STUDIECENTRUM VOOR ECONOMISCH EN SOCIAAL ONDERZOEK

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- no 2 -

ARSTU
User's Manual

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Deze software is tot stand gekomen mede dank zij de steun van de Christelijke Centrale van Houtbewerkers en Bouwvakarbeiders.

We thank Mr. H. Pauwels. Without his help this program would never have been achieved.

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Abstract

ARSTU is a computer program for the estimation of Auto Regressive Models for STationary Time Series in the Univariate case.

Two modelling procedures are available : a Minimum AIC procedure and a Quasi Bayesian procedure. In order to evaluate the estimated AR-model the computation of the characteristic roots of the polynomial in the lag operator and an estimation of the spectrum based on the estimated AR-coefficients are included as additional features.

The input data file for ARSTU can also be used for the TSERS, PACK (PACK 1, PACK 2 & Pack 3), MULTISTOCH and MULTITRAN programs.

TABLE OF CONTENTS

1. GENERAL DESCRIPTION

ARSTU is a computer program for the estimation of

Auto
Regressive Models for
STationary Time Series in the
Univariate case

Two modelling procedures are available:

1. Minimum AIC Procedure

   The method of LEAST SQUARES is applied to estimate the parameters of a particular model and an information criterion is computed by

$$\text{AIC} = -2 \ln \left(\begin{matrix}\text{innovation}\\ \text{variance}\end{matrix}\right) + 2 \left(\begin{matrix}\text{number of}\\ \text{parameters}\end{matrix}\right)$$

   The model which attains the minimum of the AIC's among various models is selected as the final choice.

2. Quasi Bayesian Procedure

   The quasi BAYESIAN procedure is obtained by taking the average of the models, with the parameters estimated by the method of LEAST SQUARES, with respect to the "posterior" probability which is by definition proportional to

$$e^{-\frac{1}{2}\,\text{AIC}} \times \text{(prior probability of the model)}.$$

   It is expected that this Bayesian type model fitting procedure will reduce the risk of adopting too high order models by the simple minimum AIC procedure.

Apart from the LEAST SQUARES estimation method also the MAXIMUM LIKELIHOOD estimation method is incorporated in the ARSTU program. This feature makes it possible to use the autoregressive coefficients, estimated by Least Squares, as (good) starting values for the exact GAUSSIAN LIKELIHOOD computation of the AR model.

In order to evaluate the estimated AR model two additional features are included in the ARSTU program.

1. Characteristic Roots

   If requested the stability of the AR operator can be checked by computing the characteristic roots of the polynomial in the lag operator.

2. Spectrum estimation

   To facilitate the understanding of the behaviour of the time series considered the estimated AR coefficients can be used to derive the spectrum of that time series.

The ARSTU program is mainly based on three programs from the TIMSAC-78 computer package, implemented by H. AKAIKE from the Institute of Statistical Mathematics (Tokyo). These three programs are

1. UNIMAR (TIMSAC 78.1.1)

   UNIvariate Case of
   Minimum AIC Method of
   AR Model Fitting

2. UNIBAR (TIMSAC 78.1.2)

   UNIvariate Case of
   Bayesian Method of
   AR Model Fitting

3. EXSAR (TIMSAC 78.5.1)

   EXact Maximum Likelihood Method of
   Scalar
   AR Model Fitting

For more details on the TIMSAC-78 computer package, which is a continuation of the former packages TIMSAC and TIMSAC-74, see |7| and also |8|, |5| & |6|.

2. ALGORITHMIC AND COMPUTATIONAL ASPECTS

In this section the most important algorithmic and computational aspects of ARSTU will be discussed.

2.1. Subroutine REDATA

This subroutine, which is a TIMSAC-78 subroutine, is used for the loading of the original data and deletion of the mean value.

The data are loaded through the device specified by MT, i.e. through a datafile on disk (MT = 1) or the terminal keyboard (MT = 0).

For further details about the input of the time series data see Section 3.2.

2.2. Subroutine REDUCT

This subroutine, which is a TIMSAC-78 subroutine, first sets up a data matrix X by augmenting the successively shifted original data vector Z and then transforms X into triangular form by the HOUSEHOLDER transformation. This transformation is applied directly to the data matrix to avoid the computation of sample covariances.

This procedure allows a very simple revision of models when a new additional set of data is added to the original one.

For further information on the use of the HOUSEHOLDER transformation in this kind of applications see |12|.

2.3. Subroutine ARMFIT

This is the basic subroutine for the UNIvariate case of Minimum AIC method of AR model fitting.

This subroutine fits autoregressive models of successively higher orders by the method of LEAST SQUARES and realized through HOUSEHOLDER's transformation |12|.

First the data matrix X is generated where X depends on the model to be fitted. For the fitting of a regression model

$$x_n = a_1 y_{n1} + a_2 y_{n2} + \dots + a_M y_{nM} + \varepsilon_n$$

where

ε_n is a zero mean white noise process

and

$y_{nj} = x_{n-j}$

the X matrix is defined by

$$X = \begin{vmatrix} y_{1,1} & \cdots & y_{1,M} & x_1 \\ y_{2,1} & & y_{2,M} & x_2 \\ \vdots & & & \\ y_{N,1} & \cdots & y_{N,M} & x_N \end{vmatrix}$$

where the observations of M regressors constitute the first M columns and those of the regress and constitute the last column.

By applying the HOUSEHOLDER transformation X is transformed into an upper triangular matrix S

$$S = \begin{vmatrix} S_{1,1} & \cdots & S_{1,M} & S_{1,M+1} \\ & \ddots & & \\ 0 & & S_{M,M} & S_{M,M+1} \\ & & & S_{M+1,M+1} \end{vmatrix}$$

The AIC of the model, defined with the first m regressors, is obtained by

$$AIC(m) = N \ln|\hat{\sigma}^2(m)| + 2(m + 1)$$

where

$$\hat{\sigma}^2(m) = \sum_{i=m+1}^{M+1} S_{i,M+1}^2$$

The regression coefficients of the model with m regressors are then given by

$$a_m = S_{m,M+1} / S_{m,m}$$

$$a_i = |S_{i,M+1} - \sum_{j=i-1}^{m} a_j S_{j,M+1}| / S_{i,i} \qquad (i = m - 1, \ldots ,1)$$

For further details about the LEAST SQUARES method and the AIC procedure see |13| and |3|.

2.4. Subroutine ARBAYS

This is the basic subroutine for the UNI variate Bayesian method of AR model fitting (UNIBAR).

This subroutine fits an autoregressive model by a BAYESIAN procedure.

The posterior weight of the m^{th} order model is given by

$$w(m) = C\ \pi(m)\ c(m)$$

where the prior weight $\pi(m)$ is given by

$$\pi(m) = \frac{1}{m+1}$$

$c(m)$ is defined as

$$c(m) = \exp\{-\frac{1}{2}\ |AIC(m) - MAIC|\}$$

where MAIC is the minimum of AIC(m) and the normalizing constant is given by

$$C^{-1} = \sum_{m=0}^{M} \pi(m)\ c(m)$$

The partial autoregression coefficients are then computed as

$$b^*(m) = b(m)\ d(m)$$

where $b(m)$ is the least squares estimate and the integrated Bayesian weight $d(m)$ is defined as

$$d(m) = \sum_{i=m}^{M+1} c(i)$$

The autoregressive coefficients of the Bayesian model are then derived as a conversion from the set of partial autoregression coefficients $(b^*(m))$ to the autoregressive coefficients $a_i (i = 1,2,\ldots,M)$.

For evaluation purposes the innovation variance of the Bayesian model can be computed as

$$\sigma_B^2 = \frac{1}{N} \sum_{n=1}^{N} (x_n - \sum_{i=i}^{M} a_i y_{ni})^2$$

For the Bayesian model an equivalent AIC is defined as

$$AICB = N \ln|\sigma_B^2| + 2\ EK$$

where the equivalent number of parameters EK is given by

$$EK = \sum_{m=1}^{M} d(m)^2 + 1$$

For further details about this Bayesian procedure see |4| and |1|.

2.5. Subroutine ARMLE

This is the basic subroutine for the EXact maximum likelihood method of Scalar AR-model fitting (EXSAR).

This subroutine produces exact MAXIMUM LIKELIHOOD estimates of the parameters of a scalar AR-model.

Given the data $(x_1, x_2, \ldots x_N)$ the likelihood of the autoregressive model

$$x_n = a_1\ x_{n-1} + \ldots + a_m\ x_{n-m} + \varepsilon_n$$

where

$$\varepsilon_n \sim N(0, \sigma^2)$$

is obtained by

$$p(x_1, \ldots, x_N) \mid a_1, \ldots, a_m, \sigma^2)$$

$$= p(x_1, \ldots, x_m \mid a_1, \ldots, a_m, \sigma^2)$$

$$\prod_{i=m+1}^{N} p(x_i | x_{i-1}, \ldots, x_{i-m}, a_1, \ldots, a_m, \sigma^2)$$

where

$$p(x_i | x_{i-1}, \ldots, x_{i-m}, a_1, \ldots, a_m, \sigma^2)$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \exp \{- \frac{1}{2\sigma^2} (x_n - a_1 x_{n-1} - \ldots a_m\ x_{n-m})^2\}$$

and

$$p(x_1,...,x_m \mid a_1,...,a_m,\sigma^2)$$

is evaluated by using the theoretical autocovariance function $\rho_0,\rho_1,...,\rho_m$ determined by $a_1,...,a_m$ and σ^2.

For the minimization the DAVIDON-FLETCHER-POWELL procedure is used. (See |9| and |11|).

2.6. Subroutine ARMACR

This subroutine is mainly based on the TIMSAC-78 subroutine CHROOT.

The subroutine ARMACR finds the characteristic roots of

- autoregressive operator - all coefficients
- autoregressive operator - subset of coefficients
- AR.MA operators

The output consists of

- real part of the root (R)
- immaginary part of the root (I)
- modulus (SQRT (R ** 2 + I ** 2))
- phase in radii (ARCTAN (I/R))
- phase in degrees

Most of the calculations are performed by subroutine POLYRT. This subroutine computes the real and complex roots of a real polynomial by the NEWTON-RAPHSON iterative technique. The final iterations on each root are performed using the original polynomial rather than the reduced polynomial to avoid accumulated errors in the reduced polynomial.

Floating point overflow may occur for high order polynomials but will not affect the accuracy of the results. If the non-convergence condition occurs the error message

**** NON CONVERGENCE AT POLYRT ****

is given.

2.7. Subroutine ARMASE

This subroutine is mainly based on the TIMSAC-78 subroutine NRASPE.

The subroutine ARMASE computes the power spectrum of a
- AR-MA process
- AR process
- MA process

Outputs are given at frequencies I/(2 * H) where I = 0,1,...,H.

In computing this parametric power spectrum estimate use is made of the TIMSAC-78 subroutine FOUGER. This subroutine computes the Fourier Transform by GOERTZEL's method.

2.8. Subroutine ARMASP

This subroutine
- finds peak amplitudes
- plots all amplitudes versus frequencies
- prints peak amplitudes along with their respective periods
- prints the spectrum

Originally, the printer plot of the power spectrum was performed by the TIMSAC-78 subroutine NRASPE. However, this subroutine has been replaced by the subroutine ARSPLT which was used and referred to in |10|.

3. DESCRIPTION OF INPUT

The input of the ARSTU program consists of three parts: parameters, title or heading and time series data.

The parameters, used as arguments or as program control, are inputted by answering the questions displayed on the terminal display monitor. The sequence in which these questions are displayed cannot be changed. Without any exception these parameters are of the integer type.

The title or heading for the time series data may consist of any character; numerical, alphanumerical or special.

The time series data can be inputted in two ways: from the terminal keyboard or from a datafile.

3.1. Displayed Questions

ENTER : NPROG	GENERAL PROG. PAR. 0 = STOP 1 = LEAST SQUARES 2 = MAX. LIKELIHOOD 3 = BAYESIAN
ENTER : LU	OUTPUT DEVICE 1 = DISPLAY 6 = PRINTER
ENTER : ISWLS	PROG. PAR. FOR ARMFIT 0 = MAICE MODEL ONLY (OUTPUT SUPPRESSED) 1 = MAICE MODEL ONLY 2 = ALL AR-MODELS UP TO ORDER LAG
ENTER : ISWML	PROG. PAR. FOR ARMLE PRINT OUT CONTROL NON.LIN.OPT. PROCEDURE 0 = NO 1 = YES

ENTER : ISWBA	PROG. PAR. FOR ARBAYS 0 = TO SUPPRESS THE OUTPUTS 1 = TO PRINT OUT THE OUTPUTS	
ENTER : NCHK	CHAR. ROOTS OF THE AUTOREGR. OPERATOR 0 = NO 1 = YES	
ENTER : MT	INPUT OF DATA 0 = CONSOLE 1 = FILE	
ENTER : N	DATA LENGTH (N.LE.300)	
ENTER : LAG	UPPER LIMIT OF AR-MODEL (LAG.LE.30)	
ENTER : NSPAN	PROG. PAR. FOR ARMASE 0 = ARMASE SUPPRESSED 1 = SPECTRUM 2 = SPECTRAL DENSITIES	 (NBRL GIVEN) (UNIT GIVEN)
ENTER : NBRL	NUMBER OF LINES SPECTRUM PLOT (NBRL.LE.50) UNIT = PXX (MAX)/FLOAT (NBRL)	
ENTER : NUNIT	UNIT ON Y-AXIS DENSITY PLOT (NUNIT.LE.50) UNIT = FLOAT (NUNIT)/1000	
ENTER : TITLE	OF DATA	
ENTER : DATA	IN FREE FIELD FORMAT	
ENTER : NAME	OF DATA FILE	

3.2. Conditions, Restrictions and Comments

ISWLS

- Will only be displayed if NPROG = 1
- If ISWLS = 0 the model procedure ARMFIT will produce no output at all ! This situation only makes sense if NSPAN $\neq$ 0 in which case the spectrum or the spectral densities, based on the model fitted by ARMFIT, will be given.

ISWML	- Will only be displayed if NPROG = 2
ISWBA	- Will only be displayed if NPROG = 3 - If ISWBA = 0 the model procedure ARBAYS will produce no output at all ! This situation only makes sense if NSPAN $\neq$ 0, in which case the spectrum or the spectral densities, based on the model fitted by ARBAYS, will be given.
MT	- If MT = 0 the time series data is inputted via the terminal keyboard in FREE FIELD FORMAT. - If MT = 1 the time series data is inputted from a datafile on disk.
N	- Restriction: N $\leq$ 300
LAG	- Restriction: LAG $\leq$ 30
NSPAN	- If NSPAN = 1 the spectrum is computed. The number of lines used for the graphical representation of the spectrum is given by the parameter NBRL. The unit on the Y-axis is then given by UNIT = PXX (MAX)/FLOAT (NBRL) where PXX (MAX) stands for the maximum of the spectrum and NBRL is the number of lines. - If NSPAN = 2 the spectral densities are computed. The unit on the Y-axis is determined by the parameter NUNIT. The unit itself is given by UNIT = FLOAT (NUNIT)/1000. Since the densities are scaled between zero and one the number of lines used in the graphical representation of the spectral densities is defined as the nearest integer to NBRL = 1/UNIT
NBRL	- Will only be displayed if NSPAN = 1
NUNIT	- Will only be displayed if NSPAN = 2
TITLE	- May consist of any character; numerical, alphanumerical or special - Restriction: the maximum number of characters is 20.

DATA - Will only be displayed if MT = 0.

- Time series data is inputted via the terminal keyboard in FREE FIELD FORMAT. The input must follow immediately after the message

  ENTER: DATA IN FREE FIELD FORMAT

  has been displayed on the screen. Each observation must be separated from the other by at least one blank.

NAME - Will only be displayed if MT = 1.

- Time series data are to be read from a datafile on disk. The name of this datafile must be entered immediately after the message

  ENTER: NAME OF DATAFILE

  is displayed on the screen (see Appendix I for details about the preparation of this datafile).

4. DESCRIPTION OF OUTPUT

Each output starts with an echo of the input parameters. The sequence in which these parameters are given is: NPROG, ISWML, ISWBA, NCHK, MT, N, LAG, NSPAN, NBRL, NUNIT.

4.1. Output Minimum AIC Procedure (NPROG = 1)

The output for the Least Squares procedure consists of:
- mean and variance of the inputted time series
- order, innovations variance, AIC(M) and AIC(M) - AICMIN for all models up to order LAG
- AR-coefficients and innovation variance for all models up to order LAG
- coefficients and innovation variance for that model for which AIC(M) was minimum
- characteristic roots of the AR-operator of the MAIC model
- spectrum or spectral densities based on the coefficients and innovation variance of the MAIC model
- amplitude, frequency and period for the peak frequencies of the spectrum or spectral densities.

In many situations it might be advantageous to suppress most of the output. This can be done readily by means of the parameters ISWLS, NCHK and NSPAN.

See Section 5 for an example of the output.

4.2. Output Bayesian Procedure (NPROG = 3)

The output for the Bayesian procedure consists of:
- mean and variance of the inputted time series
- order, innovation variance, AIC(M) and AIC(M) - AICMIN for all models up to order LAG
- partial correlation coefficients, weights and integrated weights
- partial correlation coefficients for the Bayesian model, innovation variance, equivalent number of parameters and equivalent AIC
- coefficients and innovation variance for the Bayesian model
- characteristic roots of the AR-operator of the Bayesian model

- spectrum or spectral densities based on the coefficients and innovation variance of the Bayesian model
- amplitude, frequency and period for the peak frequencies of the spectrum or spectral densities.

In many situations it might be advantageous to suppress most of the output. This can be done readily by means of the parameters ISWBA, NCHK and NSPAN.

See Section 6 for an example of the output.

4.3. Output Maximum Likelihood (NPROG = 2)

Not available yet.

5. OUTPUT EXAMPLE : ARMFIT

Input:

NPROG	=	1
ISWLS	=	2
NCHK	=	1
MT	=	1
N	=	156
LAG	=	24
NSPAN	=	1
NBRL	=	25
TITLE	=	&USTRB
NAME	=	&USTRB

Resulting output: See pp. 17-20.

1 2 0 0 1 1 156 24 1 25 0

PROGRAM TIMSAC 78.1.1

AUTOREGRESSIVE MODEL FITTING (SCALAR CASE) ; LEAST SQUARES METHOD BY HOUSEHOLDER TRANSFORMATION

(AUTOREGRESSIVE MODEL)

Z(I) = A(1)*Z(I-1) + A(2)*Z(I-2) + ... + A(M)*Z(I-M) + E(I)

WHERE
M: ORDER OF THE MODEL
E(I): GAUSSIAN WHITE NOISE WITH MEAN 0 AND VARIANCE SD(M).

FITTING UP TO THE ORDER K = 24 IS TRIED
ORIGINAL DATA INPUT DEVICE MT = 1

** ORIGINAL DATA **

TITLE -- "&USTRB

MEAN = .34223525D+01 VARIANCE = .10679650D+01

MAICE PROCEDURE

ORDER M	INNOVATION VARIANCE SD(M)	AIC(M)	AIC(M)-AICMIN
0	.1196941236D+01	25.730	425.085
1	.5677142792D-01	-374.671	24.684
2	.4638072046D-01	-399.355	0.000
3	.4593079726D-01	-398.642	.713
4	.4592973596D-01	-396.645	2.710
5	.4592916546D-01	-394.646	4.709
6	.4561311683D-01	-393.558	5.797
7	.4402794058D-01	-396.227	3.128
8	.4294280743D-01	-397.521	1.834
9	.4284984885D-01	-395.807	3.548
10	.4261439242D-01	-394.534	4.821
11	.4233428370D-01	-393.405	5.950
12	.4232824520D-01	-391.424	7.931
13	.4227404960D-01	-389.593	9.762
14	.4179959059D-01	-389.083	10.272
15	.4143058217D-01	-388.253	11.102
16	.4132143808D-01	-386.601	12.754
17	.4085919079D-01	-386.086	13.269
18	.4058271636D-01	-384.983	14.373
19	.4014472994D-01	-384.415	14.940
20	.4014415803D-01	-382.417	16.938
21	.4013943480D-01	-380.432	18.923
22	.3929409100D-01	-381.242	18.113
23	.3673279400D-01	-388.139	11.216
24	.3671907067D-01	-386.189	13.166

```
  AIC(M)-AICMIN (TRUNCATED AT 40.0)
0        10        20        30        40
+---------+---------+---------+---------+
|                                       .
|                        *
*
|*
|  *
|    *
|     *
|  *
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|     *
|       *
|         *
|         *
|          *
|            *
|            *
|             *
|              *
|                *
|                  *
|                 *
|          *
|            *
```

***** MINIMUM AIC = -.3993550266D+03 ATTAINED AT M = 2 SD(M) = .4638072046D-01 *****

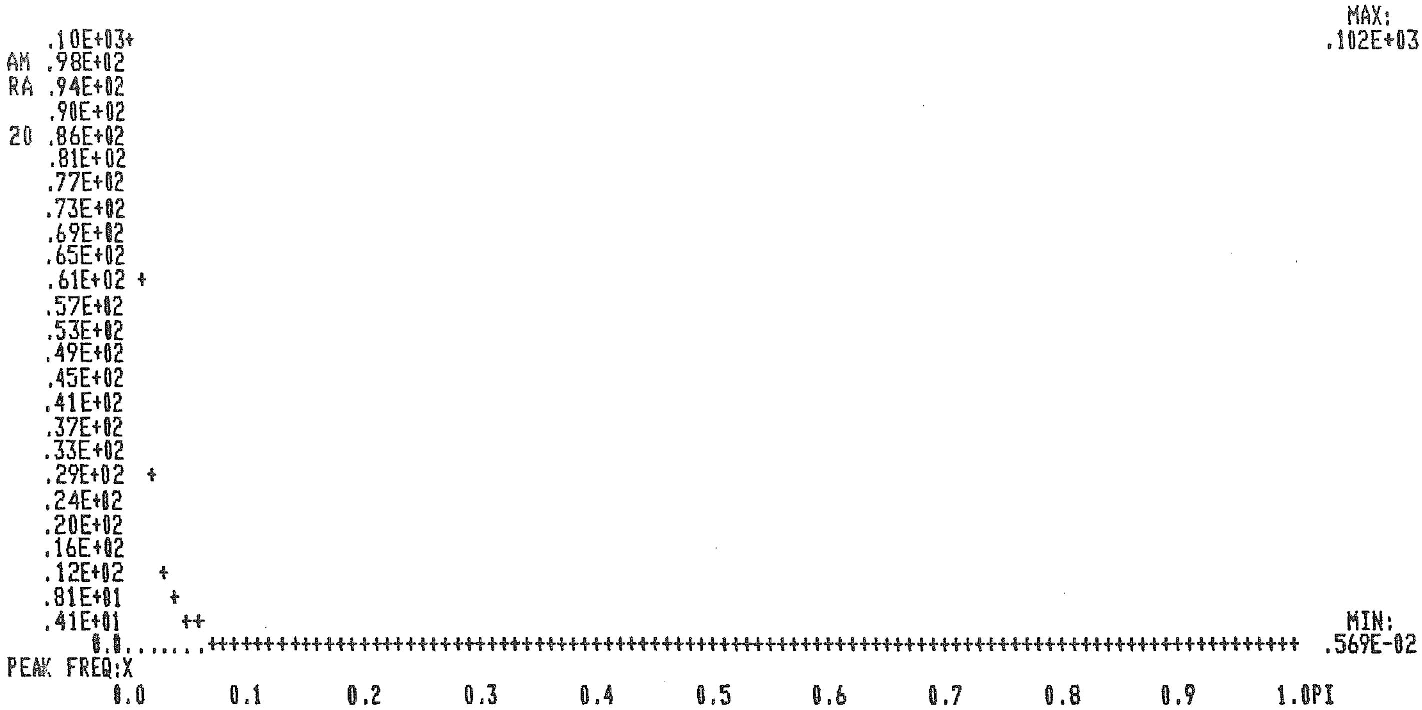
AR-COEFFICIENTS

```
M =  0        (SD(M) =  .11969412D+01 )

M =  1        (SD(M) =  .56771428D-01 )
   .99545255

M =  2        (SD(M) =  .46380720D-01 )
  1.41666652  -.43801075

M =  3        (SD(M) =  .45930797D-01 )
  1.37345367  -.29827789  -.10073993

M =  4        (SD(M) =  .45929736D-01 )
  1.37393174  -.29686312  -.10732385   .00491112

M =  5        (SD(M) =  .45929165D-01 )
  1.37390861  -.29650670  -.10625717   .00008253   .00359062

M =  6        (SD(M) =  .45613117D-01 )
  1.37574725  -.29928657  -.09691088   .02582853  -.11179439   .08563078

M =  7        (SD(M) =  .44027941D-01 )
  1.36325676  -.27513846  -.10936097   .04514482  -.05699283  -.16925663   .19169775

M =  8        (SD(M) =  .42942807D-01 )
  1.33333064  -.24618870  -.09609780   .03271021  -.03953697  -.13024245  -.01714473   .16042572

M =  9        (SD(M) =  .42849849D-01 )
  1.34137008  -.24830349  -.10214674   .03010284  -.03715665  -.13543866  -.02621163   .22071335  -.04770418

M = 10        (SD(M) =  .42614392D-01 )
  1.33844336  -.23154770  -.10607962   .02044365  -.04185084  -.13198542  -.03425007   .20629037   .04872720  -.07606236

M = 11        (SD(M) =  .42334284D-01 )
  1.33266233  -.22671649  -.08941900   .01545935  -.05244034  -.13685893  -.03177327   .19785931   .03414887   .02900272
  -.08317364

M = 12        (SD(M) =  .42328245D-01 )
  1.33392786  -.22743119  -.08985230   .01317900  -.05205824  -.13515533  -.03081204   .19773030   .03511097   .03107468
  -.09876236   .01238098

M = 13        (SD(M) =  .42274050D-01 )
  1.33402890  -.23145240  -.08772849   .01442767  -.04486019  -.13628595  -.03616470   .19475348   .03546974   .02814977
  -.10459191   .05889669  -.03714210

M = 14        (SD(M) =  .41799591D-01 )
  1.33642999  -.23675621  -.07381523   .00917804  -.04742653  -.16043857  -.02932483   .20743228   .04417776   .02761826
  -.09531661   .07623297  -.17469672   .10937447

M = 15        (SD(M) =  .41430582D-01 )
  1.32571075  -.22076291  -.08082657   .02059816  -.05112211  -.16367013  -.05221549   .21249656   .05758263   .03574304
  -.09472634   .08313544  -.15913206  -.01297116   .09709419

M = 16        (SD(M) =  .41321438D-01 )
  1.33138773  -.22199932  -.08844122   .02483575  -.05830472  -.16085670  -.04952981   .22588226   .05354876   .02811738
  -.10015664   .08357082  -.16287725  -.02055371   .16344794  -.05323454

M = 17        (SD(M) =  .40859191D-01 )
  1.33490049  -.24010538  -.08391157   .04136501  -.06532433  -.14835825  -.05268153   .21750330   .02555656   .03672186
  -.08321707   .09469060  -.16397137  -.01291626   .17867519  -.18981018   .10964825

M = 18        (SD(M) =  .40582716D-01 )
  1.32640056  -.22632390  -.09945011   .04443147  -.05259813  -.15518693  -.04282334   .21385576   .02118596   .01562526
  -.07694609   .10703624  -.15374327  -.01580505   .18453512  -.17605586   .00251990   .08514031

M = 19        (SD(M) =  .40144730D-01 )
  1.31061688  -.22000479  -.08377750   .02182042  -.04077141  -.14032093  -.05861557   .22274752   .01932090   .01068749
  -.09655663   .11251590  -.14139761  -.00213607   .17842317  -.16788333   .02366456  -.05811142   .11425424

M = 20        (SD(M) =  .40144158D-01 )
  1.31085012  -.21986212  -.08385308   .02123149  -.03980038  -.14080625  -.05912467   .22322115   .01894214   .01077111
  -.09630710   .11323785  -.14164969  -.00251784   .17787282  -.16769985   .02355692  -.05927820   .11978099  -.00421058

M = 21        (SD(M) =  .40139435D-01 )
  1.31103600  -.22073713  -.08423947   .02157062  -.03832065  -.14364885  -.05746583   .22464938   .01758962   .01187513
  -.09669609   .11251203  -.14360713  -.00186749   .17905172  -.16609494   .02290377  -.05860061   .12261764  -.01992322
   .01212079

M = 22        (SD(M) =  .39294091D-01 )
```

```
............................ M A I C E .............................
:                                                                  :
:    &USTRB                                                        :
:                                                                  :
:              COEFFICIENTS              INNOVATION VARIANCE       :
:         I        A(I)                  SD =     .4638072046D-01  :
:         1      1.4166665153                                      :
:         2      -.4380107479                                      :
:                                                                  :
:    THIS MODEL WAS FITTED TO THE DATA  ( Z(   25), ... ,Z(  156) ) :
....................................................................
```

```
------------------------------------------------------------------
CHARACTERISTIC ROOTS OF AR OPERATOR  1-A(1)*S- ... -A(M)*S**M=0
------------------------------------------------------------------
```

REAL	IMAGINARY	SQRT(R**2+I**2)	ARCTAN(I/R)	DEGREE
.45589D+00	.00000D+00	.45589D+00	.00000D+00	0.000
.96077D+00	.00000D+00	.96077D+00	.00000D+00	0.000



THE AMPLITUDE AT FREQUENCY 0.00PI IS .102E+03
THE AMPLITUDE AT FREQUENCY 1.00PI IS .569E-02

THE AMPLITUDES ARE MONOTONIC

.10181D+03	.62898D+02	.29212D+02	.15353D+02	.91638D+01	.59906D+01	.41760D+01	.30502D+01	.23074D+01	.17934D+01
.14243D+01	.11511D+01	.94380D+00	.78333D+00	.65695D+00	.55595D+00	.47423D+00	.40739D+00	.35219D+00	.30623D+00
.26767D+00	.23511D+00	.20743D+00	.18378D+00	.16347D+00	.14595D+00	.13076D+00	.11755D+00	.10600D+00	.95881D-01
.86979D-01	.79124D-01	.72171D-01	.65999D-01	.60505D-01	.55602D-01	.51214D-01	.47279D-01	.43742D-01	.40554D-01
.37676D-01	.35072D-01	.32711D-01	.30567D-01	.28615D-01	.26836D-01	.25212D-01	.23726D-01	.22365D-01	.21115D-01
.19968D-01	.18912D-01	.17939D-01	.17041D-01	.16212D-01	.15445D-01	.14735D-01	.14076D-01	.13466D-01	.12898D-01
.12371D-01	.11880D-01	.11424D-01	.10998D-01	.10601D-01	.10230D-01	.98846D-02	.95615D-02	.92596D-02	.89773D-02
.87134D-02	.84665D-02	.82356D-02	.80196D-02	.78176D-02	.76286D-02	.74521D-02	.72870D-02	.71330D-02	.69892D-02
.68552D-02	.67305D-02	.66146D-02	.65070D-02	.64074D-02	.63155D-02	.62308D-02	.61531D-02	.60822D-02	.60178D-02
.59596D-02	.59076D-02	.58615D-02	.58211D-02	.57864D-02	.57572D-02	.57334D-02	.57150D-02	.57019D-02	.56941D-02
.56915D-02									

6. OUTPUT EXAMPLE : ARBAYS

Input:

NPROG	=	3
ISWBA	=	1
NCHK	=	1
MT	=	1
N	=	156
LAG	=	24
NSPAN	=	1
NBRL	=	25
TITLE	=	&USTRB
NAME	=	&USTRB

Resulting output: See pp. 22-26.

3 0 0 1 1 1 156 24 1 25 0

PROGRAM TIMSAC 78.1.2

EXPONENTIALLY WEIGHTED BAYESIAN AUTOREGRESSIVE MODEL FITTING; SCALAR CASE

(AUTOREGRESSIVE MODEL)

Z(I) = A(1)*Z(I-1) + A(2)*Z(I-2) + ... + A(M)*Z(I-M) + E(I)

WHERE
M: ORDER OF THE MODEL
E(I): GAUSSIAN WHITE NOISE WITH MEAN 0 AND VARIANCE SD(M).

24-TH ORDER BAYESIAN MODEL IS FITTED
ORIGINAL DATA INPUT DEVICE MT = 1

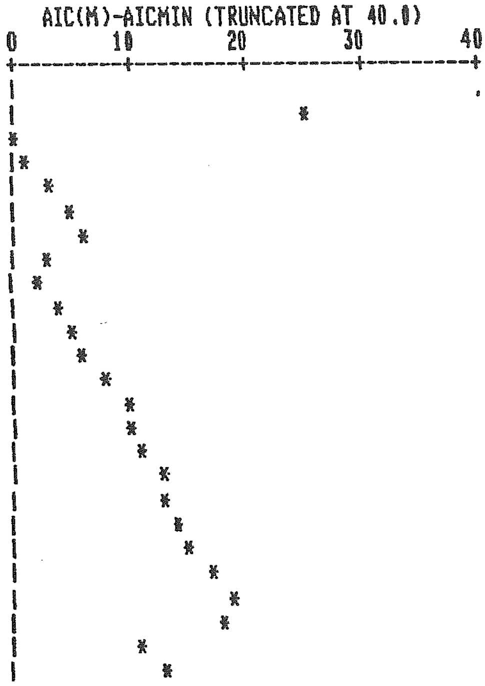
** ORIGINAL DATA **

TITLE -- "&USTRB

MEAN = .34223525D+01 VARIANCE = .10679650D+01

BAYESIAN PROCEDURE

ORDER M	INNOVATION VARIANCE SD(M)	AIC(M)	AIC(M)-AICMIN
0	.1196941236D+01	25.730	425.085
1	.5677142792D-01	-374.671	24.684
2	.4638072046D-01	-399.355	0.000
3	.4593079726D-01	-398.642	.713
4	.4592973596D-01	-396.645	2.710
5	.4592916546D-01	-394.646	4.709
6	.4561311683D-01	-393.558	5.797
7	.4402794058D-01	-396.227	3.128
8	.4294280743D-01	-397.521	1.834
9	.4284984885D-01	-395.807	3.548
10	.4261439242D-01	-394.534	4.821
11	.4233428370D-01	-393.405	5.950
12	.4232824520D-01	-391.424	7.931
13	.4227404960D-01	-389.593	9.762
14	.4179959059D-01	-389.083	10.272
15	.4143058217D-01	-388.253	11.102
16	.4132143808D-01	-386.601	12.754
17	.4085919079D-01	-386.086	13.269
18	.4058271636D-01	-384.983	14.373
19	.4014472994D-01	-384.415	14.940
20	.4014415803D-01	-382.417	16.938
21	.4013943480D-01	-380.432	18.923
22	.3929409100D-01	-381.242	18.113
23	.3673279400D-01	-388.139	11.216
24	.3671907067D-01	-386.189	13.166



***** MINIMUM AIC = -.3993550266D+03 ATTAINED AT M = 2 SD(M) = .4638072046D-01 *****

I	PARCOR B(I)	WEIGHT W(I)	INTEGRATED WEIGHT
1	.97599671	.00000318	1.00000000
2	-.42781662	.48542630	.99999682
3	-.09849188	.25485991	.51457052
4	.00480690	.07512065	.25971061
5	.00352436	.02304834	.18458996
6	.08295311	.01146356	.16154162
7	.18642060	.03809641	.15007806
8	.15699193	.06467567	.11198166
9	-.04652641	.02470610	.04730599
10	-.07412772	.01188651	.02259989
11	-.08107467	.00619443	.01071338
12	.01194314	.00212341	.00451895
13	-.03578219	.00078936	.00239554
14	.10594060	.00057087	.00160618
15	.09395765	.00035349	.00103531
16	-.05132626	.00014567	.00068181
17	.10576683	.00010634	.00053614
18	.08225885	.00005801	.00042980
19	.10388666	.00004150	.00037178
20	-.00377442	.00001455	.00033029
21	.01084697	.00000515	.00031574
22	.14512127	.00000738	.00031059
23	.25530913	.00022262	.00030320
24	-.01932870	.00008058	.00008058

```
     PARCOR (LINES SHOW +SD AND +2SD), SD=SQRT(1/N)
-1                  0                   1
+---------+---------+---------+---------+
                | | | | |               *
           *    | | | | |
                | * | | |
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                | | * | |
                | | * | |
                | | | |*|
                | | | | |*
                | | * | |
```

(BAYESIAN MODEL)

PARCOR'S

1	.97599671
2	-.42781526
3	-.05068102
4	.00124840
5	.00065056
6	.01340038
7	.02797764
8	.01758022
9	-.00220098
10	-.00167528
11	-.00086858
12	.00005397
13	-.00008572
14	.00017016
15	.00009727
16	-.00003499
17	.00005671
18	.00003535
19	.00003862
20	-.00000125
21	.00000342
22	.00004507
23	.00007741
24	-.00000156

INNOVATION VARIANCE = .45516119D-01
EQUIVALENT NUMBER OF PARAMETERS = 3.430
EQUIVALENT AIC = -400.978

BAYESIAN MODEL

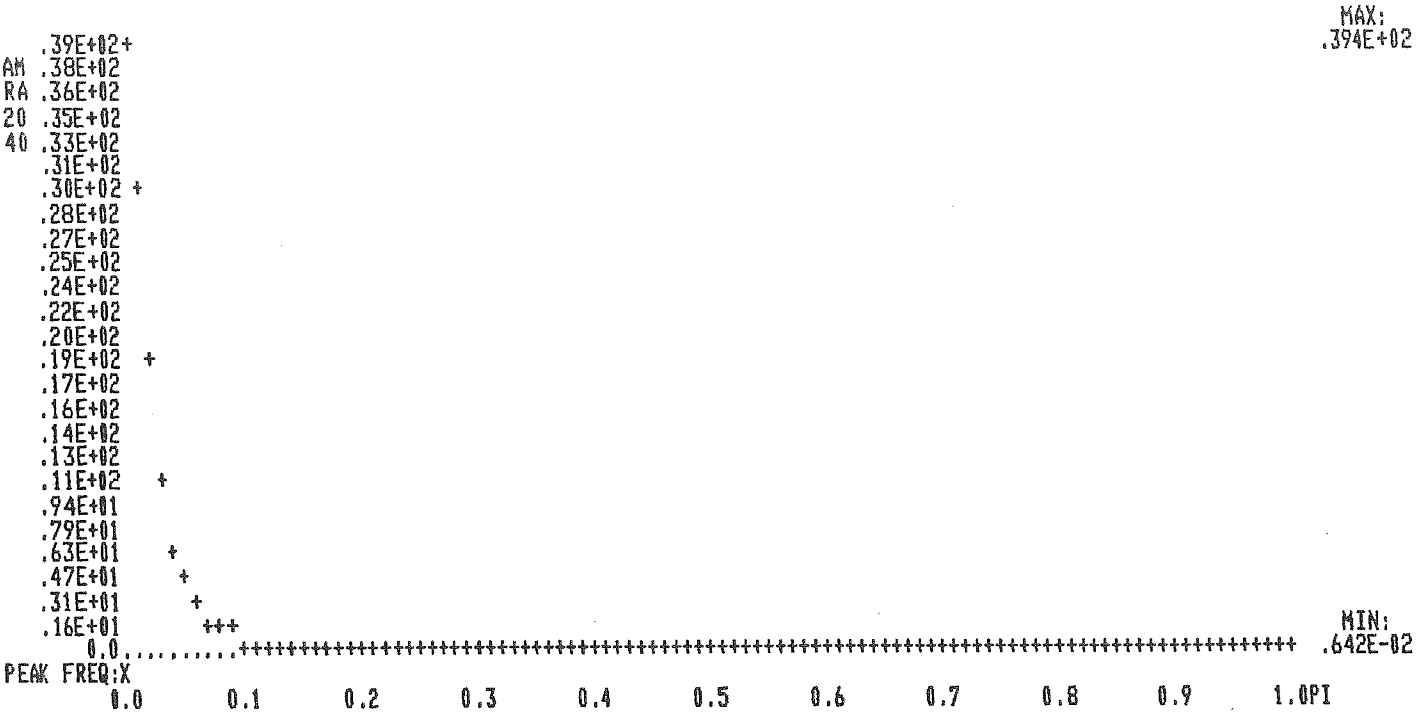
&USTRB

I	COEFFICIENTS A(I)	INNOVATION VARIANCE
		SD = .4551611868D-01
1	1.3710814565	
2	-.3557354506	
3	-.0514885340	
4	.0064151945	
5	-.0068584541	
6	-.0188306624	
7	.0029982404	
8	.0199598395	
9	-.0002076768	
10	-.0004715297	
11	-.0009649601	
12	.0002383221	
13	-.0002856324	
14	.0000263787	
15	.0001653759	
16	-.0000974985	
17	.0000236567	
18	-.0000176558	
19	.0000433645	
20	.0000140874	
21	-.0000309162	
22	-.0000616174	
23	.0000795461	
24	-.0000015576	

THIS MODEL WAS FITTED TO THE DATA (Z(25), ... ,Z(156))

CHARACTERISTIC ROOTS OF AR OPERATOR 1-A(1)*S- ... -A(M)*S**M=0

REAL	IMAGINARY	SQRT(R**2+I**2)	ARCTAN(I/R)	DEGREE
.19890D-01	.00000D+00	.19890D-01	.00000D+00	0.000
.94010D+00	.00000D+00	.94010D+00	.00000D+00	0.000
-.63213D+00	.88178D-01	.63825D+00	.30030D+01	172.059
-.63213D+00	-.88178D-01	.63825D+00	-.30030D+01	-172.059
.66746D+00	-.11703D+00	.67765D+00	-.17357D+00	-9.945
.66746D+00	.11703D+00	.67765D+00	.17357D+00	9.945
.52090D+00	-.39015D+00	.65081D+00	-.64286D+00	-36.833
.52090D+00	.39015D+00	.65081D+00	.64286D+00	36.833
-.56954D+00	-.26493D+00	.62815D+00	-.27062D+01	-155.054
-.56954D+00	.26493D+00	.62815D+00	.27062D+01	155.054
.22708D-01	-.65185D+00	.65225D+00	-.15360D+01	-88.005
.22708D-01	.65185D+00	.65225D+00	.15360D+01	88.005
.65377D+00	-.28631D+00	.71372D+00	-.41278D+00	-23.651
.65377D+00	.28631D+00	.71372D+00	.41278D+00	23.651
-.48304D+00	-.41878D+00	.63931D+00	-.24273D+01	-139.076
-.48304D+00	.41878D+00	.63931D+00	.24273D+01	139.076
.35430D+00	-.53119D+00	.63850D+00	-.98257D+00	-56.297
.35430D+00	.53119D+00	.63850D+00	.98257D+00	56.297
-.34969D+00	-.53851D+00	.64209D+00	-.21467D+01	-122.998
-.34969D+00	.53851D+00	.64209D+00	.21467D+01	122.998
.19578D+00	-.63906D+00	.66837D+00	-.12735D+01	-72.967
.19578D+00	.63906D+00	.66837D+00	.12735D+01	72.967
-.17497D+00	-.60831D+00	.63297D+00	-.18509D+01	-106.047
-.17497D+00	.60831D+00	.63297D+00	.18509D+01	106.047



THE AMPLITUDE AT FREQUENCY 0.00PI IS .394E+02
THE AMPLITUDE AT FREQUENCY 1.00PI IS .642E-02

THE AMPLITUDES ARE MONOTONIC

.39358D+02	.31327D+02	.19484D+02	.12013D+02	.78629D+01	.54755D+01	.40115D+01	.30568D+01	.23999D+01	.19266D+01
.15719D+01	.12970D+01	.10781D+01	.90004D+00	.75316D+00	.63102D+00	.52919D+00	.44435D+00	.37385D+00	.31544D+00
.26720D+00	.22742D+00	.19466D+00	.16769D+00	.14547D+00	.12714D+00	.11198D+00	.99403D-01	.88927D-01	.80156D-01
.72767D-01	.66501D-01	.61149D-01	.56539D-01	.52533D-01	.49018D-01	.45897D-01	.43091D-01	.40533D-01	.38169D-01
.35958D-01	.33870D-01	.31887D-01	.29997D-01	.28199D-01	.26490D-01	.24872D-01	.23346D-01	.21914D-01	.20576D-01
.19333D-01	.18185D-01	.17131D-01	.16170D-01	.15298D-01	.14511D-01	.13804D-01	.13170D-01	.12602D-01	.12095D-01
.11640D-01	.11233D-01	.10869D-01	.10542D-01	.10247D-01	.99795D-02	.97355D-02	.95107D-02	.93014D-02	.91046D-02
.89182D-02	.87405D-02	.85703D-02	.84067D-02	.82491D-02	.80967D-02	.79491D-02	.78060D-02	.76673D-02	.75334D-02
.74048D-02	.72823D-02	.71667D-02	.70589D-02	.69594D-02	.68686D-02	.67869D-02	.67145D-02	.66514D-02	.65977D-02
.65530D-02	.65168D-02	.64884D-02	.64667D-02	.64506D-02	.64389D-02	.64305D-02	.64247D-02	.64208D-02	.64185D-02
.64178D-02									

7. OUTPUT EXAMPLE : ARMLE

Not available yet.

APPENDIX I - INPUT DATAFILE

1. The datafile is entered, CReated or REplaced by the EDITR. Each line (record), consisting of 70 characters, must contain 7 observations. Each observation, entered in free format, is separated from the other by at least one blank.

   For an example see the input datafile EXF1 on p.29.

2. If the input datafile is entered such that
   - each line (record) is equally divided into 8 fields, each field consisting of 10 characters,
   - the first 7 of these fields are used for time series observations,
   - each observation is separated from the other by at least one blank,
   - the 8th field, i.e. the last 10 characters of the record, is used for the Identification and Sequencing of the data record

   this same input datafile can also be used for the MULTISTOCH and MULTITRAN programs.

   For an example see the input datafile EXF2 on p.29.

3. Apart from the Identification and the Sequencing the input datafile, resulting from a TSERS' <u>PUNC</u> command, is MULTISTOCH and MULTITRAN compatible. The FORMAT used is (7 (G 9.5, 1X)).

```
EXF1   T=00004 IS ON CR00007 USING 00003 BLKS R=0000

0001           1         2         3         4         5         6         7         8
0002  12345678901234567890123456789012345678901234567890123456789012345678901234567890
0003
0004  2.456 2.372 2.310 2.613 2.650 2.527 2.334
0005  2.606 2.850 2.961 3.000 3.230 3.210 3.165
0006  3.140 3.113 3.042 3.316 3.165 3.404 3.578
0007  3.591 3.337 3.102 2.598 1.562 1.354 1.126
0008  1.046 0.881 0.962 1.686 2.484 2.793 2.756
```

```
EXF2   T=00004 IS ON CR00007 USING 00004 BLKS R=0000

0001           1         2         3         4         5         6         7         8
0002  12345678901234567890123456789012345678901234567890123456789012345678901234567890
0003
0004  2.456     2.372     2.310     2.613     2.650     2.527     2.334     USTRB    01
0005  2.606     2.850     2.961     3.000     3.230     3.210     3.165     USTRB    02
0006  3.140     3.113     3.042     3.316     3.165     3.404     3.578     USTRB    03
0007  3.591     3.337     3.102     2.598     1.562     1.354     1.126     USTRB    04
0008  1.046     0.881     0.962     1.686     2.484     2.793     2.756     USTRB    05
```

APPENDIX II : USER-LEVEL FLOW CHART

FLOW   T=00004 IS ON CR00007 USING 00016 BLKS R=0000

```
                         +-------+
+----------------------->| READ  |
|                        | NPROG |
|                        +-------+
|                             |
|                             |
|                             |
|                             V
|                        +-------+  =0     +-------+
|                        | NPROG |-------->| STOP  |
|                        +-------+         +-------+
|                             |
|                             |  >0
|                             V
|                        +-------+
|                        | REDATA|
|                        +-------+
|                             |
|                             |
|                             V
|                        +-------+
|                        | REDUCT|
|                        +-------+
|                             |
|                             |
|                             V
|                        +-------+  =3
|                        | NPROG?|-------------+
|                        +-------+             |
|                             |                |
|                             |  =1,2          |
|                             V                V
|                        +-------+         +-------+
|                        | ARMFIT|         | ARBAYS|
|                        +-------+         +-------+
|                             |                |
|                             |                |
|                             V                |
|                    =2  +-------+             |
|           +------------| NPROG?|             |
|           |            +-------+             |
|           |                 |                |
|           |                 |  =1            |
|           V                 |                |
|       +-------+             |                |
|       | ARMLE |------------>+<---------------+
|       +-------+             |
|                             |
|                             V
|                        +-------+  =0
|                        | NCHK? |-------------+
|                        +-------+             |
|                             |                |
|                             |  =1            |
|                             V                |
|                        +-------+             |
|                        | ARMACR|             |
|                        +-------+             |
|                             |                |
|                             +<---------------+
|                             |
|                             V
|                    =0  +-------+
+<-----------------------| NSPAN?|
|                        +-------+
|                             |
|                             |  =1
|                             V
|                        +-------+
|                        | ARMASE|
|                        | ARMASP|
|                        +-------+
|                             |
|                             |
|                             V
+<----------------------------+
```

APPENDIX III : SUBROUTINE MAP

1. <u>REDATA</u>
   - AXAGET
     - CODE
     - OPENF
     - READF

2. <u>REDUCT</u>
   - HUSHLD
   - SETX1

3. <u>ARMFIT</u>
   - COMAIC
   - MAICE
   - PRINTA
   - RECOEF

4. <u>ARBAYS</u>
   - ARCOEF
   - BAYSPC
   - BAYSWT
   - COMAIC
   - MAICE
   - PRINTA
   - SDCOMP

5. <u>ARMLE</u>

```
| DAVIDN
|       | FUNCT
|       |       | ARCOEF
|       |       | PARCOR
|       |       | SUBDET
|       | HESSIAN
|       |         INVDET
|       | LINEAR
|       |         FUNCT
|       |               | ARCOEF
|       |               | PARCOR
|       |               | SUBDET
| PRINTA
```

6. <u>ARMACR</u>

    POLYRT

7. <u>ARMASE</u>

    FOUGER

8. <u>ARMASP</u>

```
| PEAKS
| PLOTA
| PRNT
```

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