	(0.3714)	(0.4011)	(0.2623)	(0.2518)	(0.3621)	(0.4941)	(0.5316)	(0.6423)
1963-74	1.3378	1.2338	1.4439	2.3720	1.9415	1.8142	1.7731	1.5736	1.2565
	(0.4279)	(0.4053)	(0.4326)	(0.3217)	(0.3081)	(0.3982)	(0.5489)	(0.5551)	(0.6017)
1975-86	0.9181	0.8805	0.8457	1.7263	1.5553	1.3126	1.8803	1.7664	1.4907
	(0.3589)	(0.3472)	(0.3319)	(0.4196)	(0.4076)	(0.5117)	(0.5821)	(0.5977)	(0.5741)

NOTE : 1. See Note 1 Table 3.

2. The formulas used to calculate RTS, P1 and P2 are simple nonlinear functions of the estimated cost function parameters, see equations (14), (19) and (20). The asymptotic standard errors were approximated using the first-order approximation given in Klein (1953, p. 258).

Table 5 : Estimated scale economies and productivity growth (asymptotic standard errors in parentheses)

FOOTNOTES

1 Comparing alternative functional forms was not the primary purpose of the paper by Diewert and Wales (1987), however. The major contribution of their analysis was the development of procedures to empirically impose global curvature restrictions, as required by economic theory.

2 A recent exception is Livernois and Ryan (1989). They estimated a generalized Leontief variable profit function corresponding to a multiple output production process by extending the specification suggested by Hall (1973).

3 With respect to cost models, see e.g. Wales (1977), Barnett and Lee (1985) and Diewert and Wales (1987). On the consumer demand side reference should be made of Jorgenson, Lau and Stoker (1982) and Lewbel (1989). The latter developed a consumer demand system that has both the AIDS system and the translog nested within it.

4 Note that in total there are (N+K-R) factors of production. To keep notation as simple as possible we do not use different symbols for outputs and fixed factors.

5 All derivations are available from the author on request.

6 It is well-known that the Box-Cox transformation

$$\left(\frac{x^{\Lambda} - 1}{\Lambda}\right)$$

of x converges to lnx for $\Lambda \to 0$.

7 In the case of a variable cost function the inverse of the sum of output cost elasticities is sometimes used as a short-run equivalent of returns to scale, i.e., a measure of the effects of a proportional increase in outputs made possible by a proportional increase in all variable inputs

$$D = 1/\sum_{k=1}^{R} \varepsilon_k .$$

This short-run equivalent of RTS is sometimes referred to as economies of density. With respect to the transportation literature this terminology is somewhat misleading as D is not necessarily related to the network density.

8 Expressions (19) and (20) can be simplified in case we are dealing with a total cost function. In that case R = K and we get $P1 = - \varepsilon_t * RTS$ and $P2 = - \varepsilon_t$, the rate of total cost diminution. The former is, in the terminology of Ohta (1974), called the primal rate of total factor productivity, the latter the dual rate of total cost diminution.

9 The data are 'rich' compared to several other countries. That does not mean that there were no data problems, see below. Also note that the availability of railroad data has previously been exploited by De Borger and Deloddere (1982) and Van de Voorde (1985).

[10] There is still another reason for preferring variable cost minimization over total cost minimization. There are strong suggestions in the literature that railroads in the short-run do not optimally adjust their capital stock but operate with considerable excess capacity. For this reason variable cost minimization is the standard behavioral hypothesis in the railroad literature, see the references given in the first paragraph of this section.

[11] In related work in progress we found, using a different theoretical framework and a simple translog cost model based on shadow input prices, that economy-wide unemployment as well as the political composition of the government significantly affected labor demand by the railroad company. However, it was also found that introducing shadow prices for the variable inputs hardly affected the estimates of the economic characteristics we are interested in in the present paper (scale econmies, substitution possibilities, technical change).

[12] Also note that treating rolling stock as variable together with the assumption of variable cost minimization at observed factor prices would be unrealistic because of the government's subsidization of investment in rolling stock.

[13] The assumption of weak separability of group (K, L, E) from materials M implies that marginal rates of substitution between labor and energy are independent of the quantity of materials used. Assuming weak separability in the major categories of K, L and E allows us to construct aggregate price indices for labor and energy and a capital stock index. Note that especially the assumption of capital-energy-separability is quite restrictive as energy substitution has been accompanied by energy-specific capital investments. A formal test of capital-energy separability within the GBC framework is outside the scope of this paper.

[14] Other measures such as man-hours were not available.

[15] Together with the physical output measures the operating characteristics tried to capture as much as possible of three distinct effects of transport output on cost, viz. cost effects due to distance, due to carrying passengers (or freight) and due to variations in load-factors which reflect the effect of network characteristics and scheduling operations by the company. For more details, and for the estimation results with respect to the parameters of the aggregator functions, see De Borger (1989).

[16] Note that we did not estimate the parameters of the aggregator functions by inserting (21) and (22) into the GBC model and estimating the a_v and b_w jointly with the GBC cost parameters. Although this would have been preferable from a theoretical viewpoint the complexity of the GBC model induced us to choose a simpler alternative. We therefore estimated the aggregates in a translog framework and used the resulting F and R as independent variables in all estimations reported in this paper.

[17] Note that in theory the unobservability of equilibrium cost C can be avoided by substituting expression (11) for C on the right-hand side of (13). The resulting equation system is extremely complicated and unlikely to be empirically tractable. One obtains

$$X_i + \frac{1}{p_i}\left[\frac{2}{\Lambda}\sum_i\sum_j \gamma_{ij}\, p_i^{\Lambda/2}\, p_j^{\Lambda/2}\right]^{1/\Lambda}\left[\prod_k Z_k^{\beta_k(Z,P)}\right]\left[e^{T(Z,P)}\right]$$

$$* \{\sum_k \phi_{ik} \ln Z_k + \tau_i t + [\frac{2}{\Lambda}\sum_j \gamma_{ij}(\frac{p_j}{p_i})^{\Lambda/2}]\ [p_i^{\Lambda}]\ [\frac{2}{\Lambda}\sum_i\sum_j \gamma_{ij}\, p_i^{\Lambda/2}\, p_j^{\Lambda/2}]$$

[18] In fact, to make sure that the results were robust with respect to the choice of instruments we experimented with different sets of instrumental variables. The results were very similar in all relevant respects.