STUDIECENTRUM VOOR ECONOMISCH EN SOCIAAL ONDERZOEK

ALTERNATIVE HOUSING CONCEPTS AND THE BENEFITS OF PUBLIC HOUSING PROGRAMS*

Bruno L. DE BORGER

rapport 84/161

August 1984

*This paper was written while I was at the University of Wisconsin-Madison. I am grateful to the participants in the Public Economics Workshop for comments. In particular, E.O. Olsen and J. Goddeeris provided useful suggestions. Errors are mine.

Please send correspondence to : Bruno De Borger, Department of Economics, SESO, University of Antwerp, UFSIA, Prinsstraat 13, B-2000, Antwerp, Belgium.

Universitaire Faculteiten St.-Ignatius
Prinsstraat 13 - 2000 Antwerpen
D/1984/1169/12

Abstract

Almost all previous studies analyzing the benefits and consumption effects of public housing programs have used aggregation theorems in order to construct composite goods "housing" and "all other goods". In this paper we show that, if it is more realistically assumed that households have preferences defined on housing characteristics, benefits estimated using the composite approach are upward biased. Some empirical work suggests that the bias is large. We therefore strongly advise to take into account the composition of the bundle of housing attributes provided under a public housing program, when evaluating the program's economic effects.

0. Introduction

kan det?

Almost all previous studies of the benefits and consumption effects of public housing programs have used the homogeneous housing concept introduced by R. Muth (1960).[1] Consumption is measured in terms of unobservable units called "housing services", assumed to have a constant unit price at each location. Consequently, housing is treated as a composite and unidimensional good, ~~which~~ eliminat~~es~~ing the distinction between quality and quantity aspects of the bundle of housing attributes. This simplification is extremely convenient for applied research. Studies dealing with the implications of public housing programs typically assume that households buying all goods and services on the private market maximize a utility function defined on two composite commodities "housing" and "all other goods", subject to a linear budget constraint.

Despite its intuitive appeal the housing services approach is of somewhat limited use to describe and analyze many interesting microeconomic problems. A quite different housing concept emerged from the urban economics literature. It is recognized that housing is a heterogeneous good, differentiated by structural attributes (such as space, structural quality), location and neighborhood characteristics (such as air quality, provision of public services, proximity to employment, etc.). The development of the new consumer demand theory (see e.g., Lancaster (1971)) and increasing popularity of hedonic pricing techniques have created a widespread interest in the demand for the utility-bearing

---

[1]Examples include Clemmer (1983), Cronin (1982), Murray (1975), Olsen (1972) and Olsen and Barton (1983). The only exception we found in the literature is Quigley (1982). The latter assumes a nonlinear budget constraint, unlike the other studies.

housing characteristics. Applications can be found in e.g. King (1976), Wheaton (1977), Awan, Odling-Smee and Whitehead (1982).

The use of apparently different housing concepts in urban economics and in the literature on housing programs is surprising. As the estimation of benefits and consumption effects requires more information and is slightly more difficult if the composition of the attribute bundle is taken into account, a natural question to ask is whether these complications are worth the effort. If the use of different housing concepts has no sizeable impact on the results, then the "housing services" approach retains its attractiveness as an analytical tool.[2]

The purpose of this paper is to investigate the consequences of using different approaches to the estimation of the welfare effects of public housing programs. The method prevalent in the existing literature will be compared with an alternative procedure, in which preferences are defined on housing attributes and "all other goods". In order to focus on the specification of the housing commodity, it is assumed that the latter can be grouped in a composite good X. Moreover, we assume that the budget constraint is linear, which implies a linear hedonic relation between rent or house value and housing characteristics. Although there exists a voluminous empirical literature suggesting that this relation is likely to be nonlinear, this assumption was necessary to keep the problem theoretically manageable. It implies that the

---

[2]Of course, if one believes that demand functions for housing attributes provide relevant information in addition to calculated benefits, one may decide not to use the homogeneous housing good, even if this decision has no impact on the final results.

tools of neoclassical economics, i.e., expenditure and indirect utility functions, can be used.[3]

The paper is organized as follows: in the first section we review the restrictions on preferences that have to be imposed in order to justify the use of composite commodities housing and all other goods.[4] In Section 2 we assume that these conditions are satisfied and compare the benefits calculated on the basis of the two alternative procedures. An empirical example is provided in Section 3, with which we hope to illustrate the theoretical findings. A final section contains a summary of the main conclusions.

[3]For a summary of all other assumptions underlying the procedures to estimate benefits, see Olsen and Barton (1983, p. 301-302).

[4]It is also possible to justify composite goods by imposing restrictions on relative prices, using Hicks aggregation theorem. As the constancy of relative prices for housing attributes is likely to be violated (Anas and Eum (1984)), we focus on restrictions on preferences in this paper.

1. The existence of composite commodities: restrictions on preferences.

Suppose households care about a set of housing attributes h_i and a set of other goods x_j. Consider the problem

$$\text{Max } U(h_1, h_2, \ldots\ldots, h_n, x_1, x_2, \ldots\ldots, x_m)$$

$$\text{subject to} \quad \sum_{i=1}^{n} p_i h_i + \sum_{j=1}^{m} q_j x_j = y$$

where y is income, and p_i and q_j are the implicit attribute prices and the unit prices of other goods, respectively. The usual approach to estimating the benefits of public housing has been to assume that composite goods $H(h_1, \ldots., h_n)$ and $X(x_1, \ldots., x_m)$ exist.[5] In this section we review the restrictions on preferences that justify the construction of these composites, i.e., under what conditions is it possible to analyze the allocation of expenditures to the aggregates "housing" and "all other goods" using a single price and quantity index for each commodity group.

---

[5]A composite commodity may more formally be defined as follows: suppose we have a vector $\underset{\sim}{Z}$ of n goods z_i $(i = 1, \ldots, n)$ with corresponding prices k_i. A composite commodity of the first ℓ goods $D(z_1, \ldots, z_\ell)$ with price $R(k_1, \ldots, k_\ell)$ is said to exist if, for a utility function $u(\underset{\sim}{Z})$ there exists a function $v(D, z_{\ell+1}, \ldots z_n)$ such that maximization of $u(\underset{\sim}{Z})$ subject to $y = \sum_{i=1}^{n} k_i z_i$ and maximization of $v(D, z_{\ell+1}, \ldots z_n)$ subject to $y = RD + \sum_{i=\ell+1}^{n} k_i z_i$ lead to the same total expenditures on the first ℓ goods (i.e., $RD = \sum_{i=1}^{\ell} k_i z_i$) and yield the same demand for all other goods $z_{\ell+1}, \ldots, z_n$.

The answer to the previous question has been given by Gorman (1959) in a more general context.[6] Suppose the utility function is weakly separable in housing and other goods, i.e.,

$$u(g_H(h_1, \ldots., h_n), g_X(x_1, \ldots., x_m))$$

where the $g_i(\cdot)$ define subutility functions on goods in group i. Two sets of additional restrictions guarantee the existence of composite goods H and X. They are sufficient conditions for a two stage budgeting process in which households first allocate expenditures to housing and other goods using group price and quantity indices.[7] In the second stage households may be assumed to determine the optimal consumption of commodities within each group, taking the optimal expenditures of the previous allocation problem as given.

First, assume each group expenditure function $e_i(g_i, \underset{\sim}{p}_i)$ can be written as $[\theta_i(g_i)][b_i(\underset{\sim}{p}_i)]$, where $\theta_i(\cdot)$ is a monotonically increasing function, $b_i(\cdot)$ is a function homogeneous of degree one and $\underset{\sim}{p}_i$ is the vector of prices of goods in group i. This is just saying that the subutility functions are homothetic. In that case $\theta_i(g_i)$ and $b_i(\underset{\sim}{p}_i)$ can be treated as aggregate quantity and price indices for group i.[8]

---

[6]An excellent overview is in Deaton and Muellbauer (1980, p. 129-133).

[7]If there are only two composite goods these restrictions are sufficient but cannot be shown to be necessary. However, Gorman (1959, p. 478) was unable to find an example of a utility function that yields composite goods without satisfying the restrictions.

[8]In a defense of the use of a composite good housing Murray (1978) recently arrived at a similar conclusion. Moreover, he showed that if in addition to the conditions previously stated, it is the case that the joint production function transforming physical inputs into housing attributes is linearly homogeneous, then it is possible to define a composite good housing as a function of the physical inputs. Murray uses this result to criticize the definition of submarkets on the basis of differences in hedonic prices.

Although the separability assumption may not be unreasonable, homotheticity of the subutility functions is extremely restrictive because it implies that all goods within each composite have the same expenditure elasticity. However, a second set of sufficient conditions is available. Let the utility function $u(\cdot)$ have the additive form

$$u(h_1, \ldots, h_n, x_1, \ldots, x_m) = g_H(h_1, \ldots, h_n) + g_X(x_1, \ldots, x_m)$$

and assume that the group indirect utility functions take the Gorman generalized polar form

$$v_i(e_i, \underset{\sim}{p}_i) = K_i \left(\frac{e_i}{b_i(\underset{\sim}{p}_i)} \right) + c_i(\underset{\sim}{p}_i)$$

where $K_i(\cdot)$ is a monotonically increasing function. It is not hard to see that it is possible to treat $\left(\frac{e_i}{b_i(\underset{\sim}{p}_i)} \right)$ and $b_i(\underset{\sim}{p}_i)$ as quantity and price indices of composite good i.

It might be argued that both sets of restrictions on preferences are unduly strong. However, many common utility functions are consistent with the stated conditions. For example, assuming a general $u(\cdot)$ function and a Cobb-Douglas or CES specification for the subutility functions satisfies the first set of restrictions. Using any explicitly additive form for $u(\cdot)$ and a Cobb-Douglas or Stove-Geary specification for the subutility functions is consistent with the second set of conditions.

It is important for the next section to formalize the implications of constructing composite goods within the framework of two stage budgeting. First, note that the aggregates housing and other goods can be written as functions of the respective $g_i(\cdot)$ functions, i.e.,

$$H = f_H(g_H(\cdot))$$

$$X = f_X(g_X(\cdot))$$

Then, since utility maximizing behavior is implied by the definition of the composites, it will be the case that

$$u(g_H(h_1^*, \ldots, h_n^*), g_X(X_1^*, \ldots, x_m^*)) = u(f_H^{-1}(H^*), f_X^{-1}(X^*))$$

where starred variables represent optimal quantities. Moreover, the h_i^* and x_j^* solve the second stage problems

$$\text{Max } g_H(h_1, \ldots, h_n) \qquad \text{Max } g_X(x_1, \ldots, x_m)$$

$$\text{s.t. } \sum_{i=1}^{n} p_i h_i = e_H \qquad \text{s.t. } \sum_{j=1}^{m} q_j x_j = e_X$$

where e_H and e_X are the expenditures on housing and other goods determined in the first stage.

2. The benefits of public housing using alternative housing concepts: a theoretical comparison.

In this section we focus on the implications of using different housing concepts for the calculus of benefits of public housing. We only consider Hicks equivalent variation, the most frequently used benefit measure. We assume that the conditions stated in the previous section are satisfied so that it is legitimate to define composite goods "housing" and "nonhousing".

Consider the utility function

$$u(g_H(h_1, \ldots, h_n), X)$$

where X is a composite "all other goods". A housing aggregate H can be constructed as $H = f(g_H(\cdot))$ with unit price p_H. Consequently we have

$$u(g_H(h_1^*, \ldots, h_n^*), X^*) = u(f^{-1}(H^*), X^*)$$

where starred values are utility maximizing quantities.

Suppose a housing program offers eligible households a housing unit containing h_i^s of attribute i and charges a rent PR. The quantity of other goods consumed under the program is clearly

$$X^s = \frac{y - PR}{p_X} .$$

The housing unit occupied under the program will have a certain market value, say M. If we assume that the introduction of the program does not affect market prices, we have[9]

---

[9]This assumption is made in all previous studies, see e.g., Olsen and Barton (1983, p. 301).

$$M = \sum_{i=1}^{n} p_i h_i^s .$$

Moreover, studies using the composite good housing typically assume

$$M = p_H H^s ,$$

which implicitly determines the amount of housing consumed under the program.

In general, the bundle of housing attributes and other goods consumed under the program will not be a utility maximizing choice. Most programs impose quantity constraints on housing consumption so that observed amounts will correspond to points off the Marshallian demand curves. More specifically, it is of crucial importance to investigate under what conditions it will be the case that

$$g_H(h_1^s, \ldots, h_n^s) = f^{-1}(H^s)$$

Only if this equality holds we will have that

$$u(g_H(h_1^s, \ldots, h_n^s), X^s) = u(f^{-1}(H^s), X^s)$$

It follows from the discussion in the previous section that these relations will hold if and only if the observed attribute quantities under the program h_i^s are the solution to the problem

$$\text{Max } g_H(h_1, \ldots, h_n)$$

$$\text{s.t. } \sum_{i=1}^{n} p_i h_i = M$$

If the program provides households with their desired combination of attributes, for a given market value of the public unit, then previous equalities will hold.

Since this will only occur by chance, we will have in general

$$u(g_H(h_1^s, \ldots, h_n^s), X^s) < u(f^{-1}(H^s), X^s)$$

Our conclusion thus far is that, if the restrictions on preferences are satisfied to justify the construction of composite goods housing and nonhousing, the utility level under the program using the composite good H will be greater than the utility level defined on housing attributes. Equality would only result if the program happened to offer the optimal attribute bundle in the hypothetical situation where households can freely choose their desired combination of attributes, under the restriction that it has the same market value as the public unit. In other words, using a composite good "housing" is equivalent to assuming that the previous optimal quantities are provided under the program.

The situation is illustrated on figure 1 for the case where people only care about two attributes. The program offers the bundle (h_1^s, h_2^s, X^s), which has a total market value of, say, Q. Consumption under the program is indicated by point A' on the plane YWZ which contains all commodity bundles with market value Q. All combinations of goods that contain X^s and have value Q are on K'L'. Correspondingly, all bundles of housing attributes with the same market value as the public unit are situated on KL.

It is possible to depict the indifference curves of the subutility function $g(h_1, h_2)$, see Figures 1 and 2. The attribute combination offered by the program yields subutility level g^s. However, the most preferred combination of attributes with the same market value as the public unit is (h_1^m, h_2^m), yielding a subutility level g^m, see point B. Unless $h_i^s = h_i^m$ it is possible to improve households well-being by optimizing the composition of the attribute bundle.

The composite commodity approach assumes households are indifferent between all combinations of goods on K'L', or alternatively, between all combinations of housing characteristics on KL, having the same market rent. Moreover, we showed that the approach implicitly assumes that the subutility level for housing under the program is g^m, the optimal utility level for the market value of the public unit.

These findings suggest that benefits - i.e., the equivalent variation - calculated using the composite housing concept will provide an upper bound to the "true" benefit based on a specification of housing as a bundle of characteristics. It turns out that this is indeed the case. In the first case benefits are

$$EV1 = e(p_H, p_X, u(f^{-1}(H^s), X^s)) - y$$

$$= \underset{H,X}{\text{Min}} \{p_H H + p_X X \mid u \geq u(f^{-1}(H^s), X^s))\} - y$$

In the latter case the equivalent variation is defined as

$$EV2 = e(p_1, \ldots p_n, p_X, u(g_H(h_1^s, \ldots h_n^s), X^s)) - y$$

$$= \underset{h_1,\ldots h_n, X}{\text{Min}} \{ \sum_{i=1}^{n} p_i h_i + p_X X \mid u \geq u(g_H(h_1^s, \ldots h_n^s), X^s)\} - y$$

Using the fact that $f^{-1}(H^s) \geq g_H(h_1^s, \ldots, h_n^s)$ and remembering that under optimizing conditions $H = f(g_H(\cdot))$ it is straightforward to show that $EV1 \geq EV2$,

FIGURE 1

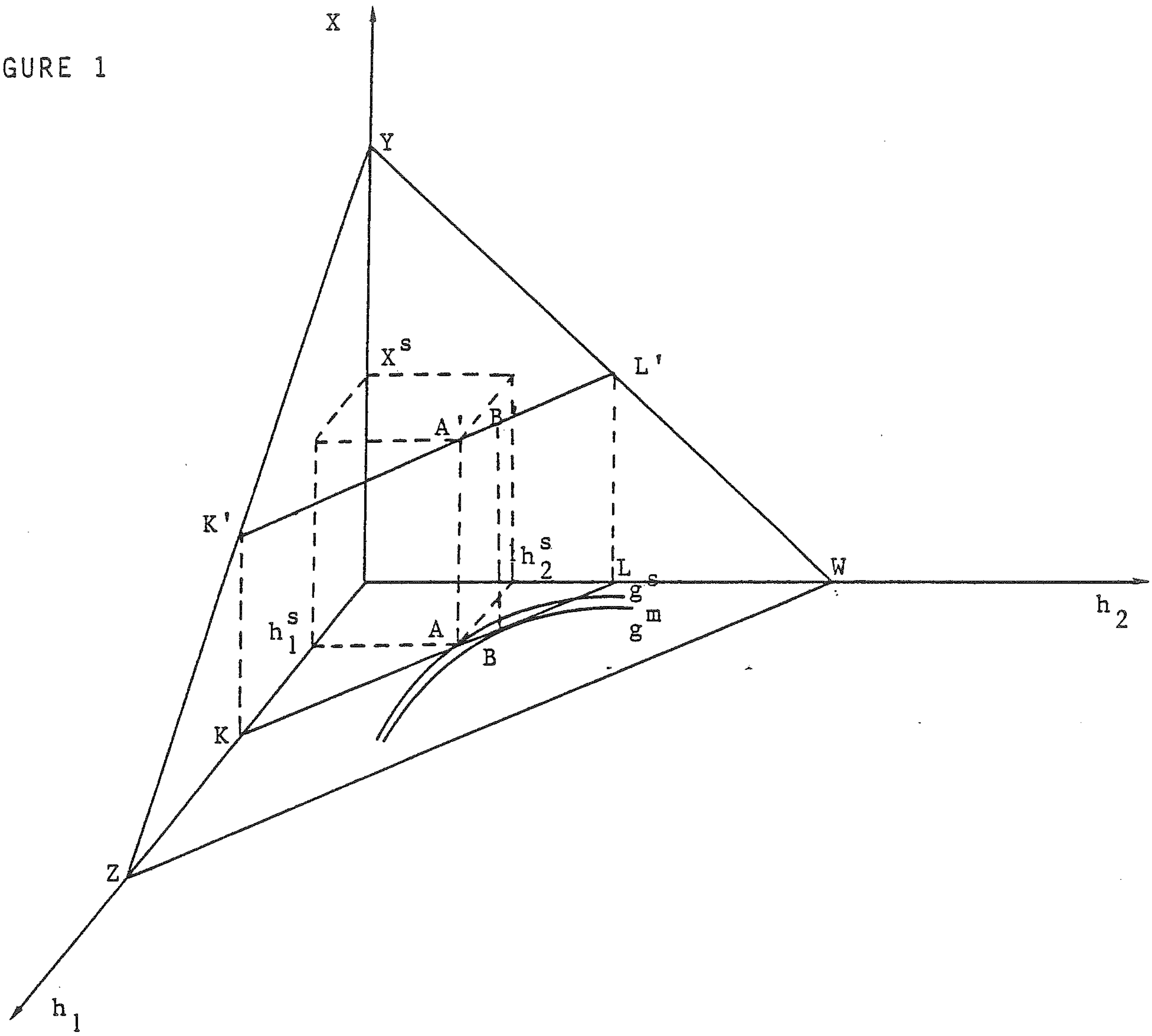
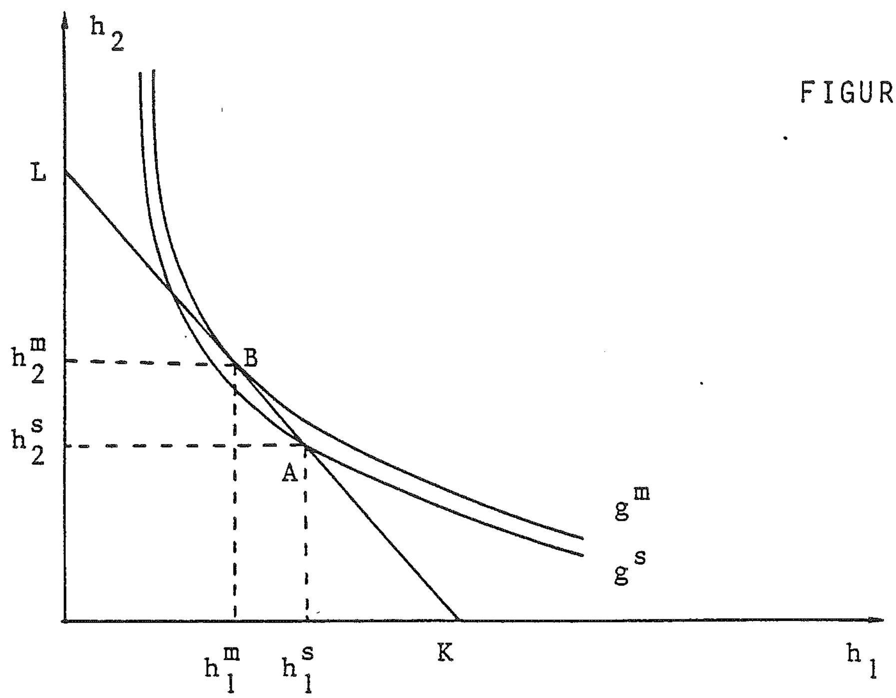


FIGURE 2



where equality only holds if $h_i^s = h_i^m$.[10] Consequently, the benefits using a composite good housing will in general be upward biased unless the condition $h_i^s = h_i^m$ holds, which is extremely unlikely in practice.

To conclude this section we present a simple example that will clarify the results. Consider the following utility function defined on housing attributes and other goods

$$u(g_H(h_1, h_2), X) = h_1^{\alpha_1} h_2^{\alpha_2} X^{1-\alpha_1-\alpha_2}$$

As this Cobb-Douglas specification satisfies the first set of conditions stated in the previous section we can apply Gorman's results. Assuming utility maximization a composite housing H exists, viz.

$$H = h_1^{\frac{\alpha_1}{\alpha}} h_2^{\frac{\alpha_2}{\alpha}} \qquad \alpha = \alpha_1 + \alpha_2$$

with unit price $p_H = \alpha \left[\left(\frac{p_1}{\alpha_1} \right)^{\frac{\alpha_1}{\alpha}} \left(\frac{p_2}{\alpha_2} \right)^{\frac{\alpha_2}{\alpha}} \right]$

---

[10] A formal proof of this intuitive assertion might go as follows. Denote the solution values of

$$\min_{H,X} \{p_H H + p_X X \mid u \geq u(f^{-1}(H^s), X^s)\}$$

by H' and X'. Similarly, let the quantities solving

$$\min_{h_1,\ldots h_n X} \{ \sum_{i=1}^{n} p_i h_i + p_X X \mid u \geq u(g_H(h_1^s, \ldots h_n^s), X^s)\}$$

be h_1'', h_2'', ..., h_n'', X''. If we define $H'' = f(g_H(h_1'', \ldots, h_n''))$ then it follows $p_H H'' = \sum_{i=1}^{n} p_i h_i''$. Consequently, it will be the case that

$p_H H' + p_X X' > p_H H'' + p_X X''$ as long as $f^{-1}(H^s) > g_H(h_1^s, \ldots, h_n^s)$, which implies EV1 > EV2. If, however, $h_i^s = h_i^m$ then we have $f^{-1}(H^s) = g_H(h_1^s, \ldots h_n^s)$ and $p_H H' + p_X X' = p_H H'' + p_X X''$, which yields EV1 = EV2.

Letting starred variables denote optimal quantities we have[11]

$$(h_1^*)^{\alpha_1} (h_2^*)^{\alpha_2} (X^*)^{1-\alpha_1-\alpha_2} = (H^*)^{\alpha} (X^*)^{1-\alpha}$$

Suppose that a housing program offers h_1^s, h_2^s and charges PR which leaves $p_X X^s = y - PR$ for other goods. The market value of the unit is

$$M = p_H H^s = p_1 h_1^s + p_2 h_2^s .$$

Consider the composite commodity approach. The utility level under the program is $u(H^s, X^s)$ and benefits are easily shown to be

$$EV_C = \left(\frac{p_H H^s}{\alpha}\right)^{\alpha} \left(\frac{p_X X^s}{1-\alpha}\right)^{1-\alpha} - y$$

Using the alternative approach the utility level under the program is $u(g_H(h_1^s, h_2^s), X^s)$ and benefits equal

$$EV_A = \left(\frac{p_1 h_1^s}{\alpha_1}\right)^{\alpha_1} \left(\frac{p_2 h_2^s}{\alpha_2}\right)^{\alpha_2} \left(\frac{p_X X^s}{1-\alpha_1-\alpha_2}\right)^{1-\alpha_1-\alpha_2} - y$$

First we show that $EV_C = EV_A$ if the attribute bundle under the program is the households optimal choice given the market value M of the public unit. In that case the h_i^s solve the problem

$$\text{Max } h_1^{\alpha_1/\alpha} h_2^{\alpha_2/\alpha} \quad \text{subject to } p_1 h_1 + p_2 h_2 = M.$$

---

[11] It is easy to check this result using the definitions of α, p_H and H. It follows that both the left-hand and right-hand side equal

$$\left(\frac{\alpha_1}{p_1}\right)^{\alpha_1} \left(\frac{\alpha_2}{p_2}\right)^{\alpha_2} \left(\frac{1-\alpha_1-\alpha_2}{p_x}\right)^{1-\alpha_1-\alpha_2} - y .$$

This implies $h_i^s = \frac{\alpha_i}{\alpha}\frac{M}{p_i}$ which yields

$$(h_1^s)^{\frac{\alpha_1}{\alpha}} \; (h_2^s)^{\frac{\alpha_2}{\alpha}} = \left(\frac{\alpha_1 M}{\alpha p_1}\right)^{\frac{\alpha_1}{\alpha}} \left(\frac{\alpha_2 M}{\alpha p_2}\right)^{\frac{\alpha_2}{\alpha}}$$

$$= \left(\frac{\alpha_1}{p_1}\right)^{\frac{\alpha_1}{\alpha}} \left(\frac{\alpha_2}{p_2}\right)^{\frac{\alpha_2}{\alpha}} \frac{M}{\alpha}$$

$= H^s$, using the definition of p_H and the fact that $M = p_H H^s$.

Obviously, this is precisely the case where $u(H^s, X^s)$ and $u(g_H(h_1^s, h_2^s), x^s)$ are equal. We also have in this case

$$EV_A = \left(\frac{M}{\alpha}\right)^{\frac{\alpha_1}{\alpha}} \left(\frac{M}{\alpha}\right)^{\frac{\alpha_2}{\alpha}} \left(\frac{p_X X^s}{1-\alpha_1-\alpha_2}\right)^{1-\alpha_1-\alpha_2} - y$$

$$= \left(\frac{p_H H^s}{\alpha}\right)^{\alpha} \left(\frac{p_X X^s}{1-\alpha}\right)^{1-\alpha} - y$$

$$= EV_C$$

It is clear that if the h_i^s do not solve previous maximization problems then

$H^s > (h_1^s)^{\frac{\alpha_1}{\alpha}} (h_2^s)^{\frac{\alpha_2}{\alpha}}$. Straightforward algebra shows that in that case $EV_C > EV_A$.[12]

The "bias" of using the composite commodity approach to calculate benefits is clearly $EV_C - EV_A$:

$$\left(\frac{p_X X^s}{1-\alpha}\right)^{1-\alpha} \left[\left(\frac{p_H H^s}{\alpha}\right)^{\alpha} - \left(\frac{p_1 h_1^s}{\alpha_1}\right)^{\alpha_1} \left(\frac{p_2 h_2^s}{\alpha_2}\right)^{\alpha_2}\right]$$

Some intuition suggests that the bias will be determined by - apart from the parameters of the utility function - the deviations of the attribute quantities h_i^s from the desired quantities h_i^m. We believe the relevance of the bias is largely an empirical matter.[13]

---

[12] $$EV_C = \left(\frac{p_H H^s}{\alpha}\right)^{\alpha} \left(\frac{p_X X^s}{1-\alpha}\right)^{1-\alpha} - y$$

$$> \left(\frac{p_H}{\alpha}\right)^{\alpha} (h_1^s)^{\alpha_1} (h_2^s)^{\alpha_2} \left(\frac{p_X X^s}{1-\alpha}\right)^{1-\alpha} - y$$

$$> \left(\frac{p_1}{\alpha_1}\right)^{\alpha_1} \left(\frac{p_2}{\alpha_2}\right)^{\alpha_2} \left(\frac{p_X X^s}{1-\alpha_1-\alpha_2}\right)^{1-\alpha_1-\alpha_2} - y$$

$$> EV_A$$

[13] For a numerical illustration, suppose $p_1 = 0.4$, $p_2 = 0.625$, $\alpha_1 = \alpha_2 = 0.1$, (therefore $p_H = 1$), $y = 100$, $h_1^s = 15$, $h_2^s = 20$, $PR = 10$. Then $EV_C = 8.18$, $EV_A = 6.76$ which implies the bias is approximately 20% of EV_C.

3. An empirical illustration

In this section we present an empirical example to illustrate the relevance of the bias due to the use of a composite good housing. Our line of reasoning here is obviously somewhat different than in the theoretical part of the paper. There we started out with a utility function defined on attributes and other goods and used the parameters to construct a composite good housing. We then showed that the benefits of a public housing program using this composite will overestimate the benefits based on the original utility function. In this section, however, we independently estimated the parameters of two utility functions - one defined on housing attributes, the other on the housing composite-, calculated benefits in each case and compared the empirical results.

The data were derived from a household survey conducted in Liege, Belgium, in the early seventies. The sample is extremely small : we used 221 households who reported to live in multi-family rental housing. In total, 162 families occupied uncontrolled units whereas as few as 59 households reported to be public housing beneficiaries [14]. Given the sample size, it should be obvious that the results are, at best, to be considered as illustrations of the theory.

As our data did not allow us to observe price variation over the sample, we selected Cobb-Douglas and Stone-Geary specifications for the utility function. Both have been widely used to estimate benefits of housing programs based on the composite commodity approach. The specifications used in this paper are summarized in Table 1. Note that they were also chosen so as to be consistent with the construction of a composite good housing.

In each case we allowed the parameters of the utility function to be different for families with different observed characteristics in order to capture some variations in taste. Relevant household traits included age, professional status, family size and education.

---

[14] The data are described in detail in De Borger (1984,a)).

$u(H,X)$	COBB-DOUGLAS	$u = H^{\alpha} X^{1-\alpha}$
	STONE-GEARY	$u = (H - \beta_H)^{\gamma} (X - \beta_X)^{1-\gamma}$
$u(h_1,\ldots h_u,X)$	COBB-DOUGLAS	$u = [\prod_{i=1}^{n} h_i^{\alpha_i}] X^{1-\sum_{i=1}^{n} \alpha_i}$
	STONE-GEARY	$u = [\prod_{i=1}^{n} (h_i - \beta_i)^{\gamma_i}][(X - \beta_X)^{1-\sum_{i=1}^{n} \gamma_i}]$

Table 1: Utility Function Specifications

The composite commodity approach required the prediction of the market value of the public housing unit by hedonic pricing techniques. The parameters of the utility functions were determined on the basis of an estimated budget share equation for housing. We slightly modified the procedures developed by Olsen and Barton (1983) and Murray (1975) in order to introduce the household characteristics. Estimation results can be found in De Borger (1984 a), p. 42 and 48) for the Stone-Geary and Cobb-Douglas specifications, respectively.

Derivation of the parameters of the utility functions defined on housing attributes required estimation of a system of demand equations for housing characteristics and other goods. In De Borger (1984 b)) it is explained how to derive such a system that is linear in the parameters and includes household traits as explicit explanatory variables from the Stone-Geary utility function. A straight-forward simplification, i.e. $\beta_i = \beta_x = 0$ for all i, yields the Cobb-Douglas demand system. Efficient estimation of the demand equations required the use of a modification of Zellner's technique for seemingly unrelated regressions in order to exploit the singularity of the variance-covariance matrix of the disturbances.

The housing attributes used in the analysis were constructed as suggested by King (1976). On the basis of the available information five rather crudely defined attributes were constructed : 'SANITARY QUALITY', STRUCTURAL QUALITY', 'SPACE', 'APARTMENT TYPE' and an attribute capturing all remaining, unobserved features 'ALL OTHER CHARACTERISTICS'.

The construction of the attributes and the estimation results for the Stone-Geary demand system are presented in De Borger (1984 c)). The estimated Cobb-Douglas demand equations are given in an appendix to the present paper.

Once the parameters of the utility functions have been estimated using the subsample of households living in uncontrolled apartments, the calculus of the benefits of public housing for the beneficiaries is a relatively simple exercise. The relevant formulas for the equivalent variation are summarized in table 2.

The theory presented in the previous section suggested that benefits based on the composite commodity approach would overestimate the 'true' benefits calculated using the bundle of attributes as the appropriate housing concept. Mean benefits reported in table 3 are consistent with this prediction. On average benefits based on $u(\bar{H}, X)$ are approximately 50 % and 85 % higher for the Cobb-Douglas and Stone-Geary specifications, respectively[15]. Admittedly, for several households in the sample benefits estimated using $u(h_1, \ldots, h_n, X)$ exceeded those based on $u(\bar{H}, X)$. We do not consider this to be a surprising finding, however, since the parameters of the respective utility functions are obviously estimated with error and quite

[15]It should be noted that both for the composite commodity approach and its alternative based on housing attributes a priori information on one subsistence parameter is needed to identify all remaining β_i's of the Stone-Geary utility function. The figures reported in table 3 were obtained using extraneous information on β_H (composite approach) and the subsistence parameter for 'APARTMENT TYPE' (attribute approach). The results were slightly different when other a priori assumptions were made. The main conclusions were in no way affected, however.

$u(H,X)$	COBB-DOUGLAS	$EV = \left(\frac{p_H H^s}{\alpha}\right)^{\alpha} \left(\frac{p_X X^s}{1-\alpha}\right)^{1-\alpha} - y$
	STONE-GEARY	$EV = \left[\frac{p_H(H^s-\beta_H)}{\gamma}\right]^{\gamma} \left[\frac{p_X(X^s-\beta_X)}{1-\gamma}\right]^{1-\gamma} + p_H\beta_H + p_X\beta_X - y$
$u(h_1,..,h_n,X)$	COBB-DOUGLAS	$EV = \left[\prod_{i=1}^{n}\left(\frac{p_i h_i^s}{\alpha_i}\right)^{\alpha_i}\right]\left[\frac{p_X X^s}{1-\sum_{i=1}^{n}\alpha_i}\right]^{1-\sum_{i=1}^{n}\alpha_i} - y$
	STONE-GEARY	$EV = \left[\prod_{i=1}^{n}\left[\frac{p_i(h_i^s-\beta_i)}{\gamma_i}\right]^{\gamma_i}\right]\left[\frac{p_X(X^s-\beta_X)}{1-\sum_{i=1}^{n}\gamma_i}\right]^{1-\sum_{i=1}^{n}\gamma_i} + \sum_{i=1}^{n} p_i\beta_i + p_X\beta_X - y$

Table 2: Benefit Formulas for Alternative Specifications

		Mean Benefit	Observed Standard Deviation
$u(H, X)$	COBB-DOUGLAS	8.842	4.846
	STONE-GEARY	8.325	3.589
$u(h_1,...,h_n,X)$	COBB-DOUGLAS	5.947	5.032
	STONE-GEARY	4.428	5.240

Table 3: Mean benefit and calculated standard deviation over the sample for alternative specifications (in 10^2 Belgian francs for month).

different estimation procedures were used. Moreover, the set of household characteristics entering the demand equations was different for the two alternative approaches. For more details, see the references given above[16].

A second proposition of the theory was that, if a housing program would offer the optimal quantities of attributes conditional on the market value of the public unit, both approaches would lead to the same result. Since no housing program can ever be designed to fulfill this condition for all households, the described situation is obviously purely hypothetical. It does suggest a second crude test of the theory, which yields some interesting additional information as well.

Using the estimated parameters of the utility functions $u(h_1, \ldots h_n, x)$ we first predicted for each household in public housing the desired combination of attributes, provided that the bundle had the same market value as the public unit. As previously indicated, these 'optimal' quantities h_i^m are the solution to the problem

$$\text{Max } u(h_1, \ldots, h_n, x)$$

$$\text{s.t. } p_X X + PR = y$$

$$\sum_{i=1}^{n} p_i h_i = M$$

[16]Note also that the benefits based on a composite good housing and a Stone-Geary utility function are slightly different from those reported in De Borger (1984 a), p. 52). The latter results were calculated using both apartments and single-family units (65 observations), whereas the former only used the subsample of households in apartments (59 observations).

It is straightforward to show that the h_i^m are given by the following expressions, for the Cobb-Douglas and Stone-Geary utility functions, respectively :

$$h_i^m = \frac{\alpha_i}{\alpha} \frac{M}{p_i} \text{, where } \alpha = \sum_{i=1}^{n} \alpha_i$$

$$h_i^m = \beta_i + \frac{\gamma_i}{\gamma} \left(\frac{M - \sum_{i=1}^{n} p_i \beta_i}{p_i} \right) \text{, where } \gamma = \sum_{i=1}^{n} \gamma_i$$

Mean attribute consumption under the program is compared with the predicted h_i^m in table 4. If households were allowed to select a housing unit with the same market value as the public unit they occupy and pay a rent PR, they would on average choose more spacious units and give up a little bit of quality. Although this space-quality trade-off may seem somewhat surprising, it is a direct consequence of the characteristics of most public units. We found earlier (De Borger 1984 c)) that many public housing beneficiaries would consume more space and less quality in the absence of the program, reflecting the fact that housing programs provide in general relatively high quality but relatively little space.

The predicted h_i^m were finally inserted in the benefit formulas so as to obtain the equivalent variation of the hypothetical, 'ideal' housing program that offers the desired bundle of attributes. The following results were obtained :

	Mean Benefit	Standard Deviation
Cobb-Douglas	8.627	5.332
Stone-Geary	8.224	5.189

	Consumption under the program h_i^s		Predicted optimal consumption, conditional on the market value of the public unit, M. h_i^m			
			Cobb-Douglas		Stone-Geary	
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
SANITARY QUALITY	6.512	3.073	5.468	2.045	5.946	2.431
STRUCTURAL QUALITY	9.676	2.685	9.917	3.291	9.008	1.857
SPACE	7.363	1.891	9.486	3.278	8.382	0.988
APARTMENT TYPE	6.245	5.718	4.305	1.836	4.894	2.289
ALL OTHER CHARACTE-RISTICS	- 1.881	0	- 1.256	2.243	- 0.310	3.679
TOTAL HOUSING	27.920	9.696	27.920	9.696	27.920	9.696

table 4 : observed attribute consumption, h_i^s, and optimal consumption given the market value of the public unit (*).

(*) Again, the results for the Stone-Geary are based on using extraneous information on the subsistence parameter for 'APARTMENT TYPE'. Different, but very similar results were obtained for other a priori assumptions.

As predicted by the theory, mean benefits of the hypothetical program come extremely close to those calculated using the composite commodity approach, see table 3. Both for the Cobb-Douglas and Stone-Geary specifications mean benefit is within 3 % of the numbers reported in table 3. Obviously, due to estimation errors and differences in estimation procedures individual benefits were not always that close. Still we believe that the very similar result obtained for mean benefit is a remarkable finding that supports our earlier theoretical work.

We think the results of this paper have some important empirical implications. We have argued that the composite commodity approach to benefit estimation is not entirely satisfactory because it implicitly assumes that households are indifferent between all combinations of housing attributes with the same market value[17]. Both the theory and the empirical analysis suggest that the bundle of attributes provided under a public housing program should be taken into account when evaluating the program's benefits [18].

---

[17] A quite similar argument is used in the literature to reject the use of the subsidy, i.e. the difference between the market value of the public unit and the rent actually paid, as a measure of consumer benefit. This would be unsatisfactory because households are not indifferent between all bundles of housing and other goods with the same total market value.

[18] We only considered the implications of using different housing concepts without even mentioning other sources of possible bias in estimates of mean benefit. For a discussion of selection and aggregation bias, see Olsen and Barton (1983) or De Borger (1984 a), p. 28-29).

4. Summary and Conclusion.

In this paper we have investigated the implications of using different concepts of "housing" to calculate the benefit of public housing programs. We compared the classical composite commodity approach with an obvious alternative, in which housing is defined as a bundle of attributes.

In the theoretical part of the paper we specify a utility function on housing attributes and other goods and analyze what kind of restrictions on preferences have to be imposed in order to justify the construction of a composite good housing with a constant unit price. We then show that, if these conditions are satisfied, the benefits using the composite commodity approach provide an upper bound to the benefits calculated on the basis of the alternative housing concept. The reason for the bias due to the former method was indicated to be the fact that households are unable to choose the optimal combination of attributes, conditional on the market value of the public unit. Both procedures will only give the same result in the hypothetical situation of a housing program that provides for each household its most preferred bundle of attributes, given the market rent of the unit they occupy. In other words, the composite commodity approach implicitly assumes that this optimal bundle is provided.

An empirical example strongly suggested the relevance of our results in practice. Comparing the two alternative methods using Cobb-Douglas and Stone-Geary utility functions, we found that mean benefit using the composite commodity approach exceeded mean benefit based on the alternative procedure by almost 50% and 85%, respectively. This indicates that the composition of the bundle of housing characteristics should be taken into account in the process of estimating benefits of public housing programs.

Appendix : regression results Cobb-Douglas demand system

For completeness sake, we report here the estimated demand functions for housing attributes derived from the Cobb-Douglas utility function. Results are in table A1. (*) The dependent variables are $\frac{p_i h_i}{y}$, where y is income.

table A1 : see page 27

(*) Definition of the variables is as follows (also see De Borger (1984 C)) :

CH12, CH3+ : dummies equal to 1 if the household contains 1 or 2, 3 or more children, respectively.

ED1, ED2 : dummies for education, ED1 = 1 if highest educational attainment was a high school degree, ED2 = 1 if it was a higher degree.

PROCLM : dummy variable equal to one if husband is white-collar.

PRACTW : dummy variable equal to one if wife has some professional activity.

	SANITARY QUALITY	STRUCTURAL QUALITY	SPACE	APARTMENT TYPE	ALL OTHER CHARACTERISTICS
CONSTANT	0.033 (2.58)	0.066 (3.18)	0.0529 (3.27)	0.0318 (1.98)	- 0.022 (- 1.74)
CH12			0.0108 (2.77)	- 0.0138 (- 1.18)	
CH3+			0.0186 (4.10)	- 0.0065 (- 1.76)	
ED1	- 0.00295 (- 0.78)	- 0.0067 (- 1.74)	- 0.0082 (- 1.91)	0.0094 (1.88)	0.0160 (1.79)
ED2	- 0.00231 (- 1.54)	- 0.0063 (- 1.51)	- 0.0101 (- 2.18)	0.0038 (0.68)	0.0121 (1.34)
PROCLM	0.0112 (3.56)	0.0023 (1.72)	- 0.004 (- 1.11)	0.0055 (1.26)	- 0.0056 (1.75)
PRACTW	- 0.0108 (- 3.77)	- 0.0158 (- 5.38)	- 0.0125 (- 3.69)	- 0.0138 (- 3.34)	0.0077 (2.12)
R^2 =	0.16	0.19	0.28	0.13	0.11

table A1 : regression results Cobb-Douglas system (t-statistics in parentheses)

References

1. Anas, A. and S.J. Eum, Hedonic Analysis of a Housing Market in Disequilibrium, _Journal of Urban Economics_, 1984, p. 87-106.

2. Awan, K., Odling-Smee, J.C. and C.M.G. Whitehead, Household Attributes and the Demand for Private Rental Housing, _Economica_, 1982, p. 183-200.

3. Clemmer, R.B., Measuring Welfare Effects of In-Kind Transfers, _Journal of Urban Economics_, 1984, p. 46-65.

4. Cronin, F., The Household's Decision to Accept or Reject a Conditional Transfer Offer, _Southern Economic Journal_, 1982, p. 218-234.

5. Deaton, A. and J. Muellbauer, _Economics and Consumer Behavior_, Cambridge University Press, 1980.

6. De Borger, B., Housing Demand and the Benefits of Public Housing Programs, the Composite Commodity Approach, SESO-report 84/151, February 1984 a).

7. De Borger, B., Household Attributes and the Demand for Private Rental Housing : Comment, _Economica_, 1984 b), forthcoming.

8. De Borger, B., The Demand for Housing Attributes and the Benefits of Public Housing Programs, unpublished, 1984 c).

9. Gorman, W.M., Separable Utility and Aggregation, _Econometrica_, 1959, p. 469 - 481.

10. King, A.T., The Demand for Housing : A Lancastrian Approach, Southern Economic Journal, 1976, p. 1077-1087.

11. Lancaster, K., Consumer Demand : A New Approach, Columbia University Press, New York, 1971.

12. Murray, M.P., The Distribution of Tenant Benefits in Public Housing, Econometrica, 1975, p. 771-788.

13. Murray, M.P., Hedonic Prices and Composite Commodities, Journal of Urban Economics, 1978, p. 188-197.

14. Muth, R.F., The Demand for non-Farm Housing, in : Harberger (ed.), The Demand for Durable Goods, Chicago, 1960.

15. Olsen, E.O., An Econometric Analysis of Rent Control, Journal of Political Economy, 1972, p. 1081-1100.

16. Olsen, E.O, and D.M. Barton, The Benefits and Costs of Public Housing in New York City, Journal of Public Economics, 1983, p. 299-332.

17. Quigley, J.M., Nonlinear Budget Constraints and Consumer Demand; An Application to Public Programs for Residential Housing, Journal of Urban Economics, 1982, p. 177-201.