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COMPOSITE COMMODITIES, HOUSING
CHARACTERISTICS AND THE HICKSIAN
SURPLUS MEASURES OF WELFARE CHANGE

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Abstract

The purpose of this paper is to elaborate on previous research that deals with the implications of alternative housing concepts for the estimation of the welfare effects of housing programs.

We compare one housing concept, which defines housing in terms of a composite good housing services, with an alternative in which housing is treated as a bundle of attributes. We extend the analysis in the literature by concentrating on welfare measures that are applicable to a much broader class of government programs than those that have been discussed before. We focus on the Hicksian surplus measures which are particularly suited to handle situations in which constraints on quantity are imposed.

In a theoretical section we present an argument that suggests that the measures of welfare change based on the composite good housing services will be biased whenever a housing program imposes restrictions on the consumption of attributes. The direction of the bias depends on the nature of the program. Empirical work confirms the predictions of the theoretical analysis.

0. Introduction

In a recent paper (De Borger, Forthcoming) we investigated the consequences of using alternative housing concepts for the evaluation of the welfare implications of a particular type of government program, viz. public housing. Within a highly stylized framework we showed that the equivalent variation measure of consumer benefit based on the housing services approach would provide an upper bound to the 'true' benefits that take into account the composition of the attribute bundles consumed under the program and in the absence of the program.

The purpose of this paper is to find out whether the result of the previous analysis can be extended to other welfare measures and to other types of housing programs. We will focus on welfare measures that are independent of prices. As a consequence, they can be used even in situations where market prices do not reflect consumers' marginal willingness to pay (e.g., in the case of rationing) or when market prices simply do not exist (e.g., in the case of public goods). The measures considered here, Hicks' compensating and equivalent surplus, are particularly suited to make welfare comparisons in quantity-constrained regimes. In the context of housing program analysis they have been used by, e.g., Quigley (1982, 1986).

Organisation of the paper is as follows. In the first section we briefly review the restrictions on preferences that have to be satisfied in order to justify the composite goods housing and all other goods, and point out some important implications. In Section 2 we assume that the required restrictions are satisfied and compare, within a highly stylized theoretical framework, the Hicksian surplus measures of consumer benefit that would be calculated using the composite commodity approach with those that would result from application of the alternative approach based on housing attributes and other goods. A recently proposed distance function will turn out to be extremely useful for this comparison. It will be shown that in most cases the welfare measures based

on the composite commodity approach are biased. The direction of the bias depends on the precise characteristics of the program under consideration. In Section 3 we investigate the implications of the theoretical results for empirical work. We estimate the Hicksian surplus measures for two housing programs using each of the two alternative approaches, and compare the empirical results. For each of the two programs, empirical results confirm the theoretical analysis. Finally, in Section 4 we review the main conclusions of this paper.

1. A stylized theoretical framework¹

In this section we review the highly stylized theoretical framework that will allow us to compare the composite commodity approach to benefit estimation in quantity constrained regimes with the obvious alternative in which preferences are defined in terms of housing attributes and other goods.

Assume that households, in the absence of any constraints on quantity, behave as if they implicitly solve the problem²

$$\text{Max } u(h_1, h_2, \dots, h_n, x_1, x_2, \dots, x_m) \quad (1)$$

$$\text{subject to } \sum_{i=1}^n p_i h_i + \sum_{j=1}^m q_j x_j = y \quad (2)$$

where h_i, x_j are the quantities of housing characteristics and other goods, respectively;

p_i, q_j are the implicit attribute prices and the unit prices of other goods, respectively;

y is income.

¹ This section is to a large extent taken from De Borger (forthcoming).

² The surplus measures to be studied in the next section can obviously be used when households would face smoothly nonlinear budget constraints, due to, e.g., a nonlinear hedonic relation between house rent and attributes. An example can be found in Quigley (1982). The reason we assume linear budget constraints is simply that, as far as we know, the restrictions on preferences necessary to justify the existence of composite goods when budget constraints are nonlinear have not been derived in the literature. If these were to be derived it would be a straightforward exercise to generalize the proof in this paper to nonlinear constraints.

The approach common in the literature has been to assume that composite goods housing (H) and nonhousing (X) exist with respective unit prices p_H and p_X . It is well-known, however, that analyzing the consumer's problem of allocating expenditures to broad commodity groups in terms of a single price and quantity index is not generally possible. Within the framework of two-stage budgeting Gorman (1959) derived the appropriate restrictions on preferences that guarantee the existence of composite goods.³ One set of sufficient conditions is to assume that preferences are weakly separable and that the subutility functions $g_k(\cdot)$ defined on commodities in group k are homothetic. Alternatively, the existence of broad aggregated can be shown if the utility function is explicitly additive and the group indirect utility functions take the Gorman generalized polar form.⁴

It is important to formalize the implications of constructing composite commodities housing and other goods within the framework of two stage budgeting.

Consider a consumer facing a given set of prices for housing attributes and other goods. Letting starred variables denote utility maximizing quantities the composites can be written as, see De Borger (Forthcoming),

$$H^* = f_H\{g_H(h_1^*, \dots, h_n^*)\} \quad (3)$$

$$X^* = f_X\{g_X(x_1^*, \dots, x_m^*)\} \quad (4)$$

with respective unit prices $p_H = b_H(p_1, \dots, p_n)$ and $p_X = b_X(q_1, \dots, q_m)$, where the $f_i\{\cdot\}$ are monotonically increasing functions and the $b_i\{\cdot\}$ are functions homogeneous of degree one. Equations (3) and (4) imply that

$$u(g_H(h_1^*, \dots, h_n^*), g_X(x_1^*, \dots, x_m^*)) = u(f_H^{-1}(H^*), f_X^{-1}(X^*)) \quad (5)$$

³ There are other ways to justify the use of composite goods, e.g., through the use of Hicks' composite commodity theorem, which imposes restrictions on relative prices. In this paper we focus on restrictions on preferences, however.

⁴ For more details about Gorman (generalized) polar forms, see, e.g. Deaton and Muellbauer (1980: 130 - 131, 144 - 145).

Moreover, the h_i^* and x_j^* maximize the subutility functions $g_H(h_1, \dots, h_n)$ and $g_X(x_1, \dots, x_m)$ subject to the respective constraint

$$\sum_{i=1}^n p_i h_i = p_H H^* \quad \text{and} \quad \sum_{j=1}^m q_j x_j = p_X X^*.$$

These results have important implications for the use of composite goods housing and nonhousing when restrictions on quantity are imposed, as may be the case under housing programs. To see this, consider a consumer having a utility function defined on housing characteristics and other goods for which Gorman's conditions are satisfied and, as a consequence, composite goods H and X can be defined, using (3) and (4). Suppose the consumer participates in a housing program. It is not necessary at this moment to specify the exact nature of this program. Suppose that the consumer, under the program, is observed to live in a housing unit offering h_i' of housing characteristic i and is paying a rent equal to R , which leaves $(y-R)$ to be allocated to other goods. Suppose he is observed to consume x_j' of commodity j . The total market value of housing and nonhousing goods consumed under the program is

$$\sum_{i=1}^n p_i h_i' = N \quad \text{and} \quad \sum_{j=1}^m q_j x_j' = M, \quad \text{respectively, where } (N+M) \text{ will}$$

generally exceed income y .

Assume that we are interested in evaluating the welfare effects of this housing program and that we, rather than using the original utility function, specify preferences in terms of the composite goods H and X. Suppose we follow the procedure prevalent in the literature and implicitly determine the quantities consumed under the program, H' and X' , on the basis of the market value of housing and other goods:

$$p_H H' = N (= \sum_{i=1}^n p_i h_i')$$

$$p_X X' = M (= \sum_{j=1}^m q_j x_j')$$

Under what conditions will the quantities H' and X' so determined satisfy the equations

$$H' = f_H\{g_H(h'_1, \dots, h'_n)\} \quad (6)$$

$$X' = f_X\{g_X(x'_1, \dots, x'_m)\} \quad (7)$$

These equalities would imply that the two alternative approaches would indicate the same utility level under the program. Indeed we would then have:

$$u(f_H^{-1}(H'), f_X^{-1}(X')) = u(g_H(h'_1, \dots, h'_n), g_X(x'_1, \dots, x'_m)) \quad (8)$$

A corollary of the implications of constructing composite goods within the framework of two-stage budgeting is that, under the stated conditions, equations (6), (7) and (8) will only hold if the quantities h'_i and x'_j consumed under the program maximize the respective subutility functions $g_H(\cdot)$ and $g_X(\cdot)$ subject to the respective constraints $\sum_{i=1}^n p_i h'_i = p_H H' = N$ and $\sum_{j=1}^m q_j x'_j = p_X X' = M$. In other words, if the combination of housing attributes consumed under the program is the consumer's most desired choice given the market rent of the housing unit, and if the individual is, in addition, allowed to freely allocate his nonhousing expenditures $(Y-R)$ among different other goods x_j , then the composite commodity approach and the alternative approach would lead to the same utility level under the housing program. In all other cases, in which the h'_i and x'_j fail to maximize the respective subutility functions, we have the inequalities

$$H' > f_H\{g_H(h'_1, \dots, h'_n)\}$$

$$X' > f_X\{g_X(x'_1, \dots, x'_m)\}$$

and, by implication,

$$u(f_H^{-1}(H'), f_X^{-1}(X')) > u(g_H(h'_1, \dots, h'_n), g_X(x'_1, \dots, x'_m))$$

Most housing programs do not impose restrictions on the consumption of other goods x_j . Participating households can reasonably be assumed to freely allocate their nonhousing expenditures $(y-R)$ to different goods and services. Therefore, if one assumes that Gorman's restrictions on preferences hold, evaluating the welfare effects of housing programs using a composite X is theoretically acceptable. In the next section we therefore focus on the housing concept used. We will compare the use of the composite good housing services with the use of the bundle of attributes h_i .

2. Alternative housing concepts and the Hicksian surplus measures of welfare change

In this section we focus on the theoretical implications of using different housing concepts for the estimation of the equivalent and compensating surplus of housing programs. We assume that the restrictions on preferences stated in the previous section are satisfied, so that it is theoretically legitimate to define composite goods housing and other goods. For reasons spelled out before, our only concern is with the specification of the housing commodity.

Consider the utility function $u(g_H(h_1, \dots, h_n), X)$, where X is all other goods. Without loss of generality we treat X as a numeraire good with unit price equal to one. Assuming utility maximizing behavior a composite H can be defined as

$$H^* = f_H(g_H(h_1^*, \dots, h_n^*))$$

which implies $u(g_H(h_1^*, \dots, h_n^*), X^*) = u(f_H^{-1}(H^*), X^*)$

We now consider the consumer in each of two different states. The initial situation, state 0, is characterized by a vector of housing attributes and other goods $(h_1^0, \dots, h_n^0, X^0)$ and corresponding utility level $v_A^0 = u(g_H(h_1^0, \dots, h_n^0), X^0)$, where the subscript A refers to specification of utility on attributes. It should be emphasized that state 0 is not required to be a utility maximizing

situation given prevailing prices p_i^0 and income y due to, e.g., constraints on the quantity of housing attributes consumed. Similarly, state 1 is characterized by a vector $(h_1^1, \dots, h_n^1, X^1)$ and corresponding utility level $v_A^1 = u(g_H(h_1^1, \dots, h_n^1), X^1)$. Again this situation need not be optimal for the consumer but may correspond to quantity rationing. As in most applied studies on housing programs it is assumed that market prices and income do not change due to the transition from state 0 to state 1.

We are interested in evaluating the change in welfare between states 0 and 1. Consider a consumer in state 0. Hicks' equivalent surplus ES_A is defined as the increase in the numeraire good X required to bring the consumer to utility level v_A^1 , reached in state 1, assuming that the quantities of housing attributes remain fixed at h_i^0 . It is implicitly defined by the equation

$$u(g_H(h_1^0, \dots, h_n^0), X^0 + ES_A) = v_A^1 \quad (9)$$

Similarly, for a consumer in state 1 Hicks' compensating surplus, CS_A , is defined as the decrease in X required to bring the consumer back to the initial utility level v_A^0 , assuming that the quantities of housing attributes remain fixed at h_i^1 . It is obtained as the solution of the implicit equation

$$u(g_H(h_1^1, \dots, h_n^1), X^1 - CS_A) = v_A^0 \quad (10)$$

Extremely simple expressions for the Hicksian surplus measures can be derived through the use of a distance function recently proposed by Pauwels (1985), and extensively used by De Borger and Pauwels (1985).⁵ Corresponding to the utility function $u(g_H(h_1, \dots, h_n), X)$ this distance function gives, for any vector of housing attributes, the quantity of the numeraire commodity X required for the consumer to reach a given utility level v . In other words it is the solution of the implicit equation $u(g_H(h_1, \dots, h_n), X) = v$ for X . Given the separable structure of the direct utility function the distance function can be written as $D(g_H(h_1, \dots, h_n), v)$. It follows that $u(g_H(h_1, \dots, h_n), X) = v$ if and

⁵ This distance function is similar to, but distinctly different from, the one proposed and discussed by Deaton (1979).

only if $X = D(g_H(h_1, \dots, h_n), v)$. Assuming positive marginal utilities for h_i and X it can easily be shown that the distance function $D(\cdot)$ is decreasing in the quantity of housing goods and increasing in utility v .⁶

Using equation (9) and the definition

$$u\{g_H(h_1^0, \dots, h_n^0), D(g_H(h_1^0, \dots, h_n^0), v_A^1)\} = v_A^1$$

we immediately derive a simple expression for ES_A , viz.

$$ES_A = D(g_H(h_1^0, \dots, h_n^0), v_A^1) - X^0 \quad (11)$$

In a similar way the compensating surplus defined in (10) can be written in terms of the distance function as follows

$$CS_A = X^1 - D(g_H(h_1^1, \dots, h_n^1), v_A^0). \quad (12)$$

At a theoretical level, suppose that, rather than using the utility function $u(g_H(h_1, \dots, h_n), X)$, one prefers to specify preferences in terms of the corresponding utility function defined on composite goods housing and other goods, $u(f_H^{-1}(H), X)$. The corresponding distance function would, as a consequence, be defined in terms of the housing composite as well, viz. $D(f_H^{-1}(H), v)$. Following current practice in the literature state 0 would be characterized by quantities H^0 and X^0 , where $p_H H^0 = \sum_{i=1}^n p_i h_i^0$. The utility level would be $v_C^0 = u(f_H^{-1}(H^0), X^0)$, where the subscript C refers to the composite commodity approach.

In a similar way, state 1 would be characterized by quantities H^1 and X^1 , where the equality $p_H H^1 = \sum_{i=1}^n p_i h_i^1$ would hold. The consumer would be assumed to attain a utility level $v_C^1 = u(f_H^{-1}(H^1), X^1)$.

The equivalent surplus measure ES_C for the transition from state 0 to state 1 in composite commodity space would be obtained by solving the equation

$$u(f_H^{-1}(H^0), X^0 + ES_C) = v_C^1$$

⁶ See Pauwels (1985, p.5).

which yields

$$ES_C = D(f_H^{-1}(H^0), v_C^1) - x^0. \quad (13)$$

Similarly, the compensating surplus measure CS_C would be calculated to be

$$CS_C = x^1 - D(f_H^{-1}(H^1), v_C^0). \quad (14)$$

Within this highly idealized framework we can compare the surplus measures of the transition from state 0 to state 1 that would result using the composite commodity approach (equations (13) and (14) with those one would obtain using the specification on housing attributes and other goods (equations (11) and (12)). Obviously, different housing programs impose different constraints on consumption in either of the states 0 and 1 so that different cases will have to be considered. We will show that in most cases the use of the composite commodity approach leads to biased results as compared to the true welfare effect that takes into account the composition of the bundle of housing attributes in each of the two states.

Case 1

Suppose that both the initial and the final state correspond to situations where households are consuming their most desired combination of housing attributes, given the market rent of the housing unit they occupy. This would be the case, e.g., for a pure price subsidy program. Some housing allowances (see, e.g., Cronin (1983) or Reeder (1983)) in which the consumer is restricted in terms of the market rent of the unit occupied under the program, but in which he is allowed to freely choose his most preferred bundle of attributes as long as it satisfies the rent restrictions, would also be consistent with the case being considered here.

Under the stated conditions it is easy to show that the two alternative approaches to benefit estimation lead to the same result. Indeed, using the argument developed in the previous section we have in this case the following equalities

11.

$$H^0 = f_H(g_H(h_1^0, \dots, h_n^0)) \quad (15)$$

$$H^1 = f_H(g_H(h_1^1, \dots, h_n^1)) \quad (16)$$

which immediately implies, using the definitions of v_C^0 and v_A^0 ,

$$v_C^0 = v_A^0. \quad (17)$$

Appropriately using (15), (16) and (17) in the definitions of the surplus measures as given in (11), (12), (13) and (14) yields

$$ES_C = ES_A$$

$$CS_C = CS_A.$$

Case 2

Consider the case where in the initial situation the consumer can freely allocate housing expenditures among different attributes. In state 1, however, a housing program imposes restrictions on the quantities of attributes consumed so that participating households are unable, given the market rent of the unit, to choose their most preferred combination of attributes.

There are at least two major housing programs that are consistent with this description. Both public housing programs (see, e.g., Clemmer (1984), Schwab (1985)) and rent control in the U.S. (see Olsen 1972) can be interpreted as corresponding to a transition from an uncontrolled situation in the absence of the program to a situation in which families face an all-or-nothing choice to consume a given bundle of housing attributes at a specified rent. This fixed bundle rarely, if ever, corresponds to households' most desired combination for the given market value of the unit. Moreover, some housing allowance programs impose explicit restrictions on the consumption of housing attributes, usually in the form of required minimum levels.

It easily follows that under the described conditions welfare measures based on the composite approach will be upward biased.

As state 0 imposes no restrictions on consumption of housing characteristics we have the equalities $H^0 = F_H(g_H(h_1^0, \dots, h_n^0))$ and

$$v_C^0 = v_A^0.$$

However, the suboptimality of attribute consumption in state 1 implies, see previous section,

$$H^1 > f_H(g_H(h_1^1, \dots, h_n^1))$$

from which it follows that $v_A^1 < v_C^1$.

Using these results appropriately in expressions (11) - (14), and taking into account that the distance function is decreasing in housing and increasing in utility we find $ES_C > ES_A$

$$CS_C > CS_A.$$

In other words, if one believes that the proper specification of preferences is in terms of housing attributes and other goods, the composite commodity approach overestimates the change in welfare due to, e.g., public housing or rent control.

The situation is illustrated on figure 1. There we present a simple indifference curve diagram in which we measure both the quantity $f_H^{-1}(H)$ and the quantity $(g_H(h_1, \dots, h_n))$ on the horizontal axis. It is easily seen that the inequalities $v_A^1 < v_C^1$ and $H^1 > f_H(g_H(h_1^1, \dots, h_n^1))$ imply differences in welfare measures. On the graph ES_C is indicated as the distance AC, ES_A as the distance BC, CS_C as the distance DF and, finally, CS_A as the distance EG.

Case 3

For theoretical purposes only we consider briefly the case where constraints on consumption of housing attributed are imposed in the initial situation but not in the final situation. This type of transition is unlikely to be very important in practice. It might occur, e.g., if we would be interested in evaluating the welfare change that would result from replacing a program discussed in case 2, say public housing, by another subsidy scheme that does not impose constraints on attribute consumption, such as a pure price subsidy. Such drastic changes are rarely, if ever, conceived in reality.

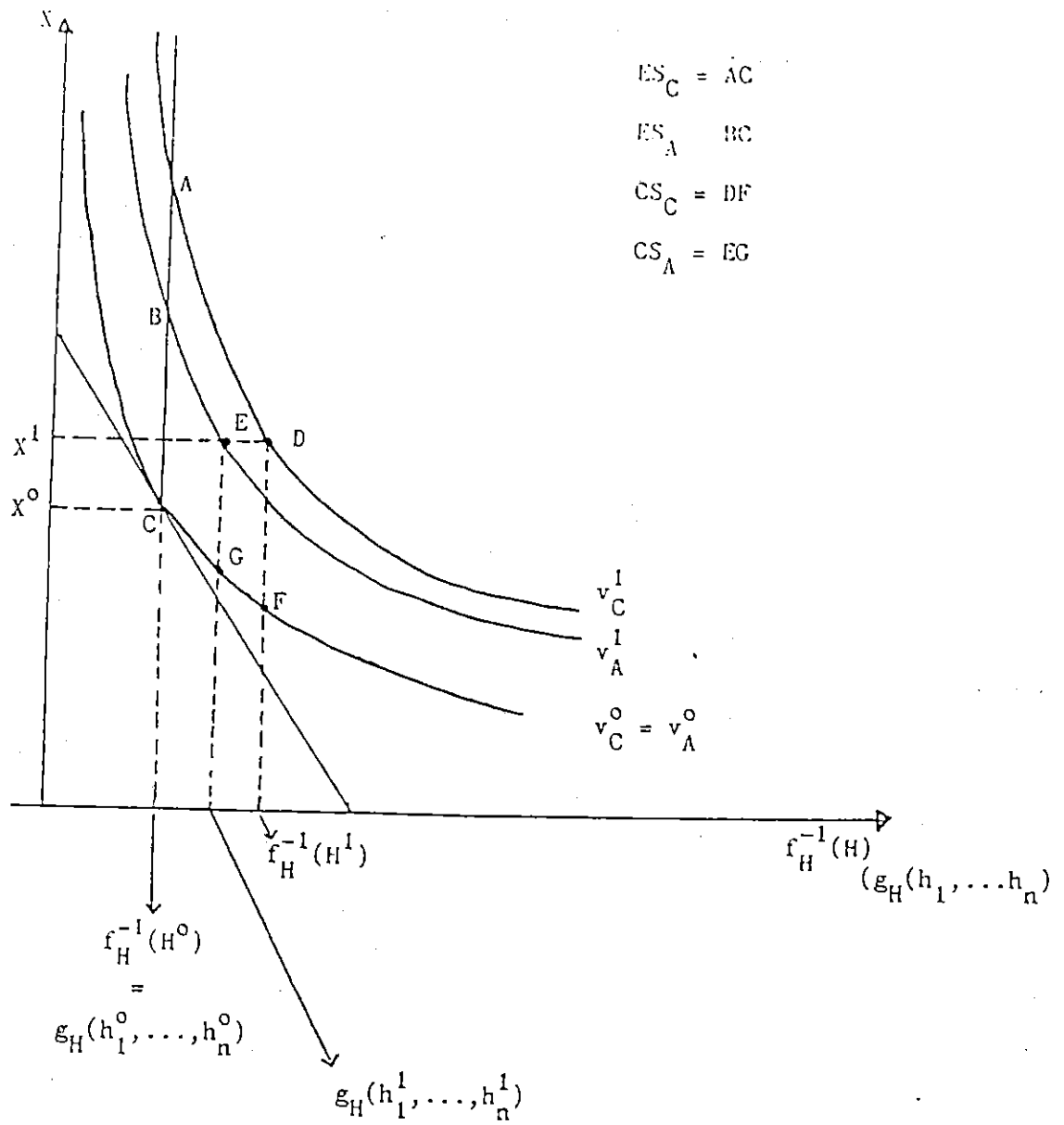


Figure 1 : Comparison of Hicksian surplus measures (Case 2).

For a transition of state 0 to 1 under the stated assumptions we would have that $H^0 > f_H(g_H(h_1^0, \dots, h_n^0))$ and $v_C^0 > v_A^0$. However the equalities $H^1 = f_H(g_H(h_1^1, \dots, h_n^1))$ and $v_C^1 = v_A^1$ corresponding to state 1 would hold. Then using the definitions of the surplus measures in (11) - (14) and the properties of the distance function it immediately follows

$$ES_C < ES_A$$

$$CS_C < CS_A.$$

In other words, a transition of type 3 would imply a downward bias in measures of welfare change based on the composite commodity approach. This is not surprising given the results obtained for case 2, since the equivalent surplus of a transition of state 0 to 1 is the compensating surplus of the transition of state 1 to state 0. Figure 2 illustrates the downward bias in the composite commodity approach for case 3.

Case 4

Consider finally a transition from state 0 to state 1 where both states are characterized by restrictions on the consumption of housing attributes. This situation may arise if one were evaluating the change in welfare resulting from changes in program rules of existing housing subsidy programs that impose constraints on attribute consumption, i.e., programs considered in case 2. An example of this would be the upgrading of public housing units, possibly accompanied by a change in rent charged. Changing the restriction on attribute consumption under a housing allowance scheme would be another simple example.

Under the previous conditions it is easy to show that the composite commodity approach results in biased welfare measures but, unfortunately, the direction of the bias cannot be determined a priori. To show this proposition note that in this case the following inequalities hold

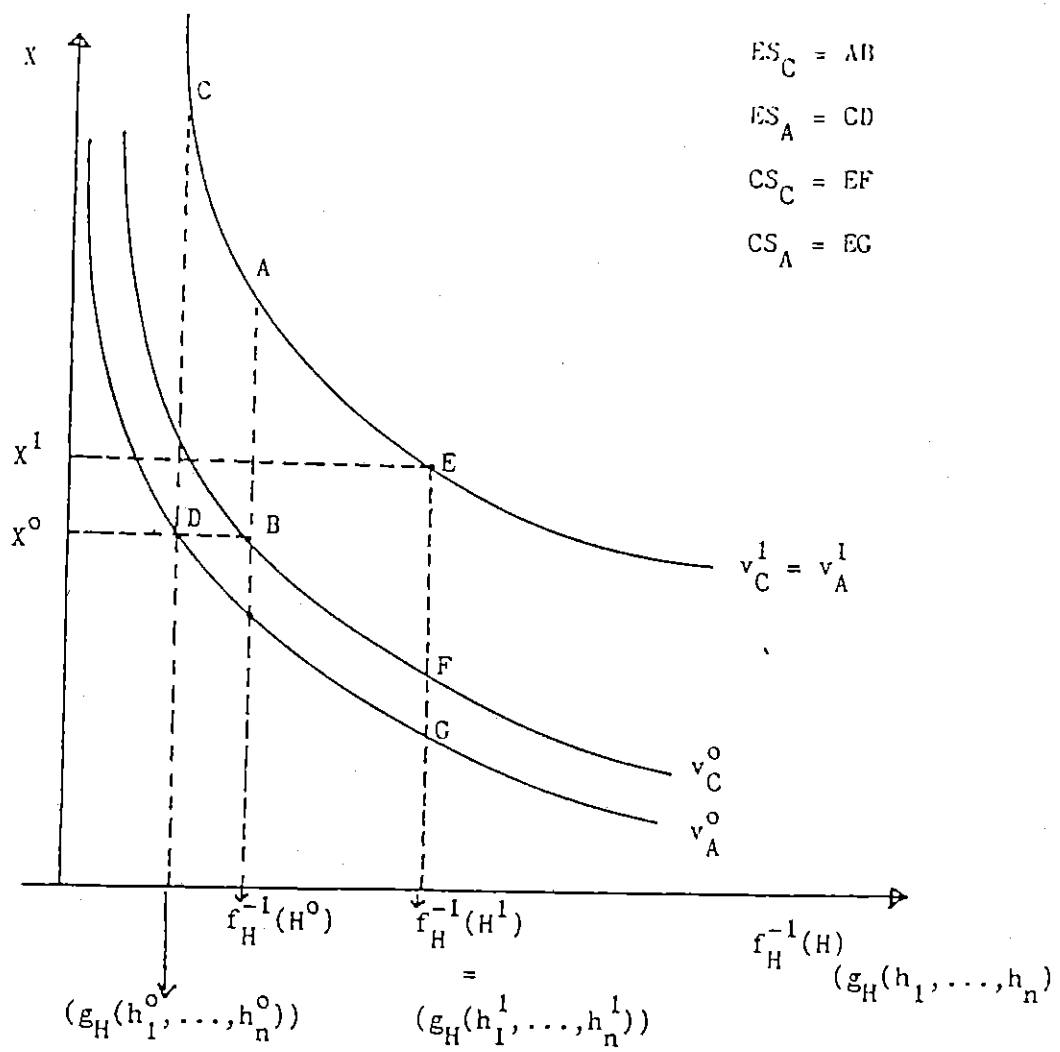


figure 2 : Comparison of Hicksian surplus measures (Case 3).

$$H^0 > f_H(g_H(h_1^0, \dots, h_n^0))$$

$$H^1 > f_H(g_H(h_1^1, \dots, h_n^1))$$

$$v_C^0 > v_A^0$$

$$v_C^1 > v_A^1$$

Using these inequalities in (11) -(14) and applying the economic properties of the distance function it immediately follows that, normally, $ES_C \neq ES_A$ and $CS_C \neq CS_A$. However, whether the outcomes based on the composite commodity approach will exceed those based on the housing characteristics approach depends on previous inequalities and on the exact structure of preferences. One would expect an upward bias if the attribute combination consumed in state 0 is close to the optimal combination, given the market rent. Alternatively, a downward bias is to be expected if consumption of attributes is close to the optimum in the final situation. The indeterminacy of the comparison of the two approaches to benefit estimation is illustrated on figure 3.

3. Empirical illustration

In the previous section we showed that the Hicksian surplus measures of welfare change calculated on the basis of the composite commodity approach may, depending on the precise characteristics of the housing program under consideration, overestimate or underestimate the true welfare effects resulting from a more realistic specification of the housing good as a bundle of attributes.

The purpose of this section is to illustrate the implications of the theoretical results for empirical work. Our approach is obviously somewhat different from the theoretical section. We consider two programs, public housing and a demand-oriented subsidy, to be described below. In each case we estimate, for a sample of participating households, the Hicksian surplus measures

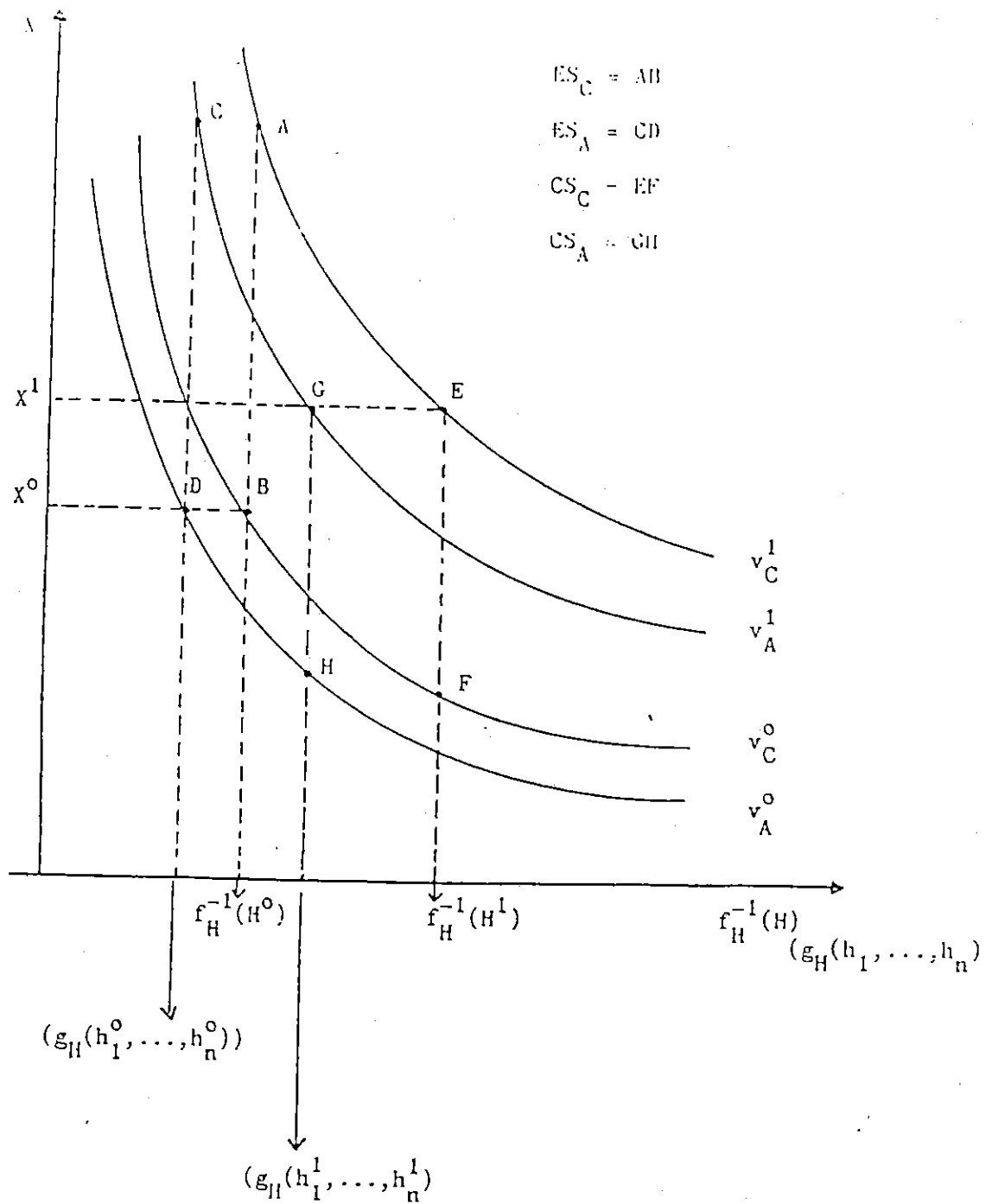


figure 3 : Comparison of Hicksian surplus measures (Case 4).

of welfare change due to the introduction of these programs. For each program the empirical exercise is performed twice, using each of the two alternative approaches. We then compare the empirical results in order to find out whether they are consistent with the theoretical predictions of the previous section.

We use Cobb-Douglas and Stone-Geary specifications for the utility function, both of which satisfy the restrictions on preferences necessary to construct composite good. The utility functions are summarized in tabel 1. The corresponding expressions for Hicks' equivalent and compensating surplus are given in table 2. Note that the quantities consumed in state 0 refer to consumption in the absence of the housing program under consideration, whereas state 1 corresponds to the situation under the program.

The data used for this study were derived from a budget survey conducted by the Center for Population and Family Studies which is affiliated with the Belgian Ministry of Health and the Family. The survey was conducted in the early seventies and restricted to a relatively homogeneous area, viz. the central city of Liège. The sample was small: it contained 218 useful observations, of which 59 referred to households in public housing.

To estimate the welfare effects of public housing using the composite commodity approach we predicted the market value of public units by hedonic pricing techniques. The parameters of the respective utility functions were obtained from estimates of the appropriate budget share equations for housing. Applications of the attribute approach to benefit estimation required estimates of the parameters of the system of demand functions for housing attributes and other goods that corresponds to each of the two specifications of preferences. In each case we allowed the utility function parameters to be different for families with different observed characteristics so as to account for some variations in tastes. The estimated demand functions were also used to predict the quantities of housing and other goods public housing tenants would have consumed in the absence of the program. Once estimates for all inputs in the formulas given in table 2 have been derived,

Table 1 : utility function specifications

$u(H, X)$	Cobb-Douglas $u(H, X) = H^\alpha X^{1-\alpha}$
	Stone-Geary $u(H, X) = (H - \beta_H)^\gamma (X - \beta_X)^{1-\gamma}$
$u(h_1, \dots, h_n, X)$	Cobb-Douglas $u(h_1, \dots, h_n, X) = \left[\prod_{i=1}^n h_i^{\alpha_i} \right] X^{1 - \sum_{i=1}^n \alpha_i}$
	Stone-Geary $u(h_1, \dots, h_n, X) = \left[\prod_{i=1}^n (h_i - \beta_i)^{\gamma_i} \right] (X - \beta_X)^{1 - \sum_{i=1}^n \gamma_i}$

Table 2 : Hicks' equivalent and compensating surplus for alternative specifications of the utility function

	$ES_C = \left(\frac{H^I}{H^O} \right)^{\frac{\alpha}{1-\alpha}} X^I - X^O$
	<p>Cobb-Douglas</p> $CS_C = X^I - \left(\frac{H^O}{H^I} \right)^{\frac{\alpha}{1-\alpha}} X^O$
$u(H, X)$	$ES_C = \left[\frac{H^I - \beta_H}{H^O - \beta_H} \frac{Y}{1-\gamma} (X^I - \beta_X) \right] + \beta_X - X^O$
	<p>Stone-Geary</p> $CS_C = X^I - \beta_X + \left[\frac{H^O - \beta_H}{H^I - \beta_H} \frac{Y}{1-\gamma} (X^O - \beta_X) \right]$
	$ES_A = \left[\prod_{i=1}^n \frac{h_i^I}{h_i^O} \frac{\alpha_i}{1-\sum \alpha_i} \right] X^I - X^O$
	<p>Cobb-Douglas</p> $CS_A = X^I - \left[\prod_{i=1}^n \frac{h_i^O}{h_i^I} \frac{\alpha_i}{1-\sum \alpha_i} \right] X^O$
$u(h_1, \dots, h_n)$	$ES_A = \left[\prod_{i=1}^n \frac{h_i^I - \beta_i}{h_i^O - \beta_i} \frac{\gamma_i}{1-\sum \gamma_i} (X^I - \beta_X) \right] + \beta_X - X^O$
	<p>Stone-Geary</p> $CS_A = X^I - \beta_X + \left[\prod_{i=1}^n \frac{h_i^O - \beta_i}{h_i^I - \beta_i} \frac{\gamma_i}{1-\sum \gamma_i} (X^O - \beta_X) \right]$

it is a straightforward exercise to evaluate the surplus measures of welfare change for each household in public housing.

Public housing programs offer selected eligible households an all-or-nothing choice to consume a given bundle of attributes at a specified rent. They are clear examples of the second case considered in the previous section. The prediction was that the surplus measured based on the composite commodity approach would be upward biased. Empirical results summarized in table 3 are consistent with this prediction. Mean benefits based on the composite good housing are between 45 % and 55 % higher than those estimated using the attribute approach. This result consistently holds both for the equivalent and compensating surplus and for both specifications of preferences. It confirms the upward bias, and it suggests that its magnitude may be non-trivial when evaluating the implications of existing housing programs that impose restrictions on attribute consumption.

Estimates of the parameters of the utility functions for different types of households also made it possible to simulate the welfare effects of a demand-oriented housing program. The second program considered in this paper combines a constrained price subsidy with a system of unrestricted cash grants. The program rules may be summarized as follows.

Let y_{Li} be the income limit for eligibility for families of type i and let R_{Li} be the average housing expenditures for families of this type with income y_{Li} . Then the program provides a price subsidy to all eligible families of type i who consume housing with a market rent larger than a required minimum R_{Mi} but smaller than R_{Li} . The fraction of the rent that would be paid by the government would depend on the household's income relative to the upper income limit for eligibility. Obviously, the minimum R_{Mi} would reflect the government's view with respect to minimal acceptable housing standards. Families of type i who prefer to consume housing with a market rent in excess of R_{Li} would not explicitly be subsidized to consume even better housing but would instead receive a cash grant, the amount again depending on their income relative to y_{Li} .

Table 3 : Estimated surplus measures of public housing using alternative specifications : mean and observed standard deviation (in 10^2 Belgian francs per month)

		Mean Benefit	Observed standard deviation over the sample	
u(H,X)	Cobb-Douglas	ES _C	5.02	5.12
		CS _C	8.06	5.17
	Stone-Geary	ES _C	5.74	5.22
		CS _C	5.02	5.06
u(h ₁ ,...,h _n ,X)	Cobb-Douglas	ES _A	5.83	5.32
		CS _A	5.29	5.41
	Stone-Geary	ES _A	5.67	4.98
		CS _A	5.23	5.38

More specifically, the program would provide subsidies as follows:

$$\begin{aligned}
 \text{(price subsidy)} \quad S &= R \left(\frac{Y_{Li} - Y}{Y_{Li}} \right) && \text{if } R_{Mi} \leq R \leq R_{Li} \\
 \text{(cash grant)} \quad S &= R_{Li} \left(\frac{Y_{Li} - Y}{Y_{Li}} \right) && \text{if } R > R_{Li} \\
 S &= 0 && \text{if } R < R_{Mi}
 \end{aligned}$$

where R is the market rent of the unit under the program and y is the household's income. The program has many desirable features, which are extensively discussed in De Borger (1986).

We simulated the effects of the proposed program using both the composite commodity approach and the alternative specification based on housing as a bundle of attributes. To this purpose we first designed a set of income limits for eligibility and a set of minimum market rents for each family type.

With this information we determined, using the estimated Cobb-Douglas and Stone-Geary utility function parameters, the optimal consumption bundle under the program for each eligible family and for both specifications of the housing commodity.

Given the consumption bundles under the program it was a straightforward exercise to evaluate the Hicksian surplus measures of consumer benefit using the expressions given in table 2.

The program described above does impose minimum rent requirements but does not explicitly restrict the consumption of housing attributes. Under the program households can choose the most desired combination of housing characteristics for any market rent satisfying the minimum restriction. As a consequence, within the framework presented above, the theoretical prediction derived from the previous section (case 1) is that the composite commodity approach and the approach based on attributes will lead to the same result. For applied work we would expect the two approaches to yield very similar, though obviously not identical, results. The estimated surpluses summarized in table 4 indicate that, on average, this hypothesis is strongly confirmed. Mean benefits based on the composite commodity approach are extremely close to

Table 4 : Estimated surplus measures of demand-oriented housing program using alternative specifications : mean and observed standard deviation (in 10^2 Belgian francs per month).

		Mean Benefit	Observed standard deviation over the sample	
$u(H,X)$	Cobb-Douglas	ES_C	5.12	6.36
		CS_C	4.71	6.11
	Stone-Geary	ES_C	5.22	6.81
		CS_C	4.72	6.18
$u(h_1, \dots, h_n, X)$	Cobb-Douglas	ES_A	5.30	5.91
		CS_A	4.82	6.23
	Stone-Geary	ES_A	5.08	5.80
		CS_A	4.59	5.67

the corresponding figures for the attribute approach. This result holds both for the equivalent and compensating surplus measure and for the Cobb-Douglas and Stone-Geary utility functions. In all cases, differences in mean benefit due to a different specification of the housing commodity are less than 3.5 %.

Both the analysis of public housing and the demand-oriented program produced results that are consistent, on average, with the theoretical predictions based on the highly stylized model presented in the previous section. Although no data were available that would allow us to estimate the welfare implications of programs classified under cases 3 and 4, the empirical evidence obtained for the two housing programs considered is at least quite remarkable. Admittedly, the results should be confirmed by other studies using larger data sets before any definite conclusions can be drawn with respect to the magnitude of the potential bias implied in the composite commodity approach. The results of this paper strongly suggest, however, that it is important, in order to obtain reliable welfare results, to take into account the composition of the bundle of housing characteristics when evaluating programs that impose restrictions on the quantity of attributes consumed. For programs that allow households to freely choose their most preferred attribute bundle conditional on the market rent of the unit, the evidence suggests that the composite commodity and attribute approaches to benefit estimation will lead to fairly similar results.

Summary and conclusion

The purpose of this paper was to elaborate on previous research dealing with the implications of using alternative housing concepts for the evaluation of the benefits of housing programs. In one approach housing is defined in terms of a composite good housing services, whereas the alternative approach treats housing as a bundle of characteristics. The welfare measures considered in this paper, Hicks' equivalent and compensating surplus, are generally applicable. They are particularly suited to analyse welfare changes in quantity constrained regimes. As such, they can be used to evaluate the effects of a much broader class of housing programs than other welfare measures that have been discussed elsewhere in the literature.

In the theoretical part of the paper we assumed that the restrictions on preferences necessary to justify the construction of composite goods housing and all other goods were satisfied. Under those assumptions we then compared, for different types of housing programs, the Hicksian surplus measures of welfare change based on the composite commodity approach with those that would result from application of the alternative approach based on attributes. Treating a housing program as a transition from an initial situation, state 0, to a final state, state 1, it was shown that whenever restrictions on the quantities of housing attributes are imposed in at least one of the two states, the two approaches will lead to different results. If one believes that preferences are to be defined on housing attributes and other goods, this implies that the composite commodity approach leads to systematically biased welfare implications. It was shown that the direction of the bias depends on the exact nature of the housing program under consideration.

In the empirical section of the paper we considered two different housing programs, estimated the Hicksian surplus measures for a sample of participating households using each of the two alternative approaches and Cobb-Douglas and Stone-Geary specifications for the utility functions, and compared the empirical results. For

the first program, public housing, the empirical findings strongly confirmed the prediction derived from the theoretical analysis, which suggested that the composite commodity approach would overestimate the true benefit obtained using the approach based on housing attributes. The upward bias in mean benefit implied by the use of the composite good housing was estimated to be in the range of 45 % to 55 % for our sample.

Empirical results for the second program, a combination of a price subsidy with cash grants which does not impose restrictions on the consumption of attributes, again confirmed the theoretical prediction. The two alternative specifications of the housing commodity in this case yielded negligible differences in mean benefit.

References

- Clemmer, R.B., 1984, Measuring Welfare Effects of In-Kind Transfers, Journal of Urban Economics, 14, 46-65.
- Cronin, F.J., 1983, The Efficiency of Demand-Oriented Housing Programs, Journal of Human Resources, 18, 100-125.
- Deaton, A.S., 1979, The Distance Function in Consumer Behavior with Application to Index Numbers and Optimal Taxation, Review of Economic Studies, XLVI, 391-405.
- Deaton, A.S. and J. Muellbauer, 1980, Economics and Consumer Behavior, Cambridge, Cambridge University Press.
- De Borger, B., Forthcoming, Alternative Housing Concepts and the Benefits of Public Housing Programs, Journal of Urban Economics.
- De Borger, B., 1986, In-Kind Redistribution and Demand-Oriented Housing Subsidies: Some Micro-Simulation Results, Public Finance Quarterly, 14, 235-261.
- De Borger, B. and W. Pauwels, 1985, Some Implications of a New Distance Function for Applied Welfare Economics, Antwerp, SESO, Paper 85/179.
- Gorman, W., 1959, Separable Utility and Aggregation, Econometrica, 27, 469-481.

- Muth, R.F., 1960, The Demand for Non-Farm Housing, in Harberger, A. (ed., The Demand for Durable Goods, Chicago, University of Chicago Press.
- Olsen, E.O., 1969, A Competitive Theory of the Housing Market, American Economic Review, 59, 612-622.
- Olsen, E.O., 1972, An Econometric Analysis of Rent Control, Journal of Political Economy, 80, 1081-1100.
- Pauwels, W., 1985, Measuring Welfare Changes in Quantity-Constrained Regimes, Antwerp, SESO, Paper 85/174.
- Quigley, J.M., 1979, What have we learned about housing markets, pp.391-429 in Mieszkowski, P. and M.R. Straszheim, eds., Recent Issues in Urban Economics, Baltimore, The John Hopkins University Press.
- Quigley, J.M., 1982, Nonlinear Budget Constraints and Consumer Demand: An Application to Public Programs for Residential Housing, Journal of Urban Economics, 12, 177-201.
- Quigley, J.M., 1986, The Evaluation of Complex Urban Policies, Regional Science and Urban Economics, 16, 31-42.
- Reeder, W.J., 1983, An Economic Evaluation of the Section 8 Existing Housing Program, Unpublished Ph. D. dissertation, Charlottesville, University of Virginia.
- Schwab, R.M., 1985, The Benefits of In-Kind Government Programs, Journal of Public Economics, 27, 195-210.