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COMPOSITE COMMODITY THEOREMS AND THE BENEFITS OF GOVERNMENT PROGRAMS

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Composite commodity theorems and the benefits of government programs

1. Introduction

Few government programs subsidize the consumption of a homogeneous good. Instead they subsidize a broad class of goods (e.g., food) or a good that can possess different attributes to different extents (e.g., housing).[1] In some cases, the recipient of the subsidy is free to choose any bundle of subsidized goods or attributes with the same market value. In other cases, participation in the program implies consumption of particular quantities of some goods or attributes. Among the many examples of programs of this type are public housing, public schools, and government-operated hospitals.

In analyzing the benefits and consumption effects of both types of programs, it is common to aggregate all goods into a small number of composite commodities, where one composite contains all goods being subsidized, and to define a utility function over composites. For example, a typical housing program analysis divides all goods into two categories, housing and all other goods (see, for example, Mayo, 1986; Olsen and Barton, 1983, Rosen, 1979). Similarly, in the few existing welfare analyses of the public school system one defines a utility function over a limited number of composite goods, where education is one of the composites (see, for example, Landford, 1985; Sonstelie, 1982). These studies typically assume that all individuals living in a particular area would face the same set of prices in the absence of the government program under consideration and that unsubsidized individuals face this set of prices in the presence of the program. The value of the quantities of goods in each commodity group at these prices is typically used as an index of the quantity of the corresponding composite.

Implicitly the authors of the preceding welfare analyses are appealing to a composite commodity theorem. The purpose of this paper is to show that, under the assumptions of the best known composite commodity theorems, the approach common in the literature will lead to an overestimate of recipient benefit unless the bundle of goods consumed in each group is preferred to all other bundles in that group with the same market value.[2] In Section 1 we formally show this proposition for the case where composite goods are constructed on the basis of Hicks' aggregation theorem, which relies on restrictions on relative prices within each group. In Section 2 we prove the same proposition for the alternative case where restrictions on preferences are used to justify the construction of composite commodities (Gorman, 1959). We summarize our results and discuss their implications in the final section.

2. Hicks's Aggregation Theorem and the Benefits of Government Programs

Among program analysts Hicks's theorem is probably the best known composite commodity theorem. Since the market value of the goods in each group is typically used as an index of the quantity of the composite commodity in program analyses and is an index of quantity under Hicks's assumptions, this may be the composite commodity theorem that many program analysts have in mind. To show that using this index of the quantity of each composite commodity and a utility function defined over Hicksian composites will often lead to a calculated recipient benefit that is too large and never lead to one that is too small, we begin with a statement of Hicks's theorem in the case of two composites. Nothing is gained by considering more.

Let $u(\tilde{q}_A, \tilde{q}_B)$ be the value of a utility index for an individual, where $\tilde{q}_A$ and $\tilde{q}_B$ are vectors of quantities of goods in group A and B consumed by this individual. Assume that the prices of goods in a group change proportionally so that relative prices of goods within each group stay the same. That is, we limit ourselves to price vectors $\tilde{p}_A$ and $\tilde{p}_B$ satisfying

(1) $$\tilde{p}_A = \theta_A \tilde{p}_A^0 \text{ and } \tilde{p}_B = \theta_B \tilde{p}_B^0$$

where θ_A and θ_B are scalars and $\tilde{p}_A^0$ and $\tilde{p}_B^0$ are fixed price vectors. Hicks (1946, pp.312-313) has shown that there exists a utility function U for each $\tilde{p}_A^0$ and $\tilde{p}_B^0$ such that if the solution to

(2) $$\max_{\tilde{q}_A, \tilde{q}_B} u(\tilde{q}_A, \tilde{q}_B) \text{ s.t. } \theta_A \tilde{p}_A^0 \cdot \tilde{q}_A + \theta_B \tilde{p}_B^0 \cdot \tilde{q}_B = Y$$

occurs at

(3) $$\tilde{q}_A^* = \tilde{d}_A(\theta_A \tilde{p}_A^0, \theta_B \tilde{p}_B^0, Y)$$
$$\tilde{q}_B^* = \tilde{d}_B(\theta_A \tilde{p}_A^0, \theta_B \tilde{p}_B^0, Y)$$

and the solution to

$$\max_{Q_A, Q_B} U(Q_A, Q_B; \tilde{p}_A^0, \tilde{p}_B^0) \text{ s.t. } \theta_A Q_A + \theta_B Q_B = Y \tag{4}$$

occurs at

$$\begin{aligned} Q_A^* &= D_A(\theta_A, \theta_B, Y; \tilde{p}_A^0, \tilde{p}_B^0) \\ Q_B^* &= D_B(\theta_A, \theta_B, Y; \tilde{p}_A^0, \tilde{p}_B^0), \end{aligned} \tag{5}$$

then

$$Q_A^* = \tilde{p}_A^0 \cdot \tilde{q}_A^* \text{ and } Q_B^* = \tilde{p}_B^0 \cdot \tilde{q}_B^* \tag{6}$$

$$u(\tilde{q}_A^*, \tilde{q}_B^*) = U(Q_A^*, Q_B^*; \tilde{p}_A^0, \tilde{p}_B^0). \tag{7}$$

That is, under Hicks's assumption that budget frontiers are hyperplanes involving the same relative prices within each group, the scalars θ_A and θ_B can be thought of as the prices of composite commodities whose quantities Q_A and Q_B are the market values of the individual goods in these groups at the fixed prices $\tilde{p}_A^0$ and $\tilde{p}_B^0$.

Suppose one is interested in evaluating the welfare effects of a particular government program. Consider a participant in the program. Assume that the participant's consumption bundle under the program is $(\tilde{q}_A', \tilde{q}_B')$ and that $\tilde{p}_A^0$ and $\tilde{p}_B^0$ are the prices facing all consumers in the absence of the program and unsubsidized consumers in its presence. It is not necessarily the case that the program changes the prices of all goods in each group by the same percentage and imposes no other constraints. Indeed, participation in many government programs requires that particular quantities of certain goods be consumed.

Let $e(\tilde{p}_A, \tilde{p}_B, \bar{u})$ be the expenditure necessary at prices $\tilde{p}_A$ and $\tilde{p}_B$ to attain a value of the underlying utility index $\bar{u}$ and $E(\theta_A, \theta_B, \bar{U}; \tilde{p}_A^0, \tilde{p}_B^0)$ be the level of expenditure necessary at composite prices θ_A and θ_B to attain a value of the utility index $\bar{U}$ when the prices underlying the composite utility function are $\tilde{p}_A^0$ and $\tilde{p}_B^0$. Hereafter it is to be understood that the utility, expenditure, and demand functions in composite commodity space are those based on the fixed price vectors $\tilde{p}_A^0$ and $\tilde{p}_B^0$. Given the participant's consumption bundle under the program, $(\tilde{q}_A', \tilde{q}_B')$, the equivalent variation measure of the benefit of the program to the consumer is

$$B = e(\tilde{p}_A^0, \tilde{p}_B^0, u(\tilde{q}_A', \tilde{q}_B')) - Y, \tag{8}$$

where Y is the participant's income.

If the utility function defined over Hicksian composite commodities is used to calculate the equivalent variation, the result is

$$B_C = E(1,1,U(Q_A', Q_B')) - Y, \tag{9}$$

where $Q_A' = \tilde{p}_A^0 \cdot \tilde{q}_A'$ and $\tilde{Q}_B' = \tilde{p}_B^0 \cdot \tilde{q}_B'$.

We want to show that

$$B < B_C \tag{10}$$

unless the bundles $\tilde{q}_A'$ and $\tilde{q}_B'$ consumed under the program are preferred by the consumer to any other bundles $\tilde{q}_A$ and $\tilde{q}_B$ with the same market value, that is, bundles for which $\tilde{p}_A^0\ \tilde{q}_A = \tilde{p}_A^0\ \tilde{q}_A'$ and $\tilde{p}_B^0\ \tilde{q}_B' = \tilde{p}_B^0\ \tilde{q}_B'$. If the bundles $\tilde{q}_A'$ and $\tilde{q}_B'$ do happen to be the consumer's most preferred choices given the respective market values,then we show that $B=B_C$.

In order to prove these propositions,first note that, in light of Hicks's theorem,

$$e(\tilde{p}_A^0, \tilde{p}_B^0, \bar{u}) = E(1,1,\bar{u}) \tag{11}$$

for any $\tilde{p}_A^0, \tilde{p}_B^0$ and $\bar{u}$. Therefore, inequality (10) will hold if and only if

$$(12) \qquad u(\tilde{q}_A', \tilde{q}_B') < U(\tilde{p}_A^0 \tilde{q}_A', \tilde{p}_B^0 \tilde{q}_B')$$

Observe that, provided that the composite utility function is quasi concave, there exist prices of the composite goods θ_A^* and θ_B^* such that the maximum of the composite utility function subject to the linear budget constraint associated with these prices and income Y occurs at quantities of the composites $\tilde{p}_A^0 \tilde{q}_A'$ and $\tilde{p}_B^0 \tilde{q}_B'$. That is, there exist prices θ_A^* and θ_B^* such that

$$(13) \qquad U(\tilde{p}_A^0 \tilde{q}_A', \tilde{p}_B^0 \tilde{q}_B') = \max_{Q_A, Q_B} U(Q_A, Q_B) \quad \text{s.t. } \theta_A^* Q_A + \theta_B^* Q_B = Y$$

It follows that the prices θ_A^*, θ_B^* satisfy the equations

$$(14) \qquad D_A(\theta_A^*, \theta_B^*, Y) = \tilde{p}_A^0 \tilde{q}_A'$$

$$(15) \qquad D_B(\theta_A^*, \theta_B^*, Y) = \tilde{p}_B^0 \tilde{q}_B'$$

Now let the bundle (q_A'', q_B'') be the solution to the problem

$$(16) \qquad \max_{\tilde{q}_A, \tilde{q}_B} u(q_A, q_B) \quad \text{s.t. } \theta_A^* p_A^0 q_A + \theta_B^* p_B^0 q_B = Y$$

Then by Hicks's theorem

$$(17) \qquad \tilde{p}_A^0 \tilde{q}_A'' = \tilde{p}_A^0 \tilde{q}_A'$$

$$(18) \qquad \tilde{p}_B^0 \tilde{q}_B'' = \tilde{p}_B^0 \tilde{q}_B'$$

$$(19) \qquad u(\tilde{q}_A'', \tilde{q}_B'') = U(\tilde{p}_A^0 \tilde{q}_A', \tilde{p}_B^0 \tilde{q}_B').$$

Assuming that $u(\tilde{q}_A,\tilde{q}_B)$ is strictly quasi concave the bundle $(\tilde{q}''_A,\tilde{q}''_B)$ is the unique solution to problem (16). Note, however, that (17) and (18) imply

$$(20) \qquad \theta^*_A \tilde{p}^0_A \tilde{q}'_A + \theta^*_B \tilde{p}^0_B \tilde{q}'_B = Y$$

In other words, the bundle observed under the program satisfies the constraint in (16). It follows that

$$(21) \qquad u(\tilde{q}'_A,\tilde{q}'_B) \leq u(\tilde{q}''_A,\tilde{q}''_B)$$

where equality holds if and only if $(\tilde{q}'_A,\tilde{q}'_B) = (\tilde{q}''_A,\tilde{q}''_B)$.

Combining (19) and (21) we find that

$$(22) \qquad u(\tilde{q}'_A,\tilde{q}'_B) < U(\tilde{p}^0_A\tilde{q}'_A, \tilde{p}^0_B\tilde{q}'_B)$$

and hence

$$(23) \qquad B < B_C$$

unless $(\tilde{q}'_A,\tilde{q}'_B) = (\tilde{q}''_A,\tilde{q}''_B)$, in which case $B = B_C$.

We have shown that the equivalent variation measure of benefit based on two Hicksian composites will overstate the benefit obtained using the observed commodity bundles $\tilde{q}'_A$ and $\tilde{q}'_B$ unless the condition $(\tilde{q}'_A,\tilde{q}'_B) = (\tilde{q}''_A,\tilde{q}''_B)$ holds. In order to see the economic interpretation of this result, note that $(\tilde{q}''_A,\tilde{q}''_B)$ is also the unique solution to the problem,

$$(24) \qquad \max_{\tilde{q}_A,\tilde{q}_B} u(\tilde{q}_A,\tilde{q}_B) \quad \text{s.t.} \quad \tilde{p}^0_A\,\tilde{q}_A = \tilde{p}^0_A\,\tilde{q}'_A \quad \text{and} \quad \tilde{p}^0_B\,\tilde{q}_B = \tilde{p}^0_B\,\tilde{q}'_B$$

because $(\tilde{q}''_A,\tilde{q}''_B)$ satisfies the constraints of this problem (see equations (17) and (18), any other $(\tilde{q}_A,\tilde{q}_B)$ that satisfies these constraints also satisfies the constraints of problem (16) (see equations 20), and (q''_A,q''_B) is the unique solution to problem (16).

This implies a very natural interpretation for the bundle $(\tilde{q}''_A, \tilde{q}''_B)$. It gives the consumer's preferred quantities of goods in groups A and B among all bundles with the same market value as the bundles $\tilde{q}'_A$ and $\tilde{q}'_B$ consumed under the program. We have shown, therefore, that if the quantities provided under the government program are not the consumer's most preferred combinations of goods among bundles with the same market values, an analysis based on Hicksian composite goods will overstate the benefit the consumer derives from the program. Only if the bundle $(\tilde{q}'_A, \tilde{q}'_B)$ does solve (24) will an approach based on composites lead to the same benefit measure as an approach based on the bundle of individual goods.

Our main finding is illustrated on Figure 1 for the case where the bundle $(\tilde{q}'_A, \tilde{q}'_B)$ does not solve (24). In that case we showed that $u(\tilde{q}'_A, \tilde{q}'_B) < U(\tilde{p}^0_A\ \tilde{q}'_A,\ \tilde{p}^0_B\ \tilde{q}'_B)$ and, consequently, $B < B_C$.

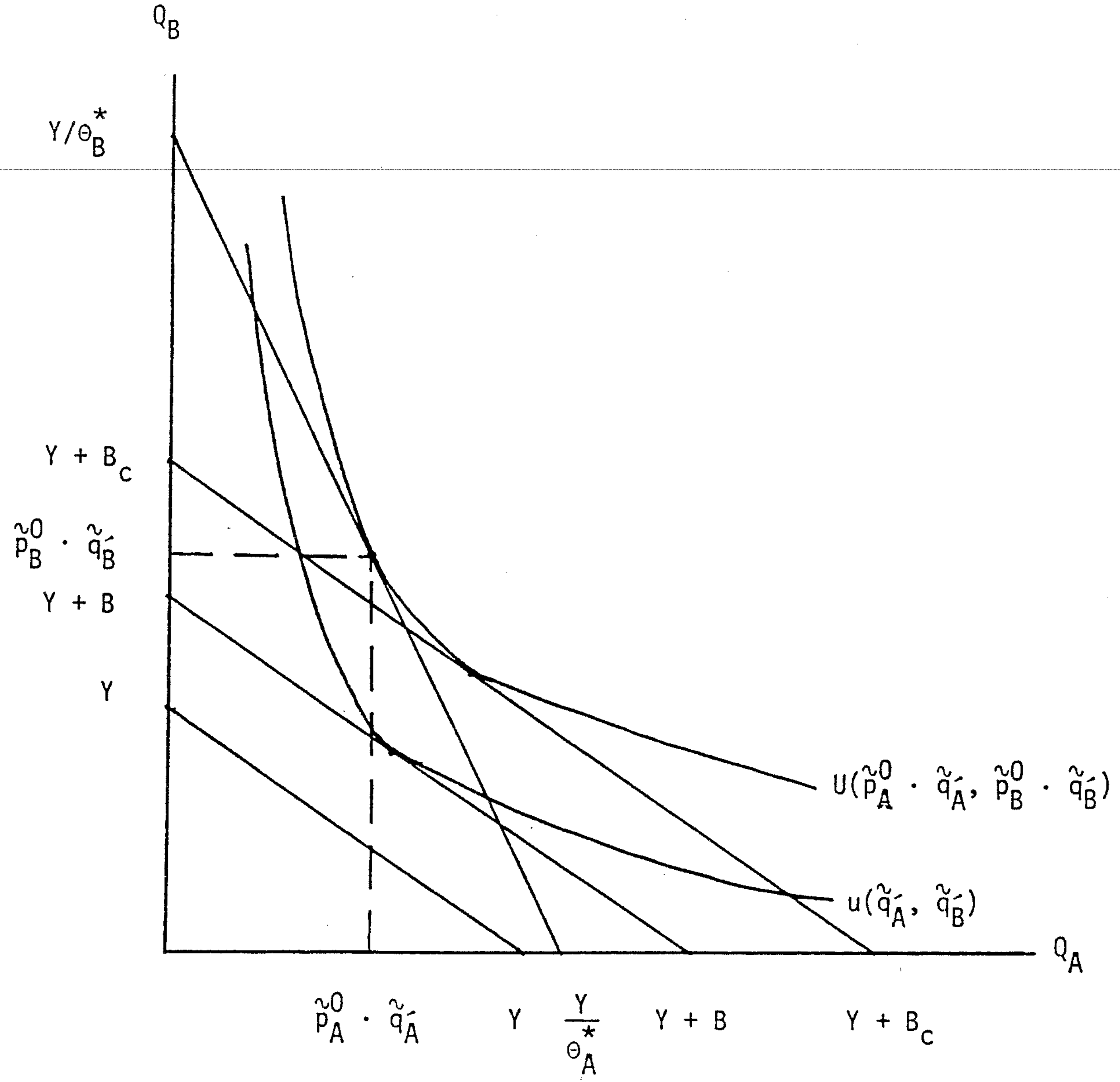

Figure 1

3. Gorman's Aggregation Theorems and the Benefits of Government Programs

Two composite commodity theorems that many consider to be more useful for empirical analysis than Hicks's theorem (see, for example, Deaton and Müellbauer, 1980, Chapter 5) are due to Gorman (1959, pp.476-478). These theorems are based on assumptions about preferences rather than how budget frontiers change. Since a number of the indifference maps widely used in program analysis satisfy the assumptions of one of Gorman's theorems, it is tempting to try to justify the typical program analysis based on composites by reference to these theorems. It will be shown that the usual analysis cannot be so justified unless the bundle of goods consumed in each group is preferred to all other bundles in that group with the same market value and that, in cases where it cannot be justified, the usual approach will yield calculated benefits that are too large.

We show this proposition under the assumptions of Gorman's first theorem. The proof for the case of his second theorem is almost completely analogous and therefore presented in the appendix to this paper. In addition to the axioms of the theory of consumer choice, Gorman's first theorem is based on the assumptions that the individual's utility function is weakly separable in the goods involved in the different composites and the subutility functions are linear homogeneous.[3] Since nothing is gained by considering more than two composites, we limit ourselves to this case. Assume that a household

$$(25) \qquad \max_{\tilde{q}_A,\tilde{q}_B} \; u(v_A(\tilde{q}_A),v_B(\tilde{q}_B)) \quad \text{s.t.} \quad \tilde{p}_A\cdot\tilde{q}_A + \tilde{p}_B\cdot\tilde{q}_B = Y.$$

Since the subutility functions are assumed tc be linear homogeneous,

$$(26) \qquad e_G(v_G(\tilde{q}_G),\tilde{p}_G) = b_G(\tilde{p}_G)v_G(\tilde{q}_G) \quad \text{for} \quad G = A,B,$$

where $e_G(v_G(\tilde{q}_G),\tilde{p}_G)$ is the expenditure on goods in composite

G necessary at prices $\tilde{p}_G$ to reach subutility level $v_G(\tilde{q}_G)$, and b_G is a linear homogeneous function. Gorman has shown that we can treat the $v_G(\tilde{q}_G)$ as the quantities composite goods with corresponding unit prices $b_G(\tilde{p}_G)$. More specifically, define $Q_G = v_G(\tilde{q}_G)$ and $P_G = b_G(\tilde{p}_G)$. Let the solution to (25) be given by the bundle $(\tilde{q}_A^*, \tilde{q}_B^*)$. Gorman showed that if the bundle (Q_A^*, Q_B^*) solves the problem

$$(27) \qquad \max_{Q_A, Q_B} \; u(Q_A, Q_B) \quad \text{s.t.} \quad P_A Q_A + P_B Q_B = Y,$$

it follows that

$$(28) \qquad P_G Q_G^* = \tilde{p}_G \tilde{q}_G^* \qquad (G = A, B)$$

$$(29) \qquad u(v_A(\tilde{q}_A^*), v_B(\tilde{q}_B^*)) = u(Q_A^*, Q_B^*) = u\left(\frac{\tilde{p}_A \tilde{q}_A^*}{\tilde{b}_A(\tilde{p}_A)}, \frac{\tilde{p}_B \tilde{q}_B^*}{\tilde{b}_B(\tilde{p}_B)}\right).$$

Provided that the restrictions on preferences are satisfied, this theorem justifies the use of expenditures on goods in each group divided by an appropriate price index as quantity indices of composite goods in standard consumption analysis. However, the theorem cannot be invoked to justify the use of composite goods in many analyses of the effects of government programs. The reason is that the typical analysis miscalculates the quantity of at least one composite under the program, as we now proceed to show.

Assume that a participant in a government program consumes the bundle $(\tilde{q}_A', \tilde{q}_B')$ under the program and attains a level of well-being given by $u(v_A(\tilde{q}_A'), v_B(\tilde{q}_B'))$. Let the vectors of market prices of goods in groups A and B be given by $\tilde{p}_A^0$ and $\tilde{p}_B^0$ respectively. Now suppose one follows the procedure implicit in the typical program analysis and defines composite goods where the quantities of the composites consumed under the program, Q_A' and Q_B', are given by the market value of the goods consumed in each group divided by the appropriate price index[4]

$$(30) \qquad Q_A' = \frac{\tilde{p}_A^0 \cdot \tilde{q}_A'}{b_A(\tilde{p}_A^0)} \qquad Q_B' = \frac{\tilde{p}_B^0 \cdot \tilde{q}_B'}{b_B(\tilde{p}_B^0)}$$

We want to show that this procedure prevalent in the literature will overstate the utility level attained under the program and, as a consequence, the benefit the consumer derives from participating in the program, unless the bundle consumed in each group is preferred to all other bundles with the same market value.

To see this, suppose that the solution to

$$(31) \qquad \max_{\tilde{q}_A, \tilde{q}_B} \; u(v_A(\tilde{q}_A), v_B(\tilde{q}_B)) \qquad \text{s.t. } \tilde{p}_A^0 \tilde{q}_A = \tilde{p}_A^0 \tilde{q}_A', \quad \tilde{p}_B^0 \tilde{q}_B = \tilde{p}_B^0 \cdot \tilde{q}_B'$$

is given by $(\tilde{q}_A'', \tilde{q}_B'')$. Since u is a positive monotonic function of its arguments $\tilde{q}_A''$ and $\tilde{q}_B''$ are also the solutions to the respective problems

$$(32) \qquad \max_{\tilde{q}_A} \; v_A(\tilde{q}_A) \qquad \text{s.t. } \tilde{p}_A^0 \cdot \tilde{q}_A = \tilde{p}_A^0 \cdot \tilde{q}_A'$$

and

$$(33) \qquad \max_{\tilde{q}_B} \; v_B(\tilde{q}_B) \qquad \text{s.t. } \tilde{p}_B^0 \cdot \tilde{q}_B = \tilde{p}_B^0 \cdot \tilde{q}_B' .$$

Suppose that $\tilde{q}_A'' \neq \tilde{q}_A'$. Then it follows that

$$(34) \qquad v_A(\tilde{q}_A') < v_A(\tilde{q}_A'')$$

and, since $b_A(\tilde{p}_A^0)$ is positive,

$$(35) \qquad b_A(\tilde{p}_A^0)\; v_A(\tilde{q}_A') < b_A(\tilde{p}_A^0)\; v_A(\tilde{q}_A'').$$

By equation (26) and the dual of problem (32) we also have

$$(36) \qquad b_A(\tilde{p}_A^0)\; v_A(\tilde{q}_A'') = e_A(v_A(\tilde{q}_A''), \tilde{p}_A^0) = \tilde{p}_A^0 \tilde{q}_A' .$$

Combining (35) and (36) we see that

$$(37) \qquad v_A(q_A') < \frac{\tilde{p}_A^0 \cdot \tilde{q}_A'}{b_A(\tilde{p}_A^0)}.$$

The term on the LHS of this inequality is the quantity of the composite good constructed on the basis of Gorman's first theorem that corresponds to the vector $\tilde{q}_A'$ consumed under the program. The term on the RHS is the quantity of the composite good used in the typical program analysis, namely the market value of the bundle of goods in the composite divided by an index of market price. The inequality implies that if the vector $\tilde{q}_A'$ provided under the program is not the participant's most desired bundle of goods in group A among all bundles with the same market value (i.e. $\tilde{q}_A' \neq \tilde{q}_A''$) then the typical analysis miscalculates the quantity of composite A consumed under the program.

By a similar argument it follows that if $\tilde{q}_B' \neq \tilde{q}_B''$ then we have

$$(38) \qquad v_B(\tilde{q}_B') < \frac{\tilde{p}_B^0 \ \tilde{q}_B'}{b_B(\tilde{p}_B^0)}$$

Using the results in (37) and (38) it is clear that if $\tilde{q}_A' \neq \tilde{q}_A''$ or $\tilde{q}_B' \neq \tilde{q}_B''$ then

$$(39) \qquad u(v_A(\tilde{q}_A'), v_B(\tilde{q}_B')) < u(\frac{\tilde{p}_A^0 \cdot \tilde{q}_A'}{b_A(\tilde{p}_A^0)}, \frac{\tilde{p}_B^0 \cdot \tilde{q}_B'}{b_B(\tilde{p}_B^0)}).$$

Consequently,

$$(40) \qquad B = e(\tilde{p}_A^0, \tilde{p}_B^0, u(v_A(\tilde{q}_A'), v_B(\tilde{q}_B'))) - Y$$

$$< E(b_A(\tilde{p}_A^0), b_B(\tilde{p}_B^0), u(\frac{\tilde{p}_A^0 \ \tilde{q}_A'}{b_A(\tilde{p}_A^0)}, \frac{\tilde{p}_B^0 \ \tilde{q}_B'}{b_B(\tilde{p}_B^0)})) - Y = B_C,$$

where e and E are the expenditure functions associated with the utility function defined on individual goods and on composites, respectively.

We have shown that if the bundle $(\tilde{q}_A', \tilde{q}_B')$ consumed under the program does not solve problem (31), that is, if the bundle of goods consumed in each group is not preferred to all other bundles in that group with the same market value, the typical approach overstates the level of well-being attained under the program and hence its benefit to the participant. If, however,

$$(41) \qquad u(v_A(\tilde{q}_A'), v_B(\tilde{q}_B')) = \max_{\tilde{q}_A, \tilde{q}_B} u(v_A(\tilde{q}_A), v_B(\tilde{q}_B)) \text{ s.t. } \begin{array}{l} \tilde{p}_A^0 q_A = \tilde{p}_A^0 q_A' \\ \tilde{p}_B^0 q_B = \tilde{p}_B^0 q_B' \end{array}$$

then inequalities in (34), (35), (37), (38), (39) and (40) are converted to equalities. That is, benefit is correctly calculated by using the utility function defined over Gorman composites and market values of the consumption bundles in each group as indices of the quantities of the composite commodities.

4. Implications and Conclusion

Most analyses of government programs treat groups of goods as composite commodities. This paper shows that, under the assumptions of the best known aggregation theorems, using utility functions defined over composites will lead to an overestimate of the net benefit of the program to a participant unless the bundle of goods consumed in each category is preferred to all other bundles in that category with the same market value. This result is important because it implies that the many analyses of government programs, for which the required conditions were not satisfied, have overestimated the benefits participating families derive from these programs.

Almost all government programs subsidize a broad category of goods (e.g., food), or a good that can possess different characteristics to different extents (e.g., education and housing). In addition, however, many programs do not allow participants to consume their most preferred combination of particular goods or attributes belonging to the subsidized group. Some in-kind subsidy programs, for example, offer eligible households an all-or-nothing choice to consume given quantities of goods or attributes in the subsidized group at a specified price. Examples include public housing, public schools and rent control. Other programs such as food stamps and certain housing allowance programs impose restrictions on the quantities of particular goods or attributes in the subsidized group. The results of this paper show that analyses of all these programs would lead to overestimates of the benefit to participating families. Among the many other examples of programs whose benefits would be overestimated using the standard composite commodity approach are government-operated hospitals, public mass transit and public recreational facilities.

The only available estimates of the extent of the bias suggest that it can be substantial. De Borger (1987) found that mean benefit of public housing in Belgium calculated using two composite commodities, housing and other goods, exceeded mean benefit based on a more disaggregated approach by 50 to 80 percent. Our theoretical results together with this empirical finding suggest the desirability of obtaining additional estimates of the magnitude of the bias in cases where composite commodity theorems have been misapplied and using a more disaggregated approach in these cases.

If the bundle of goods consumed in each category under the government program is preferred to all other bundles in that category with the same market value, there will be no bias in estimating net benefit of the government program on account of the results shown in this paper. This condition will be satisfied if, for example, the government gives individuals cash grants conditional on spending at least X on a particular group of goods or offers to sell vouchers that can only be used to purchase certain types of goods. This generalizes Hicks's and Gorman's composite commodity theorems to cases where budget frontiers are not hyperplanes.

Appendix

Gorman's second theorem is based on the assumption that the individual's utility function is additively separable in the goods involved in the different composites and that the indirect subutility functions can be written in Gorman polar form. Specifically, assume that a household

(A1) $$\max_{\tilde{q}_A,\tilde{q}_B} f_A(v_A(\tilde{q}_A)) + f_B(v_B(\tilde{q}_B)) \quad \text{s.t. } \tilde{p}_A\cdot\tilde{q}_A + \tilde{p}_B\cdot\tilde{q}_B = Y,$$

where f_A and f_B are positive monotonic transformations of the subutility functions v_A and v_B, and that

(A2) $$w_G(\varepsilon_G,\tilde{p}_G) = h_G(\varepsilon_G/b_G(\tilde{p}_G)) + c_G(\tilde{p}_G) \quad \text{for } G=A,B$$

where $w_G(\varepsilon_G,\tilde{p}_G)$ is the maximum value of the subutility function v_G attainable if ε_G is spent on goods in composite G and the prices of these goods are the components of $\tilde{p}_G$. If we let ε_G in (A2) be equal to $e_G(v_G(\tilde{q}_G),\tilde{p}_G)$, we get

(A3) $$v_G(\tilde{q}_G) = h_G(e_G(v_G(\tilde{q}_G),\tilde{p}_G/b_G(\tilde{p}_G)) + c_G(\tilde{p}_G) \quad \text{for } G=A,B.$$

Gorman shows that if we define $Q_G \equiv e_G(v_G(\tilde{q}_G),p_G)/b_G(\tilde{p}_G)$ and $p_G \equiv b_G(\tilde{p}_G)$ for G = A,B and if $(\tilde{q}^*_A,\tilde{q}^*_B)$ is the solution to the maximization problem (A1) and (Q^*_A,Q^*_B) is the solution to

(A4) $$\max_{Q_A,Q_B} f_A(h_A(Q_A) + c_A(\tilde{p}_A)) + f_B(h_B(Q_B) + c_B(\tilde{p}_B))$$
$$\text{s.t. } P_AQ_A + P_BQ_B = Y.$$

then $f_A(v_A(\tilde{q}^*_A)) + f_B(v_B(\tilde{q}^*_B)) = f_A(h_A(Q^*_A) + c_A(\tilde{p}_A)) + f_B(h_B(Q^*_B) + c_B(\tilde{p}_B))$, and $P_GQ^*_G = \tilde{p}_G\cdot\tilde{q}^*_G$ (G=A,B) for any $\tilde{p}_A,\tilde{p}_B$, and Y.[5]

The reason that this theorem cannot be used to justify the typical program analysis is the same as for the first theorem presented in the text, namely that the typical analysis miscalculates the quantity of at least one composite. A necessary condition for using this theorem to justify a program analysis involving composite commodities is that the bundle of goods consumed in each group is preferred to all other bundles in that group with the same market value. The proofs of these propositions are similar to the proofs given in the text.

Suppose that the consumption bundle under the program is $(\tilde{q}'_A, \tilde{q}'_B)$ and that the solution to

$$\text{(A5)} \qquad \max_{\tilde{q}_A, \tilde{q}_B} f_A(v_A(\tilde{q}_A)) + f_B(v_B(\tilde{q}_B)) \quad \text{s.t.} \quad \begin{array}{l} \tilde{p}^0_A \cdot \tilde{q}_A = \tilde{p}^0_A \cdot \tilde{q}'_A \\ \tilde{p}^0_B \cdot \tilde{q}_B = \tilde{p}^0_B \cdot \tilde{q}'_B \end{array}$$

is $(\tilde{q}''_A, \tilde{q}''_B)$. Since f_A and f_B are positive monotonic functions $v_A(\tilde{q}_A)$ and $v_B(\tilde{q}_B)$ respectively, $\tilde{q}''_A$ and $\tilde{q}''_B$ are also the solution to

$$\text{(A6)} \qquad \max_{\tilde{q}_A} v_A(\tilde{q}_A) \quad \text{s.t.} \quad \tilde{p}^0_A \cdot \tilde{q}_A = \tilde{p}^0_A \cdot \tilde{q}'_A$$

$$\text{(A7)} \qquad \max_{\tilde{q}_B} v_B(\tilde{q}_B) \quad \text{s.t.} \quad \tilde{p}^0_B \cdot \tilde{q}_B = \tilde{p}^0_B \cdot \tilde{q}'_B .$$

Suppose, for example, that $\tilde{q}''_A \neq \tilde{q}'_A$. Then

$$\text{(A8)} \qquad v_A(\tilde{q}'_A) < v_A(\tilde{q}'') .$$

By inequality (A8) and the dual of the problem (A6),

$$\text{(A9)} \qquad e_A(v_A(\tilde{q}'_A), \tilde{p}^0_A) < e_A(v_A(\tilde{q}''_A), \tilde{p}^0_A) = \tilde{p}^0_A \cdot \tilde{q}'_A .$$

Since $b_A(\tilde{p}^0_A)$ is positive,

$$\text{(A10)} \qquad e_A(v_A(\tilde{q}'_A), \tilde{p}^0_A)/b_A(\tilde{p}^0_A) < \tilde{p}^0_A \cdot \tilde{q}'_A / b_A(\tilde{p}^0_A) .$$

The term on the LHS of this inequality is the quantity of the composite good consumed under the program if the analysis is to be based on Gorman's second theorem. The term on the RHS of this inequality is the quantity of the composite good used in the typical program analysis. By a similar argument,

$$\text{(A11)} \qquad e_B(v_B(\tilde{q}_B'),\tilde{p}_B^0)/b_B(\tilde{p}_B^0) \leq \tilde{p}_B^0 \cdot \tilde{q}_B'/b_B(\tilde{p}_B^0).$$

Therefore,

$$\text{(A12)} \qquad U(e_A(v_A(\tilde{q}_A'),\tilde{p}_A^0)/b_A(\tilde{p}_A^0),e_B(v_B(\tilde{q}_B'),\tilde{p}_B^0)/b_B(\tilde{p}_B^0))$$

$$< \; U(\tilde{p}_A^0\cdot\tilde{q}_A'/b_A(\tilde{p}_A^0),\tilde{p}_B^0\cdot\tilde{q}_B'/b_b(\tilde{p}_B^0)).$$

That is, the typical approach overstates the level of well-being attained under the program when the bundle of goods consumed in each group is not preferred to all other bundles in that group with the same market value and hence overestimates benefits.

If $\tilde{q}_A''=\tilde{q}_A'$ and $\tilde{q}_B''=\tilde{q}_B'$, the inequalities in (A8) through (A12) become equalities. That is, benefit is correctly calculated by using the utility function defined over composites and market values of the consumption bundles in each group as indices of the quantities of the composite commodities.

Footnotes

1. These two cases are not really different since we can always consider goods with different characteristics to be different goods.

2. This result was suggested, but not proven, by Olsen (1972, p.1096). De Borger (1987) has sketched proofs for Gorman's composite commodity theorems; our proofs are more general and straightforward. To avoid a confusion shared by several previous readers, let us state explicitly a theorem that is not proven here, namely if a government program provides a proportional subsidy to all commodities in a group at the same rate but contains binding constraints on how much of at least some of these goods can be consumed, the individual will not be as well off as in the absence of these constraints. This theorem is obvious and well-known.

3. We could equally well assume that the subutility functions are homothetic because, for any weakly separable utility function with homothetic subutility functions, there exists a weakly separable utility function with linear homogeneous subutility functions that has the same indifference surfaces.

4. We can define units of measurement for goods in the composites so that $b_G(\tilde{p}_G^0)$ is equal to one, and this is implicit in most analyses.

5. Notice that the values of the composite quantity indices and utility functions depend on prices. This creates no problem for our analysis because the change in an individual's well-being is evaluated at some one set of prices.

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