A GENERAL DYNAMIC PORTFOLIO MODEL
- With Application on Belgian Private Sector Data -

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Abstract

In this paper we search for a system of asset demand equations that is both theoretically founded and easily empirically verifiable at the same time.

We first revisit the basic Tobin-portfolio model and investigate its Slutsky properties. Next, the portfolio behaviour is viewed from a Hicksian perspective by allowing for transaction costs in securities markets; it is argued that the existence of such costs adds a new (time) dimension to the wealth allocation problem because of the precautionary motive coming into play. Therefore, the basic model is only regarded as descriptive w.r.t. the investor's long run behaviour and we provide it with a general dynamic specification comprising the partial adjustment scheme and the static formulation as special cases. Such a specification allows to impose the Slutsky conditions exclusively in terms of the long run coefficients.

The model was FIML – estimated with Belgian Private Sector data (59-84, yearly); both the static and the frequently used partial adjustment specifications came out inferior w.r.t. the general dynamic framework.
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1. Introduction

In his seminal 1935 article, 'A suggestion for Simplifying the Theory of Money', John Hicks called for a Marginal Revolution in monetary theory. Up till then, marginal utility analysis had mainly been the tool to tackle flow problems (which are for instance dealt with in consumer and producer demand theory), but, this analysis being a general theory of choice, Hicks saw no objection as to applying it to the stock or balance-sheet problems which are the main subject of monetary theory.

Under the heading 'Portfolio Analysis', this suggestion started being developed in the 50's and 60's, either along the expected utility approach (a direct application of the rational choice theory), or with the help of the mean-variance model (which can be shown to be consistent with the former method under particular conditions); a host of authors contributed to these developments, the ones the most referred to being Markowitz (1952) and Tobin (1958). Besides its microeconomic and normative applications, the theory of portfolio behaviour also entered the macroeconomic field where it became more and more customary to conceptualise the monetary part of the economy as a general accounting framework (cf. Tobin, 1969). Such a framework consists of rows relating to assets/liabilities, and columns representing the different sectors or agents of the economy. Thus, each column can be regarded as a balance sheet or as sources and uses of funds account (depending on whether the matrix entries denote stocks or flows of assets), while a row reflects the positions or movements in a particular asset market.

This framework, if appropriately disaggregated in both dimensions, can provide answers to questions that could hardly be raised in the traditional LM models of the 'money-bonds' type. Indeed, different sources of monetary changes can have different implications for wealth and portfolio behaviour of different sectors.
To bring this framework 'to life', as Tobin would call it, one needs to explain the behaviour of the sectors w.r.t. asset holdings; this is where portfolio theory enters the picture.

This paper was written in the context of the construction of a monetary model, which will form part of a larger macro-economic model that aims at describing both real and financial aspects of the Belgian economy. In the tradition of other monetary econometric (sub)models for Belgium and the Netherlands¹, it is conceived as a general accounting framework in which six sectors are represented: the central bank, deposit or commercial banks, non-monetary financial institutions, the private sector, central & local government, and the rest of the world (cf. Schroyen, 1988).

The search for an adequate modelling methodology to describe (some of) these sectors' portfolio management, constitutes the purpose of the present paper. Section 2 deals with the derivation of the asset demand model that is associated with a negative exponential utility function and normal distributed returns. The model is called the pure, or basic, portfolio model because of the stringent market assumptions that surround it. Already in his 'Simplifying' paper, but even much more in later writings, Hicks draw attention to the serious implications of frictions and transaction costs on portfolio behaviour. Section 3 therefore investigates how the introduction of market imperfections affects the nature of the pure portfolio model. One of the basic findings is that portfolio behaviour really becomes a dynamic phenomenon once transaction costs are allowed for.

The dynamic modelling of asset demand systems is the subject of section 4. Though most empirical studies of portfolio behaviour have recourse to a generalised partial adjustment mechanism, use will be made of a more general dynamic specification which was adapted to singular demand systems (like share systems) in the early 80's by Anderson and Blundell, and which has already proven to be successful in
the context of consumer and factor demand systems. In section 5, this methodology is applied for the estimation and testing, of an asset demand model of the Belgian private sector. Finally, section 6 summarises the main conclusions of this paper.
2. The basic portfolio model

Let us consider a risk averse investor who is faced with the problem of allocating his wealth in an optimal way among \( n \) assets. The first of these assets is assumed to be a riskless one, yielding a return known with certainty; in contrast, the returns on the \( n-1 \) remaining assets, hereafter called risky assets or securities, are supposed to be of an uncertain nature, which is characterised in the investor's mind by a subjective density function.

Before tackling the formal analysis of the problem, it is instructive to sketch the precise context in which this investment decision is made (cf. Rousseas, 1972). In particular, we will regard asset markets to be perfect in the sense of Tobin (1965, p. 3); that is, all assets are 'fully liquid' (i.e. convertible without delay into currency at full market value, by sale, redemption, or pledging as loan collateral), 'perfectly reversible' (i.e. capable of being both purchased and sold at every moment of time, and at the same price for both buyer and seller) and 'completely divisible' (i.e. capable of being purchased, sold, and held in any quantity, no matter how small).

In addition, the investor is assumed to have an immediate accumulation objective, so that there is only one investment period to be considered and that we can abstract from a sequence-of-periods analysis. These two assumptions, the singularity of the decision period on the one hand, and the perfectness of asset markets, often typed as 'absence of transaction costs', on the other, are not independent and we will come back on it extensively in the next section. For the time being, however, the investment decision period is kept self-contained.

Finally, we make the assumption that the investor is operating on perfect competitive markets: he will be unable to affect the prices of the different securities by his own behaviour. Let us now introduce the following symbols in the analysis:
\( W_0 \) : initial wealth to be allocated;
\( z \) : (n-1) vector of risky asset levels;
\( \zeta \) : amount of wealth invested in the riskless asset;
\( a \) : (n-1) vector of risky asset shares in the total portfolio, i.e. \( a_i = z_i/W_0 \);
\( \alpha \) : share of riskless asset in the total portfolio, i.e. \( \alpha = \zeta/W_0 \);
\( r \) : (n-1) vector of random returns on the risky assets, behaving according the (subjective) multivariate distribution function \( F(r) \);
\( \rho \) : (n-1) vector of expected returns on the risky assets, i.e. \( \rho = E(r) \);
\( \Psi \) : (n-1)x(n-1) (subjective) covariance matrix of returns;
\( r^* \) : the (fixed) return on the riskless asset;
\( W \) : end-of-period wealth, calculated as \( z'(1+r) + z(1+r^*) \), where \( 1 \) denotes the vector of units;
\( \pi \) : the overall portfolio rate of return, defined as \( (W-W_0)/W_0 \).

With the wealth constraint \( (1'z+\zeta=W_0) \) in mind, the overall portfolio rate of return \( \pi \) can be computed as a weighted average of the individual asset returns, viz.

\[
(2.1) \quad \pi = a'r + \alpha r^*.
\]

Next, we will assume that the investor's utility function, the expected value of which he is going to maximise, is defined in terms of this overall portfolio rate of return\(^2\), and that the subjective distribution function \( F(r) \) can be approximated by the normal distribution \( F_N(r; \rho, \Psi) \). The former assumption implies that the system of asset demands will be linear homogeneous in initial wealth which is often considered as appropriate for equations representing the behaviour of aggregate categories of investors in a time series context (cf. De Leeuw, 1965, and Brainard & Tobin, 1968). The normality hypothesis, on the other hand, ensures that the utility maximisation procedure can take place along the lines of the mean-variance approach\(^3\).
Since $\pi$ is defined as in (2.1), it will be normally distributed with mean $\mu_\pi = a'\rho + \alpha \mathbf{r}^*$ and variance $\sigma_{\pi}^2 = a'\Psi a$, i.e. $\pi \sim f_N(\pi; \mu_\pi, \sigma_{\pi}^2)$, where $f_N(.)$ denotes the normal density function.

Expected utility is therefore given by

$$
(2.2) \quad E[U(\pi)] = \int_{-\infty}^{+\infty} U(\pi) \ f_N(\pi; \mu_\pi, \sigma_{\pi}^2) \ d\pi.
$$

Assuming next that the investor's behaviour is characterised by constant absolute risk aversion, the implied utility function $U(\pi)$ will be of the negative exponential type, i.e.

$$
(2.3) \quad U(\pi) = \delta_1 - \delta_2 \ e^{-\eta \pi}, \ \delta_2, \ \eta > 0,
$$

where $\eta$ denotes the degree of absolute risk aversion according the Pratt-Arrow definition. It can be shown that the expression for expected utility, (2.2), becomes

$$
(2.4) \quad E[U(\pi)] = \delta_1 - \delta_2 \ \exp \left[-\eta(\mu_\pi - \frac{\eta \sigma_{\pi}^2}{2})\right],
$$

which is solely in terms of the mean and variance of the overall portfolio rate of return. Since $\eta$ is by assumption constant and positive, maximisation of (2.4) is equivalent to maximising the expression

$$
(2.5) \quad \bar{U} = \mu_\pi - \frac{\eta}{2} \sigma_{\pi}^2.
$$

The Lagrangian of the investor's constrained optimisation problem thus looks like

$$
(2.6) \quad \mathcal{L} = (a'\rho + \alpha \mathbf{r}^*) - \frac{\eta}{2} a'\Psi a + \theta (1 - \mathbf{r}'a - \alpha),
$$

yielding the following set of first-order conditions:
\textbf{(2.7a)} \quad \frac{\partial l}{\partial a} = \rho - \eta \psi a - \theta l = 0,

\textbf{(2.7b)} \quad \frac{\partial l}{\partial \alpha} = r^* - \theta = 0,

\textbf{(2.7c)} \quad \frac{\partial l}{\partial \theta} = 1 - \psi' a - \alpha = 0.

These conditions will be necessary as well as sufficient for a maximum, in view of our assumptions on \eta. Eq. (2.7b) directly provides us with the optimal value of \theta, the Lagrangian multiplier, which may be inserted in (2.7a) to obtain optimal risky asset demands, \(a^0\). The optimal share of the riskless asset in the portfolio, \(\alpha^0\), then follows from the wealth constraint (2.7c). So, the optimal asset demand system can be written in matrix notation as

\textbf{(2.8a)} \quad \begin{bmatrix} \alpha^0 \\ a^0 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{\eta} \psi^{-1} & -\frac{1}{\eta} \psi^{-1} \\ \frac{\psi^{-1}}{\eta} & 1 & \frac{1}{\eta} \psi^{-1} \end{bmatrix} \begin{bmatrix} 1 \\ r^* \\ \rho \end{bmatrix}

or,

\textbf{(2.8b)} \quad \begin{bmatrix} \alpha^0 \\ a^0 \end{bmatrix} = \begin{bmatrix} 1 & \kappa & k' \\ \kappa & \kappa & K \end{bmatrix} \begin{bmatrix} 1 \\ r^* \\ \rho \end{bmatrix},

with obvious definitions for \(\kappa, k\) and \(K\).

Since the model has been derived from an explicit optimisation problem, it is possible to infer several properties which make it bear close resemblance to the standard consumer demand system under certainty. In particular, the adding-up requirement of the portfolio shares to unity imply that
(2.9) \[ \mathbf{K}' = \mathbf{Q}', \]

where $\mathbf{K}$ is a shorthand for the matrix of interest effects

\[
\begin{bmatrix}
K & k' \\
 k & K \\
\end{bmatrix}
\]

The homogeneity property results from the fact that

(2.10) \[ \mathbf{K}t = \mathbf{0} \]

i.e. an increase of all (expected) interest rates with the same percentage point, will leave optimal asset shares unaffected. It is easy to check that the cross effects of interest rates are characterised by symmetry; therefore

(2.11) \[ \mathbf{K} = \mathbf{K}'. \]

Moreover, the strict concavity of the utility function amounts to the positivity condition that

\[
\begin{align*}
(2.12a) & \quad \kappa > 0, \text{ and} \\
(2.12b) & \quad x'Kx > 0, \quad \forall x \in \mathbb{R}^{n-1}, \ x \neq 0.
\end{align*}
\]

Notice that this last condition does not apply to the entire matrix of interest effects, but only to the block diagonals $K$ and $K$.

These properties seem to suggest that there exists an analogy between the matrix of interest effects, $K$, and the substitution matrix in standard demand theory. However, as shown by Roley (1983), necessary and sufficient conditions for a symmetric Jacobian matrix $\partial(\alpha, a) / \partial(\pi^*, \rho)'$ are

\[
\partial^2 \bar{U} / \partial (\mu_\pi)^2 = \partial^2 \bar{U} / \partial (\sigma_\pi^2) \partial (\mu_\pi) = 0,
\]
because only then the wealth effect in each asset Slutsky equation vanishes so that one is left with the substitution term; it can be easily checked that these conditions apply in our case. Finally, it is important to mention that on the asset demand system (2.8), the separation theorem is effective, because the structure of the risky part of the portfolio is independent of initial wealth, $W_0$, and the preference parameter $\eta$. Indeed, the mix of securities within the amount decided to invest in risky opportunities, $a^0W_0/(1'a^0)W_0$, is given by the expression

$$\frac{\Psi^{-1}(\rho-1r^*)}{1'\Psi^{-1}(\rho-1r^*)}$$

in which the 'investor typical' parameters to the allocation problem, $W_0$ and $\eta$, do not appear.\(^5\)

To summarise, working with a negative exponential utility function and normal returns, we derived the optimal portfolio strategy for an investor with a one period horizon who is operating on perfect markets. This strategy is described by a system of asset equations, homogeneous of degree one in wealth and degree zero in (expected) rates of interest, with symmetric cross interest effects and positive influence being exerted by the own rates of return. Moreover the system appeared to possess the separation property. 'But', says Hicks (1982, essay 19, p. 248), 'the self containedness of the decision period is a great simplification and it is important to notice that we are able to make it because we are neglecting the costs of transactions'. The purpose of the next section will therefore be to give a sketch of the portfolio behaviour in a world where transaction costs are no longer negligible.
3. Portfolio theory in a Hicksian perspective

With the results of the preceding section in mind, at least three negative conclusions can be drawn w.r.t. the demand for money. We should first notice that, as long as there exists the possibility of investing in a riskless asset with a positive rate of return, there is no place for a demand for money in the framework of the pure portfolio model. Money is then said to be dominated by that particular asset. Secondly, the separation theorem implies that one can treat all risky assets as a bundle. The effect of a change in risk aversion simply results in a substitution between the riskless asset on the one hand, and the bundle of risky assets on the other, leaving the proportions within the bundle intact. Within this perspective, the demand for money (if not dominated by another riskless asset) is determined in the same manner as the liquidity preference choice in Keynes (1936, ch. 13) and much of Keynesian theory where all non-money assets are treated as 'bonds'.

Our second negative result is therefore that the pure portfolio model offers no scope, at least when it comes to analysing the effects of changes in risk attitudes, to consider on entire spectrum of assets, which was the subject of Keynes (1930, vol. II, ch. 25) and Hicks (1935). A third negative conclusion may be drawn from the fact that in the perfect market embedding of the basic model, a precautionary demand for money makes no sense at all (cf. Tsiang, 1969). In that context it is unnecessary to look forward for more than one period; whether the target date, i.e. the date when the investor intends to need his invested wealth, is near or distant, or uncertain at all, does not matter, for it is only the (expected) returns immediately ahead that are relevant for the portfolio decision. So the question arises:
Why then should anyone hold money, except for the very instant he
receives a payment or is about to make one? The necessary and
sufficient conditions, as Hicks rightly pointed out, is that out-of-
pocket costs and the effort required in moving from cash to bonds and
back to cash exceed the yield.

(Modigliani, 1968, p. 398)

Indeed, as Hicks pointed out in his second Two Triads
Lecture (1967, p. 31), 'it is fatal to leave out the cost of
making transactions when one's subject is money'.
If we introduce transaction costs in the very broad sense,
by allowing not only for direct pecuniary charges but also
non-pecuniary costs associated with bothering and costs
experienced due to the occurrence of indivisibilities,
almost the entire perfect market setting (in Tobin's sense)
is undermined with serious consequences for the results
derived within it.
First, let us see what happens when we keep the decision
period self contained. The allocations possible at the
beginning of the single decision period are pictured in
Figure 3.1. Because we restrict ourselves to one decision
period, total wealth, held in no matter what form, is to be
transformed into money (which is supposed not to be
dominated by another riskless asset, i.e. $\xi = m$), the only
means of exchange.

Suppose that the investor starts off with his entire wealth
held in money form and that the optimal investment strategy
requires the operations (I) and (II). The part that is kept
in money form ($m$) is not subjected to any transaction costs
while the investment in risky assets ($z_i$) is not possible
without paying the amounts (or charges in percent of face
value) $\tau_{mi}$ and $\tau_{im}$, corresponding to the twofold conversion.
The expected net returns will thus be situated below the
original ones. Unfortunately, we can make no general
statements how the optimal investment strategy is affected\(^6\),
unless we restrict ourselves to the case where all rates of
Figure 3.1. Reallocations and transaction costs

\[ \text{Beginning of period} \quad \rightarrow \quad \text{End of period} \]

I  \[ m \quad \overset{\tau_{mi}}{\longrightarrow} \quad z_i \quad \overset{\tau_{im}}{\longrightarrow} \quad m \]

II  \[ m \quad \rightarrow \quad m \]

III  \[ z_i \quad \overset{\tau_{im}}{\rightarrow} \quad m \]

IV  \[ z_i \quad \overset{\tau_{im}}{\rightarrow} \quad m \]

V^a  \[ z_i \quad \overset{\tau_{im}}{\rightarrow} \quad z_j \quad \overset{\tau_{jm}}{\rightarrow} \quad m \]

\[ ^a \text{it is assumed that direct exchange of asset } i \text{ for asset } j \text{ is impossible.} \]
return are uncorrelated; for the optimal position in a risky asset is then given by

\[ a_i^o = \frac{1}{\eta} \frac{1}{\psi_{ii}} (\rho_i - r^*) \]

and will be smaller when the \( \rho_i \) is corrected downwards.

Eventually, the net return on the risky asset may even fall below the safe yield, \( r^* \); if this occurs, and short sales, i.e. negative asset holdings are not allowed, the risky asset drops out of the investor's portfolio. Spread among risky assets is then limited. This will surely be the case when investment costs are of a large fixed nature (i.e. invariable with the amount invested) or when indivisibilities are present; the possibility to divide his wealth in small portions, thus spreading his risks, is then closed to the investor who cannot raise sufficient amounts of money to overcome these frictions in a profitable way (cf. Hicks, 1935).

The picture becomes, however, even more blurred when we consider situations where the investor holds his initial capital in a diversified shape. In that case, the reallocation associated with the optimal investment of the new period, takes place by several or perhaps all the transactions pictured in Fig. 3.1. Even when we take the simplified model with only one risky asset besides the riskless one, it is hard to make some general statements as to the effect of transaction costs; too much depends on the structure of them. What we can say is that, given a diversified initial position, the introduction of costs of transaction will have asymmetrical effects, depending on the position of the initial portfolio relative to the position which would be optimal in the absence of these costs. This asymmetry comes from the fact that going out of the risky asset, whether at the beginning of the period to invest in money (III), or at the end of it when wealth is realised (IV) involves the same kind of transaction cost (\( \tau_{im} \)). On the other hand, taking up a riskier position than the
initial one requires two transactions on the part of the risky asset (I), with associated costs $\tau_{mi}$ and $\tau_{im}$, whereas the marginal return on money is not altered. This would mean that transaction costs hamper the investor more when he is trying to expand his portfolio towards the risky asset, than when securing his position by investing more in the riskless asset, money.

So far, the analysis remained in the one-period framework and is therefore only partial. The reason why we were allowed to investigate the wealth allocation problem for just one single period was precisely the absence of transaction costs; then the target date of the investor does not matter at all. But, once these costs are introduced, the timing of the investor's accumulation objectives becomes relevant and an extra dimension is added to the analysis. Tobin (1965) outlines this problem with the help of a two-period two-asset model in a world of complete certainty on the future of both assets. Without costs of transactions, the optimal policy to follow for the investor is to maximise the return over the period immediate ahead, no matter whether the target date lies at the end of the first or second period. However, once (dis)investment costs enter the picture, his investment strategy, i.e. his portfolio sequence and therefore the immediate portfolio choice (which is the first step in the sequence), becomes dependent on the timing of the investor's accumulation goal. And even though asset returns are certain in Tobin's model, portfolio diversification takes place because of diversity in the investor's timing of his accumulation goals.

One may go one step further and allow for the fact that the target date, i.e. the date of planned realisation, becomes uncertain. In this case, the investor has to make sure that he does not lock himself in, i.e. that his portfolio consists of sufficient funds which are easy realisable at the (uncertain) moment when realisation becomes necessary. It is at this point in the analysis that liquidity comes into play, for
it (...) not only mean(s) that (the funds) must be held in securities that are readily marketable; it also means that they must be held in such a form that the value of the portfolio is not likely to vary too much over time. The value, at whatever date there is to be realisation, must be much the same. A particular asset which has this property, is surely what is meant by a liquid asset.

(Hicks, 1982, p. 260, original italics)

But, unlike the pure portfolio dichotomy of a riskless asset on the one side, and a bundle of securities on the other, the distinction between liquid and illiquid assets is not a clearcut one. In fact, there are several degrees of liquidity, one asset being more liquid than another when it is 'more realisable at short notice without loss' (Keynes, 1930, vol. II, p. 67). This means that a spectrum of assets (a liquidity spectrum) should be considered. When an investor feels confident about the future, in that he estimates the likelihood of being obliged to realise part of his portfolio on short term small, he will move to the right along the spectrum. On the other hand, if he thinks requirements will be very probable in the near future, the same investor will allocate his wealth more to the left side of the 'liquidity axis'. The demand for money that arises from such a leftward shift, one should see as a precautionary demand for money.

So, where the role of the speculative motive is stressed in the pure portfolio model, the admission of transaction costs introduces, almost automatically, the precautionary motive as an equivalent, if not superior modus operandi in the individual's portfolio behaviour. The third element in the familiar triad, the transaction motive, is nicely fit into the model by recognising the need for money as a means of payment.
To this motive-triad, Hicks (1967, Lecture III, and 1975, ch. 2) proposed a corresponding classification of the assets on the individual's balance sheet into three categories: running, reserve and investment assets.

To the first category, Hicks reckons assets that are required for the current running of a business or a household. Real assets of this kind can be easily thought of: goods in production, fixed equipment in so far that it is used in the production process, etc. Financial running assets can only be money or near money, in so far that these are required for current business, that is, serving as a means of payments. Other financial assets are excluded from this class since they do not share the medium of exchange property\(^9\). The requirement for money as a running asset is thus a transaction requirement\(^10\) with a rather complementary character to the business process.

Money will, however, also appear among financial reserve assets. These assets are held for emergencies that may arise in the near future. The demand for reserve assets is therefore governed by the precautionary motive and it can be satisfied by any financial asset that possesses some degree of liquidity, that is, realisable at short notice without loss. Financial assets held for this purpose will usually be close substitutes and are subjected to a liquidity preference substitution for one another when their rate of return structure changes, given the investor's state of confidence in the future. Finally, there are investment assets. Financial assets that are classified in this category, are held for their yield and, unlike reserve assets, there is no liquidity requirement. If the investor is one who faces huge transaction costs, his portfolio will only be managed at the margin, i.e. new savings will be allocated into investment assets and the rate of diversification among them will be governed by the degree of certainty the investor attaches to the different returns these assets are expected to yield in the future. On the other hand, when the costs of reallocating the existing portfolio do not render speculative behaviour unprofitable,
a speculative demand for a non-interest bearing money will emerge. But then we should agree with Modigliani that this speculative money demand will be replaced by a demand for a liquid interest-bearing asset if such exist:

(...) Keynes' theory of the speculative demand suffers from his excessive concentration on long term bonds as the alternative to cash, to the neglect of short term instruments. The proposition that people will flee from bonds when the price of bonds is deemed to be intenably high seems valid enough, but the obvious abode for funds accruing from moving out of long term bonds should be short term ones, not cash.

(Modigliani, 1968, p. 399)

With these considerations in mind, it looks as if a large part of the investor's liquidity preference behaviour can be explained in terms of the precautionary motive, with the speculative behaviour pertaining only the demand for assets in so far these assets can be ranked to the investment category. Therefore, it seems we have drifted a long way from the pure portfolio model of the previous section and it is not clear how the standard properties pertaining to demand systems are affected when introducing transaction costs.

What we can say about the modified model is that it has a dynamic flavour and that it will at least consist of the same explanatory variables as the pure model does (since the latter can be seen as nested within the former); also, a general substitution in favor of an asset whose rate of return has increased, is to be expected, just as in the pure model, although this substitution will now be governed by speculative as well as precautionary motives.

Instead of setting off towards a stochastic multiperiod model, which seems the natural way of formalising Hicks' modifications, but with few chance of arriving at any tractable equations that can easily be verified empirically, we choose to take a different route in the next section. This route will be inspired by the basic insights of the
previous and present sections, combined with a sense of pragmatism or *ad hoc* modelling which will lead us to some flexible model that is tractable at the same time.
4. Dynamic portfolio models

4.1. Introducing dynamics

In this section, we will consider the asset demand system (2.8) arrived at in section 2, as the representation of the investor's portfolio behaviour in the long run, i.e. when the investor is given plenty of time to overcome all kinds of frictions and externalities that are likely to influence his short run financial allocations. Therefore, system (2.8) is regarded as the equilibrium response model. How fast the equilibrium response will come about, then depends on the costs of adjustment which have to be incurred. These costs are closely related to the costs of transaction that were the subject of the previous section, and, as already noted there, they have to be interpreted in a very broad way, ranging from direct pecuniary charges (like bid/ask spreads, brokerage fees) towards more indirect costs due to bothering, market inadequacies, and indivisibilities; also the habit to invest wealth in typical familiar assets and imperfect information on the market characteristics of certain assets may not provide the individual investor with sufficient stimuli to exploit all market opportunities to the full.

Besides their direct effect on the profitability of particular investment, these costs will bring liquidity considerations into play which will in turn have their impact on the wealth allocation problem. Hence, a dynamic treatment of portfolio behaviour urges itself and this is also the lesson to be drawn from empirical studies. For instance, Parkin, Gray and Barret (1970) estimated a static allocation model for commercial banks of the U.K. in which the demand for loans, treasury bills, commercial bills and government bonds were linearly related to the respective interest rates and net wealth; they imposed homogeneity and symmetry restrictions. Ten years later, Berndt, McCurdy and Rose (1980) reestimated the unconstrained as well as constrained model using FIML-tech-
niques. The likelihood ratio tests they carried out rejected both homogeneity and symmetry restrictions. However, when appending a first order autoregressive error process to the model, the rejection of both null hypotheses was significantly weakened. From this evidence, we may conclude that a dynamic specification of a portfolio model is crucial if one wants to use the equilibrium response model as a maintained structure of time series data.

Once one has recognised the need for a proper dynamic specification, two courses can be taken. Either one proceeds with a formal modelling of the adjustment behaviour by deriving optimal rules of adjustment; this direction was taken by Sharpe (1974), Christofides (1976) and Hunt and Upsher (1979) who formulate a quadratic cost minimisation problem which outweighs the costs of divergence from the optimal portfolio with the costs of transition towards it. The solution to this problem is then a generalised partial adjustment scheme which has been very frequently applied in empirical portfolio studies. The other course is a more pragmatic one in that it does not undertake any formal adjustment modelling, but rather formulates a dynamic framework which exhibits enough flexibility to catch most of the adjustment imperfections. The partial adjustment scheme then appears as a special case which will have to be tested w.r.t. the maintained model. It is along these lines that we will proceed in this and next section.

4.2. A general dynamic system

In a series of articles, Anderson and Blundell (1982, 1983 and 1984), have developed and tested a general dynamic system of demand equations. The starting considerations of these authors where that (i) empirical testing of demand systems often leads to the rejection of restrictions, imposed on by economic theory; (ii) these empirical studies often show serious serially correlated residuals, reflecting
an inadequate specification of the dynamic structure; and
(iii) singular demand systems where a predetermined aggre-
gate is allocated among an exhaustive list of items, need a
'system approach' when it comes to specifying the dynamics.
It is precisely this last point that was strongly emphasised
by Brainard and Tobin (1968) in the context of asset demand
equations.

The methodology Anderson and Blundell (hereafter A & B)
suggest is a vector time series model that can be repara-
terised in such a way so as to reveal the long run or
equilibrium response structure, without straining the
general dynamic nature of the system. This approach seems
apt to follow in the context of this paper. On the one
side, the dynamics of portfolio behaviour appears to be
determined by a vague and complex interaction of pecuniary,
non-pecuniary and liquidity constraints, and therefore
requires a modelling which leaves enough degrees of freedom,
while on the other hand, we have a fairly clear idea how the
wealth allocation process happens in equilibrium; to use this
a priori information seems appropriate for two reasons.
Firstly, it will lead to an increased efficiency of the
parameter estimates (this is no superflues luxery when most
of the exogenous variables, in casu interest rates, are
severely correlated). Secondly, the incorporation of a
priori restrictions in the estimation will guarantee a kind
of 'well behaviour' of the model outside the sample period,
i.e. perverse outcomes are ruled out; this is desirable in
view of simulation exercises.

Formally, let us rewrite the static asset demand system
(2.8) in a more compact way as

\[(4.1) \quad w^0(t) = \Pi x(t),\]

where \(w^0(t)\) denotes an \(n\)-vector of optimal asset shares in
the total portfolio and \(x(t)\) represents an \(h\)-vector composed
from an intercept term as first element, \(n\) interest rates,
and \((h-n-1)\) other explanatory variables. The similarity between the first \(n+1\) columns of \(\Pi\) and the coefficient matrix in (2.8) is then obvious. Relation (4.1) pictures the long run relationship according to which the portfolio allocation operates. The equilibrium response to the exogenous variables is given by the \((nxh)\)-matrix \(\Pi\) and we assume that the properties of the interest effects, like homogeneity and positivity are operative on the relevant part of \(\Pi\). A & B (1982) postulate that short run changes in \(w(t)\) are reactions to anticipated and unanticipated changes in the \(x\) vector in an attempt to maintain a long run relation_ship represented by (4.1). Using the lag operator \(L\), such a pattern may be written as

\[
(4.2) \quad B^*(L)w(t) = \Gamma^*(L)x(t) + u(t),
\]

where \(B^*(L)\) and \(\Gamma^*(L)\) denote matrix polynomials of the \(p\)th and \(q\)th order resp., i.e.

\[
(4.3a) \quad B^*(L) = I + B_1^*L + B_2^*L^2 + \ldots + B_p^*L^p,
\]

\[
(4.3b) \quad \Gamma^*(L) = \Gamma_0^* + \Gamma_1^*L + \Gamma_2^*L^2 + \ldots + \Gamma_q^*L^q,
\]

and \(u(t)\) is a \(n\)-vector of random errors, assumed to be independently and identically distributed over time with covariance matrix \(\Omega\).

The singularity of the demand system, due to the adding-up condition \(1'w(t) = 1\), implies certain adding-up requirements on the matrix polynomials and disturbance vector (cf. A & B, 1982), i.e.

\[
(4.4a) \quad 1'B^*_j = g_j 1', \quad j=1, \ldots, p,
\]

\[
(4.4b) \quad \sum_{j=1}^{p} g_j = g
\]

\[
(4.4c) \quad 1'\Gamma^*_0 = [1+g, 0, \ldots, 0]
\]
\[(4.4d) \quad \iota' \Gamma_j^* = Q' \quad , \quad j = 1, \ldots, q, \]

\[(4.4e) \quad \iota'u(t) = 0. \]

It appears from these conditions, as well as later in the stability analysis, that there is a degree of arbitrariness in system (4.2). But, as Bewley (1986, ch. 2) argues, it is of a spurious nature and presents no inconvenience to further analysis.

When the model is stable, the long run structure can be retrieved as

\[(4.5) \quad \Pi = B^*(1)^{-1} \Gamma^*(1) = \left(\sum_{j=1}^{p} B_j^*\right)^{-1} \left(\sum_{j=0}^{q} \Gamma_j^*\right). \]

And since also in the long run, adding-up is in force, we have

\[(4.6) \quad \iota'\Pi = [1, 0, \ldots, 0]. \]

Though system (4.2) can be reparameterised in several ways, the spectrum of choice is limited if we want to focus attention on the equilibrium response matrix \(\Pi\). A & B (1982) derive the following observationally equivalent set of equations\(^{12}\):

\[(4.7) \quad \Delta w(t) = -B(L)\Delta w(t) + \Gamma(L)\Delta x(t) + A[\Pi x(t-q) - w(t-p)] + u(t), \]

where

\[B(L) = \sum_{j=1}^{p} \left(\sum_{j=0}^{j} B_j^*\right)L_j, \quad p > 1, \]

\[= 0, \quad p \leq 1, \]

\[\Gamma(L) = \sum_{j=0}^{q-1} \left(\sum_{j=0}^{j} \tilde{\Gamma}_j^*\right)L_j, \quad q \geq 1, \]

\[A = \sum_{j=0}^{p} B_j^*. \]
and the tilde above the $\Gamma_j^*$ and $\Delta x(t)$ points to the
deletion of the intercept term which clearly becomes
redundant when $x(t)$ is written in first differences. The
corresponding adding-up conditions are:

$$1' B_j = m_j 1' \quad , \quad m_j = 1 + \sum_{i=1}^{j} q_i \quad , \quad j=1, ..., p-1,$$

$$1' \Gamma_i = 0' \quad , \quad i=1, ..., q-1,$$

$$1'A = (1+g)1',$$

$$1' \Pi = [1, 0, ..., 0];$$

and, of course, conditions (4.4e) and (4.6). $B_j$ and $\Gamma_i$ now
denote the $j$th and $l$th coefficient matrix in the matrix
polynomials $B(L)$ and $\Gamma(L)$, resp..

Besides the earlier mentioned redundant character of the
intercept term in $\Delta x(t)$, system (4.7) exhibits another
potential redundant variable problem. Indeed, the appearance
of the full vector of portfolio shares as explanatory
variables, either in levels or in first differences, gives
rise to collinearity, because information on the $(n-1)$
elements of $w(t)$ or $\Delta w(t)$, implicitly says everything about
the $n$th share and its change. This problem can be overcome
by deleting one element, say the $n$th, of $w(t)$ and $\Delta w(t)$.

Though the consequence of this operation is that also the
$n$th row of the equilibrium response matrix has to be
dropped, it implies no loss of information on the long run
structure, as this row can always be retrieved by making use
of the adding-up property (4.6). Hence, if we denote the
deletion of the last row of a matrix with the subscript $n$, and a modified $nx(n-1)$ matrix with a superscript $n$, model
(4.7) can be restated in the form (cf. Appendix A):

$$\text{(4.8) } \Delta w(t) = -B^n(L)w_n(t) + \Gamma(L) \Delta x(t) + A^n [\Pi_n x(t-q) - w_n(t-p)] + u(t),$$

where the matrix $A^n$ and every matrix of the polynomial
$B^n(L)$, are build up from the first $(n-1)$ columns of the
original matrix, each column, however, being diminished with the nth column; the matrix $A^n$, for instance, takes the form of

$$
A^n = \begin{bmatrix}
  a_{11} - a_{1n} & a_{12} - a_{1n} & \cdots & a_{1,n-1} - a_{1n} \\
  a_{n1} - a_{nn} & a_{n2} - a_{nn} & \cdots & a_{n,n-1} - a_{nn}
\end{bmatrix}
$$

With the previous adding-up conditions in mind, we can say that all rows of $A^n$ and the element matrixes of $B^n(L)$ and $\Gamma(L)$ will sum to zero.

Finally, it should be mentioned that without a priori information on the dynamic structure, there is a loss of identification since the elements of the original $B_j$ and $A$ matrices cannot be retrieved from the associated truncated matrices $B^n_j$ and $A^n$; but, as proven in A & B (1982), no identification problem pertains to $\Pi$, the matrix of long run multipliers.

In what follows, we will concentrate ourselves on the first order general dynamic model $(p=1, q=1)$, i.e.

$$
(4.9) \Delta w(t) = \Gamma \Delta \hat{X}(t) + A^n [\Pi_n x(t-1) - w_n(t-1)],
$$

which we will consider as the maintained hypothesis and which encompasses two familiar cases$^{13}$, the popular generalised partial adjustment model $(p=1, q=0)$:

$$
(4.10) \Delta w(t) = A^n [\Pi_n x(t) - w_n(t-1)] + u(t),
$$

and the static formulation $(p=0, q=0)$:

$$
(4.11) w(t) = \Pi x(t) + u(t).
$$
According to the assumptions made earlier in this section, the covariance structure of the disturbance vector \( u(t) \) is given by

\[
E[u(t) \, u(s)'] = \Omega \quad , \quad t=s \\
= 0 \quad , \quad \text{otherwise}
\]

In view of the deterministic nature of the sum of disturbances (cf. eq. (4.4e)), the matrix with contemporary covariances, \( \Omega \), is singular, since \( t'\Omega = E[t' u(t) u(t)'] = 0 \), and one has to delete one equation of the model to make system estimation possible. As shown by Barten (1969), it is immaterial which equation is dropped. If we therefore choose to eliminate the \( nth \) equation, we arrive at an estimable form of the first order general dynamic model, i.e.

\[(4.12) \Delta w_n(t) = \Gamma_n \Delta x(t) + A_n^n [\Pi_n x(t-1) - w_n(t-1)] + u_n(t),\]

in which the subscript \( n \), as before, points at the deletion of the final row (element) of a matrix (vector); this row of the coefficient matrices \( \Gamma \) and \( A^n \) can always be retrieved by means of the adding-up properties.

4.3. Stability analysis

The share system (4.7) thus reduces to

\[(4.13) \Delta w(t) = \Gamma \Delta x(t) + A[\Pi x(t-1) - w(t-1)] + u(t)\]

in the case of a first order general dynamic model \( p=q=1 \), and becomes the equations set (4.9) when one eliminates the earlier mentioned collinearity, inherent in it.

Assuming away the disturbance vector and defining \( \tilde{\Gamma} \) as the extended coefficient matrix \( [\tilde{\Omega}, \Gamma] \), this model can be written as
\[(4.14) \quad [I-(I-A)L]w(t) = [I-(I-A\Pi)L]x(t)\].

Hence, stability requires all eigenvalues of the matrix \((I-A)\) to lie within the unit circle. These eigenvalues are found by solving for all \(\lambda\) in the characteristic equation

\[(4.15) \quad |(I-A)-\lambda I| = 0\].

But, as shown in appendix B, this is equivalent to solving the equation

\[(4.16) \quad (\lambda+g) \quad |(I^n_n - A^n_n) - \lambda \quad I^n_n| = 0\],

where \(I^n_n\) denotes the unit matrix with the \(n\)th row and column dropped and \(A^n_n\) is built up from the first \((n-1)\) rows of the previously defined matrix \(A^n\).

Two assessments w.r.t. the stability of the first order general dynamic model are now in order.

First, it would seem that this stability depends on the parameter \(g\), the arbitrary constant in the adding-up conditions (4.3). At the bottom, however, this is not the case because the appearance of \(g\) in the original system can be shown to be of a spurious nature.

Secondly, it becomes clear that, apart from the eigenvalue \(\lambda = -g\), the roots of \((I-A)\) are identical to those of \((I^n_n-A^n_n)\), which is the adjustment matrix of the estimable version (4.12), the system after solving the collinearity problem and eliminating the last equation to overcome singularity of the disturbance covariance matrix.

From these considerations, it follows that the stability of the first order general dynamic model is guaranteed when all eigenvalues of \((I^n_n-A^n_n)\) lie within the unit circle. If this is the case, the final form of (4.13) exists, i.e. the equilibrium response values of \(w(t)\) will be achieved when the vector \(x\) stabilises to some constant value over time; for the first \((n-1)\) shares (and thus implicitly for the \(n\)th), it is given by
\[ (4.17) \quad w_n(t) = \left[ \Gamma_n + \sum_{j=1}^{n-1} (I_n - A_n)^{j-1} A_n^T (\Pi_n - \Gamma_n) L_j \right] x(t), \]

where \( \Gamma_n \) corresponds to \( \Gamma \) with the nth row deleted. The impact and interim multiplier matrices are resp. given by

\[ (4.18) \quad \frac{\partial w_n(t)}{\partial x(t-j)} = \begin{cases} \Gamma_n & , j = 0, \\ (I_n - A_n)^{j-1} A_n^T (\Pi_n - \Gamma_n) & , j \geq 1. \end{cases} \]

We may obtain an idea about the speed at which the equilibrium values are reached by studying the different cumulated interim multipliers:

\[ (4.19) \quad \sum_{j=0}^{J-1} \frac{\partial w_n(t)}{\partial x(t-j)} = \Gamma_n + \sum_{j=1}^{J-1} (I_n - A_n)^{j-1} A_n^T (\Pi_n - \Gamma_n), \quad J \geq 1. \]

If the stability condition is satisfied, the RHS expression can be rewritten as

\[
\{I_n - (A_n)^{-1} \left[ I_n - (I_n - A_n)^{J} A_n^T \right] \Gamma_n\} \Gamma_n
\]

\[
+ \{A_n^{-1} \left[ I_n - (I_n - A_n)^{J} A_n^T \right] A_n^T \Pi_n
\]

so as to reveal more clearly how the cumulated multipliers are constructed as a weighted combination of the impact multipliers \( \Gamma_n \) and the final form ones \( \Pi_n \); as \( J \) goes to infinity, the expression indeed converges to the latter. When we replace \( (I_n - A_n^T) \) by its spectral decomposition

\[ V \Lambda V^{-1}. \]

where \( \Lambda \) is the diagonal matrix of eigenvalues and \( V \) the matrix of corresponding eigenvectors, we observe how the
'weights' will depend on the number of considered lags as well as on the dominant eigenvalue of the adjustment matrix.
5. Application on the Belgian Private Sector

5.1. Parameterisation: a monetary framework of the Belgian Private Sector

In Table 5.1, a version of the Belgian private sector's balance sheet is given for the end of 1984\textsuperscript{14}. This sector is defined as the aggregate of households, business firms which, for statistical reasons, also comprise nationalised enterprises, and the 'institutions of life and labour accident insurance and pension funds'.

Depending on which assets/liabilities are grouped together, an allocation model can be defined on the remaining assets and liabilities, the latter being treated as negative assets.

To keep the model manageable, we choose to consider the private sector's allocation of total wealth (WP) over four broad classes of assets, i.e. liquidities\textsuperscript{15} (LIQP), total short term time deposits (TSTP), assets on long term (ASLP) and net foreign assets (NAFP). This amounts to reshuffling the original balance sheet to the one pictured in Table 5.2.\textsuperscript{16}

The vector of asset shares, $w$, can thus be constructed as

$$w = \begin{bmatrix} w_{LI} & w_{SI} & w_{AS} & w_{NA} \end{bmatrix} = \frac{1}{WP} \begin{bmatrix} LIQP & TSTP & ASLP & NAFP \end{bmatrix},$$

with obvious definitions for $w_i$.

The evolution of the portfolio structure, as defined by $w$, is visualised in Figure 5.1. for the sample period '59-'84. Besides showing the weakening interest of the private sector in liquidities (in the strict sense) and foreign assets, the figure reveals the growing importance of liquidities in the large sense (i.e. $w_{LI} + w_{TS}$) up till '76 (with a period of stabilisation between '66 and '70); afterwards, this
### Table 5.1: Balance sheet Belgian Private Sector (end of 1984)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Symbol</th>
<th>Amount ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Currency</td>
<td>CUMP</td>
<td>385.4</td>
</tr>
<tr>
<td>Demand deposits with monetary authorities</td>
<td>DDMP</td>
<td>83.1</td>
</tr>
<tr>
<td>Demand deposits with deposit banks</td>
<td>DDBP</td>
<td>376.2</td>
</tr>
<tr>
<td>Demand deposits with NMFI</td>
<td>DDNP</td>
<td>105.8</td>
</tr>
<tr>
<td>Time &amp; saving deposits on short term&lt;sup&gt;c&lt;/sup&gt; with deposit banks</td>
<td>TSBP</td>
<td>909.6</td>
</tr>
<tr>
<td>Time &amp; saving deposits on short term with NMFI</td>
<td>TSNP</td>
<td>1133.7</td>
</tr>
<tr>
<td>Time &amp; saving deposits on long term with deposit banks</td>
<td>TLBP</td>
<td>421.7</td>
</tr>
<tr>
<td>Time &amp; saving deposits on long term with NMFI</td>
<td>TLNP</td>
<td>1485.4</td>
</tr>
<tr>
<td>Government debt certificates on short term</td>
<td>CSGP</td>
<td>6.7</td>
</tr>
<tr>
<td>Government debt certificates on long term</td>
<td>CLGP</td>
<td>874.8</td>
</tr>
<tr>
<td>Net foreign assets</td>
<td>NAFP</td>
<td>283.3</td>
</tr>
<tr>
<td><strong>Total assets</strong></td>
<td></td>
<td>6065.7</td>
</tr>
</tbody>
</table>

### Liabilities

<table>
<thead>
<tr>
<th>Liabilities</th>
<th>Symbol</th>
<th>Amount ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans on short term (initially) granted by deposit banks</td>
<td>LSPB</td>
<td>1168.1</td>
</tr>
<tr>
<td>Loans on short term granted by NMFI</td>
<td>LSPN</td>
<td>103.2</td>
</tr>
<tr>
<td>Loans on long term granted by deposit banks</td>
<td>LLPB</td>
<td>57.2</td>
</tr>
<tr>
<td>Loans on long term granted by NMFI</td>
<td>LLPN</td>
<td>1485.6</td>
</tr>
<tr>
<td>Net other liabilities (net financial wealth)</td>
<td>NAPO</td>
<td>3251.6</td>
</tr>
<tr>
<td><strong>Total liabilities</strong></td>
<td></td>
<td>6065.7</td>
</tr>
</tbody>
</table>

<sup>a</sup> billions of BF  
<sup>b</sup> NMFI = non-monetary financial institutions  
<sup>c</sup> short term ≤ 1 year  
long >

### Table 5.2: Reorganised balance sheet Belgian Private Sector (end of 1984)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Symbol</th>
<th>Amount ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Liquidity</strong></td>
<td>LIQP</td>
<td>950.5</td>
</tr>
<tr>
<td>(= CUMP + DDMP + DDBP + DDNP)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total short term deposits</strong></td>
<td>TSTP</td>
<td>2043.3</td>
</tr>
<tr>
<td>(= TSBP + TSNP)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Assets on long term</strong></td>
<td>ASLP</td>
<td>2781.9</td>
</tr>
<tr>
<td>(= TLBP + TLNP + CLGP)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Net foreign assets</strong></td>
<td>NAFP</td>
<td>283.3</td>
</tr>
<tr>
<td><strong>Liabilities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total wealth</strong></td>
<td>WP</td>
<td>6059.0</td>
</tr>
<tr>
<td>(= LSPB + LSPN + LLPB + LLPN + NAPO - CSGP)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> in billions of BF
Figure 5.1. Structure financial portfolio of the Belgian Private Sector ('59-'84)

Table 5.3. Average yearly rate of change\textsuperscript{a} of the different portfolio components for 5 subperiods

<table>
<thead>
<tr>
<th>Subperiod</th>
<th>Liquidities (in the large sense)</th>
<th>Illiquidity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_0 - t_1 )</td>
<td>( w_{LI} )</td>
<td>( w_{TS} )</td>
</tr>
<tr>
<td>59-66</td>
<td>-0.06 %</td>
<td>+0.94 %</td>
</tr>
<tr>
<td>66-70</td>
<td>-0.80 %</td>
<td>+0.01 %</td>
</tr>
<tr>
<td>70-76</td>
<td>-0.48 %</td>
<td>+0.95 %</td>
</tr>
<tr>
<td>76-82</td>
<td>-1.54 %</td>
<td>-0.77 %</td>
</tr>
<tr>
<td>82-84</td>
<td>-0.88 %</td>
<td>-0.78 %</td>
</tr>
</tbody>
</table>

\( \text{a} \) the rates of change are absolute rates, as a % of the total portfolio, i.e.

\[
\frac{w_{i,t_1} - w_{i,t_0}}{t_1 - t_0} \times 100.
\]
tendency is strongly reversed and seems to temper around '82 at early sample period values. A more detailed account of this portfolio structure evolution is given in Table 5.3; for the five identified subperiods, the table reports on average yearly changes of the different portfolio components.

During the first subperiod, the growing liquid position of the 'investor', almost exclusively due to the increase of short term time deposits \( w_{TS} \), is more than financed by the release of funds invested in foreign assets \( w_{NA} \), allowing for a strengthening position in domestic long term assets \( w_{AS} \) as well. Since the second half of the 60s, however, the share of liquidities in the strict sense \( w_{LI} \) starts falling at a steady rate, necessitating a flee from long term assets in the early 70s to keep the overall degree of liquidity sufficiently high. Liquidity requirements are severely relaxed during the second half of the 70s and early 80s, and the long term position can again be expanded; since '82 this is also true w.r.t. the foreign position.

The vector of exogenous variables, \( x \), consists of an intercept, the expected rates of return, and additional explanatory variables. As to the modelling of the mechanism which generates the expectations on rates of return, a common practise in the literature on portfolio behaviour will be pursued here by replacing the expected rates by the observed ones\(^1\). The eventual existence of expectation formation lags can then be regarded as an additional argument to the introduction of dynamics (cf. Friedman, 1977).

The four rates of return, relevant to the portfolio model defined above, are\(^2\):

- **ILIQ**: interest rate on liquidities (strict sense), defined as a weighted average of the interest paid on demand deposits with deposit banks, non-monetary financial institutions and the monetary authorities (mainly postal deposits), being resp. 0.5, 0.5 and zero per cent over the entire sample period;
ITS : interest rate on short term deposits, defined as the rate paid on 3-month deposits;
IASL : interest rate on long term assets, defined as a weighted average of the rate paid on long term time deposits with deposit banks and non-monetary financial institutions (yield on 5-year certificates & bonds issued by public financial institutions) and the interest rate on long term government debt (yield of bonds issued by central government, maturing over 5 years or more);
IFC : the foreign interest rate, i.e. the interest rate on 3-month US $ deposits in the London Euro-Currency market, covered by the premium on the 3-month forward exchange rate of BF vs. US $.

Two additional explanatory variables were included: GDP at current market prices (YU) scaled by the private sector's total wealth (WP) and the growth rate of this wealth ($\Delta WP/WP_{-1}$). The former variable is a good proxy for the current nominal value of transactions which determines the requirement of an asset, in particular (near) money, as a running asset. As such, this variable is treated in the same way as the rates of return and belongs fully to the vector of long run determinants, $x^{19}$.

The introduction of wealth growth allows for a relaxation of the linear homogeneity in total wealth, which may appear as a rather strong property in the short run. By its nature, this growth rate can never play in the long run and is only allowed to exert its influence in the same ways as the other explanatory variables do in the very short run, i.e. through the matrix $\Gamma$.

Figure 5.2. gives an idea on the evolution of these six exogenous variables over the sample period '59-'84.
Figure 5.2. Evolution of the exogenous variables ('59-'84)
5.2. Estimation results

The first order general dynamic model (4.12) and two of its specialisations, the partial adjustment and static models, were estimated over the sample period '59-'84 (yearly) with Full Information Maximum Likelihood. The homogeneity restriction on the long run multipliers was imposed throughout rather than tested because this seemed the least one could do to alleviate the multicollinearity problem that arises from the strong interest correlations. On the other hand, the symmetry of long run cross interest effects has tested by reestimating every specification subject to this constraint. Because the ranking of the estimated long run income effects on the holdings of the three domestic assets was contrary to a priory expectations, the entire exercise was repeated without introducing the income variable as a long run determinant of portfolio behaviour. This leads altogether to 12 specifications of which the estimation results are summarised in Table 5.420.

At least three questions are to be answered: which is the most appropriate dynamic formulation? is symmetry rejected by the data? and, do we restrict the influence of the income variable to the short run? As to the first question, the log likelihood ratio test pointed unanimously towards the general dynamic specification21. In Table 5.5. the results of this test are reported for different assumptions regarding symmetry and long run income effects.

Though the DW-statistic looses its exact meaning because lagged dependent variables occur in the specifications, it is illustrative how the very low DW-values that accompany the static formulation improve considerably when dynamics are introduced via a partial adjustment mechanism. Still, this mechanism is not satisfactory either. First, it leaves us with a long run structure which contradicts economic theory w.r.t. positivity22. Secondly, the adjustment matrix of this mechanism has a dominant eigenvalue very close to
<table>
<thead>
<tr>
<th>Specification</th>
<th>Log Lik</th>
<th>SE</th>
<th>DW</th>
<th>Coef. Signif.</th>
<th>Positivity</th>
<th>LR income effect on $w_{li}$</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>GEN. DYN.</td>
<td>350.3</td>
<td>.0029</td>
<td>.0039</td>
<td>.0041</td>
<td>6.621 6.766 6.394</td>
<td>71 %  40 %</td>
</tr>
<tr>
<td></td>
<td>PART. ADJ.</td>
<td>317.6</td>
<td>.0048</td>
<td>.0057</td>
<td>.0048</td>
<td>1.866 2.439 2.204</td>
<td>63 %  25 %</td>
</tr>
<tr>
<td></td>
<td>STATIC</td>
<td>241.4</td>
<td>.0133</td>
<td>.0262</td>
<td>.0133</td>
<td>1.041 .991 .454</td>
<td>87 %  67 %</td>
</tr>
<tr>
<td></td>
<td>GEN. DYN.</td>
<td>346.1</td>
<td>.0032</td>
<td>.0039</td>
<td>.0045</td>
<td>6.468 6.231 2.415</td>
<td>72 %  36 %</td>
</tr>
<tr>
<td></td>
<td>PART. ADJ.</td>
<td>215.9</td>
<td>.0160</td>
<td>.0351</td>
<td>.0191</td>
<td>.429 .429 .330</td>
<td>92 %  67 %</td>
</tr>
<tr>
<td></td>
<td>STATIC</td>
<td>201.0</td>
<td>.0136</td>
<td>.0465</td>
<td>.0218</td>
<td>1.064 .256 .672</td>
<td>78 %  11 %</td>
</tr>
<tr>
<td></td>
<td>GEN. DYN.</td>
<td>341.4</td>
<td>.0037</td>
<td>.0040</td>
<td>.0043</td>
<td>2.702 3.820 2.424</td>
<td>81 %  50 %</td>
</tr>
<tr>
<td></td>
<td>PART. ADJ.</td>
<td>300.2</td>
<td>.0059</td>
<td>.0071</td>
<td>.0078</td>
<td>1.915 1.628 1.491</td>
<td>56 %  22 %</td>
</tr>
<tr>
<td></td>
<td>STATIC</td>
<td>201.0</td>
<td>.0136</td>
<td>.0465</td>
<td>.0218</td>
<td>1.064 .256 .672</td>
<td>78 %  11 %</td>
</tr>
</tbody>
</table>

** = % of all directly estimated coefficients whose point estimate in absolute value is larger than once their corresponding standard error.

*** = twice

b) when symmetry is not imposed, no. of own interest effects (4 in total) and no. of cross interest effects (12 in total) is given that are resp. positif and negatif; when symmetry is imposed, it is investigated whether all Choleski values of the matrix $\tilde{K}(K)$ are non negative.

c) point estimate (asymptotic standard error) of $R_{15}$.

d) dominant eigenvalue or dominant modulus of conjugate pair of complex eigenvalues of the matrix $(I_n - \Lambda_{\Omega})$.

e) in order to make the Log Lik, SE and DW statistics comparable with other specifications, the static model was estimated as $\Delta w_{ni}(t) = z_{ni}(t) - w_{ni}(t-1) + \omega_{ni}(t)$.

* this dominant root is of the real type.
Table 5.5. Log likelihood ratio tests: dynamic structure

<table>
<thead>
<tr>
<th>Specification</th>
<th>Hypotheses</th>
<th>No. of constraints</th>
<th>$\chi^2$</th>
<th>Critical Value (5 %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR Y-EFFECT</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NO SYMMETRY</td>
<td>$H_0^o$: Part. Adj. vs $H_1^o$: Gen. Dyn.</td>
<td>18</td>
<td>65.3</td>
<td>28.9</td>
</tr>
<tr>
<td></td>
<td>$H_0^o$: Static vs $H_1^o$: Part. Adj.</td>
<td>9</td>
<td>152.4</td>
<td>16.9</td>
</tr>
<tr>
<td>NO SYMMETRY</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$H_0^o$: Part. Adj. vs $H_1^o$: Gen. Dyn.</td>
<td>18</td>
<td>80.8</td>
<td>28.9</td>
</tr>
<tr>
<td></td>
<td>$H_0^o$: Static vs $H_1^o$: Part. Adj.</td>
<td>9</td>
<td>191.8</td>
<td>16.9</td>
</tr>
<tr>
<td>SYMMETRY</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$H_0^o$: Part. Adj. vs $H_1^o$: Gen. Dyn.</td>
<td>18</td>
<td>82.3</td>
<td>28.8</td>
</tr>
<tr>
<td></td>
<td>$H_0^o$: Static vs $H_1^o$: Part. Adj.</td>
<td>9</td>
<td>198.3</td>
<td>16.9</td>
</tr>
</tbody>
</table>

Table 5.6. Log likelihood ratio tests: symmetry constraint

<table>
<thead>
<tr>
<th>Specification</th>
<th>Hypotheses</th>
<th>No. of constraints</th>
<th>$\chi^2$</th>
<th>Critical Value (5 %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR Y-EFFECT</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEN. DYN.</td>
<td>$H_0^o$: symmetry vs $H_1^o$: no symmetry</td>
<td>3</td>
<td>8.3</td>
<td>50.8</td>
</tr>
<tr>
<td>STATIC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NO LR Y-EFFECT</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEN. DYN.</td>
<td>$H_1^o$: no symmetry</td>
<td>3</td>
<td>4.9</td>
<td>7.8</td>
</tr>
<tr>
<td>PART. ADJ.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>STATIC</td>
<td></td>
<td></td>
<td></td>
<td>13.0</td>
</tr>
</tbody>
</table>
unity, pointing at an extremely slow convergence of the system towards the equilibrium response. Moreover, the numerical estimation problems that were encountered when imposing the symmetry condition (specification V) are a third indication that this adjustment mechanism exhibits insufficient flexibility to capture the dynamics of portfolio behaviour.

Table 5.6 reports on the log likelihood ratio tests of the symmetry hypothesis. The findings of A & B (1982) and (1983, 1984) where symmetry was tested in factor and consumer demand models resp., were such that this hypothesis could not be rejected when the demand system was formulated in a general dynamic way. Our results are in line with these findings up to a certain degree only. First, notice that, if one takes the static model as maintained hypothesis (though this is in conflict with the dynamic structure tests carried out above), the imposition of symmetry on the interest rate coefficients reduces the log likelihood value in a significant way; a similar reduction was found in the earlier mentioned study of Berndt et. al. (1980). The same exercise, repeated with the partial adjustment model as maintained hypothesis (again, incorrectly) does not lead to a symmetry rejection at 5% significance level. However, we do not give much credit to this model either, as it is almost characterised by instability (cf. the high dominant eigenvalue); moreover, though the substitution matrices $\bar{K}$ and $K$ both satisfy the positivity condition, their smallest Choleski values (with the exception of the obvious zero for $\bar{K}$) are too close to zero to be really convincing.

When the symmetry test is carried out within the general dynamic structure, a twofold picture emerges: if the income effect is ruled out in the long run, there is no rejection of the hypothesis at the 5% significance level, while this is the case when we do allow for such income effects. This absence of unanimity puts us to the choice.
In Section 2, we referred to Roley (1983) where it is shown in a mean-variance context, that the symmetry restriction is not a general property of asset demand functions, but requires zero wealth effects as a necessary and sufficient condition. In other words, this restriction imposes a behavioural assumption on the investor which is complementary to the set of rationality axioms portfolio theory starts off; therefore, there are no reasons to believe that it holds a priori and one should rely on the data for a decisive answer.

On the other hand, imposing the symmetry hypothesis not only sharpens the precision of the point estimates, it also enhances the attractiveness of the model for simulation purposes, i.e. the well-behaviour argument. As we believe these elements to be of great importance w.r.t. the aim of this study, the symmetry property will be accepted in the remainder of the text as a maintained hypothesis.

Finally, we have to make a choice w.r.t. the presence of (scaled) nominal income as a long run determinant of wealth allocation. Economic theory is rather straightforward on this matter: the transaction motive reallocates funds in the direction of the running asset when current business activities swell. So, a priori, one would expect the income variable, YU/WP to appear with a positive coefficient in the liquidity equation (\\( \omega_{L1} \)) which is then compensated by negative income coefficients in (some of) the remaining demand equations.

The estimation results of the general dynamic specification with symmetry imposed and with a long run income effect allowed for (cf. IV) are presented in Table 5.7. The coefficients related to the dynamics are given in the upper section, whereas the long run coefficient matrix \( \Pi \) is pictured in the second part of the table. The coefficients on the last row of each matrix, and their corresponding standard errors, were retrieved by means of the adding-up conditions. The matrix of impact multipliers is given by \( \Gamma \) and it is recalled that the structure of this short run
Table 5.7. Estimation of general dynamic model, symmetry imposed, with long run income effect

\[ \Delta w(t) = \Gamma \Delta w(t) + A_n^{\alpha}[w_n^{o}(t-1) - w_n(t-1)] = \Gamma \Delta w(t) + A_n^{\alpha}[\Delta x(t-1) - w_n(t-1)] \]

<table>
<thead>
<tr>
<th>[\Delta w_{L1}(t)]</th>
<th>[\Delta w_{TS}(t)]</th>
<th>[\Delta w_{AS}(t)]</th>
<th>[\Delta w_{RA}(t)]</th>
<th>[\Delta \text{LIQ}(t)]</th>
<th>[\Delta \text{ITS}(t)]</th>
<th>[\Delta \text{IASL}(t)]</th>
<th>[\Delta \text{IFC}(t)]</th>
<th>[\Delta \text{Yu}(t)]</th>
<th>[\Delta \text{WP}(t)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.45** (.1834)</td>
<td>.1402 (.1669)</td>
<td>-.0281 (.0596)</td>
<td>.2336 (.0645)</td>
<td>-.1390 (.0772)</td>
<td>[\Delta \text{LIQ}(t)]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-22.42 (.2264)</td>
<td>.4519 (.2067)</td>
<td>-.3323 (.0768)</td>
<td>.1127 (.0814)</td>
<td>.3698 (.0974)</td>
<td>[\Delta \text{ITS}(t)]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-28.10 (.3175)</td>
<td>-.2873 (.2847)</td>
<td>.6267 (.0935)</td>
<td>-.2530 (.1147)</td>
<td>-.4322 (.1219)</td>
<td>[\Delta \text{IASL}(t)]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.3707 (.1822)</td>
<td>.7193 (.1596)</td>
<td>-.6346 (.0513)</td>
<td>.1684 (.0682)</td>
<td>.2846 (.0659)</td>
<td>[\Delta \text{IFC}(t)]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[\begin{align*}
&= \\
&= \begin{bmatrix}
.4175** (.1269) & .0970* (.0517) & .1109* (.0800) \\
-.4277** (.1615) & .0291* (.6409) & .1286* (.1014) \\
.2309 (.2563) & .0362 (.0966) & .3411* (.1145) \\
-.2207 (.1637) & .1623* (.0616) & -.3233* (.0575)
\end{bmatrix}
\]

\[\begin{align*}
\Delta w_{L1}(t) - w_{L1}(t-1) \\
\Delta w_{TS}(t) - w_{TS}(t-1) \\
\Delta w_{AS}(t) - w_{AS}(t-1)
\end{align*}\]

Eigenvalues \([I_n - A_n^{\alpha}] : .5497 \text{ and } .8312 + .0679 \text{ (modulus : .8340)}\)

\[\begin{align*}
\Delta w_{L1}(t-1) & = \begin{bmatrix}
.2899** (.0802) & 1.946** (.345) & -.8752** (.4604) & -.1792** (.576) & .7283** (.2463) & .1071* (.0982) \\
.1172 (.2377) & 2.989** (.1161) & -.1992* (.1104) & .1222 (.5799) & .1119 (.3297)
\end{bmatrix}
\end{align*}\]

\[\begin{bmatrix}
1 \\
\text{ILIQ}(t-1) \\
\text{ITS}(t-1) \\
\text{IASL}(t-1) \\
\text{IFC}(t-1) \\
\text{Yu}(t-1) \\
\text{WP}(t-1)
\end{bmatrix}\]

Choleski values \(\tilde{\chi} : 1.946, 2.596, .3074, \text{ and } 0; \chi : 2.989, 3.645, \text{ and } .1425\)

\[\begin{align*}
\Delta w_{L1}^{o}(t) & = \begin{bmatrix}
.0872 (.2361) & 4.972** (.1961) & -.1189 (.633) & .2862 (.2714)
\end{bmatrix}
\end{align*}\]

\[\begin{align*}
\Delta w_{TS}^{o}(t) & = \begin{bmatrix}
.5059** (.1365) & .5901* (.3435) & -.5051* (.1888)
\end{bmatrix}
\end{align*}\]

\(\text{a }\) asymptotic standard errors in parentheses;

\(\ast\) point estimate in absolute value larger than one (twice) the standard error.
behaviour was in no way restricted. As far as the interest coefficients concern, own short-run effects are all positive and most of the cross-effects bear a negative sign. The relative strong impact of the yield on liquidities (which characterise all estimations of a general dynamic specification) is striking, but not alarming since the changes in the ILIQ variable will lever be of such magnitude so as to produce share changes outside the interval (0,1). The long run demand system is homogeneous and symmetric in interest rates. The own effect of the rate on liquidities is considerably tempered, while the reverse holds for the own effects of the remaining rates of return. An examination of the Choleski values of this interest coefficient matrix reveals that the positivity condition is respected. The point estimates that are encountered for the short run income effects, suggest that in a first instant a reallocation takes place from funds invested in foreign assets towards liquidities (in the strict sense); the relative holdings of the two other assets are merely affected. Still, an entirely different picture is sketched by the final form income parameters: in the long run, investors would react to an income increase by reallocating wealth towards long term assets (+.2862) and, to a lesser extent, towards liquidities in the large sense (+.1070 +.1119), at the expense of foreign asset holdings (-.5051). Before proceeding, it should be noted however that these coefficients measure only a partial effect of an income increase. The effect is indeed incomplete, because it ignores the increase in wealth that is generated through the channel of financial savings. Mathematically, the long run demand for asset $i$, say $z_i$, is given by

$$z_i = w_i^0 W P,$$

where $w_i^0$ is the desired long run share of asset $i$, viz.
\[ w^o_i = \Pi_{i0} + \sum_{j=1}^{i} \Pi_{ij} x_j + \Pi_{i5} \frac{YU}{WP}. \]

Hence, differentiation of \( z_i \) w.r.t. YU yields

\[ \frac{\partial z_i}{\partial YU} = \Pi_{i5} \left( 1 - \frac{\partial WP}{\partial YU} \frac{YU}{WP} \right) + w^o_i \frac{\partial WP}{\partial YU}, \]

where the second and third RHS terms denote the aforementioned ignored effect through wealth: the savings quote. Though the effect on liquidities seems appropriate, doubts have to be raised w.r.t. the strong impact in the long run on long term asset holdings.

Since our time perspective is the long run, we are dealing with the effects of a sustained higher level in nominal income, in stead of a temporary shock. This means that the need for a means of payment can be anticipated and that the problem is not any longer one of reallocation towards the medium of exchange but rather one of finding a suitable temporary abode for means intended to be used for payment of goods and services. It is evident that with anticipated payments, this 'intermediary' function cannot only be performed by money, but by any asset that exhibits a sufficient degree of liquidity for then, i.e., when it is 'realisable at short notice without loss', it can be converted into the medium of exchange almost at the instant of payment, precisely because this payment was anticipated. So, the fact that our estimates suggest an increased interest in an asset, in casu bonds, that cannot serve as a means of payment, should not be disturbing. What is disturbing, however, is that this asset, its market value being strongly subjected to fluctuations, cannot properly function as a temporary abode for means intended for later payment, that is, it is an illiquid asset. Because the precision of 3 out of the 4 point estimates of the long run income effect is rather low, the entire system was reesti-
Table 5.8. Estimation of general dynamic model, symmetry imposed, without long run income effect

\[
\Delta w(t) = \Gamma \Delta x(t) + A_n^0 \{ w_n^{0 \prime}(t-1) - w_n(t-1) \} = \Gamma \Delta x(t) + A_n^0 \{ \lambda_n x(t-1) - w_n(t-1) \}
\]

\[
\begin{bmatrix}
\Delta w_{LI}(t) \\
\Delta w_{TS}(t) \\
\Delta w_{AS}(t) \\
\Delta w_{NA}(t)
\end{bmatrix} =
\begin{bmatrix}
5.22 & -0.8225 & 0.3304 & -0.0564 & 0.2044 & -0.0266 \\
(13.11) & (0.1835) & (0.1477) & (0.0528) & (0.0551) & (0.0637) \\
-21.09 & 0.3197 & -0.3247 & 0.1130 & 0.0286 & 0.3072 \\
(14.47) & (0.2202) & (0.1850) & (0.0616) & (0.0606) & (0.0714) \\
-20.53 & 0.0743 & 0.4461 & -0.1715 & -0.0501 & -0.3781 \\
(15.57) & (0.2237) & (0.2060) & (0.0660) & (0.0667) & (0.0777) \\
36.39 & 0.4285 & -0.4518 & 0.1149 & -0.1931 & 0.0975 \\
(8.13) & (0.1149) & (0.1120) & (0.0379) & (0.0345) & (0.0388) \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta LIQ(t) \\
\Delta ITS(t) \\
\Delta IASL(t) \\
\Delta IFCT(t) \\
\Delta WP(t) \\
\Delta WP(t-1)
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.2646 & 0.0711 & -0.0735 \\
(0.0928) & (0.0419) & (0.0763) \\
-0.2813 & 0.0679 & -0.1222 \\
(0.1019) & (0.0470) & (0.0868) \\
-0.0286 & -0.0620 & 0.3149 \\
(0.1093) & (0.0500) & (0.0908) \\
0.0453 & -0.0770 & -0.2661 \\
(0.0559) & (0.0269) & (0.0475)
\end{bmatrix}
\]

Eigenvalues \( I_n^0 - A_n^0 \): .6493 and .8517 + .1055i (modulus: .8582)

\[
\begin{bmatrix}
\omega^{0 \prime}_{LI}(t-1) \\
\omega^{0 \prime}_{TS}(t-1) \\
\omega^{0 \prime}_{AS}(t-1) \\
\omega^{0 \prime}_{NA}(t-1)
\end{bmatrix} =
\begin{bmatrix}
0.3503 & 1.875** & -1.051 & -1.367** \\
(0.0185) & (0.306) & (0.486) & (0.235) & (0.2045) \\
0.1609 & 2.153** & -0.5364 & -0.5658 \\
(0.0462) & (1.008) & (1.4515) & (0.5210) \\
0.3569 & 2.380** & -0.4758 & -0.4758 \\
(0.0176) & (0.368) & (0.1874) \\
0.1318 & 0.4982 & 0 \\
(0.0435) & (0.4760)
\end{bmatrix}
\]

Choleski values: \( K : 1.875, 1.564, .2971, \) and \( G : 2.153, 2.458 \) and .1802

\(^a\) see notes to Table 5.7.
mated with exclusion of these effects. The results are shown in Table 5.8.

The general impression one got from the previous estimation, also applies here: the difference in magnitude of long run vs. short run interest coefficients, positivity of the substitution matrix, plausible impact income multipliers, short run tendencies to invest additional wealth in short term time deposits ($w_{TS}$) and foreign assets ($w_{NA}$). On the other hand, when comparing the two versions, a number of differences draw attention:

(i) the fall in the log likelihood value when ruling out the long run income effects is not sharp, but still strong enough to reject the restricted model as a null hypothesis ($\chi^2 = 9.4$, c.v.(3d.f.) = 7.8 at 5%);

(ii) 13 out of 16 short run interest coefficients and 9 of the 10 reported long run ones become, in absolute value, smaller, when the long run income effect is absent;

(iii) although the coefficients of the adjustment matrix $A^N$ do not lend themselves easily for interpretation (cf. the identification problem discussed in section 4.2.), eigenanalysis of the 3x3 upper sub matrix reveals everything on the stability of the system. In both versions a conjugate pair of complex eigenvalues yield a dominant modulus inside the unit circle, though in the income restricted case, this modulus takes a higher value, pointing at a slightly slower adjustment process;

(iv) in the income restricted version, 81% of all directly estimated coefficients have a standard error lower than their point estimate, and 50% are larger than twice their standard error; in the unrestricted version these percentages are 72 and 36 resp.

The slow rate of adjustment, in either case, implies that on short and even medium term, the long run response to the income variable (whether zero or not) remains relatively
Table 5.9. Cumulated impact & final form income multipliers

<table>
<thead>
<tr>
<th>lag</th>
<th>w_{LI}</th>
<th>w_{TS}</th>
<th>w_{AS}</th>
<th>w_{NA}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U</td>
<td>R</td>
<td>U</td>
<td>R</td>
</tr>
<tr>
<td>0</td>
<td>.234</td>
<td>.204</td>
<td>-.011</td>
<td>.029</td>
</tr>
<tr>
<td>1</td>
<td>.218</td>
<td>.152</td>
<td>.018</td>
<td>.078</td>
</tr>
<tr>
<td>2</td>
<td>.200</td>
<td>.108</td>
<td>.046</td>
<td>.112</td>
</tr>
<tr>
<td>3</td>
<td>.182</td>
<td>.072</td>
<td>.070</td>
<td>.134</td>
</tr>
<tr>
<td>4</td>
<td>.167</td>
<td>.043</td>
<td>.089</td>
<td>.146</td>
</tr>
<tr>
<td>5</td>
<td>.154</td>
<td>.021</td>
<td>.104</td>
<td>.150</td>
</tr>
<tr>
<td>6</td>
<td>.143</td>
<td>.003</td>
<td>.116</td>
<td>.148</td>
</tr>
<tr>
<td>7</td>
<td>.134</td>
<td>-.010</td>
<td>.123</td>
<td>.141</td>
</tr>
<tr>
<td>8</td>
<td>.127</td>
<td>-.019</td>
<td>.129</td>
<td>.132</td>
</tr>
<tr>
<td>9</td>
<td>.121</td>
<td>-.025</td>
<td>.132</td>
<td>.121</td>
</tr>
<tr>
<td>10</td>
<td>.116</td>
<td>-.029</td>
<td>.133</td>
<td>.108</td>
</tr>
<tr>
<td>∞</td>
<td>.107</td>
<td>0</td>
<td>.112</td>
<td>0</td>
</tr>
</tbody>
</table>

*a row sums may differ slightly from zero due to rounding-off errors;
R: income restricted case;
U: income unrestricted case.
unimportant (cf. the expression for cumulated interim multipliers at the end of subsection 4.3). Table 5.9 gives the cumulated impact multipliers for YU/WP up to 10 lags, together with the final form multiplier to which the former will converge at infinity.

In the light of what has been said above, the pattern generated in the restricted case seems more plausible than in the other case: in the very short and short run, an increase in the nominal value of economic activity, which still has a temporary feature, necessitates the use of liquidities (in the strict sense) which are mainly obtained at the expense of foreign assets. However, when the rise in running activity gets a more permanent character, we see how the importance of short term assets grows while the emphasis on liquidities (in the strict sense) is relaxed; the effect on long term assets remains negligible. This substitution of short term assets for (near) monies is justified on the basis of the high degree of liquidity of these former assets which are interest-bearing at the same time. Because long term assets lack this liquidity property, the scenario depicted by the unrestricted model looses its credibility. To sum up, though the income restricted version of the model does not pass the log likelihood ratio test, it is not rejected with a large margin either, its point estimates have a greater degree of precision and its implied reaction to changes in income can be theoretically supported. These considerations, and the fact that in a dynamic simulation over the sample period it performs equally well as the unrestricted version, made our preference go to the income restricted model.

A full sketch of this model's dynamics is given in Figure 5.3, where the cumulated interim multipliers of the six exogenous variables are graphically represented. The slow convergence to the long run solution, already 'announced' by the high dominant eigenvalue, strikes the eye; the oscillat-
ing movement is due to the occurrence of the conjugate pair of complex roots.

To obtain a more accurate idea on the speed (or should we rather say the slowness) of adjustment, one would like to have recourse to measures such as the mean or median lag. However, the first concept is only defined when the lag scheme can be said to be normalised; whether this is the case with a multivariate lag scheme, is not trivial to investigate (cf. Bewley, 1984, pp. 48-50). The second measure, the median lag, is also surrounded with definitional problems when the lag scheme is a multivariate one; at least when it is characterised by complex roots, for then the oscillating behaviour of the multipliers around the long run response can preclude one to say when 50% of the long run effect has taken place.

Nevertheless, in order to get some notion of the speed of convergence, we calculated for each long run interest multiplier the 50% zone, viz. \[ i \leq \Pi_{ij} \leq 1.5 \Pi_{ij}, \quad i, j = 1, \ldots, 4, \]
and searched for the number of periods that a shock should sustain for the cum. interim effect to fall within this range. These lag numbers are given in parentheses to the right of the variable legends in the figure. For instance, the cum. interim multiplier of ITS on TSTP enters the 50% zone after 4 lags (approximately). The definition problem arose only in two cases: the effect of ILIQ on \( w_{NA} \) and of IASL on \( w_{TS} \).

Let us first study the reaction speeds of the different asset holdings to changes in a particular rate of interest (i.e. looking at all reaction curves in one figure at a time). Then we observe that the speeds of adjustment to changes in ILIQ are very low, whereas the adjustments to movements in IFC take place very fast; a mixed picture is obtained for reactions to the ITS and IASL rates (slow reactions on the part of \( w_{NA} \) and \( w_{NA} \) and \( w_{TS} \) resp.).

Concentrating on the speed of adjustment of one single asset share to changes in the different rates of return (by
Figure 5.3. Cumulated interim multipliers (Lag 50% zone within brackets)
Figure 5.3. Cumulated interim multipliers (cont.)
looking at the same multiplier curve across the different figures), we may term liquidities ($w_{LI}$) and long term assets ($w_{AS}$) as first adjusters (ignoring in both cases the slow reaction to ILIQ); the short term ($w_{TS}$) and foreign ($w_{NA}$) assets, on the other hand, perform a very slow process of convergence to their equilibrium values, although they too react fast w.r.t. movements in their own rates of return which is therefore an overall property of the model. It would have been an interesting exercise to assess the contribution of the different exogenous variables to the evolution of the portfolio structure over, say, the last ten years of the sample period, by truncating the final form of the demand system (cf. eq. (4.17)) at a finite number of lags such that the associated cumulated interim multipliers are relatively close to the final ones. However, from the convergence analysis above, it is clear that our sample period is too limited to carry out such an ex post analysis in a sensible way.

The general impression one gets from these model dynamics, is a very slow portfolio adjustment behaviour of the Belgian private sector to the long run response values. But this is something different than to say that the private sector agents manage their portfolios in a 'lazy' way. Sometimes, they react very violently to exogenous shocks; it only takes them a long way to attune their portfolio composition to the long run structure.

We conclude this empirical section by having a look at the long run interest elasticities. These are given by

$$\varepsilon_{ij} = \frac{\partial w_i / w_i}{\partial x_j / x_j} = \Pi_{ij} \frac{x_j}{w_i}, \ i, \ j=1, \ldots, 4.$$ 

Though this concept denotes the percentual change in the portfolio share of a particular asset, it may also be interpreted as the percentual change in the absolute holdings of that particular asset if one ignores the effect of
the change in the interest rate on total wealth. An even more interpretable concept can be obtained by considering percentual changes in asset shares (or levels) due to changes in interest rates of one percentage point (e.g. from .08 to .09); these changes are then given by the semi-elasticities \( v_{ij} \), i.e.

\[
v_{ij} = \frac{\partial w_i / w_i}{\partial x_j} = \Pi_{ij}/w_i, \quad i, j = 1, \ldots, 4.
\]

Both reaction measures are depicted in Table 5.10 and 5.11 resp. where use was made of the long run coefficient estimates of Table 5.8 and the sample averages of \( x_j \) and \( w_i \).

All interest elasticities are fairly small in magnitude, a finding that typifies most empirical asset demand studies. For short and long term placements, all competing assets appear to be substitutes.

Apart for liquidities, assets respond the most elastically to changes in the own rates of return. The positive interest elasticities of (net) foreign assets vs liquidities points at a complementarity between both assets, for which no immediate explanation is available. A similar relationship was initially encountered in the MORKMON-model (De Nederlandsche Bank, 1984) where an allocation mechanism was estimated over four asset categories that comprise broadly the same assets as the aggregates of the present study. But all complementary relationships where subsequently suppressed in MORKMON by restricting positive long run coefficients to zero; this may explain why the final form non-zero elasticities in that study are of a slightly higher magnitude than the ones in table 5.10.

Looking at the semi-elasticities learns us that all asset holdings will, in the long run, be raised with 6 - 8 % when their own rates of return go up with one percentage point. Most cross semi-elasticities are smaller than the own ones, except for foreign assets holdings (NAFP) which are reduced with almost 7.5 % when the rate on short term deposits experiences a 1 % increase; an asymmetric reaction is found
Table 5.10. Long run interest elasticities\(^a\)

<table>
<thead>
<tr>
<th></th>
<th>1. ILIQ</th>
<th>2. ITS</th>
<th>3. IASL</th>
<th>4. IFC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. LIQP</td>
<td>.029</td>
<td>-.210</td>
<td>-.464</td>
<td>.189</td>
</tr>
<tr>
<td>2. TSTP</td>
<td>-.013</td>
<td>.339</td>
<td>-.143</td>
<td>-.155</td>
</tr>
<tr>
<td>3. ASLP</td>
<td>-.013</td>
<td>-.066</td>
<td>.499</td>
<td>-.015</td>
</tr>
<tr>
<td>4. NAFP</td>
<td>.026</td>
<td>-.356</td>
<td>-.509</td>
<td>.545</td>
</tr>
</tbody>
</table>

\(^a\) \(\bar{\epsilon}_{ij} = \bar{\pi}_{ij} \cdot \bar{x}_j / \bar{w}_i\), where \(\bar{x}_j\) and \(\bar{w}_i\) denote sample period averages.

Table 5.11. Long run semi interest elasticities\(^a\)

<table>
<thead>
<tr>
<th></th>
<th>1. ILIQ</th>
<th>2. ITS</th>
<th>3. IASL</th>
<th>4. IFC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. LIQP</td>
<td>7.893</td>
<td>-4.425</td>
<td>-5.756</td>
<td>2.288</td>
</tr>
<tr>
<td>2. TSTP</td>
<td>-3.481</td>
<td>7.132</td>
<td>-1.777</td>
<td>-1.874</td>
</tr>
<tr>
<td>3. ASLP</td>
<td>-3.551</td>
<td>-1.393</td>
<td>6.180</td>
<td>-.183</td>
</tr>
<tr>
<td>4. NAFP</td>
<td>7.199</td>
<td>-7.496</td>
<td>-6.304</td>
<td>6.600</td>
</tr>
</tbody>
</table>

\(^a\) \(\bar{\nu}_{ij} = \bar{\pi}_{ij} / \bar{w}_i\), where \(\bar{w}_i\) denotes sample period average.
on behalf of short term deposits of which only 1.9% is disposed of.

If one knows that these four rates of return changed, on average, either up- or downwards with .006, .85, .68 and 2.52% resp., one cannot deny that the portfolio behaviour of the Belgian private sector exhibits a certain degree of flexibility.
6. Conclusion

In its search for an adequate portfolio modelling strategy, this paper started off by investigating the properties of the wealth allocation mechanism in a (Tobin-) perfect market world. Apart from adding-up, this basic model exhibited homogeneity, symmetry and positivity in rates of returns which made it resemble very much to the structure of consumer and factor demand systems. Moreover, we observed that the optimal allocation within the risky part of the portfolio was independent of the parameters that typify the particular investor, i.e. his initial wealth and his aversion to risk (the separation theorem).

But the perfect market setting seemed too remote from real world surroundings to give much credit to the explanatory power of the basic model in the short run; this we learned when studying the allocation mechanism from a Hicksian perspective. Though we were not able to keep the analysis always as formal as before, it became clear that transaction costs have a strong impact on the mechanism as a whole, for not only have rates of return to be adjusted downwards in the optimisation problem, the problem itself gets an entirely new dimension: it now stretches further than just the period immediate ahead. In this way the introduction of transaction costs automatically brings liquidity considerations into play. These are identified with a new *modus operandi* of portfolio behaviour: the precautionary motive, which operates not just on a twofold 'money-bonds' choice set, but rather on the entire liquidity *spectrum*.

With these new elements in mind, we took up thread of the formal analysis by embedding the basic portfolio model in a flexible dynamic framework, adapted to singular demand systems by Anderson and Blundell. In this way, the basic model was 'consigned' to the long run.

To keep the model within limits, we regarded a first order general dynamic specification as the maintained hypothesis. The empirical results, based on data of the Belgian private sector, revealed the following:
(i) the partial adjustment mechanism, though frequently applied in empirical asset demand studies, is rejected by the data as a dynamic specification;

(ii) though the evidence found against the symmetry hypothesis was sufficient, it could not be called overwhelming either; using a well-behaviour argument, it was decided to accept symmetric interest effects as a long run property of the allocation model;

(iii) while the impact income multipliers implied reasonable short term reactions, perverse effects of income were generated in the long run. Although in a log likelihood ratio test, the restriction of zero long run income effects was proven to be too strong, an examination of the time path of the cumulated interim multipliers made us choose for the restricted version as a maintained hypothesis;

(iv) further analysis of these multipliers indicated at a very slow tendency of the model to converge to its equilibrium solution; and finally,

(v) the long run interest elasticities, evaluated at sample averages, were found to be low, a feature which most empirical portfolio studies come across with. Still, on the basis of the more interpretable semi-elasticities, it is hard to characterise the private sector agent as an 'apathic' investor who does not respond to stimuli from the market.

Further research on this topic should develop at least in two directions.

At the theoretical level, there remains work to be done on the analytical implementation of the precautionary motive in the basic (speculative) portfolio model. This would be a welcome step towards non-walrasian economics, for the uncertainty underlying the precautionary behaviour may stem from the ignorance as to which particular regime will prevail in the near future; and, the presence of sufficient
liquid wealth enlarges the scope for a household to react when confronted with market rationing (the usually studied flow reactions are extended with stock reactions, cf. Korliras, 1983).

At the empirical level, the model studied in this paper may benefit from a more rigorous specification of the mechanism generating the interest rate expectations. The use we made of observed rates as proxies is only a second best solution which should be weighted against rational or autoregressive formation rules. In the latter case, recourse will have to be taken to specifications even more general than the one stucked to in this paper; if this specification then appears not to be general enough the slow convergence results could be traced back to a shortage of degrees of freedom in the modelling of dynamics.
Notes


2 Throughout the present paper we neglect the implications that arise when the real portfolio rate of return is substituted for the nominal one as an argument in the investor's utility function. The aspects of expected inflation in the theory and estimation of portfolio models have only recently been dealt with in the literature; see e.g. Courakis (1988).

3 Alternatively, one could have postulated a utility function with end-of-period wealth \( W \), in stead of the overall return \( \pi \), as argument. In this case, Friedman and Roley (1980) show that the assumption of constant relative risk aversion, together with the joint normally distributed asset return assessments are jointly sufficient conditions to derive asset demand functions that are linear homogeneous in wealth (and linear in expected returns); the same result is achieved when starting from isomorphic assumptions.

4 Expression (2.4) in the text is arrived at on basis of similarity between the exponential part of the utility function (2.3) and the moment generating function of the normal distribution.

5 Since the asset demand system is linear homogeneous in initial wealth by construction, this parameter will a fortiori not exert any influence on the risky asset structure. However, if we had defined utility in terms of terminal wealth, \( W \), in stead of overall portfolio return, \( \pi \), the separation theorem w.r.t. initial wealth would still hold, despite the fact that the demand for (risky) assets is no longer linear homogeneous in this aggregate.
When transaction costs are present, the vector of net expected returns, e.g. \( \rho_N \), will be lower than the original one, viz. \( \rho_N \leq \rho \). Whether the total fraction of the portfolio, allocated towards risky assets, \( \Theta_N^0 \), is smaller than when transaction costs are absent, depends on the sign of

\[
1' \Psi^{-1} (\rho_N - \rho)
\]
on which no general statements can be made.

Imagine a two-asset spectrum, A and B, with the following visualised (expected) return structure over two months:

<table>
<thead>
<tr>
<th>A</th>
<th>4%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1%</td>
</tr>
<tr>
<td>1</td>
<td>2.97%</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B</th>
<th>2%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2%</td>
</tr>
<tr>
<td>1</td>
<td>0%</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

The superiority of A in the second month, even though it makes this asset the highest yielding on the 2-month period, is irrelevant for the portfolio choice at the outset of the first month. Indeed, investing the first month in asset B, and switching over to asset A at the end of it, leads to an overall 2-month return of 5.03\%, far better than the 4\% earned by investing solely in A. Thus, whether the investor wants to liquidate his wealth after two months, or immediately after the first one, makes no difference; the best initial portfolio choice is to invest in B.

This is no longer true when the switch from B to A after one month necessitates a charge of 1.5\% (on the portfolio value), as this lowers the overall return of the B-A strategy just below the 4\% of the A-A strategy (3.98\% to be precise). In this case, it is worth considering whether the
funds are needed only after two months or earlier. (cf. Tobin, 1965, pp. 3-6)

8 'A demand for money that springs from this cause, I think we shall agree, is precisely what Keynes meant by the precautionary motive' (Hicks, 1967, p. 34).

9 With the exception of trade credit.

10 Hicks avoids speaking of a transaction demand, to stress the fact that transaction balances are held because of a necessity, rather than on voluntary basis.

11 Of the 19 empirical on portfolio behaviour reviewed by Owen (1986), 10 make use of this multivariate partial adjustment mechanism in one way or another. Both the use of this scheme has not been restricted to portfolio modelling; see Nadiri and Rosen (1969) for an application of it to factor demand modelisation.

12 System (4.7) in the text can be labelled as the pseudo structural form of (4.2), a term introduced by Bewley (1979).

13 A third model, nested within the first order general dynamic system (4.9) of the text, is the autoregressive specification which is arrived at by restricting all systematic responses to take place immediately, viz. $\Gamma = \Pi$. It can be written as

$$w(t) = \Pi x(t) + v(t),$$

$$v(t) = P^n v_n(t-1) + \varepsilon(t),$$

where the sub- and superscript interpretation is the same as in the text.
All variables were taken from a database describing the monetary accounting framework for Belgium (cf. Section 1) from '59 till '84. The sources and exact definitions of the variables used in this section, are listed in appendix C.

I.e. liquidities in the strict sense, not to be confused with what was termed 'liquidities' or 'liquid assets' in section 3.

Government debt certificates on short term (CSGP) enter the private sector's balance sheet because they are held by nationalised enterprises and by 'institutions of life and labour accident insurance and pension funds'; private persons and business firms, however, are not allowed to subscribe to these debt certificates. Since these latter categories constitute an overwhelming majority of the 'private sector', the CSGP variable was taken exogenous and therefore subtracted from total wealth available for allocation.


All rates of return were stored as percentage points.

At least initially. As will appear in section 5.2, the effects of this income variable were confined to the short run.

It may be recalled from section 2 that the intercept entered the asset demand system in a special way (cf. eq. (2.8) in the text). Prior estimations with this particular structure imposed upon, led to dynamic demand systems which failed to meet the stability conditions and with log
likelihood values far below those obtained and reported in this section. Consequently, this restriction was dropped. Probably, the constant term catches many more aspects of portfolio behaviour which were neglected in section 2.

21 Observe that the testing of the general dynamic specification vs. the partial adjustment scheme implicitly also means a test of the short run wealth effect which operates in the former specification through the matrix $\Gamma$, but is totally absent in the latter model.

22 With the exception of specification XI; but more will be said in the sequel.

23 W.r.t. specification XI in table 5.3, the Cholesky values of the matrix $K$ are .8541, 4.767, .0188 and 0; those of the matrix $K$ are 5.115, 1.695 and .0088.

24

I.e. $\frac{1}{T} \sum_{t}^{T} | \Delta x_{jt} | . 100$
References


Appendix A

The transformed dynamic system of demand equations is given by (cf. eq. (4.7) in the text)

\[(A.1) \Delta w(t) = -B(L)\Delta w(t) + \Gamma(L)\Delta x(t) + A[\Pi x(t-q) - w(t-p)] + u(t).\]

Let us introduce the following partitionings:

\[B_i = [B_{1i}, B_{2i}], \ A = [A_1, A_2],\]

\[\text{nx}(n-1) \quad \text{nx}1 \quad \text{nx}(n-1) \quad \text{nx}1\]

\[
\Pi = \begin{bmatrix}
\Pi_1^{(n-1)xh} \\
\Pi_2^{1xh}
\end{bmatrix}
\quad w(t) = \\
\begin{bmatrix}
w_1(t) \\
w_2(t)
\end{bmatrix}^{(n-1)x1}
\]

The adding up restriction (4.6) of the text, and the fact that shares sum to unity, allow us to write \(\Pi_2\) and \(w_2\) as

\[(A.2) \quad \Pi_2 = e_1' - l_n' \Pi_1',\]

\[(A.3) \quad w_2 = 1 - l_n' w_1(t),\]

where \(e_1' = [1, 0, \ldots, 0]\) and \(l_n\) denotes an \((n-1)\) vector of units. Incorporation of the identities (A.2) and (A.3) in the partitionned version of (A.1) yields:
\[(A.4) \Delta w(t) = - \sum_{i=1}^{p} [B_{1i}B_{2i}] \begin{bmatrix} \Delta w_1(t-1) \\ -t_n^i \Delta w_1(t-1) \end{bmatrix} + \Gamma(L)\Delta x(t) \]

\[+ [A_1, A_2] \begin{bmatrix} \Pi_1 \\ -t_n^i \Pi_1 \end{bmatrix} x(t-q) - \begin{bmatrix} w_1(t-p) \\ -t_n^i w_1(t-p) \end{bmatrix} + u(t), \]

since \( e_1^\top x(t-q) = 1. \)

This version can now be rewritten as

\[(A.5) \Delta w(t) = - \sum_{i=1}^{p} (B_{1i} - B_{2i} t_n^i) \Delta w_1(t-1) + \Gamma(L)\Delta x(t) \]

\[+ (A_1 - A_2 t_n^i) [\Pi_1 x(t-q) - w_1(t-p)] + u(t). \]

The equivalence with system (4.8) in the text is then obvious.
Appendix B

The characteristic equation \(|(I-A)-\lambda I| = 0\) can be rearranged as

\[(B.1) \quad |(I-\lambda I)-A| = 0.\]

Let us partition the identity matrix and matrix \(A\) so as to isolate the \(n\)th row and column, viz.

\[
I = \begin{bmatrix} I^n & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},
\]

with obvious notations for \(I^n\) and \(A_{ij}\), \(i, j = 1, 2\).

Hence (A1) can be rewritten as

\[(B.2) \quad \begin{vmatrix} (1-\lambda) I^n - A_{11} & -A_{12} \\ -A_{21} & (1-\lambda)-A_{22} \end{vmatrix} = 0\]

Next, we follow Hunt's (1981) suggestion, and modify the matrix of which the determinant value is taken in the LHS of (B2) by elementary row operations which add the sum of the first \((n-1)\) rows to the last row; as is well known, these operations leave the determinant value unaltered, so that expression (B2) is equivalent to

\[(B.3) \quad \begin{vmatrix} (1-\lambda) I^n - A_{11} & -A_{12} \\ (1-\lambda)I^n - I^n A_{11} - A_{21} & -I^n A_{12} + (1-\lambda)-A_{22} \end{vmatrix} = 0\]

which, in view of the adding-up condition, \(I'A = (1+g)I'\), can be restated as

\[(B.4) \quad \begin{vmatrix} (1-\lambda) I^n - A_{11} & -A_{12} \\ (1-\lambda)I^n - (1+g) I^n & (1-\lambda)-(1+g) \end{vmatrix} = 0.\]
Analogously, elementary column operations can be applied to subtract the last column of each of the (n-1) preceding columns. This yields the expression

\[
(B.5) \quad \begin{vmatrix}
(1-\lambda) \ I_n^0 - (A_{11} - A_{12} \ I_n^0) & - A_{12} \\
\bar{Q} & (1-\lambda) - (1+g)
\end{vmatrix} = 0.
\]

Evaluating the determinant via the nth row, and rearranging, amounts to the equation

\[
(B.6) \quad (\lambda+g) \ |I_n^0 - (A_{11} - A_{12} \ I_n^0) - \lambda I_n^0| = 0
\]

in which the second LHS term is nothing but the characteristic equation of \((I_n^0 - A_n^0)\), \(A_n^0\) being the matrix consisting of the first (n-1) rows of \(A^n\), which was defined in section 4.2 of the text.
Appendix C : Data Sources

All variables were taken from a database describing the asset/liability position of six sectors in the Belgian economy [monetary authorities, deposit banks, non-monetary financial institutions (NMFI), private sector, government and rest of the world] w.r.t. each other, as well as the relevant rates of interest and some real sector variables on a yearly basis from '53 (sometimes '57) till '84 (cf. Schroyen, 1988). This database leans heavily on the two following sources:

(a) the subperiod till '80:

(b) the subperiod '81-'84:
   NATIONALE BANK VAN BELGIE, Tijdschrift van de Nationale Bank van België, Brussels (several issues).

In the following list of variables we refer to the numbers of tables and accounts as published in source (b), unless another reference is indicated; all assets/liabilities are of course held by, or claimed against the private sector:

CUMP (currency) : XIII.2 & 4b
DDMP (demand deposits with monetary authorities):XIII.2 & 4b
DDBP (demand deposits with deposit banks) : XIII.2 & 4b
DDNP (demand deposits with NMFI) : XII.1
TSBP (time & saving deposits on short term with deposit banks) : XIII.2 & XV.3b
TLBP (time & saving deposits on long term with deposit banks) : XIII.2 & XV.5b
TSNP (time & saving deposits on short term with NMFI) :
   XII.1
TLNP (time & saving deposits on long term with NMFI) : XII.1
CSGP (government debt certificates on short term) : XII.1
CLGP (government debt certificates on long term) : XII.1
NAFP (net foreign assets) : constructed using the accumulation rule $\text{NAFP}_t = \text{NAFP}_{t-1} + \Delta \text{NAFP}_t$ where $\Delta \text{NAFP}_t$ was taken from IX.1 and the benchmark figure for '63 computed from QUINTYN, M. (1986), De Uitvoerbaarheid van een Geldgroeibeleid, Doctoral dissertation, R.U. Gent, page 348
LSPB (loans on short term initially granted by deposit banks) : XIII.2
LLPB (loans on long term granted by deposit banks) : XIII.2
LSPN (loans on short term granted by NMFI) : XII.1
LLPN (loans on long term granted by NMFI) : XII.1
NAPO (net other liabilities) : balance account identity
ITS (interest rate on short term time & saving deposits) : XIX.5, col. 4
ITL (interest rate on long term time & saving deposits) : XIX.8, col. 3
ICL (interest rate on long term government debt certificates) : XIX, col. 3
FP (forward premium on the 3-month forward exchange of the BF vs the US $) : X.3
IFC (foreign interest rate, covered) : = IFU + FP
YU (gross domestic product at market prices in current prices) : Eurostat, National Accounts ESA, Aggregates
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1. Introduction

In his seminal 1935 article, 'A suggestion for Simplifying the Theory of Money', John Hicks called for a Marginal Revolution in monetary theory. Up till then, marginal utility analysis had mainly be the tool to tackle flow problems (which are for instance dealt with in consumer and producer demand theory), but, this analysis being a general theory of choice, Hicks saw no objection as to applying it to the stock or balance-sheet problems which are the main subject of monetary theory.

Under the heading 'Portfolio Analysis', this suggestion started being developed in the 50's and 60's, either along the expected utility approach (a direct application of the rational choice theory), or with the help of the mean-variance model (which can be shown to be consistent with the former method under particular conditions); a host of authors contributed to these developments, the ones the most referred to being Markowitz (1952) and Tobin (1958).

Besides its microeconomic and normative applications, the theory of portfolio behaviour also entered the macroeconomic field where it became more and more customary to conceptualise the monetary part of the economy as a general accounting framework (cf. Tobin, 1969). Such a framework consists of rows relating to assets/liabilities, and columns representing the different sectors or agents of the economy. Thus, each column can be regarded as a balance sheet or as sources and uses of funds account (depending on whether the matrix entries denote stocks or flows of assets), while a row reflects the positions or movements in a particular asset market.

This framework, if appropriately disaggregated in both dimensions, can provide answers to questions that could hardly be raised in the traditional LM models of the 'money-bonds' type. Indeed, different sources of monetary changes can have different implications for wealth and portfolio behaviour of different sectors.
To bring this framework 'to life', as Tobin would call it, one needs to explain the behaviour of the sectors w.r.t. asset holdings; this is where portfolio theory enters the picture.

This paper was written in the context of the construction of a monetary model, which will form part of a larger macro-economic model that aims at describing both real and financial aspects of the Belgian economy. In the tradition of other monetary econometric (sub)models for Belgium and the Netherlands, it is conceived as a general accounting framework in which six sectors are represented: the central bank, deposit or commercial banks, non-monetary financial institutions, the private sector, central & local government, and the rest of the world (cf. Schroyen, 1988).

The search for an adequate modelling methodology to describe (some of) these sectors' portfolio management, constitutes the purpose of the present paper. Section 2 deals with the derivation of the asset demand model that is associated with a negative exponential utility function and normal distributed returns. The model is called the pure, or basic, portfolio model because of the stringent market assumptions that surround it. Already in his 'Simplifying' paper, but even much more in later writings, Hicks draw attention to the serious implications of frictions and transaction costs on portfolio behaviour. Section 3 therefore investigates how the introduction of market imperfections affects the nature of the pure portfolio model. One of the basic findings is that portfolio behaviour really becomes a dynamic phenomenon once transaction costs are allowed for. The dynamic modelling of asset demand systems is the subject of section 4. Though most empirical studies of portfolio behaviour have recourse to a generalised partial adjustment mechanism, use will be made of a more general dynamic specification which was adapted to singular demand systems (like share systems) in the early 80's by Anderson and Blundell, and which has already proven to be successful in
the context of consumer and factor demand systems. In section 5, this methodology is applied for the estimation and testing, of an asset demand model of the Belgian private sector. Finally, section 6 summarises the main conclusions of this paper.
2. The basic portfolio model

Let us consider a risk averse investor who is faced with the problem of allocating his wealth in an optimal way among n assets. The first of these assets is assumed to be a riskless one, yielding a return known with certainty; in contrast, the returns on the n-1 remaining assets, hereafter called risky assets or securities, are supposed to be of an uncertain nature, which is characterised in the investor's mind by a subjective density function.

Before tackling the formal analysis of the problem, it is instructive to sketch the precise context in which this investment decision is made (cf. Rousseas, 1972). In particular, we will regard asset markets to be perfect in the sense of Tobin (1965, p. 3); that is, all assets are 'fully liquid' (i.e. convertible without delay into currency at full market value, by sale, redemption, or pledging as loan collateral), 'perfectly reversible' (i.e. capable of being both purchased and sold at every moment of time, and at the same price for both buyer and seller) and 'completely divisible' (i.e. capable of being purchased, sold, and held in any quantity, no matter how small).

In addition, the investor is assumed to have an immediate accumulation objective, so that there is only one investment period to be considered and that we can abstract from a sequence-of-periods analysis. These two assumptions, the singularity of the decision period on the one hand, and the perfectness of asset markets, often typed as 'absence of transaction costs', on the other, are not independent and we will come back on it extensively in the next section. For the time being, however, the investment decision period is kept self-contained.

Finally, we make the assumption that the investor is operating on perfect competitive markets: he will be unable to affect the prices of the different securities by his own behaviour. Let us now introduce the following symbols in the analysis:
\( W_0 \) : initial wealth to be allocated;
\( z \) : \((n-1)\) vector of risky asset levels;
\( \zeta \) : amount of wealth invested in the riskless asset;
\( a \) : \((n-1)\) vector of risky asset shares in the total portfolio, i.e. \( a_i = z_i/W_0 \);
\( \alpha \) : share of riskless asset in the total portfolio, i.e. \( \alpha = \zeta/W_0 \);
\( r \) : \((n-1)\) vector of random returns on the risky assets, behaving according to the (subjective) multivariate distribution function \( F(r) \);
\( \rho \) : \((n-1)\) vector of expected returns on the risky assets, i.e. \( \rho = E(r) \);
\( \Psi \) : \((n-1)\times(n-1)\) (subjective) covariance matrix of returns;
\( r^* \) : the (fixed) return on the riskless asset;
\( W \) : end-of-period wealth, calculated as \( z'(1+r) + z(1+r^*) \), where \( l \) denotes the vector of units;
\( \pi \) : the overall portfolio rate of return, defined as \( (W-W_0)/W_0 \).

With the wealth constraint \((l'z+\zeta=W_0)\) in mind, the overall portfolio rate of return \( \pi \) can be computed as a weighted average of the individual asset returns, viz.

\[
(2.1) \quad \pi = a'r + \alpha r^*.
\]

Next, we will assume that the investor's utility function, the expected value of which he is going to maximise, is defined in terms of this overall portfolio rate of return\(^2\), and that the subjective distribution function \( F(r) \) can be approximated by the normal distribution \( F_N(r; \rho, \Psi) \). The former assumption implies that the system of asset demands will be linear homogeneous in initial wealth which is often considered as appropriate for equations representing the behaviour of aggregate categories of investors in a time series context (cf. De Leeuw, 1965, and Brainard & Tobin, 1968). The normality hypothesis, on the other hand, ensures that the utility maximisation procedure can take place along the lines of the mean-variance approach\(^3\).
Since \( \pi \) is defined as in (2.1), it will be normally distributed with mean \( \mu_\pi = \alpha' \rho + \alpha x^* \) and variance \( \sigma_\pi^2 = \alpha' \Psi a \), i.e. \( \pi \sim f_N(\pi; \mu_\pi, \sigma_\pi^2) \), where \( f_N(.) \) denotes the normal density function.

Expected utility is therefore given by

\[
E[U(\pi)] = \int_{-\infty}^{+\infty} U(\pi) f_N(\pi; \mu_\pi, \sigma_\pi^2) \, d\pi.
\]

Assuming next that the investor's behaviour is characterised by constant absolute risk aversion, the implied utility function \( U(\pi) \) will be of the negative exponential type, i.e.

\[
U(\pi) = \delta_1 - \delta_2 e^{-\eta \pi}, \quad \delta_2, \eta > 0,
\]

where \( \eta \) denotes the degree of absolute risk aversion according to the Pratt-Arrow definition. It can be shown that the expression for expected utility, (2.2), becomes

\[
E[U(\pi)] = \delta_1 - \delta_2 \exp \left[ -\eta (\mu_\pi - \frac{\eta}{2} \sigma_\pi^2) \right],
\]

which is solely in terms of the mean and variance of the overall portfolio rate of return. Since \( \eta \) is by assumption constant and positive, maximisation of (2.4) is equivalent to maximising the expression

\[
\bar{U} = \mu_\pi - \frac{\eta}{2} \sigma_\pi^2.
\]

The Lagrangian of the investor's constrained optimisation problem thus looks like

\[
\mathcal{L} = (\alpha' \rho + \alpha x^*) - \frac{\eta}{2} a' \Psi a + \theta (1 - \theta' a - \alpha),
\]

yielding the following set of first-order conditions:
\begin{align}
(2.7a) \quad & \frac{\partial \lambda}{\partial \theta} = \rho - \eta \Psi a - \theta = 0, \\
(2.7b) \quad & \frac{\partial \lambda}{\partial \alpha} = \alpha^* - \theta = 0, \\
(2.7c) \quad & \frac{\partial \lambda}{\partial \theta} = 1 - \alpha^* - \alpha = 0.
\end{align}
These conditions will be necessary as well as sufficient for a maximum, in view of our assumptions on $\eta$. Eq. (2.7b) directly provides us with the optimal value of $\theta$, the Lagrangian multiplier, which may be inserted in (2.7a) to obtain optimal risky asset demands, $a^o$. The optimal share of the riskless asset in the portfolio, $\alpha^o$, then follows from the wealth constraint (2.7c). So, the optimal asset demand system can be written in matrix notation as

\begin{align}
(2.8a) \quad & \begin{bmatrix}
\alpha^o \\
\alpha^o
\end{bmatrix} = \\
& \begin{bmatrix}
1 & \frac{1}{\eta} \psi^{-1} \psi & -\frac{1}{\eta} \psi^{-1} \\
0 & -\frac{1}{\eta} \psi^{-1} \psi & \frac{1}{\eta} \psi^{-1}
\end{bmatrix} \\
& \begin{bmatrix}
1 \\
r^*
\end{bmatrix}
\end{align}

or,

\begin{align}
(2.8b) \quad & \begin{bmatrix}
\alpha^o \\
\alpha^o
\end{bmatrix} = \\
& \begin{bmatrix}
1 & \kappa & k' \\
0 & k & K
\end{bmatrix} \\
& \begin{bmatrix}
1 \\
r^*
\end{bmatrix}
\end{align}

with obvious definitions for $\kappa$, $k$ and $K$.

Since the model has been derived from an explicit optimisation problem, it is possible to infer several properties which make it bear close resemblance to the standard consumer demand system under certainty. In particular, the adding-up requirement of the portfolio shares to unity imply that
(2.9) \quad \bar{\kappa} = \bar{\kappa}'

where \bar{\kappa} is a shorthand for the matrix of interest effects

\[
\begin{bmatrix}
\kappa & \kappa' \\
\kappa & \kappa
\end{bmatrix}
\]

The homogeneity property results from the fact that

(2.10) \quad \bar{\kappa} \mathbf{1} = 0

i.e. an increase of all (expected) interest rates with the same percentage point, will leave optimal asset shares unaffected. It is easy to check that the cross effects of interest rates are characterised by symmetry; therefore

(2.11) \quad \bar{\kappa} = \bar{\kappa}'.

Moreover, the strict concavity of the utility function amounts to the positivity condition that

(2.12a) \quad \kappa > 0, \text{ and}

(2.12b) \quad x'\kappa x > 0, \quad \forall x \in \mathbb{R}^{n-1}, \quad x \neq 0.

Notice that this last condition does not apply to the entire matrix of interest effects, but only to the block diagonals \kappa and \kappa.

These properties seem to suggest that there exists an analogy between the matrix of interest effects, \kappa, and the substitution matrix in standard demand theory. However, as shown by Roley (1983), necessary and sufficient conditions for a symmetric Jacobian matrix \partial (\alpha, a)/\partial (r^*, \rho)' are

\[
\partial^2 \bar{u}/\partial (\mu^2) = \partial^2 \bar{u}/\partial (\sigma^2) = 0,
\]
because only then the wealth effect in each asset Slutsky equation vanishes so that one is left with the substitution term; it can be easily checked that these conditions apply in our case. Finally, it is important to mention that on the asset demand system (2.8), the separation theorem is effective, because the structure of the risky part of the portfolio is independent of initial wealth, $W_0$, and the preference parameter $\eta$. Indeed, the mix of securities within the amount decided to invest in risky opportunities, $a_0 W_0 / (1' a_0) W_0$, is given by the expression

$$\frac{\Psi^{-1}(\rho-\lambda^*)}{1' \Psi^{-1}(\rho-\lambda^*)}$$

in which the 'investor typical' parameters to the allocation problem, $W_0$ and $\eta$, do not appear$^5$.

To summarise, working with a negative exponential utility function and normal returns, we derived the optimal portfolio strategy for an investor with a one period horizon who is operating on perfect markets. This strategy is described by a system of asset equations, homogeneous of degree one in wealth and degree zero in (expected) rates of interest, with symmetric cross interest effects and positive influence being exerted by the own rates of return. Moreover the system appeared to possess the separation property. 'But', says Hicks (1982, essay 19, p. 248), 'the self containedness of the decision period is a great simplification and it is important to notice that we are able to make it because we are neglecting the costs of transactions'. The purpose of the next section will therefore be to give a sketch of the portfolio behaviour in a world where transaction costs are no longer negligible.
3. Portfolio theory in a Hicksian perspective

With the results of the preceding section in mind, at least three negative conclusions can be drawn w.r.t. the demand for money. We should first notice that, as long as there exists the possibility of investing in a riskless asset with a positive rate of return, there is no place for a demand for money in the framework of the pure portfolio model. Money is then said to be dominated by that particular asset. Secondly, the separation theorem implies that one can treat all risky assets as a bundle. The effect of a change in risk aversion simply results in a substitution between the riskless asset on the one hand, and the bundle of risky assets on the other, leaving the proportions within the bundle intact. Within this perspective, the demand for money (if not dominated by another riskless asset) is determined in the same manner as the liquidity preference choice in Keynes (1936, ch. 13) and much of Keynesian theory where all non-money assets are treated as 'bonds'.

Our second negative result is therefore that the pure portfolio model offers no scope, at least when it comes to analysing the effects of changes in risk attitudes, to consider on entire spectrum of assets, which was the subject of Keynes (1930, vol. II, ch. 25) and Hicks (1935).

A third negative conclusion may be drawn from the fact that in the perfect market embedding of the basic model, a precautionary demand for money makes no sense at all (cf. Tsiang, 1969). In that context it is unnecessary to look forward for more than one period; whether the target date, i.e. the date when the investor intends to need his invested wealth, is near or distant, or uncertain at all, does not matter, for it is only the (expected) returns immediately ahead that are relevant for the portfolio decision. So the question arises:
Why then should anyone hold money, except for the very instant he receives a payment or is about to make one? The necessary and sufficient conditions, as Hicks rightly pointed out, is that out-of-pocket costs and the effort required in moving from cash to bonds and back to cash exceed the yield.

(Modigliani, 1968, p. 398)

Indeed, as Hicks pointed out in his second Two Triads Lecture (1967, p. 31), 'it is fatal to leave out the cost of making transactions when one's subject is money'. If we introduce transaction costs in the very broad sense, by allowing not only for direct pecuniary charges but also non-pecuniary costs associated with bothering and costs experienced due to the occurrence of indivisibilities, almost the entire perfect market setting (in Tobin's sense) is undermined with serious consequences for the results derived within it.

First, let us see what happens when we keep the decision period self contained. The allocations possible at the beginning of the single decision period are pictured in Figure 3.1. Because we restrict ourselves to one decision period, total wealth, held in no matter what form, is to be transformed into money (which is supposed not to be dominated by another riskless asset, i.e. $\zeta=m$), the only means of exchange.

Suppose that the investor starts off with his entire wealth held in money form and that the optimal investment strategy requires the operations (I) and (II). The part that is kept in money form (m) is not subjected to any transaction costs while the investment in risky assets ($z_i$) is not possible without paying the amounts (or charges in percent of face value) $t_{mi}$ and $t_{im}$ corresponding to the twofold conversion. The expected net returns will thus be situated below the original ones. Unfortunately, we can make no general statements how the optimal investment strategy is affected, unless we restrict ourselves to the case where all rates of
Figure 3.1. Reallocations and transaction costs

\[\begin{array}{c}
\text{Beginning of period} \\
\text{End of period}
\end{array}\]

\[\begin{array}{c}
\text{I} \\
\text{II} \\
\text{III} \\
\text{IV} \\
\text{V}^a
\end{array}\]

\[\begin{array}{c}
m \rightarrow z_i \tau_{mi} \\
m \rightarrow m \\
z_i \rightarrow m \tau_{im} \\
z_i \rightarrow m \\
z_i \rightarrow z_j \tau_{im} \rightarrow m \tau_{m,j}
\end{array}\]

\[\begin{array}{c}
z_i \rightarrow m \\
\rightarrow m \\
z_i \rightarrow m \\
z_i \rightarrow m \\
z_j \rightarrow m \tau_{jm}
\end{array}\]

\[\text{a it is assumed that direct exchange of asset } i \text{ for asset } j \text{ is impossible.}\]
return are uncorrelated; for the optimal position in a risky asset is then given by

$$a_i^o = \frac{1}{\eta} \frac{1}{\Psi_{1i}} (\rho_i - r^*)$$

and will be smaller when the $\rho_i$ is corrected downwards.

Eventually, the net return on the risky asset may even fall below the safe yield, $r^*$; if this occurs, and short sales, i.e. negative asset holdings are not allowed, the risky asset drops out of the investor's portfolio. Spread among risky assets is then limited. This will surely be the case when investment costs are of a large fixed nature (i.e. invariable with the amount invested) or when indivisibilities are present; the possibility to divide his wealth in small portions, thus spreading his risks, is then closed to the investor who cannot raise sufficient amounts of money to overcome these frictions in a profitable way (cf. Hicks, 1935).

The picture becomes, however, even more blurred when we consider situations where the investor holds his initial capital in a diversified shape. In that case, the reallocation associated with the optimal investment of the new period, takes place by several or perhaps all the transactions pictured in Fig. 3.1. Even when we take the simplified model with only one risky asset besides the riskless one, it is hard to make some general statements as to the effect of transaction costs; too much depends on the structure of them. What we can say is that, given a diversified initial position, the introduction of costs of transaction will have asymmetrical effects, depending on the position of the initial portfolio relative to the position which would be optimal in the absence of these costs. This asymmetry comes from the fact that going out of the risky asset, whether at the beginning of the period to invest in money (III), or at the end of it when wealth is realised (IV) involves the same kind of transaction cost ($t_{im}$). On the other hand, taking up a riskier position than the
initial one requires two transactions on the part of the risky asset (I), with associated costs $\tau_{mi}$ and $\tau_{im}$, whereas the marginal return on money is not altered. This would mean that transaction costs hamper the investor more when he is trying to expand his portfolio towards the risky asset, than when securing his position by investing more in the riskless asset, money.

So far, the analysis remained in the one-period framework and is therefore only partial. The reason why we were allowed to investigate the wealth allocation problem for just one single period was precisely the absence of transaction costs; then the target date of the investor does not matter at all. But, once these costs are introduced, the timing of the investor's accumulation objectives becomes relevant and an extra dimension is added to the analysis. Tobin (1965) outlines this problem with the help of a two-period two-asset model in a world of complete certainty on the future of both assets. Without costs of transactions, the optimal policy to follow for the investor is to maximise the return over the period immediate ahead, no matter whether the target date lies at the end of the first or second period. However, once (dis)investment costs enter the picture, his investment strategy, i.e. his portfolio sequence and therefore the immediate portfolio choice (which is the first step in the sequence), becomes dependent on the timing of the investor's accumulation goal\textsuperscript{7}. And even though asset returns are certain in Tobin's model, portfolio diversification takes place because of diversity in the investor's timing of his accumulation goals.

One may go one step further and allow for the fact that the target date, i.e. the date of planned realisation, becomes uncertain. In this case, the investor has to make sure that he does not lock himself in, i.e. that his portfolio consists of sufficient funds which are easy realisable at the (uncertain) moment when realisation becomes necessary. It is at this point in the analysis that liquidity comes into play, for
it (...) not only mean(s) that (the funds) must be held in securities that are readily marketable; it also means that they must be held in such a form that the value of the portfolio is not likely to vary too much over time. The value, at whatever date there is to be realisation, must be much the same. A particular asset which has this property, is surely what is meant by a liquid asset.

(Hicks, 1982, p. 260, original italics)

But, unlike the pure portfolio dichotomy of a riskless asset on the one side, and a bundle of securities on the other, the distinction between liquid and illiquid assets is not a clearcut one. In fact, there are several degrees of liquidity, one asset being more liquid than another when it is 'more realisable at short notice without loss' (Keynes, 1930, vol. II, p. 67). This means that a spectrum of assets (a liquidity spectrum) should be considered. When an investor feels confident about the future, in that he estimates the likelihood of being obliged to realise part of his portfolio on short term small, he will move to the right along the spectrum. On the other hand, if he thinks requirements will be very probable in the near future, the same investor will allocate his wealth more to the left side of the 'liquidity axis'. The demand for money that arises from such a leftward shift, one should see as a precautionary demand for money.

So, where the role of the speculative motive is stressed in the pure portfolio model, the admission of transaction costs introduces, almost automatically, the precautionary motive as an equivalent, if not superior modus operandi in the individual's portfolio behaviour. The third element in the familiar triad, the transaction motive, is nicely fit into the model by recognising the need for money as a means of payment.
To this motive-triad, Hicks (1967, Lecture III, and 1975, ch. 2) proposed a corresponding classification of the assets on the individual's balance sheet into three categories: running, reserve and investment assets.

To the first category, Hicks reckons assets that are required for the current running of a business or a household. Real assets of this kind can be easily thought of: goods in production, fixed equipment in so far that it is used in the production process, etc. Financial running assets can only be money or near money, in so far that these are required for current business, that is, serving as a means of payments. Other financial assets are excluded from this class since they do not share the medium of exchange property. The requirement for money as a running asset is thus a transaction requirement with a rather complementary character to the business process.

Money will, however, also appear among financial reserve assets. These assets are held for emergencies that may arise in the near future. The demand for reserve assets is therefore governed by the precautionary motive and it can be satisfied by any financial asset that possesses some degree of liquidity, that is, realisable at short notice without loss. Financial assets held for this purpose will usually be close substitutes and are subjected to a liquidity preference substitution for one another when their rate of return structure changes, given the investor's state of confidence in the future. Finally, there are investment assets. Financial assets that are classified in this category, are held for their yield and, unlike reserve assets, there is no liquidity requirement. If the investor is one who faces huge transaction costs, his portfolio will only be managed at the margin, i.e. new savings will be allocated into investment assets and the rate of diversification among them will be governed by the degree of certainty the investor attaches to the different returns these assets are expected to yield in the future. On the other hand, when the costs of reallocating the existing portfolio do not render speculative behaviour unprofitable,
a speculative demand for a non-interest bearing money will emerge. But then we should agree with Modigliani that this speculative money demand will be replaced by a demand for a liquid interest-bearing asset if such exist:

(...) Keynes' theory of the speculative demand suffers from his excessive concentration on long term bonds as the alternative to cash, to the neglect of short term instruments. The proposition that people will flee from bonds when the price of bonds is deemed to be intenably high seems valid enough, but the obvious abode for funds accruing from moving out of long term bonds should be short term ones, not cash.

(Modigliani, 1968, p. 399)

With these considerations in mind, it looks as if a large part of the investor's liquidity preference behaviour can be explained in terms of the precautionary motive, with the speculative behaviour pertaining only the demand for assets in so far these assets can be ranked to the investment category. Therefore, it seems we have drifted a long way from the pure portfolio model of the previous section and it is not clear how the standard properties pertaining to demand systems are affected when introducing transaction costs.

What we can say about the modified model is that it has a dynamic flavour and that it will at least consist of the same explanatory variables as the pure model does (since the latter can be seen as nested within the former); also, a general substitution in favor of an asset whose rate of return has increased, is to be expected, just as in the pure model, although this substitution will now be governed by speculative as well as precautionary motives.

Instead of setting off towards a stochastic multiperiod model, which seems the natural way of formalising Hicks' modifications, but with few chance of arriving at any tractable equations that can easily be verified empirically, we choose to take a different route in the next section. This route will be inspired by the basic insights of the
previous and present sections, combined with a sense of pragmatism or ad hoc modelling which will lead us to some flexible model that is tractable at the same time.
4. Dynamic portfolio models

4.1. Introducing dynamics

In this section, we will consider the asset demand system (2.8) arrived at in section 2, as the representation of the investor's portfolio behaviour in the long run, i.e. when the investor is given plenty of time to overcome all kinds of frictions and externalities that are likely to influence his short run financial allocations. Therefore, system (2.8) is regarded as the equilibrium response model. How fast the equilibrium response will come about, then depends on the costs of adjustment which have to be incurred. These costs are closely related to the costs of transaction that were the subject of the previous section, and, as already noted there, they have to be interpreted in a very broad way, ranging from direct pecuniary charges (like bid/ask spreads, brokerage fees) towards more indirect costs due to bothering, market inadequacies, and indivisibilities; also the habit to invest wealth in typical familiar assets and imperfect information on the market characteristics of certain assets may not provide the individual investor with sufficient stimuli to exploit all market opportunities to the full.

Besides their direct effect on the profitability of particular investment, these costs will bring liquidity considerations into play which will in turn have their impact on the wealth allocation problem. Hence, a dynamic treatment of portfolio behaviour urges itself and this is also the lesson to be drawn from empirical studies. For instance, Parkin, Gray and Barret (1970) estimated a static allocation model for commercial banks of the U.K. in which the demand for loans, treasury bills, commercial bills and government bonds were linearly related to the respective interest rates and net wealth; they imposed homogeneity and symmetry restrictions. Ten years later, Berndt, McCurdy and Rose (1980) reestimated the unconstrained as well as constrained model using FIML-tech-
niques. The likelihood ratio tests they carried out rejected both homogeneity and symmetry restrictions. However, when appending a first order autoregressive error process to the model, the rejection of both null hypotheses was significantly weakened. From this evidence, we may conclude that a dynamic specification of a portfolio model is crucial if one wants to use the equilibrium response model as a maintained structure of time series data.

Once one has recognised the need for a proper dynamic specification, two courses can be taken. Either one proceeds with a formal modelling of the adjustment behaviour by deriving optimal rules of adjustment; this direction was taken by Sharpe (1974), Christofides (1976) and Hunt and Upsher (1979) who formulate a quadratic cost minimisation problem which outweighs the costs of divergence from the optimal portfolio with the costs of transition towards it. The solution to this problem is then a generalised partial adjustment scheme which has been very frequently applied in empirical portfolio studies\textsuperscript{11}.

The other course is a more pragmatic one in that it does not undertake any formal adjustment modelling, but rather formulates a dynamic framework which exhibits enough flexibility to catch most of the adjustment imperfections. The partial adjustment scheme then appears as a special case which will have to be tested w.r.t. the maintained model. It is along these lines that we will proceed in this and next section.

4.2. A general dynamic system

In a series of articles, Anderson and Blundell (1982, 1983 and 1984), have developed and tested a general dynamic system of demand equations. The starting considerations of these authors where that (i) empirical testing of demand systems often leads to the rejection of restrictions, imposed on by economic theory; (ii) these empirical studies often show serious serially correlated residuals, reflecting
an inadequate specification of the dynamic structure; and (iii) singular demand systems where a predetermined aggregate is allocated among an exhaustive list of items, need a 'system approach' when it comes to specifying the dynamics. It is precisely this last point that was strongly emphasised by Brainard and Tobin (1968) in the context of asset demand equations.

The methodology Anderson and Blundell (hereafter A & B) suggest is a vector time series model that can be reparameterised in such a way so as to reveal the long run or equilibrium response structure, without straining the general dynamic nature of the system. This approach seems apt to follow in the context of this paper. On the one side, the dynamics of portfolio behaviour appears to be determined by a vague and complex interaction of pecuniary, non-pecuniary and liquidity constraints, and therefore requires a modelling which leaves enough degrees of freedom, while on the other hand, we have a fairly clear idea how the wealth allocation process happens in equilibrium; to use this a priori information seems appropriate for two reasons. Firstly, it will lead to an increased efficiency of the parameter estimates (this is no superfluous luxury when most of the exogenous variables, in casu interest rates, are severely correlated). Secondly, the incorporation of a priori restrictions in the estimation will guarantee a kind of 'well behaviour' of the model outside the sample period, i.e. perverse outcomes are ruled out; this is desirable in view of simulation exercises.

Formally, let us rewrite the static asset demand system (2.8) in a more compact way as

\[(4.1) \quad w^o(t) = \Pi x(t),\]

where \(w^o(t)\) denotes an \(n\)-vector of optimal asset shares in the total portfolio and \(x(t)\) represents an \(h\)-vector composed from an intercept term as first element, \(n\) interest rates,
and \((h-n-1)\) other explanatory variables. The similarity between the first \(n+1\) columns of \(\Pi\) and the coefficient matrix in (2.8) is then obvious. Relation (4.1) pictures the long run relationship according to which the portfolio allocation operates. The equilibrium response to the exogenous variables is given by the \((nxh)\)-matrix \(\Pi\) and we assume that the properties of the interest effects, like homogeneity and positivity are operative on the relevant part of \(\Pi\). A & B (1982) postulate that short run changes in \(w(t)\) are reactions to anticipated and unanticipated changes in the \(x\) vector in an attempt to maintain a long run relation_ship represented by (4.1). Using the lag operator \(L\), such a pattern may be written as

\[
(4.2) \quad B^*(L) w(t) = \Gamma^*(L) x(t) + u(t),
\]

where \(B^*(L)\) and \(\Gamma^*(L)\) denote matrix polynomials of the \(p\)th and \(q\)th order resp., i.e.

\[
(4.3a) \quad B^*(L) = I + B^*_1 L + B^*_2 L^2 + \ldots + B^*_p L^p,
\]

\[
(4.3b) \quad \Gamma^*(L) = \Gamma^*_0 + \Gamma^*_1 L + \Gamma^*_2 L^2 + \ldots + \Gamma^*_q L^q,
\]

and \(u(t)\) is a \(n\)-vector of random errors, assumed to be independently and identically distributed over time with covariance matrix \(\Omega\).

The singularity of the demand system, due to the adding-up condition \(t'w(t) = 1\), implies certain adding-up requirements on the matrix polynomials and disturbance vector (cf. A & B, 1982), i.e.

\[
(4.4a) \quad t'B^*_j = g^*_j t', \quad j = 1, \ldots, p,
\]

\[
(4.4b) \quad \sum_{j=1}^{p} g^*_j = g,
\]

\[
(4.4c) \quad t'\Gamma^*_0 = [1+g, 0, \ldots, 0]
\]
(4.4d) \[ 1' \Gamma_j^* = 0' \]
, \( j=1, \ldots, q \),

(4.4e) \[ 1'u(t) = 0. \]

It appears from these conditions, as well as later in the stability analysis, that there is a degree of arbitrariness in system (4.2). But, as Bewley (1986, ch. 2) argues, it is of a spurious nature and presents no inconvenience to further analysis.

When the model is stable, the long run structure can be retrieved as

(4.5) \[ \Pi = B^* (1)^{-1} \Gamma^*(1) = \left[ \Sigma_{j=0}^{q} \Gamma_j^* \right]^{-1} \left[ \Sigma_{j=1}^{p} B_j^* \right]. \]

And since also in the long run, adding-up is in force, we have

(4.6) \[ 1' \Pi = [1, 0, \ldots, 0]. \]

Though system (4.2) can be reparameterised in several ways, the spectrum of choice is limited if we want to focus attention on the equilibrium response matrix \( \Pi \). A & B (1982) derive the following observationally equivalent set of equations\(^1\)\(^2\):

(4.7) \[ \Delta w(t) = -B(L) \Delta w(t) + \Gamma(L) \Delta \bar{x}(t) + A[\Pi x(t-q) - w(t-p)] + u(t), \]

where

\[ B(L) = \Sigma_{i=1}^{p} \left( \Sigma_{j=0}^{i} B_j^* \right) L, \quad p > 1 \]
\[ = 0, \quad p \leq 1, \]

\[ \Gamma(L) = \Sigma_{i=0}^{q-1} \left( \Sigma_{j=0}^{i} \Gamma_j^* \right) L, \quad q > 1, \]

\[ A = \Sigma_{j=0}^{p} B_j^* \]
and the tilde above the $\Gamma_j^*$ and $\Delta x(t)$ points to the
delection of the intercept term which clearly becomes
redundant when $x(t)$ is written in first differences. The
corresponding adding-up conditions are:
\[
\begin{align*}
1'B_j &= m_j' \\
1'\Gamma_l &= b_l' \\
1'A &= (1+g)1' \\
1'\Pi &= [1, 0, \ldots, 0]
\end{align*}
\]

and, of course, conditions (4.4e) and (4.6). $B_j$ and $\Gamma_l$ now
denote the $j$th and $l$th coefficient matrix in the matrix
polynomials $B(L)$ and $\Gamma(L)$, resp..

Besides the earlier mentioned redundant character of the
intercept term in $\Delta x(t)$, system (4.7) exhibits another
potential redundant variable problem. Indeed, the appear-
ance of the full vector of portfolio shares as explanatory
variables, either in levels or in first differences, gives
rise to collinearity, because information on the $(n-1)$
elements of $w(t)$ or $\Delta w(t)$, implicitly says everything about
the $n$th share and its change. This problem can be overcome
by deleting one element, say the $n$th, of $w(t)$ and $\Delta w(t)$.

Though the consequence of this operation is that also the
$n$th row of the equilibrium response matrix has to be
dropped, it implies no loss of information on the long run
structure, as this row can always be retrieved by making use
of the adding-up property (4.6). Hence, if we denote the
deletion of the last row of a matrix with the subscript $n$,
and a modified $n \times (n-1)$ matrix with a superscript $n$, model
(4.7) can be restated in the form (cf. Appendix A):
\[
(4.8) \quad \Delta w(t) = -B^n(L)w_n(t) + \Gamma(L)\Delta \bar{x}(t) + A^n [\Pi \bar{x}(t-q) - w_n(t-p)] + u(t),
\]

where the matrix $A^n$ and every matrix of the polynomial
$B^n(L)$, are build up from the first $(n-1)$ columns of the
original matrix, each column, however, being diminished with the nth column; the matrix $A^n$, for instance, takes the form of

$$
A^n = \begin{bmatrix}
  a_{11} - a_{1n} & a_{12} - a_{2n} & \cdots & a_{1,n-1} - a_{1n} \\
  a_{n1} - a_{nn} & a_{n2} - a_{nn} & \cdots & a_{n,n-1} - a_{nn}
\end{bmatrix}
$$

With the previous adding-up conditions in mind, we can say that all rows of $A^n$ and the element matrices of $B^n(L)$ and $\Gamma(L)$ will sum to zero.

Finally, it should be mentioned that without a priori information on the dynamic structure, there is a loss of identification since the elements of the original $B_j$ and $A$ matrices cannot be retrieved from the associated truncated matrices $B_j^n$ and $A^n$; but, as proven in A & B (1982), no identification problem pertains to $\Pi$, the matrix of long run multipliers.

In what follows, we will concentrate ourselves on the first order general dynamic model ($p=1, q=1$), i.e.

$$
(4.9) \quad \Delta w(t) = \Gamma \Delta x(t) + A^n [\Pi x(t-1) - w_n (t-1)],
$$

which we will consider as the maintained hypothesis and which encompasses two familiar cases, the popular generalised partial adjustment model ($p=1, q=0$) :

$$
(4.10) \quad \Delta w(t) = A^n [\Pi x(t) - w_n (t-1)] + u(t),
$$

and the static formulation ($p=0, q=0$) :

$$
(4.11) \quad w(t) = \Pi x(t) + u(t).
$$
According to the assumptions made earlier in this section, the covariance structure of the disturbance vector \( u(t) \) is given by

\[
E[u(t)\ u(s)'] = \Omega, \ t=s \\
= 0, \ otherwise
\]

In view of the deterministic nature of the sum of disturbances (cf. eq. (4.4e)), the matrix with contemporary covariances, \( \Omega \), is singular, since \( t'\Omega = E[t'u(t)u(t)'] = 0 \), and one has to delete one equation of the model to make system estimation possible. As shown by Barten (1969), it is immaterial which equation is dropped. If we therefore choose to eliminate the \( n \)th, we arrive at an estimable form of the first order general dynamic model, i.e.

\[
(4.12) \; \Delta w_n(t) = \Gamma_n \Delta \tilde{x}(t) + A_n^\pi x(t-1) - w_n(t-1) + u_n(t),
\]

in which the subscript \( n \), as before, points at the deletion of the final row (element) of a matrix (vector); this row of the coefficient matrices \( \Gamma \) and \( A^\pi \) can always be retrieved by means of the adding-up properties.

4.3. Stability analysis

The share system (4.7) thus reduces to

\[
(4.13) \; \Delta w(t) = \Gamma \Delta \tilde{x}(t) + A[\Pi x(t-1) - w(t-1)] + u(t)
\]

in the case of a first order general dynamic model \((p=q=1)\), and becomes the equations set (4.9) when one eliminates the earlier mentioned collinearity, inherent in it.

Assuming away the disturbance vector and defining \( \tilde{\Gamma} \) as the extended coefficient matrix \([\Omega, \Gamma]\), this model can be written as
\[(4.14) \ [I-(I-A)L] w(t) = [\bar{\Gamma}-(\bar{\Gamma}-A\Pi)L] x(t).\]

Hence, stability requires all eigenvalues of the matrix \((I-A)\) to lie within the unit circle. These eigenvalues are found by solving for all \(\lambda\) in the characteristic equation

\[(4.15) \ |(I-A)-\lambda I| = 0.\]

But, as shown in appendix B, this is equivalent to solving the equation

\[(4.16) \ (\lambda + g) \ |(I_n^n - A_n^n) - \lambda I_n^n| = 0,\]

where \(I_n^n\) denotes the unit matrix with the \(n\)th row and column dropped and \(A_n^n\) is built up from the first \((n-1)\) rows of the previously defined matrix \(A^n\).

Two assessments w.r.t. the stability of the first order general dynamic model are now in order.

First, it would seem that this stability depends on the parameter \(g\), the arbitrary constant in the adding-up conditions (4.3). At the bottom, however, this is not the case because the appearance of \(g\) in the original system can be shown to be of a spurious nature.

Secondly, it becomes clear that, apart from the eigenvalue \(\lambda = -g\), the roots of \((I-A)\) are identical to those of \((I_n^n-A_n^n)\), which is the adjustment matrix of the estimable version (4.12), the system after solving the collinearity problem and eliminating the last equation to overcome singularity of the disturbance covariance matrix.

From these considerations, it follows that the stability of the first order general dynamic model is guaranteed when all eigenvalues of \((I_n^n-A_n^n)\) lie within the unit circle. If this is the case, the final form of (4.13) exists, i.e. the equilibrium response values of \(w(t)\) will be achieved when the vector \(x\) stabilises to some constant value over time; for the first \((n-1)\) shares (and thus implicitly for the \(n\)th), it is given by
\[(4.17) \quad w_n(t) = [\bar{\Gamma}_n + \sum_{j=1}^{m} (I_n - A_n) \lambda_n^{-1} A_n^n (\Pi_n - \bar{\Gamma}_n) L_j^j] x(t),\]

where \(\bar{\Gamma}_n\) corresponds to \(\bar{\Gamma}\) with the nth row deleted. The impact and interim multiplier matrices are resp. given by

\[(4.18) \quad \frac{\partial w_n(t)}{\partial x(t-j)} = \bar{\Gamma}_n, \quad j = 0,
= (I_n - A_n)^{-1} A_n^n (\Pi_n - \bar{\Gamma}_n), \quad j \geq 1.

We may obtain an idea about the speed at which the equilibrium values are reached by studying the different cumulated interim multipliers:

\[(4.19) \quad \sum_{j=0}^{\infty} \frac{\partial w_n(t)}{\partial x(t-j)} = \bar{\Gamma}_n + \sum_{j=1}^{J} (I_n - A_n)^{-1} A_n^n (\Pi_n - \bar{\Gamma}_n), \quad J \geq 1.

If the stability condition is satisfied, the RHS expression can be rewritten as

\[
\begin{align*}
(I_n - A_n)^{-1} (I_n - A_n)^{-1} A_n^n & (I_n - A_n)^{-1} \bar{\Gamma}_n \\
+ & (A_n^n)^{-1} (I_n - A_n)^{-1} A_n^n \Pi_n 
\end{align*}
\]

so as to reveal more clearly how the cumulated multipliers are constructed as a weighted combination of the impact multipliers (\(\bar{\Gamma}_n\)) and the final form ones (\(\Pi_n\)); as \(J\) goes to infinity, the expression indeed converges to the latter. When we replace \((I_n - A_n^n)\) by its spectral decomposition

\[v \Lambda v^{-1},\]

where \(\Lambda\) is the diagonal matrix of eigenvalues and \(V\) the matrix of corresponding eigenvectors, we observe how the
'weights' will depend on the number of considered lags as well as on the dominant eigenvalue of the adjustment matrix.
5. Application on the Belgian Private Sector

5.1. Parameterisation: a monetary framework of the Belgian Private Sector

In Table 5.1, a version of the Belgian private sector's balance sheet is given for the end of 1984\(^\text{14}\). This sector is defined as the aggregate of households, business firms which, for statistical reasons, also comprise nationalised enterprises, and the 'institutions of life and labour accident insurance and pension funds'.

Depending on which assets/liabilities are grouped together, an allocation model can be defined on the remaining assets and liabilities, the latter being treated as negative assets.

To keep the model manageable, we choose to consider the private sector's allocation of total wealth (WP) over four broad classes of assets, i.e. liquidities\(^\text{15}\) (LIQP), total short term time deposits (TSTP), assets on long term (ASLP) and net foreign assets (NAFP). This amounts to reshuffling the original balance sheet to the one pictured in Table 5.2.\(^\text{16}\)

The vector of asset shares, \(w\), can thus be constructed as

\[
  w = \begin{bmatrix} w_{\text{LI}} & w_{\text{TS}} & w_{\text{AS}} & w_{\text{NA}} \end{bmatrix}' = \frac{1}{\text{WP}} \begin{bmatrix} \text{LIQP} & \text{TSTP} & \text{ASLP} & \text{NAFP} \end{bmatrix}',
\]

with obvious definitions for \(w_i\).

The evolution of the portfolio structure, as defined by \(w\), is visualised in Figure 5.1. for the sample period '59-'84. Besides showing the weakening interest of the private sector in liquidities (in the strict sense) and foreign assets, the figure reveals the growing importance of liquidities in the large sense (i.e. \(w_{\text{LI}} + w_{\text{TS}}\)) up till '76 (with a period of stabilisation between '66 and '70); afterwards, this
### Table 5.1. Balance sheet Belgian Private Sector (end of 1984)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Symbol</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Currency</td>
<td>CUMP</td>
<td>385.4</td>
</tr>
<tr>
<td>Demand deposits with monetary authorities</td>
<td>DDMP</td>
<td>83.1</td>
</tr>
<tr>
<td>Demand deposits with deposit banks</td>
<td>DDBP</td>
<td>376.2</td>
</tr>
<tr>
<td>Demand deposits with NMFI</td>
<td>DDPN</td>
<td>105.8</td>
</tr>
<tr>
<td>Time &amp; saving deposits on short term(^c) with deposit banks</td>
<td>TSBP</td>
<td>909.6</td>
</tr>
<tr>
<td>Time &amp; saving deposits on short term with NMFI</td>
<td>TSNP</td>
<td>1133.7</td>
</tr>
<tr>
<td>Time &amp; saving deposits on long term with deposit banks</td>
<td>TLBP</td>
<td>421.7</td>
</tr>
<tr>
<td>Time &amp; saving deposits on long term with NMFI</td>
<td>TLNP</td>
<td>1485.4</td>
</tr>
<tr>
<td>Government debt certificates on short term</td>
<td>CSGP</td>
<td>6.7</td>
</tr>
<tr>
<td>Government debt certificates on long term</td>
<td>CLGP</td>
<td>874.8</td>
</tr>
<tr>
<td>Net foreign assets</td>
<td>NAPF</td>
<td>283.3</td>
</tr>
<tr>
<td>Total assets</td>
<td></td>
<td>6065.7</td>
</tr>
</tbody>
</table>

### Liabilities

<table>
<thead>
<tr>
<th>Liabilities</th>
<th>Symbol</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans on short term (initially) granted by deposit banks</td>
<td>LSPB</td>
<td>1168.1</td>
</tr>
<tr>
<td>Loans on short term granted by NMFI</td>
<td>LSNP</td>
<td>103.2</td>
</tr>
<tr>
<td>Loans on long term granted by deposit banks</td>
<td>LLBP</td>
<td>57.2</td>
</tr>
<tr>
<td>Loans on long term granted by NMFI</td>
<td>LLNP</td>
<td>1485.6</td>
</tr>
<tr>
<td>Net other liabilities (net financial wealth)</td>
<td>NAPF</td>
<td>3251.6</td>
</tr>
<tr>
<td>Total liabilities</td>
<td></td>
<td>6065.7</td>
</tr>
</tbody>
</table>

\(^a\) billions of BF  
\(^b\) NMFI = non-monetary financial institutions  
\(^c\) short term ≤ 1 year  
long >

### Table 5.2. Reorganised balance sheet Belgian Private Sector (end of 1984)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Symbol</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquidity</td>
<td>LIQP</td>
<td>950.5</td>
</tr>
<tr>
<td>Liquidities ( (= \text{CUMP + DDMP + DDBP + DDPN}))</td>
<td>TSTP</td>
<td>2043.3</td>
</tr>
<tr>
<td>Total short term deposits ( (= \text{TSP + TSNP}))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assets on long term ( (= \text{TLBP + TLNP + CLGP}))</td>
<td>ASLP</td>
<td>2781.9</td>
</tr>
<tr>
<td>Net foreign assets</td>
<td>NAPF</td>
<td>283.3</td>
</tr>
</tbody>
</table>

### Liabilities

<table>
<thead>
<tr>
<th>Liabilities</th>
<th>Symbol</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total wealth ( (= \text{LSPB + LSNP + LLBP + LLNP + NAPF - CSGP}))</td>
<td>WP</td>
<td>6059.0</td>
</tr>
</tbody>
</table>

\(^a\) in billions of BF
Figure 5.1. Structure financial portfolio of the Belgian Private Sector ('59-'84)

Table 5.3. Average yearly rate of change$^a$ of the different portfolio components for 5 subperiods

<table>
<thead>
<tr>
<th>Subperiod $t_0 - t_1$</th>
<th>Liquidities (in the large sense)</th>
<th>Illiquidities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_{LI}$</td>
<td>$w_{TS}$</td>
</tr>
<tr>
<td>59-66</td>
<td>-.06 %</td>
<td>+ .94 %</td>
</tr>
<tr>
<td>66-70</td>
<td>+ .01 %</td>
<td>+ .81 %</td>
</tr>
<tr>
<td>70-76</td>
<td>- .80 %</td>
<td>+ .95 %</td>
</tr>
<tr>
<td>76-82</td>
<td>- .77 %</td>
<td>- .77 %</td>
</tr>
<tr>
<td>82-84</td>
<td>- .88 %</td>
<td>+ .10 %</td>
</tr>
</tbody>
</table>

$^a$ the rates of change are absolute rates, as a % of the total portfolio, i.e. $\frac{w_{t_1} - w_{t_0}}{t_1 - t_0} \times 100$. 
tendency is strongly reversed and seems to temper around '82 at early sample period values. A more detailed account of this portfolio structure evolution is given in Table 5.3; for the five identified subperiods, the table reports on average yearly changes of the different portfolio components.

During the first subperiod, the growing liquid position of the 'investor', almost exclusively due to the increase of short term time deposits \( w_{TS} \), is more than financed by the release of funds invested in foreign assets \( w_{NA} \), allowing for a strengthening position in domestic long term assets \( w_{AS} \) as well. Since the second half of the 60s, however, the share of liquidities in the strict sense \( w_{LI} \) starts falling at a steady rate, necessitating a flee from long term assets in the early 70s to keep the overall degree of liquidity sufficiently high. Liquidity requirements are severely relaxed during the second half of the 70s and early 80s, and the long term position can again be expanded; since '82 this is also true w.r.t. the foreign position.

The vector of exogenous variables, \( x \), consists of an intercept, the expected rates of return, and additional explanatory variables. As to the modelling of the mechanism which generates the expectations on rates of return, a common practise in the literature on portfolio behaviour will be pursued here by replacing the expected rates by the observed ones\(^{17}\). The eventual existence of expectation formation lags can then be regarded as an additional argument to the introduction of dynamics (cf. Friedman, 1977).

The four rates of return, relevant to the portfolio model defined above, are\(^{18}\):

ILIQ : interest rate on liquidities (strict sense), defined as a weighted average of the interest paid on demand deposits with deposit banks, non-monetary financial institutions and the monetary authorities (mainly postal deposits), being resp. 0.5, 0.5 and zero percent over the entire sample period;
ITS : interest rate on short term deposits, defined as the rate paid on 3-month deposits;
IASL : interest rate on long term assets, defined as a weighted average of the rate paid on long term time deposits with deposit banks and non-monetary financial institutions (yield on 5-year certificates & bonds issued by public financial institutions) and the interest rate on long term government debt (yield of bonds issued by central government, maturing over 5 years or more);
IFC : the foreign interest rate, i.e. the interest rate on 3-month US $ deposits in the London Euro-Currency market, covered by the premium on the 3-month forward exchange rate of BF vs. US $.

Two additional explanatory variables were included: GDP at current market prices (YU) scaled by the private sector’s total wealth (WP) and the growth rate of this wealth ($\Delta WP/\text{WP}_{-1}$). The former variable is a good proxy for the current nominal value of transactions which determines the requirement of an asset, in particular (near) money, as a running asset. As such, this variable is treated in the same way as the rates of return and belongs fully to the vector of long run determinants, $X^{19}$.

The introduction of wealth growth allows for a relaxation of the linear homogeneity in total wealth, which may appear as a rather strong property in the short run. By its nature, this growth rate can never play in the long run and is only allowed to exert its influence in the same ways as the other explanatory variables do in the very short run, i.e. through the matrix $\Gamma$.

Figure 5.2. gives an idea on the evolution of these six exogenous variables over the sample period ’59-’84.
Figure 5.2. Evolution of the exogenous variables (’59–’84)
5.2. Estimation results

The first order general dynamic model (4.12) and two of its specialisations, the partial adjustment and static models, were estimated over the sample period '59-'84 (yearly) with Full Information Maximum Likelihood. The homogeneity restriction on the long run multipliers was imposed throughout rather than tested because this seemed the least one could do to alleviate the multicollinearity problem that arises from the strong interest correlations. On the other hand, the symmetry of long run cross interest effects has tested by reestimating every specification subject to this constraint. Because the ranking of the estimated long run income effects on the holdings of the three domestic assets was contrary to a priory expectations, the entire exercise was repeated without introducing the income variable as a long run determinant of portfolio behaviour. This leads altogether to 12 specifications of which the estimation results are summarised in Table 5.420.

At least three questions are to be answered: which is the most appropriate dynamic formulation? is symmetry rejected by the data? and, do we restrict the influence of the income variable to the short run?

As to the first question, the log likelihood ratio test pointed unanimously towards the general dynamic specification21. In Table 5.5. the results of this test are reported for different assumptions regarding symmetry and long run income effects.

Though the DW-statistic looses its exact meaning because lagged dependent variables occur in the specifications, it is illustrative how the very low DW-values that accompany the static formulation improve considerably when dynamics are introduced via a partial adjustment mechanism. Still, this mechanism is not satisfactory either. First, it leaves us with a long run structure which contradicts economic theory w.r.t. positivity22. Secondly, the adjustment matrix of this mechanism has a dominant eigenvalue very close to
Table 5.4: Summary report FIML estimation portfolio model (59-84): 12 different specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Log Lik</th>
<th>SE</th>
<th>DW</th>
<th>Coef. Signif.</th>
<th>Positivity</th>
<th>LR income</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Δw&lt;sub&gt;L&lt;/sub&gt;</td>
<td>Δw&lt;sub&gt;T&lt;/sub&gt;</td>
<td>Δw&lt;sub&gt;S&lt;/sub&gt;</td>
<td>Δw&lt;sub&gt;AS&lt;/sub&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEN. DYN.</td>
<td>350.3</td>
<td>.0029</td>
<td>.0039</td>
<td>.0041</td>
<td>2.681 2.766 2.394</td>
<td>71% 40%</td>
<td>.1681 (.0946)</td>
</tr>
<tr>
<td>II</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PART. ADJ.</td>
<td>317.6</td>
<td>.0048</td>
<td>.0057</td>
<td>.0048</td>
<td>1.866 2.439 2.204</td>
<td>63% 25%</td>
<td>-.1817 (.3170)</td>
</tr>
<tr>
<td>III</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>STATIC&lt;sup&gt;e&lt;/sup&gt;</td>
<td>241.4</td>
<td>.0133</td>
<td>.0262</td>
<td>.0133</td>
<td>1.041 .991 .484</td>
<td>87% 67%</td>
<td>.0660 (.0776)</td>
</tr>
<tr>
<td>IV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEN. DYN.</td>
<td>346.1</td>
<td>.0032</td>
<td>.0039</td>
<td>.0045</td>
<td>2.486 2.831 2.415</td>
<td>72% 36%</td>
<td>Yes (Yes)</td>
</tr>
<tr>
<td>V</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PART. ADJ.</td>
<td>------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>STATIC</td>
<td>215.9</td>
<td>.0160</td>
<td>.0351</td>
<td>.0191</td>
<td>.429 .429 .330</td>
<td>92% 67%</td>
<td>No (No)</td>
</tr>
<tr>
<td>VII</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEN. DYN.</td>
<td>343.8</td>
<td>.0034</td>
<td>.0039</td>
<td>.0042</td>
<td>2.515 2.794 2.417</td>
<td>64% 38%</td>
<td>.3 own effects pos.</td>
</tr>
<tr>
<td>VIII</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PART. ADJ.</td>
<td>303.4</td>
<td>.0059</td>
<td>.0064</td>
<td>.0073</td>
<td>1.798 1.885 1.280</td>
<td>24% 10%</td>
<td>2 own effects pos.</td>
</tr>
<tr>
<td>IX</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>STATIC</td>
<td>207.5</td>
<td>.0135</td>
<td>.0392</td>
<td>.1017</td>
<td>1.062 .464 .425</td>
<td>92% 67%</td>
<td>all own effects pos.</td>
</tr>
<tr>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEN. DYN.</td>
<td>341.4</td>
<td>.0037</td>
<td>.0040</td>
<td>.0043</td>
<td>2.707 2.820 2.424</td>
<td>81% 50%</td>
<td>Yes (Yes)</td>
</tr>
<tr>
<td>XI</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PART. ADJ.</td>
<td>300.2</td>
<td>.0059</td>
<td>.0071</td>
<td>.0078</td>
<td>1.915 1.828 1.491</td>
<td>56% 22%</td>
<td>Yes (Yes)</td>
</tr>
<tr>
<td>XII</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>STATIC</td>
<td>201.0</td>
<td>.0196</td>
<td>.0465</td>
<td>.0218</td>
<td>1.114 .266 .672</td>
<td>78% 11%</td>
<td>No (No)</td>
</tr>
</tbody>
</table>

---

* = % of all directly estimated coefficients whose point estimate in absolute value is larger than once their corresponding standard error.
** = twice

b when symmetry is not imposed, no. of own interest effects (4 in total) and no. of cross interest effects (12 in total) is given that are resp. positiv and
negatif; when symmetry is imposed, it is investigated whether all Choleski values of the matrix $\hat{R}(k)$ are non negative.

c point estimate (asymptotic standard error) of $\Delta w_t(k)$

d dominant eigenvalue or dominant modulus of conjugate pair of complex eigenvalues of the matrix $(I_n - A_n)$.

in order to make the log Lik, SE and DW statistics comparable with other specifications, the static model was estimated as $\Delta w(t) = I_n x(t) + w(t-1) + u_n(t)$.

f this dominant root is of the real type.
Table 5.5. Log likelihood ratio tests: dynamic structure

<table>
<thead>
<tr>
<th>Specification</th>
<th>Hypotheses</th>
<th>No. of constraints</th>
<th>$\chi^2$</th>
<th>Critical Value (5 %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR Y-EFFECT</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NO SYMMETRY</td>
<td>$H_0$: Part. Adj. vs $H_1$: Gen. Dyn.</td>
<td>18</td>
<td>65.3</td>
<td>28.9</td>
</tr>
<tr>
<td></td>
<td>$H_0$: Static vs $H_1$: Part. Adj.</td>
<td>9</td>
<td>152.4</td>
<td>16.9</td>
</tr>
<tr>
<td>SYMMETRY</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NO SYMMETRY</td>
<td>$H_0$: Part. Adj. vs $H_1$: Gen. Dyn.</td>
<td>18</td>
<td>80.8</td>
<td>28.9</td>
</tr>
<tr>
<td></td>
<td>$H_0$: Static vs $H_1$: Part. Adj.</td>
<td>9</td>
<td>191.8</td>
<td>16.9</td>
</tr>
<tr>
<td>NO SYMMETRY</td>
<td>$H_0$: Part. Adj. vs $H_1$: Gen. Dyn.</td>
<td>18</td>
<td>82.3</td>
<td>28.8</td>
</tr>
<tr>
<td></td>
<td>$H_0$: Static vs $H_1$: Part. Adj.</td>
<td>9</td>
<td>198.3</td>
<td>16.9</td>
</tr>
</tbody>
</table>

Table 5.6. Log likelihood ratio tests: symmetry constraint

<table>
<thead>
<tr>
<th>Specification</th>
<th>Hypotheses</th>
<th>No. of constraints</th>
<th>$\chi^2$</th>
<th>Critical Value (5 %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR Y-EFFECT</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEN. DYN.</td>
<td>$H_0$: symmetry</td>
<td>3</td>
<td>8.3</td>
<td>50.8</td>
</tr>
<tr>
<td>STATIC</td>
<td>vs $H_1$: no symmetry</td>
<td>3</td>
<td>4.9</td>
<td>7.8</td>
</tr>
<tr>
<td>NO LR Y-EFFECT</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GEN. DYN.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PART. ADJ.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>STATIC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

unity, pointing at an extremely slow convergence of the system towards the equilibrium response. Moreover, the numerical estimation problems that were encountered when imposing the symmetry condition (specification V) are a third indication that this adjustment mechanism exhibits insufficient flexibility to capture the dynamics of portfolio behaviour.

Table 5.6 reports on the log likelihood ratio tests of the symmetry hypothesis. The findings of A & B (1982) and (1983, 1984) where symmetry was tested in factor and consumer demand models resp., were such that this hypothesis could not be rejected when the demand system was formulated in a general dynamic way. Our results are in line with these findings up to a certain degree only. First, notice that, if one takes the static model as maintained hypothesis (though this is in conflict with the dynamic structure tests carried out above), the imposition of symmetry on the interest rate coefficients reduces the log likelihood value in a significant way; a similar reduction was found in the earlier mentioned study of Berndt et. al. (1980).

The same exercise, repeated with the partial adjustment model as maintained hypothesis (again, incorrectly) does not lead to a symmetry rejection at 5\% significance level. However, we do not give much credit to this model either, as it is almost characterised by instability (cf. the high dominant eigenvalue); moreover, though the substitution matrices $\bar{K}$ and $K$ both satisfy the positivity condition, their smallest Choleski values (with the exception of the obvious zero for $\bar{K}$) are too close to zero to be really convincing$^2$.

When the symmetry test is carried out within the general dynamic structure, a twofold picture emerges: if the income effect is ruled out in the long run, there is no rejection of the hypothesis at the 5\% significance level, while this is the case when we do allow for such income effects. This absence of unanimity puts us to the choice.
In Section 2, we referred to Roley (1983) where it is shown in a mean-variance context, that the symmetry restriction is not a general property of asset demand functions, but requires zero wealth effects as a necessary and sufficient condition. In other words, this restriction imposes a behavioural assumption on the investor which is complementary to the set of rationality axioms portfolio theory starts off; therefore, there are no reasons to believe that it holds a priori and one should rely on the data for a decisive answer.

On the other hand, imposing the symmetry hypothesis not only sharpens the precision of the point estimates, it also enhances the attractiveness of the model for simulation purposes, i.e. the well-behaviour argument. As we believe these elements to be of great importance w.r.t. the aim of this study, the symmetry property will be accepted in the remainder of the text as a maintained hypothesis.

Finally, we have to make a choice w.r.t. the presence of (scaled) nominal income as a long run determinant of wealth allocation. Economic theory is rather straightforward on this matter: the transaction motive reallocates funds in the direction of the running asset when current business activities swell. So, a priori, one would expect the income variable, YU/WP to appear with a positive coefficient in the liquidity equation (\(w_{LI}^{L}\)) which is then compensated by negative income coefficients in (some of) the remaining demand equations.

The estimation results of the general dynamic specification with symmetry imposed and with a long run income effect allowed for (cf. IV) are presented in Table 5.7. The coefficients related to the dynamics are given in the upper section, whereas the long run coefficient matrix \(\Pi\) is pictured in the second part of the table. The coefficients on the last row of each matrix, and their corresponding standard errors, were retrieved by means of the adding-up conditions. The matrix of impact multipliers is given by \(\Gamma\) and it is recalled that the structure of this short run
Table 5.7. Estimation of general dynamic model, symmetry imposed, with long run income effect\textsuperscript{a}

\[
\Delta w(t) = \Gamma \Delta z(t) + A_n^{h}[w_n^{o}(t-1) - w_n(t-1)] = \Gamma \Delta z(t) + A_n^{h}[\Pi_n x(t-1) - w_n(t-1)]
\]

<table>
<thead>
<tr>
<th>(\Delta w_{Li}(t))</th>
<th>(\Delta w_{TS}(t))</th>
<th>(\Delta w_{AS}(t))</th>
<th>(\Delta w_{NA}(t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.45* (-.8838** .1402 -.0281 .2336** -.1390**</td>
<td>-22.42 (.4515* -.3323 .1127* -.0107 .3968**)</td>
<td>-28.10* (-.2873 .8267** -.2530** -.0616 -.4322**)</td>
<td>37.07** (.7193** -.6346 .1684** -.2846** .2014**)</td>
</tr>
<tr>
<td>(11.84) (.1804 (.1669 (.0596 (.0645 (.0772))</td>
<td>(14.61) (.2264 (.2067 (.0768 (.0814 (.0974))</td>
<td>(16.56) (.3175 (.2847 (.0935 (.1147 (.1219))</td>
<td>(7.57) (.1822 (.1596 (.0513 (.0662 (.0658))</td>
</tr>
<tr>
<td>(\Delta LIQ(t))</td>
<td>(\Delta ITS(t))</td>
<td>(\Delta IASL(t))</td>
<td>(\Delta FC(t))</td>
</tr>
</tbody>
</table>

\[
\begin{bmatrix}
.4175** \; .0970* \; .1109* \\
(.1269) \; (.0517) \; (.0800)
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta w_{Li}(t-1) - w_{Li}(t-1) \\
\Delta w_{TS}(t-1) - w_{TS}(t-1) \\
\Delta w_{AS}(t-1) - w_{AS}(t-1) \\
\Delta w_{NA}(t-1) - w_{NA}(t-1)
\end{bmatrix}
\]

Eigenvalues (\(1_n - A_n^{h}\)) : .5497 and .8312 ± .0679i (modulus : .8340)

<table>
<thead>
<tr>
<th>(w^{o}_{Li}(t-1))</th>
<th>(w^{o}_{TS}(t-1))</th>
<th>(w^{o}_{AS}(t-1))</th>
<th>(w^{o}_{NA}(t-1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w^{o}_{Li}(t-1))</td>
<td>.2898** (.1946** -.8752** -.1792** .7283** .1071**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((.0802 (.345 (.4604 (.576 (.2463 (.0982))</td>
<td>(.2377 (.211 (.161) (.1104 (.5799 (.3297))</td>
<td>(.0872 (.2361 (.1961 (.633 (.2714 (.5051**</td>
<td></td>
</tr>
<tr>
<td>(.0802 (.345 (.4604 (.576 (.2463 (.0982))</td>
<td>(.2377 (.211 (.161) (.1104 (.5799 (.3297))</td>
<td>(.0872 (.2361 (.1961 (.633 (.2714 (.5051**</td>
<td></td>
</tr>
<tr>
<td>(\Lambda LIQ(t-1))</td>
<td>(\Lambda ITS(t-1))</td>
<td>(\Lambda IASL(t-1))</td>
<td>(\Lambda FC(t-1))</td>
</tr>
</tbody>
</table>

Choleski values \(K : 1.946, 2.596, .3074, \text{and } 0; \ K : 2.989, 3.645, \text{and } .1425\)

\textsuperscript{a} Asymptotic standard errors in parentheses.

\textsuperscript{**} Point estimate in absolute value larger than one (twice) the standard error.
behaviour was in no way restricted. As far as the interest coefficients concern, own short-run effects are all positive and most of the cross-effects bear a negative sign. The relative strong impact of the yield on liquidities (which characterise all estimations of a general dynamic specification) is striking, but not alarming since the changes in the ILIQ variable will lever be of such magnitude so as to produce share changes outside the interval \((0,1)\). The long run demand system is homogeneous and symmetric in interest rates. The own effect of the rate on liquidities is considerably tempered, while the reverse holds for the own effects of the remaining rates of return. An examination of the Choleski values of this interest coefficient matrix reveals that the positivity condition is respected.

The point estimates that are encountered for the short run income effects, suggest that in a first instant a reallocation takes place from funds invested in foreign assets towards liquidities (in the strict sense); the relative holdings of the two other assets are merely affected. Still, an entirely different picture is sketched by the final form income parameters: in the long run, investors would react to an income increase by reallocating wealth towards long term assets \((+0.2862)\) and, to a lesser extent, towards liquidities in the large sense \((+0.1070 +.1119)\), at the expense of foreign asset holdings \((-0.5051)\). Before proceeding, it should be noted however that these coefficients measure only a partial effect of an income increase. The effect is indeed incomplete, because it ignores the increase in wealth that is generated through the channel of financial savings. Mathematically, the long run demand for asset \(i\), say \(z_i\), is given by

\[
z_i = w_i^0 \cdot WP,
\]

where \(w_i^0\) is the desired long run share of asset \(i\), viz.
\[ w_1^0 = \Pi_{10} + \sum_{j=1}^{4} \Pi_{1j} x_j + \Pi_{15} \frac{YU}{WP}. \]

Hence, differentiation of \( z_1 \) w.r.t. \( YU \) yields

\[ \frac{\partial z_1}{\partial YU} = \Pi_{15} \left(1 - \frac{\partial WP}{\partial YU} \frac{YU}{WP}\right) + \omega_1 \frac{\partial WP}{\partial YU}, \]

where the second and third RHS terms denote the aforementioned ignored effect through wealth: the savings quote. Though the effect on liquidities seems appropriate, doubts have to be raised w.r.t. the strong impact in the long run on long term asset holdings.

Since our time perspective is the long run, we are dealing with the effects of a sustained higher level in nominal income, in stead of a temporary shock. This means that the need for a means of payment can be anticipated and that the problem is not any longer one of reallocation towards the medium of exchange but rather one of finding a suitable temporary abode for means intended to be used for payment of goods and services. It is evident that with anticipated payments, this 'intermediary' function cannot only be performed by money, but by any asset that exhibits a sufficient degree of liquidity for then, i.e. when it is 'realisable at short notice without loss', it can be converted into the medium of exchange almost at the instant of payment, precisely because this payment was anticipated. So, the fact that our estimates suggest an increased interest in an asset, in casu bonds, that cannot serve as a means of payment, should not be disturbing. What is disturbing, however, is that this asset, its market value being strongly subjected to fluctuations, cannot properly function as a temporary abode for means intended for later payment, that is, it is an illiquid asset. Because the precision of 3 out of the 4 point estimates of the long run income effect is rather low, the entire system was reestii-
Table 5.8. Estimation of general dynamic model, symmetry imposed, without long run income effect\(^a\)

\[
\Delta w(t) = \Gamma \Delta \bar{x}(t) + A_n^r [w_{n-1}(t-1) - \bar{w}_{n-1}(t-1)] = \Gamma \Delta \bar{x}(t) + A_n^r [w_{n-1}(t-1) - \bar{w}_{n-1}(t-1)]
\]

| \[\Delta w_{LI}(t)\] | \(.522 \quad -.8225^{**} \quad .3304^{**} \quad -.0564 \quad .2044^{**} \quad -.0266\) | \[\Delta LLIQ(t)\] |
| \[\Delta w_{TS}(t)\] | \(-21.09^* \quad .3197 \quad -.3247 \quad .1130 \quad .0286 \quad .3072\) | \[\Delta ITS(t)\] |
| \[\Delta w_{AS}(t)\] | \(-20.53^{**} \quad .0743 \quad .4461^{**} \quad -.1715 \quad -.0501 \quad -.3781^{**}\) | \[\Delta IASL(t)\] |
| \[\Delta w_{NA}(t)\] | \(.36.39^{**} \quad .4285^{**} \quad -.4518^{**} \quad .1149 \quad -.1831 \quad .0975^{**}\) | \[\Delta IFC(t)\] |

\[\Delta WP(t) - \bar{w}_{n-1}(t-1)\]

\[.2646^{**} \quad .0711^* \quad .0735\]
\[(.0928) \quad (.0419) \quad (.0763)\]
\[.2813^{**} \quad .0679 \quad .1222^{*}\]
\[(.1019) \quad (.0470) \quad (.0868)\]
\[-.0286 \quad -.0620^{*} \quad .3149^{**}\]
\[(.1093) \quad (.0500) \quad (.0906)\]
\[.0453 \quad -.0770^{**} \quad -.2661\]
\[(.0559) \quad (.0269) \quad (.0475)\]

\[w_{LI}(t-1) - \bar{w}_{LI}(t-1)\]
| \[w_{TS}(t-1) - \bar{w}_{TS}(t-1)\] |
| \[w_{AS}(t-1) - \bar{w}_{AS}(t-1)\] |

Eigenvalues \((I_n^r - \Lambda_n^r) = .6493 \text{ and } .8517 + .1055i \text{ (modulus : .8582)}\)

\[
\begin{bmatrix}
0.3503^{**} & 1.875^{**} & -1.051^{**} & -1.367^{**} & 0.5434^{**} & 0 \\
(0.0185) & (0.306) & (0.486) & (0.235) & (0.2046) & \\
0.1609^{*} & 2.153^{**} & -5.364 & -5.563 & 0 \\
(0.0462) & (1.008) & (0.4515) & (0.5210) & \\
0.3596^{**} & 2.380^{**} & -4.759 & 0 \\
(0.0176) & (0.368) & (0.1874) & \\
0.1318^{**} & 0.4982^{**} & 0 \\
(0.0435) & (0.4760) & \\
\end{bmatrix}
\]

Choleski values: \(\bar{K} : 1.875, 1.564, 0.2971, \text{ and } 0; \ K : 2.153, 2.458 \text{ and } 1.802\)

\(^a\) see notes to Table 5.7.
mated with exclusion of these effects. The results are shown in Table 5.8.

The general impression one got from the previous estimation, also applies here: the difference in magnitude of long run vs. short run interest coefficients, positivity of the substitution matrix, plausible impact income multipliers, short run tendencies to invest additional wealth in short term time deposits ($w_{TS}$) and foreign assets ($w_{NA}$). On the other hand, when comparing the two versions, a number of differences draw attention:

(i) the fall in the log likelihood value when ruling out the long run income effects is not sharp, but still strong enough to reject the restricted model as a null hypothesis ($\chi^2 = 9.4$, c.v.(3d.f.) = 7.8 at 5%);

(ii) 13 out of 16 short run interest coefficients and 9 of the 10 reported long run ones become, in absolute value, smaller, when the long run income effect is absent;

(iii) although the coefficients of the adjustment matrix $A^n$ do not lend themselves easily for interpretation (cf. the identification problem discussed in section 4.2.), eigenanalysis of the 3x3 upper sub matrix reveals everything on the stability of the system. In both versions a conjugate pair of complex eigenvalues yield a dominant modulus inside the unit circle, though in the income restricted case, this modulus takes a higher value, pointing at a slightly slower adjustment process;

(iv) in the income restricted version, 81% of all directly estimated coefficients have a standard error lower than their point estimate, and 50% are larger than twice their standard error; in the unrestricted version these percentages are 72 and 36 resp.

The slow rate of adjustment, in either case, implies that on short and even medium term, the long run response to the income variable (whether zero or not) remains relatively
Table 5.9. Cumulated impact & final form income multipliers<sup>a</sup>

<table>
<thead>
<tr>
<th>lag</th>
<th>( w_{LI} )</th>
<th>( w_{TS} )</th>
<th>( w_{AS} )</th>
<th>( w_{NA} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U</td>
<td>R</td>
<td>U</td>
<td>R</td>
</tr>
<tr>
<td>0</td>
<td>.234</td>
<td>.204</td>
<td>-.011</td>
<td>.029</td>
</tr>
<tr>
<td>1</td>
<td>.218</td>
<td>.152</td>
<td>.018</td>
<td>.078</td>
</tr>
<tr>
<td>2</td>
<td>.200</td>
<td>.108</td>
<td>.046</td>
<td>.112</td>
</tr>
<tr>
<td>3</td>
<td>.182</td>
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<td>.146</td>
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<tr>
<td>5</td>
<td>.154</td>
<td>.021</td>
<td>.104</td>
<td>.150</td>
</tr>
<tr>
<td>6</td>
<td>.143</td>
<td>.003</td>
<td>.116</td>
<td>.148</td>
</tr>
<tr>
<td>7</td>
<td>.134</td>
<td>-.010</td>
<td>.123</td>
<td>.141</td>
</tr>
<tr>
<td>8</td>
<td>.127</td>
<td>-.019</td>
<td>.129</td>
<td>.132</td>
</tr>
<tr>
<td>9</td>
<td>.121</td>
<td>-.025</td>
<td>.132</td>
<td>.121</td>
</tr>
<tr>
<td>10</td>
<td>.116</td>
<td>-.029</td>
<td>.133</td>
<td>.108</td>
</tr>
<tr>
<td>( \infty )</td>
<td>.107</td>
<td>0</td>
<td>.112</td>
<td>0</td>
</tr>
</tbody>
</table>

<sup>a</sup> Row sums may differ slightly from zero due to rounding-off errors.

R: income restricted case;
U: income unrestricted case.
unimportant (cf. the expression for cumulated interim multipliers at the end of subsection 4.3). Table 5.9 gives the cumulated impact multipliers for YU/WP up to 10 lags, together with the final form multiplier to which the former will converge at infinity.

In the light of what has been said above, the pattern generated in the restricted case seems more plausible than in the other case: in the very short and short run, an increase in the nominal value of economic activity, which still has a temporary feature, necessitates the use of liquidities (in the strict sense) which are mainly obtained at the expense of foreign assets. However, when the rise in running activity gets a more permanent character, we see how the importance of short term assets grows while the emphasis on liquidities (in the strict sense) is relaxed; the effect on long term assets remains negligible. This substitution of short term assets for (near) monies is justified on the basis of the high degree of liquidity of these former assets which are interest-bearing at the same time. Because long term assets lack this liquidity property, the scenario depicted by the unrestricted model looses its credibility. To sum up, though the income restricted version of the model does not pass the log likelihood ratio test, it is not rejected with a large margin either, its point estimates have a greater degree of precision and its implied reaction to changes in income can be theoretically supported. These considerations, and the fact that in a dynamic simulation over the sample period it performs equally well as the unrestricted version, made our preference go to the income restricted model.

A full stretch of this model's dynamics is given in Figure 5.3, where the cumulated interim multipliers of the six exogenous variables are graphically represented. The slow convergence to the long run solution, already 'announced' by the high dominant eigenvalue, strikes the eye; the oscillat-
ing movement is due to the occurrence of the conjugate pair of complex roots.

To obtain a more accurate idea on the speed (or should we rather say the slowness) of adjustment, one would like to have recourse to measures such as the mean or median lag. However, the first concept is only defined when the lag scheme can be said to be normalised; whether this is the case with a multivariate lag scheme, is not trivial to investigate (cf. Bewley, 1984, pp. 48-50). The second measure, the median lag, is also surrounded with definitional problems when the lag scheme is a multivariate one; at least when it is characterised by complex roots, for then the oscillating behaviour of the multipliers around the long run response can preclude one to say when 50 % of the long run effect has taken place. Nevertheless, in order to get some notion of the speed of convergence, we calculated for each long run interest multiplier the 50 % zone, viz. [.5 $\Pi_{ij}$, 1.5 $\Pi_{ij}$], $i,j = 1, ..., 4$, and searched for the number of periods that a shock should sustain for the cum. interim effect to fall within this range. These lag numbers are given in parentheses to the right of the variable legends in the figure. For instance, the cum. interim multiplier of ITS on TSTP enters the 50 % zone after 4 lags (approximately). The definition problem arose only in two cases: the effect of ILIQ on $w_{NA}$ and of IASL on $w_{TS}$.

Let us first study the reaction speeds of the different asset holdings to changes in a particular rate of interest (i.e. looking at all reaction curves in one figure at a time). Then we observe that the speeds of adjustment to changes in ILIQ are very low, whereas the adjustments to movements in IFC take place very fast; a mixed picture is obtained for reactions to the ITS and IASL rates (slow reactions on the part of $w_{NA}$ and $w_{TS}$ resp.). Concentrating on the speed of adjustment of one single asset share to changes in the different rates of return (by
Figure 5.3. Cumulated interim multipliers (Lag 50 % zone within brackets)
Figure 5.3. Cumulated interim multipliers (cont.)
looking at the same multiplier curve across the different figures), we may term liquidities ($w_{LI}$) and long term assets ($w_{AS}$) as first adjusters (ignoring in both cases the slow reaction to ILIQ); the short term ($w_{TS}$) and foreign ($w_{NA}$) assets, on the other hand, perform a very slow process of convergence to their equilibrium values, although they too react fast w.r.t. movements in their own rates of return which is therefore an overall property of the model. 

It would have been an interesting exercise to assess the contribution of the different exogenous variables to the evolution of the portfolio structure over, say, the last ten years of the sample period, by truncating the final form of the demand system (cf. eq. (4.17)) at a finite number of lags such that the associated cumulated interim multipliers are relatively close to the final ones. However, from the convergence analysis above, it is clear that our sample period is too limited to carry out such an ex post analysis in a sensible way.

The general impression one gets from these model dynamics, is a very slow portfolio adjustment behaviour of the Belgian private sector to the long run response values. But this is something different than to say that the private sector agents manage their portfolios in a 'lazy' way. Sometimes, they react very violently to exogenous shocks; it only takes them a long way to attune their portfolio composition to the long run structure.

We conclude this empirical section by having a look at the long run interest elasticities. These are given by

$$
e_{ij} = \frac{\partial w_i / \partial x_j}{x_j} = \Pi_{ij} \frac{x_j}{w_i}, \quad i, j = 1, \ldots, 4.$$

Though this concept denotes the percentual change in the portfolio share of a particular asset, it may also be interpreted as the percentual change in the absolute holdings of that particular asset if one ignores the effect of
the change in the interest rate on total wealth. An even more interpretable concept can be obtained by considering percentual changes in asset shares (or levels) due to changes in interest rates of one percentage point (e.g. from .08 to .09); these changes are then given by the semi-elasticities $v_{ij}$, i.e.

$$v_{ij} = \frac{\partial w_i}{\partial x_j} = \frac{\Pi_{ij}}{w_i}, \quad i, j=1, \ldots, 4.$$  

Both reaction measures are depicted in Table 5.10 and 5.11 resp. where use was made of the long run coefficient estimates of Table 5.8 and the sample averages of $x_j$ and $w_i$. All interest elasticities are fairly small in magnitude, a finding that typifies most empirical asset demand studies. For short and long term placements, all competing assets appear to be substitutes.

Apart for liquidities, assets respond the most elastically to changes in the own rates of return. The positive interest elasticities of (net) foreign assets vs liquidities points at a complementarity between both assets, for which no immediate explanation is available. A similar relationship was initially encountered in the MORKMON-model (De Nederlandsche Bank, 1984) where an allocation mechanism was estimated over four asset categories that comprise broadly the same assets as the aggregates of the present study. But all complementary relationships where subsequently suppressed in MORKMON by restricting positive long run coefficients to zero; this may explain why the final form non-zero elasticities in that study are of a slightly higher magnitude than the ones in table 5.10.

Looking at the semi-elasticities learns us that all asset holdings will, in the long run, be raised with 6 - 8% when their own rates of return go up with one percentage point. Most cross semi-elasticities are smaller than the own ones, except for foreign assets holdings (NAFP) which are reduced with almost 7.5% when the rate on short term deposits experiences a 1% increase; an asymmetric reaction is found
Table 5.10. Long run interest elasticities\(^a\)

<table>
<thead>
<tr>
<th></th>
<th>1. ILIQ</th>
<th>2. ITS</th>
<th>3. IASL</th>
<th>4. IFC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. LIQP</td>
<td>.029</td>
<td>-.210</td>
<td>-.464</td>
<td>.189</td>
</tr>
<tr>
<td>2. TSTP</td>
<td>-.013</td>
<td>.339</td>
<td>-.143</td>
<td>-.155</td>
</tr>
<tr>
<td>3. ASLP</td>
<td>-.013</td>
<td>-.066</td>
<td>.499</td>
<td>-.015</td>
</tr>
<tr>
<td>4. NAFP</td>
<td>.026</td>
<td>-.356</td>
<td>-.509</td>
<td>.545</td>
</tr>
</tbody>
</table>

\(^a\) \( \bar{e}_{ij} = \hat{\pi}_{ij} \cdot \bar{x}_j/\bar{w}_i \), where \( \bar{x}_j, \bar{w}_i \) denote sample period averages.

Table 5.11. Long run semi interest elasticities\(^a\)

<table>
<thead>
<tr>
<th></th>
<th>1. ILIQ</th>
<th>2. ITS</th>
<th>3. IASL</th>
<th>4. IFC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. LIQP</td>
<td>7.893</td>
<td>-4.425</td>
<td>-5.756</td>
<td>2.288</td>
</tr>
<tr>
<td>2. TSTP</td>
<td>-3.481</td>
<td>7.132</td>
<td>-1.777</td>
<td>-1.874</td>
</tr>
<tr>
<td>3. ASLP</td>
<td>-3.551</td>
<td>-1.393</td>
<td>6.180</td>
<td>-.183</td>
</tr>
<tr>
<td>4. NAFP</td>
<td>7.199</td>
<td>-7.496</td>
<td>-6.304</td>
<td>6.600</td>
</tr>
</tbody>
</table>

\(^a\) \( \bar{v}_{ij} = \hat{\pi}_{ij}/\bar{w}_i \), where \( \bar{w}_i \) denotes sample period average.
on behalf of short term deposits of which only 1.9% is disposed of.

If one knows that these four rates of return changed, on average, either up- or downwards with .006, .85, .68 and 2.52% resp.\textsuperscript{24}, one cannot deny that the portfolio behaviour of the Belgian private sector exhibits a certain degree of flexibility.
6. Conclusion

In its search for an adequate portfolio modelling strategy, this paper started off by investigating the properties of the wealth allocation mechanism in a (Tobin-) perfect market world. Apart from adding-up, this basic model exhibited homogeneity, symmetry and positivity in rates of returns which made it resemble very much to the structure of consumer and factor demand systems. Moreover, we observed that the optimal allocation within the risky part of the portfolio was independent of the parameters that typify the particular investor, i.e. his initial wealth and his aversion to risk (the separation theorem). But the perfect market setting seemed too remote from real world surroundings to give much credit to the explanatory power of the basic model in the short run; this we learned when studying the allocation mechanism from a Hicksian perspective. Though we were not able to keep the analysis always as formal as before, it became clear that transaction costs have a strong impact on the mechanism as a whole, for not only have rates of return to be adjusted downwards in the optimisation problem, the problem itself gets an entire new dimension: it now stretches further than just the period immediate ahead. In this way the introduction of transaction costs automatically brings liquidity considerations into play. These are identified with a new modus operandi of portfolio behaviour: the precautionary motive, which operates not just on a twofold 'money-bonds' choice set, but rather on the entire liquidity spectrum.

With these new elements in mind, we took up thread of the formal analysis by embedding the basic portfolio model in a flexible dynamic framework, adapted to singular demand systems by Anderson and Blundell. In this way, the basic model was 'consigned' to the long run. To keep the model within limits, we regarded a first order general dynamic specification as the maintained hypothesis. The empirical results, based on data of the Belgian private sector, revealed the following:
(i) the partial adjustment mechanism, though frequently applied in empirical asset demand studies, is rejected by the data as a dynamic specification;

(ii) though the evidence found against the symmetry hypothesis was sufficient, it could not be called overwhelming either; using a well-behaviour argument, it was decided to accept symmetric interest effects as a long run property of the allocation model;

(iii) while the impact income multipliers implied reasonable short term reactions, perverse effects of income were generated in the long run. Although in a log likelihood ratio test, the restriction of zero long run income effects was proven to be too strong, an examination of the time path of the cumulated interim multipliers made us choose for the restricted version as a maintained hypothesis;

(iv) further analysis of these multipliers indicated at a very slow tendency of the model to converge to its equilibrium solution; and finally,

(v) the long run interest elasticities, evaluated at sample averages, were found to be low, a feature which most empirical portfolio studies come across with. Still, on the basis of the more interpretable semi-elasticities, it is hard to characterise the private sector agent as an 'apathic' investor who does not respond to stimuli from the market.

Further research on this topic should develop at least in two directions.
At the theoretical level, there remains work to be done on the analytical implementation of the precautionary motive in the basic (speculative) portfolio model. This would be a welcome step towards non-walrasian economics, for the uncertainty underlying the precautionary behaviour may stem from the ignorance as to which particular regime will prevail in the near future; and, the presence of sufficient
liquid wealth enlarges the scope for a household to react when confronted with market rationing (the usually studied flow reactions are extended with stock reactions, cf. Korliras, 1983).

At the empirical level, the model studied in this paper may benefit from a more rigorous specification of the mechanism generating the interest rate expectations. The use we made of observed rates as proxies is only a second best solution which should be weighted against rational or autoregressive formation rules. In the latter case, recourse will have to be taken to specifications even more general than the one stucked to in this paper; if this specification then appears not to be general enough the slow convergence results could be traced back to a shortage of degrees of freedom in the modelling of dynamics.
Notes


2 Throughout the present paper we neglect the implications that arise when the real portfolio rate of return is substituted for the nominal one as an argument in the investor's utility function. The aspects of expected inflation in the theory and estimation of portfolio models have only recently been dealt with in the literature; see e.g. Courakis (1988).

3 Alternatively, one could have postulated a utility function with end-of-period wealth $W$, in stead of the overall return $\pi$, as argument. In this case, Friedman and Roley (1980) show that the assumption of constant relative risk aversion, together with the joint normally distributed asset return assessments are jointly sufficient conditions to derive asset demand functions that are linear homogeneous in wealth (and linear in expected returns); the same result is achieved when starting from isomorphic assumptions.

4 Expression (2.4) in the text is arrived at on basis of similarity between the exponential part of the utility function (2.3) and the moment generating function of the normal distribution.

5 Since the asset demand system is linear homogeneous in initial wealth by construction, this parameter will a fortiori not exert any influence on the risky asset structure. However, if we had defined utility in terms of terminal wealth, $W$, in stead of overall portfolio return, $\pi$, the separation theorem w.r.t. initial wealth would still hold, despite the fact that the demand for (risky) assets is no longer linear homogeneous in this aggregate.
When transaction costs are present, the vector of net expected returns, e.g. \( \rho_N \), will be lower than the original one, viz. \( \rho_N \leq \rho \). Whether the total fraction of the portfolio, allocated towards risky assets, \( \pi' a^0_N \), is smaller than when transaction costs are absent, depends on the sign of

\[
\pi' \Psi^{-1} (\rho_N - \rho)
\]
on which no general statements can be made.

Imagine a two-asset spectrum, A and B, with the following visualised (expected) return structure over two months:

A:

\[
\begin{array}{cccc}
0 & 1\% & 1 & 2.97\% & 2 \\
\end{array}
\]

B:

\[
\begin{array}{cccc}
0 & 2\% & 1 & 0\% & 2 \\
\end{array}
\]

The superiority of A in the second month, even though it makes this asset the highest yielding on the 2-month period, is irrelevant for the portfolio choice at the outset of the first month. Indeed, investing the first month in asset B, and switching over to asset A at the end of it, leads to an overall 2-month return of 5.03 %, far better than the 4 % earned by investing solely in A. Thus, whether the investor wants to liquidate his wealth after two months, or immediately after the first one, makes no difference; the best initial portfolio choice is to invest in B. This is no longer true when the switch from B to A after one month necessitates a charge of 1.5 % (on the portfolio value), as this lowers the overall return of the B-A strategy just below the 4 % of the A-A strategy (3.98 % to be precise). In this case, it is worth considering whether the
funds are needed only after two months or earlier. (cf. Tobin, 1965, pp. 3-6)

8 'A demand for money that springs from this cause, I think we shall agree, is precisely what Keynes meant by the precautionary motive' (Hicks, 1967, p. 34).

9 With the exception of trade credit.

10 Hicks avoids speaking of a transaction demand, to stress the fact that transaction balances are held because of a necessity, rather than on voluntary basis.

11 Of the 19 empirical on portfolio behaviour reviewed by Owen (1986), 10 make use of this multivariate partial adjustment mechanism in one way or another. Both the use of this scheme has not been restricted to portfolio modelling; see Nadiri and Rosen (1969) for an application of it to factor demand modelisation.

12 System (4.7) in the text can be labelled as the pseudo structural form of (4.2), a term introduced by Bewley (1979).

13 A third model, nested within the first order general dynamic system (4.9) of the text, is the autoregressive specification which is arrived at by restricting all systematic responses to take place immediately, viz. \( \Gamma = \Pi \). It can be written as

\[
w(t) = \Pi x(t) + v(t),
\]

\[
v(t) = P^n v_n(t-1) + \epsilon(t),
\]

where the sub- and superscript interpretation is the same as in the text.
All variables were taken from a database describing the monetary accounting framework for Belgium (cf. Section 1) from '59 till '84. The sources and exact definitions of the variables used in this section, are listed in appendix C.

I.e. liquidities in the strict sense, not to be confused with what was termed 'liquidities' or 'liquid assets' in section 3.

Government debt certificates on short term (CSGP) enter the private sector's balance sheet because they are held by nationalised enterprises and by 'institutions of life and labour accident insurance and pension funds'; private persones and business firms, however, are not allowed to subscribe to these debt certificates. Since these latter categories constitute an overwhelming majority of the 'private sector', the CSGP variable was taken exogenous and therefore subtracted from total wealth available for allocation.


All rates of return were stored as percentage points.

At least initially. As will appear in section 5.2. the effects of this income variable were confined to the short run.

It may be recalled from section 2 that the intercept entered the asset demand system in a special way (cf. eq. (2.8) in the text). Prior estimations with this particular structure imposed upon, led to dynamic demand systems which failed to meet the stability conditions and with log
likelihood values far below those obtained and reported in this section. Consequently, this restriction was dropped. Probably, the constant term catches many more aspects of portfolio behaviour which were neglected in section 2.

21 Observe that the testing of the general dynamic specification vs. the partial adjustment scheme implicitly also means a test of the short run wealth effect which operates in the former specification through the matrix \( \Gamma \), but is totally absent in the latter model.

22 With the exception of specification XI; but more will be said in the sequel.

23 W.r.t. specification XI in table 5.3, the Cholesky values of the matrix \( K \) are .8541, 4.767, .0188 and 0; those of the matrix \( K \) are 5.115, 1.695 and .0088.

24

\[
\text{I.e. } \frac{1}{T} \sum_t | \Delta x_{jt} | \leq 100
\]
References


FRIEDMAN B.M. and V.V. ROLEY (1979), 'Investors' Portfolio Behaviour Under Alternative Models of Long-Term Interest Rate Expectations: Unitary, Rational or Autoregressive', Econometrica, 47, pp. 1475-1497.


VAN LOO P.D. (1983), A Sectoral Analysis of the Dutch
Financial System (Leiden: Stenfert Kroese).
Appendix A

The transformed dynamic system of demand equations is given by (cf. eq. (4.7) in the text)

\[(A.1) \Delta w(t) = -B(L)\Delta w(t)+\Gamma(L)\Delta x(t)+A[\Pi x(t-q)-w(t-p)]+u(t).\]

Let us introduce the following partitionings:

\[B_{1} = [B_{11}, B_{21}], A = [A_{1}, A_{2}], \]
\[\begin{array}{c}
\text{n}x(n-1) \\
\text{n}x1
\end{array}
\begin{array}{c}
\text{n}x(n-1) \\
\text{n}x1
\end{array}
\]

\[\Pi = \begin{bmatrix}
\Pi_{1} \\
\Pi_{2}
\end{bmatrix}
\begin{bmatrix}
(n-1)xh \\
lxh
\end{bmatrix}
\]

\[w(t) = \begin{bmatrix}
w_{1}(t) \\
w_{2}(t)
\end{bmatrix}
\begin{bmatrix}
(n-1)x1 \\
lx1
\end{bmatrix}
\]

The adding up restriction (4.6) of the text, and the fact that shares sum to unity, allow us to write \(\Pi_{2}\) and \(w_{2}\) as

\[(A.2) \quad \Pi_{2} = e_{1}' - i_{n}' \Pi_{1},\]

\[(A.3) \quad w_{2} = 1 - i_{n}' w_{1}(t),\]

where \(e_{1}' = [1, 0, \ldots, 0]\) and \(i_{n}\) denotes an \((n-1)\) vector of units. Incorporation of the identities (A.2) and (A.3) in the partitioned version of (A.1) yields:
\( (A.4) \) \( \Delta \omega(t) = - \sum_{i=1}^{p} \left[ B_{1i}, B_{2i} \right] \begin{bmatrix} \Delta w_1(t-i) \\ -l_n' \Delta w_1(t-i) \end{bmatrix} + \Gamma(L) \Delta x(t) \)

\[ + \begin{bmatrix} [A_1', A_2'] \begin{bmatrix} \Pi_1 \\ -l_n' \Pi_1 \end{bmatrix} x(t-q) - w_1(t-p) \end{bmatrix} + u(t), \]

since \( e_1' x(t-q) = 1 \).

This version can now be rewritten as

\( (A.5) \) \( \Delta \omega(t) = - \sum_{i=1}^{p} (B_{1i} - B_{2i} l_n') \Delta w_1(t-1) + \Gamma(L) \Delta x(t) \)

\[ + (A_1 - A_2 l_n') \begin{bmatrix} \Pi_1 x(t-q) - w_1(t-p) \end{bmatrix} + u(t). \]

The equivalence with system (4.8) in the text is then obvious.
Appendix B

The characteristic equation \(|(I-A)-\lambda I| = 0\) can be rearranged as

\[ B.1 \quad |(I-\lambda I)-A| = 0. \]

Let us partition the identity matrix and matrix \(A\) so as to isolate the \(n\)th row and column, viz.

\[
I = \begin{bmatrix}
I_n & 0 \\
0 & 1
\end{bmatrix}, \quad
A = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix},
\]

with obvious notations for \(I_n\) and \(A_{ij}\), \(i, j = 1, 2\).

Hence (A1) can be rewritten as

\[ B.2 \quad \begin{vmatrix}
(1-\lambda) I_n - A_{11} & -A_{12} \\
-A_{21} & (1-\lambda) - A_{22}
\end{vmatrix} = 0\]

Next, we follow Hunt's (1981) suggestion, and modify the matrix of which the determinant value is taken in the LHS of (B2) by elementary row operations which add the sum of the first \((n-1)\) rows to the last row; as is well known, these operations leave the determinant value unaltered, so that expression (B2) is equivalent to

\[ B.3 \quad \begin{vmatrix}
(1-\lambda) I_n - A_{11} & -A_{12} \\
(1-\lambda) i_n - i_n A_{11} - A_{21} & -i_n A_{12} + (1-\lambda) - A_{22}
\end{vmatrix} = 0\]

which, in view of the adding-up condition, \(i'A = (1+g)i'\), can be restated as

\[ B.4 \quad \begin{vmatrix}
(1-\lambda) I_n - A_{11} & -A_{12} \\
(1-\lambda) i_n' - (1+g) i_n' & (1-\lambda) - (1+g)
\end{vmatrix} = 0. \]
Analogously, elementary column operations can be applied to subtract the last column of each of the (n-1) preceding columns. This yields the expression

\[
\begin{vmatrix}
(1-\lambda) I_n - (A_{11} - A_{12} I_n) & -A_{12} \\
0' & (1-\lambda)-(1+g)
\end{vmatrix} = 0.
\]

Evaluating the determinant via the nth row, and rearranging, amounts to the equation

\[
(\lambda+g) \ |I_n - (A_{11} - A_{12} I_n) - \lambda I_n| = 0
\]

in which the second LHS term is nothing but the characteristic equation of \((I_n-A_n^n)\), \(A_n^n\) being the matrix consisting of the first (n-1) rows of \(A_n^n\), which was defined in section 4.2 of the text.
Appendix C : Data Sources

All variables were taken from a database describing the asset/liability position of six sectors in the Belgian economy [monetary authorities, deposit banks, non-monetary financial institutions (NMFI), private sector, government and rest of the world] w.r.t. each other, as well as the relevant rates of interest and some real sector variables on a yearly basis from '53 (sometimes '57) till '84 (cf. Schroyen, 1988). This database leans heavily on the two following sources :

(a) the subperiod till '80 :

(b) the subperiod '81-'84 :
   NATIONALE BANK VAN BELGIE, Tijdschrift van de Nationale Bank van België, Brussels (several issues).

In the following list of variables we refer to the numbers of tables and accounts as published in source (b), unless another reference is indicated; all assets/liabilities are of course held by, or claimed against the private sector :

CUMP (currency) : XIII.2 & 4b
DDMP (demand deposits with monetary authorities): XIII.2 & 4b
DDBP (demand deposits with deposit banks) : XIII.2 & 4b
DDNP (demand deposits with NMFI) : XII.1
TSBP (time & saving deposits on short term with deposit banks) : XIII.2 & XV.3b
TLBP (time & saving deposits on long term with deposit banks) : XIII.2 & XV.5b
TSNP (time & saving deposits on short term with NMFI) : XII.1
TLNP (time & saving deposits on long term with NMFI) : XII.1
CSGP (government debt certificates on short term) : XII.1
CLGP (government debt certificates on long term) : XII.1
NAFP (net foreign assets) : constructed using the accumulation rule $\text{NAFP}_t = \text{NAFP}_{t-1} + \Delta \text{NAFP}_t$ where $\Delta \text{NAFP}_t$ was taken from IX.1 and the benchmark figure for '63 computed from QUINTYN, M. (1986), De Uitvoerbaarheid van een Geldgroeibeleid, Doctoral dissertation, R.U. Gent, page 348
LSPB (loans on short term initially granted by deposit banks) : XIII.2
LLPB (loans on long term granted by deposit banks) : XIII.2
LSPN (loans on short term granted by NMFI) : XII.1
LLPN (loans on long term granted by NMFI) : XII.1
NAPO (net other liabilities) : balance account identity
ITS (interest rate on short term time & saving deposits) : XIX.5, col. 4
ITL (interest rate on long term time & saving deposits) : XIX.8, col. 3
ICL (interest rate on long term government debt certificates) : XIX, col. 3
FP (forward premium on the 3-month forward exchange of the BF vs the US $) : X.3
IFC (foreign interest rate, covered) : = IFU + FP
YU (gross domestic product at market prices in current prices) : Eurostat, National Accounts ESA, Aggregates