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Housing demand and the benefits of public housing programs in Belgium: the composite commodity approach.

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Abstract

In this paper we estimate some aggregate consumption effects and the benefits of public housing programs in Belgium on the basis of a small sample of individual households. Treating housing and non-housing goods as two composite commodities we derive analytical expressions for Hicks equivalent variation as well as the Marshallian measure of benefits on the assumption of Stone-Geary preferences. Formulas for evaluating aggregate consumption effects of the housing programs are also developed.

Empirical implementation of the theory requires the prediction of several unknown inputs to the benefit formulas. Hedonic price regressions are used to predict the market rent of public housing units. The parameters of the specified indifference map are derived by estimating the budget share equation for housing. Allowing for some variations in taste these parameters can be different for households with different characteristics.

The results provide information on aggregate housing demand and on the benefits participating households deriven from public housing programs. With respect to the former we found the marginal propensity to spend on housing to be 0.127 on average over the sample. Price and income elasticities of housing demand were within the range of accepted values in the urban economics literature. The benefits of the programs were estimated to be relatively small approximately 6.5 % of income on average. However, the programs yield substantial aggregate consumption effects. Many public housing tenants were found to consume significantly more housing than they would have done in the absence of the programs. Moreover, a lot of these households consume more non-housing goods as well.

In the paper we further discuss the sensitivity of the final results with respect to prediction errors and we analyze the relation between the estimated benefits and a set of household traits in order to gain at least a partial insight into the distributional effects of public housing.

Introduction

Although large amounts are spend on housing subsidies every year there has been surprisingly little research on the economic effect of subsidized housing on the participating households. In this paper we try to fill a very small part of this gap by analysing the benefits and consumption effects of public programs for rental housing in Belgium on the basis of a small sample of individual households.

Until recently studies relating to the housing market almost exclusively emphasized the importance of the construction sector in the macro economic activity of our country and investigated its possible use as an intermediate policy instrument for controlling the business cycle. By far the most comprehensive study of this type is the excellent book by Van Broekhoven, Borghers and Havermans (1972). In the meantime, however, economists paid hardly any attention to public housing in general and to its impact on the beneficiaries of the programs in particular. As far as we know the study by V. De Ridder and P. Minon (1980) is the first published analysis of the economic and social consequences of a specific housing program. Although they do not particularly emphasize the effects on consumption patterns the detailed tabulation and careful interpretation of the results of a recent survey provides valuable information. More recently M. Durez-Demal (1982) has analyzed - for a particular program - a sample of participants with respect to their income and family size. Her study is of a descriptive rather than empirical nature as its main purpose is to characterize the programs in terms of the household attributes income and size of the participating families.

The previous studies and the work by sociologists (see e.g. L. Goossens (1979)) certainly provide useful and interesting information with respect to the working of public housing programs. On the other hand, it's safe to say that almost nothing is known at this moment about the benefits participating households derive from these programs. In this paper the first attempt is made to calculate the benefits and to predict the consumption effects generated by the programs. Although a lot of other topics deserve the attention of researchers as well we will only deal with rental housing programs (*). Under these programs selected eligible households are offered the opportunity to occupy a specified public unit at a specified rent. The latter is calculated by the housing authorities on the basis of the characteristics of the unit, although rent reductions are granted to large families and families with a very low income. The programs clearly offer selected households an all-or-nothing choice. In economic terms they do not result in a simple rotation of the budget constraint but rather add an additional point to the budget space i.e. they should be interpreted as quantity-constrained price subsidies.

The benefits and the consumption effects will be calculated using the classical 'housing services' approach. Although recent developments in hedonic pricing theory and modeling of discrete choices have suggested alternatives for the analysis of public housing programs we have chosen for this approach in order to be consistent with most earlier work on the subject in other countries.

Our paper is organised as follows: in the first chapter we briefly review the most important studies on public housing programs found in the literature. The survey is very specific and entirely concentrates on those papers that are concerned with the calculus of benefits. In chapter two the theoretical framework used in this paper is presented. First we explain the importance of the assumptions underlying the study and we analyse under what conditions they can be justified and used as acceptable working hypotheses. Next we investigate how explicit benefit formulas can be derived from a specified direct

(*) Public programs for the rental housing market are almost exclusively operated by the 'Nationale Maatschappij voor de Huisvesting (NMH)'.

utility function. Under the assumption of Stone-Geary preferences it is straightforward to calculate analytical expressions for both Hick's equivalent variation and Marshallian benefits. In order to evaluate these expressions, however, we have to predict several unobserved inputs to the formulas. We develop prediction methods for the market rent of public housing units, the parameters of the utility function and household expenditures on housing and other goods in the absence of the program.

Chapter 3 contains the results that were derived from our sample. In the first subsections the predicting equations for market rents and the utility function are presented together with some additional information concerning the demand for housing such as price and income elasticities. In section 3 we give the aggregate results that were obtained when calculating the benefits of the programs and some general consumption effects. We also analyse the consequences of replacing the programs with a system of direct cash transfers and compare the results with the actual situation. It's important to consider the sensitivity of our results with respect to possible prediction errors in the inputs to the benefit formula. Moreover, as will be seen below, in arriving at our final results several very restrictive assumptions had to be made. The effects of these assumptions on the calculated benefits should be investigated as well. These problems are discussed in section 4 and some sensitivity results are presented. In a final section we carefully analyse the distributional pattern of benefits i.e. how do the estimated benefits relate to observed household characteristics. The study of this relation is not unimportant and may reveal possible inconsistencies between the official program goals and the results of their actual operation.

Finally, chapter 4 reviews the main results of this paper. A description of the data is presented in appendix.

I. A summary review of the literature

The involvement of economists in public housing programs started in the early sixties. The first attempts to measure the benefits of such programs and the associated welfare loss were developed in this period. They used the 'subsidy' - i.e. the difference between the market rent of the public unit and the rent actually paid by the tenants - as a measure of benefits. The pioneering study in this field is probably Prescott (1964), whose results have been used in subsequent work by e.g. Smolensky (68) and Olsen and Prescott (1969).

Although the subsidy has been used to estimate benefits in more recent years (Aaron (1972), E. Browning (1976), it's clear that it approximates rather poorly the true welfare effect for public housing tenants. Indeed, the utility level achieved under the programs may be quite different from the level associated with a cash grant equal to the subsidy. Therefore the latter cannot be considered a reasonable measure of benefits.

Olsen (1972) tried to estimate the welfare effects of housing programs using Marshallian consumer surplus. It's well-known, however, that this concept has no clear interpretation in economic terms and that it will lead to different results than other measures based upon explicit utility functions unless some very strong and unrealistic assumptions are introduced. Only when the income elasticity of housing demand is zero will Marshallian benefit coïncide with Hicks' equivalent and compensating variation. A substantial amount of empirical evidence exists, however, that contradicts this assumption.

In a remarkable paper De Salvo (1971) convincingly suggested the estimation of indifference maps in order to derive benefits of public programs. His methodology consists of specifying an explicit utility function defined on two composite commodities 'housing' and 'other goods', empirically estimating the parameters and using them to calculate Hicks' equivalent variation. Since the publication of De Salvo's paper nearly all further housing studies have used his suggested procedure. These studies only differ with respect to the type of data they have used and the form of the specified utility function. The functional form determines the analytical difficulty with respect to estimation of the indifference map parameters and the calculus of benefits. In general researchers have to trade-off the use of realistic utility functions against simplicity of estimation procedures and ease of calculation of the cash equivalent of the housing programs.

The first attempts to estimate the equivalent variation generally used the Cobb-Douglas utility function (De Salvo (71,75),Kraft and Olsen (1977)). Its advantages are numerous: the resulting demand functions are easy to estimate and an analytical expression can be derived for the benefits. However, the implied restrictions upon price and income elasticities - all equal to one in absolute value - are indesirable, though not necessarily unreasonable in the case of housing (*).

The possible unrealistic assumptions with respect to price and income elasticities suggested the use of more general specifications. In a largely non-empirical paper Aaron and Von Furstenberg (1971) argued in favor of the CES utility function. The latter is not straightforward to estimate, however, and it doesn't yield an analytical formula for the equivalent variation. Moreover, it still imposes unitary income elasticities. It was, by the best of our knowledge, never used in applied work until very recently.

Instead Murray (1975) worked with a generalisation of the CES:

$$u = (aH^b + X^c)^d$$

where: H : housing consumption
X : consumption of other goods
a, b, c, d: parameters (d is arbitrary)

This formulation imposes no restrictions on elasticities and

(*) Indeed many studies find price and income elasticities that are not significantly different from one in absolute value. For a review of elasticity values and an analysis of econometric biases see De Leeuw (1970), Mayo (1981), Polinsky (1977) and Vaughn (1976).

reduces to the CES for $c = b = \frac{1}{d}$. Using instrumental variable procedures the first-order conditions can be estimated to yield the required parameters a, b, and c. Although the generalized CES doesn't lead to an analytical formula for benefits either, iterative numerical approximation procedures can be used to calculate benefits up to any desired level of accuracy.

It must be noted that, whereas estimation of Cobb-Douglas preferences doesn't require explicit price variation, the CES and its generalisation cannot be estimated unless explicit price indices for housing services and other goods can be constructed over the sample. It's clear that whenever benefits are calculated on the basis of samples on households living in one particular town it may be difficult to observe any price variation at all. An important step forward, therefore, was the use of the Stone-Geary utility function which relaxes several strong assumptions on price and income elasticities and remains quite acceptable with respect to analytical convenience. It implies linear demand functions that can be estimated, using a priori information, without explicit price indices for housing services and other goods (*). Although a 'thrick' is necessary to circumvent the lack of price variation in the sample (or the impossibility of observing it) this procedure makes estimation of a more general indifference map than the Cobb-Douglas feasible. Moreover an expression for the equivalent variation can be derived. It has extensively been used recently by e.g. Cronin (1979,1982), Friedman and Weinberg (1978), Olsen and Barton (1983) etc.

It's obvious that many other functional firms could be used in theory in the context of public housing program evaluation. However, general specifications such as the translog or generalized Leontieff have never been used for this purpose.

(*) It has been used for samples containing price information as well, see the unique study by Friedman and Weinberg (78). Kain and Quigley (1975) estimated the Stone-Geary demand function simply by ignoring and thus deleting price terms. It's clear that this procedure is undesirable since it doesn't allow identification of all parameters. Moreover the estimated marginal propensity to spend on housing will be biased because of specification error, as shown by Polinsky (1977).

The knowledge that the price of housing services is unobservable together with the inherent difficulties in estimating the derived demand functions make these utility functions completely unattractive for this type of applied work. Moreover it has been indicated (Wales (1977)), that, although these functional forms provide a local second order approximation to an arbitrary utility function, they might not provide a reasonable approximation over a range of observations.

In a recent doctoral dissertation by Clemmer (1981) an original but not entirely convincing approach is elaborated. He starts from the well-known observation that Hicks equivalent variation can be represented by the area under the appropriate compensated demand function. The author derives a straightforward expression for the benefit measure on the assumption that the compensated demand function, which is by definition unobservable, can be estimated on the basis of observed variables. He specifies a real-income constant demand curve relating housing services to the price of housing and income deflated by an overall cost-of-living index. The interpretation of this demand function as a compensated demand relation relies upon his assumption that the constructed real-income variable is an acceptable surrogate for unobservable utility. The way the deflator is constructed suggests that this is far from being the case. Using data on 39 towns price indices for housing and other goods are developed. These are combined into an overall price index based on averages of housing and nonhousing expenditures in each city and using mean expenditures in each category in all towns together as weights. It's clear that this income deflating cost-of-living index is of the Laspeyres type i.e. it's a weighted average of prices with fixed weights. It suffers from the classical index number problem and is therefore likely to be a rather poor cost-of-living index (*). Apart from this flaw, the author developes an ingeneous way to calculate the equivalent variation of quantity-constrained price subsidies. It should be noted, however, that the method requires the availability of price variation over the sample.

It must finally be indicated that the developments concerning hedonic procedures have suggested the estimation of demand functions defined on housing attributes (see Kain and Quigley (1969), King (1976); utility functions defined on attributes have been estimated by e.g. Galster (77), Wheaton (77), Quigley (82) and Awan, Odling-Smee and Whitehead (82)). This substantial literature and its relevance for public housing evaluation will not be discussed in this paper.

(*) The ideal cost-of-living index value for city j as compared to a town of reference 0 would be $\frac{e(p^j,u^r)}{e(p^0,u^r)}$ where r is a reference utility level and e(p,u) is the expenditure function. The procedure used by the author is far from perfect:'A cost-of-living index should adjust money incomes across areas such that a family with $ X in deflated income would be indifferent between any two areas. In other words, a true adjustment would require knowledge of individual utility functions'.(Clemmer P. 34).

II. Theoretical foundations

In this chapter we discuss the theoretical foundations of the composite commodity approach to benefit estimation. First we carefully analyse the assumptions underlying the analysis and indicate their implications for the empirical procedures to be developed. Taking into account some implicit restrictions in the working of the housing programs under investigation in this paper we present a simple graphical analysis that allows us to indicate the inputs that are necessary in the estimation procedure of benefits.

The remainder of this section deals with the empirical implementation of the theory. We analyse how to derive explicit formulas for several benefit measures on the basis of a specified direct utility function. Prediction methods for several unobserved inputs are developed and the properties of the predictors are discussed.

1. Assumptions

The procedure for estimating benefits of public housing programs is based upon a set of assumptions some of which are quite restrictive. We think it's useful to review the main underlying assumptions and to discuss the limitations they impose on the empirical analysis. This will clarify the strength as well as some apparent weaknesses of the approach and will help us to interpret the empirical results - to be derived in the next section - in an appropriate way.

It's assumed that all commodities can be aggregated into two composites 'housing' and 'other goods'. Housing is considered to be durable capital providing a flow of services per time period. Consumption is measured in terms of unobservable units called housing services (*). This concept transforms an intrinsically heterogeneous commodity into a one-dimensional homogeneous good. In long-run equilibrium a unique unit price of housing services will exist at any given location. As noted by Diamond and Smith (1981), this housing concept implicitly assumes complete divisibility of housing and costless transformation of housing services into dwelling inputs. As a consequence the unit price of housing does not depend upon the size of the bundle contained in different dwelling units.

The aggregation of all goods into two composite commodities and especially the construction of the 'housing' composite has raised objections both from economic theorists and applied urban economists. Theoretically, aggregation of goods into a small number of composites is sometimes justified on the basis of Hicks composite commodity theorem (Hicks (1936)) which states, loosely speaking, that if for a group of commodities relative prices are fixed this group can be treated as a single commodity. More specifically, if the prices of the goods within each composite move in parallel then there exists utility function and a budget constraint defined on the composite

(*) The concept 'housing services' was introduced by R. Muth (1960). It was extensively used by E. Olsen (1969) in the developments of a competitive theory of the housing market. Since its introduction most theoretical work on housing markets and many public policy studies have used the housing services approach.

commodities that yields the same expenditures on these goods as would result from a utility maximisation problem defined on all original commodities (*).

It's clear that the constancy of relative prices within the composites 'housing' and 'other goods' is a heroic and unrealistic assumption. It seems that the composite commodity theorem offers no justification for the proposed aggregation. Economists feel especially uncomfortable with the construction of a composite housing. This attitude is reinforced by the development of hedonic pricing techniques that shifted the attention of researchers towards the analyis of housing in terms of its utility bearing characteristics. Moreover, the observed heterogeneity of the housing commodity and the variation of estimated hedonic prices for housing attributes raised serious questions about the validity of constructing a homogeneous housing concept with a constant unit price.

Recent work clarifies the conditions under which it's possible to construct a single housing commodity. Analysing the traditional hedonic models Muellbauer (1974) and Pollak and Wachter (1975) suggested that restrictions on technology and preferences were required in order to make them coherent and empirically relevant. They showed that if housing

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(*) It may be instructive at this point to define a composite commodity more formally as follows. Suppose we have a vector $\underline{x}$ of n goods $x_i (i=1,\dots,n)$ with corresponding prices p_i. A composite commodity $X(x_1,\dots,x_r)$ with corresponding price $P(p_1,\dots,p_r)$ exists for the first r goods if, for a utility function $u(\underline{x})$ there exist functions $X(x_1,\dots,x_r)$ and $v(X,x_{r+1},\dots,x_n)$ such that maximisation of $u(\underline{x})$ subject to $y=\sum_{i=1}^{n} p_i x_i$ and maximisation of $v(X,x_{r+1},\dots x_n)$ subject to $y=PX+\sum_{i=r+1}^{n} p_i x_i$ lead to the same total expenditures on the first r goods (i.e. $PX=\sum_{i=1}^{r} p_i x_i$) and yield the same demand for all other goods $x_{r+1},\dots,x_n$.

characteristics are to show the properties of ordinary, though implicit, goods one of two alternative sets of conditions should be satisfied

1) the production of housing characteristics is non-joint and characterised by constant returns to scale.
2) in the case of joint production of characteristics production should be constant returns to scale and preferences homothetic.

Recently M. Murray (1978) elaborated upon these results. He showed that the latter set of restrictions on technology and preferences imply that it is possible to construct a single composite commodity 'housing'. Thus if we assume that housing characteristics are produced from physical inputs under a linear homogeneous technology and the utility function is homothetic (*) then it's possible to construct the classical composite 'housing services', as suggested by Muth-Olsen. It follows that the strong assumption of constant relative prices is not required in order to define a single housing good (**) with a constant price per unit at any given location.

Although many economists will correctly argue that the assumptions for aggregation of housing are still very restrictive we believe they are no more restrictive than those necessary to justify the analysis of consumer demand defined on broad commodity groups such as 'food', 'transportation','clothing' etc. Making abstraction of the composite commodity theorem the required conditions have been derived by Gorman (1959). A review of his results together with some other approximate justifications for the aggregation of consumer demand in broad commodity

(*)The conditions may even be weakened. It's sufficient that the subfunction defined on housing characteristics in a weakly seperable utility function is homothetic.

(**) In independent work Diamond and Smith (1981) show that a constant unit price of housing services is only possible if the production function of housing services is constant returns to scale. In that case the long-run average cost, which in equilibrium equals the unit housing price, will be constant.

groups is provided by Deaton and Muellbauer (1980, p. 129-133). Although the restrictions on preferences are very strong in applied demand analysis aggregation seems to be common practice (*).

Despite the strong assumptions underlying its construction the concept 'housing services', as previously described, has intuitive appeal and has been very useful in theoretical and applied economics. It allows us to describe a households problem as a utility maximisation problem subject to a linear budget constraint. The definition of two composite commodities implies that the simple diagrammatic expositions of standard microeconomic textbooks can be used. However, it's clear that the simplifying assumptions have their cost. Indeed, the composite commodity approach ignores the multidimensionality of housing and obscures the distinction between quantity and quality aspects. As a consequence, many interesting questions relating to the demand for housing attributes cannot be answered. As to benefit estimation the main problem with the approach seems to be the fact that it ignores important aspects of location theory. The trade-off between housing and transport costs is not explicitly incorporated. Transportation is part of the 'other goods' composite and it's assumed that no utility gains or losses are associated with changes in location (**). Fortunately this is probably not a major problem in this study as all data relate to households living within a relatively small geographical area, i.e. the central town of Liège.

A further assumption is that the public housing program under investigation has no effect on market prices for housing and other goods. Many economists will argue that this is an unrealistic assumption unless the supply of housing services

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(*) The strong objection to aggregating housing in a composite commodity is somewhat surprising. The observed heterogeneity of the good is no justification for this attitude. Indeed, the same economists seem to worry a lot less if one constructs a composite 'food', which includes both bread and caviar.

(**) This has important implications for benefit estimation. Changes in location (and their effect on utility) due to public housing programs will not be captured appropriately by the estimation procedure.

is perfectly elastic (*). Their argument is based upon a simple demand and supply analysis of the housing market. If the government introduces public housing resulting in new construction then the supply curve shifts to the right which implies a downward effect on prices. This reasoning can be found in many microeconomic textbooks and in policy studies of the housing market (e.g. Weicher (1979)). Although it is widely accepted we think the argument is not entirely convincing. It would be correct if no public construction was undertaken at all and the government decided at once to build additional housing, thereby increasing the existing housing stock by a substantial fraction. In that case private builders cannot react in the short run and the classical textbook reasoning applies, which results in a downward pressure on housing prices. However, Belgium has a long history of government intervention on the housing market so that it's more realistic to assume that the program under investigation just adds a small proportion of the housing stock (**). As a consequence, the effect on prices will probably be small. Moreover, in this case private construction will depend, among other factors, upon builders expectations of government production. If more public housing is announced by the government private builders have the opportunity to adjust production (e.g. by intensifying nonhousing construction activities) so that there needn't necessarily be an effect on the housing price level. Insofar as these adjustment processes occur in practice government construction is just replacing private building activities that would have taken place in the absence of the housing program. A study by Swan (1973) e.g. concludes that for several specific housing programs public production largely substituted private

(*) There are very few reliable estimates of the supply elasticity of housing services. Although the estimated values vary considerably many authors find elasticities substantially less than infinity see e.g. De Leeuw and Ekanen (1971), Griesai (1973), Smith (1976). Only R. Muth (1960) concludes that the supply of housing services is close to perfectly elastic.

(**) This is even more the case in this study which only deals with rental housing.

construction (*). Although there is no empirical evidence on this subject for Belgian programs it's not impossible that the same process operates to some extent on the Belgian housing market as well (**).

Taking into account previous remarks and the fact that the rental housing programs analysed in this study represent only a minor fraction of the total production of housing services the assumption underlying the analysis seems an acceptable working hypothesis.

Except for a recent dissertation by Hammond (1982) all studies of public housing programs assume intertemporal separability of the utility function. This implies that the demand for housing and other goods only depends upon contemperaneous prices and expenditures. It's further assumed that all households have to spend all their income in every successive period. There is no saving or dissaving and income rather than wealth is the relevant variable.

It's clear that the assumption of intertemporal separability is very restrictive. Although intertemporal models have been developed (see e.g. Diewert (1974)) practical estimation of intertemporal indifference map parameters requires a large number of additional assumptions and some data that are in general difficult to obtain. The necessary assumptions (concerning people's life expectancy, patterns of wealth accumulation, the discount rate) and the multitude of variables that are unobservable and have to be predicted make it difficult to judge whether benefits of housing programs are calculated more accurately using intertemporal indifference maps rather than the simple contemperaneous utility functions. This question can only be answered by further research on the subject.

(*) He found that over 85 % of government construction in the specified programs would have been undertaken by private builders in the absence of the programs.

(**) If we believe that government construction is not just replacing private activities then a countervailing force might even cause housing prices to rise. In that case increased housing construction may lead to higher input prices - under the assumption of perfectly competitive input markets -, resulting in higher housing prices.

However, as our data are ill-suited for the estimation of intertemporal indifference maps, we decided to retain the assumption of intertemporal separability.

It's also assumed that people in public housing choose the same job and work the same number of hours as they would without the program. Although some preliminary results by Murray (1980) suggest slight work disincentive effects caused by public housing his results should be confirmed by more research before strong conclusions can be reached on this subject. Therefore we make abstraction of possible effects of housing programs on work efforts.

Finally we assume that the household rather than the individual is the approppriate economic unit. This is an important assumption because it's not always clear that a households 'decision-maker' correctly takes into account the welfare of all members of the family. It has e.g. been suggested that they might have on inadequate perception of childrens preferences in making their decisions or just fail to recognize their importance.

2. Graphical analysis

Under the previous set of assumptions it's straightforward to indicate the benefit concepts we will use in this study on simple two dimensional diagrams. Indeed, if we assume that households maximize utility, defined on the composite commodities 'housing' and 'other goods', subject to a linear budget constraint the usual indifference curve analysis is applicable.

It's important to note, however, that the programs under investigation in this study do not result in a simple rotation of the budget constraint i.e. they have not the same effect as pure price subsidies. This is a direct consequence of the way the programs operate. The Housing Authorities rent their units at below market rents to selected eligible families. Although household characteristics such as family size are taken into account in the process of assigning available units to participating families, the latter are not able to optimize their housing choice at the subsidised price of housing services. It's clear that the programs, instead of rotating the budget constraint, just add one consumption bundle to the households budget space. This bundle depends upon the specific unit to which the family is assigned and the rent charged by the Housing Authorities. Consequently the programs result in an extreme form of quantity-constrained price subsidies. It's also evident that for participating families the bundle consumed under the program is not a utility maximizing choice, an observation which has important implications that will be relevant later.

Let's consider the effects of the program for an individual family. Suppose, without the program under investigation, the household faces market prices p_m^h and p_m^x for housing and other goods, respectively. The utility maximizing bundle is (Q_m^h, Q_m^x) and the household achieves a utility level U^m, as depicted in figure 1. In order to facilitate the interpretation of the diagram we have constructed it on the assumption $p_m^x = 1$, without loss of generality.

Suppose further that the family is eligible for participation and is selected by the Housing Authorities. The household faces an all-or-nothing choice to consume, say, Q_s^h and Q_s^x of housing and other goods. If the corresponding utility level U^s exceeds utility without the program U^m the household will choose to participate. This is the situation represented in figure 1. Note that there is no a priori reason to believe that households in public housing consume more housing than they would have consumed without the program. The only condition that should be satisfied is $U^s > U^m$. Consequently, as indicated by Olsen and Barton (1983), the only thing we can say a priori about the location of point S is that it's above the indifference curve containing M and below the horizontal line at A (because OA represents total income). If the rent charged by the Housing Authorities is sufficiently low it's perfectly possible that a household consumes less housing under the program than it would have done in it's absence.

The difference between the market value of all goods consumed under the program and the total market value of consumption without the program is called the subsidy S^*. It is represented on the diagram by the vertical distance AC between the original budget constraint AB and the constraint through S, the consumption bundle under the program. By definition we have

$$S^* = p_m^h\, Q_s^h + p_m^x\, Q_s^x - y \qquad \text{, where } y \text{ is income.}$$

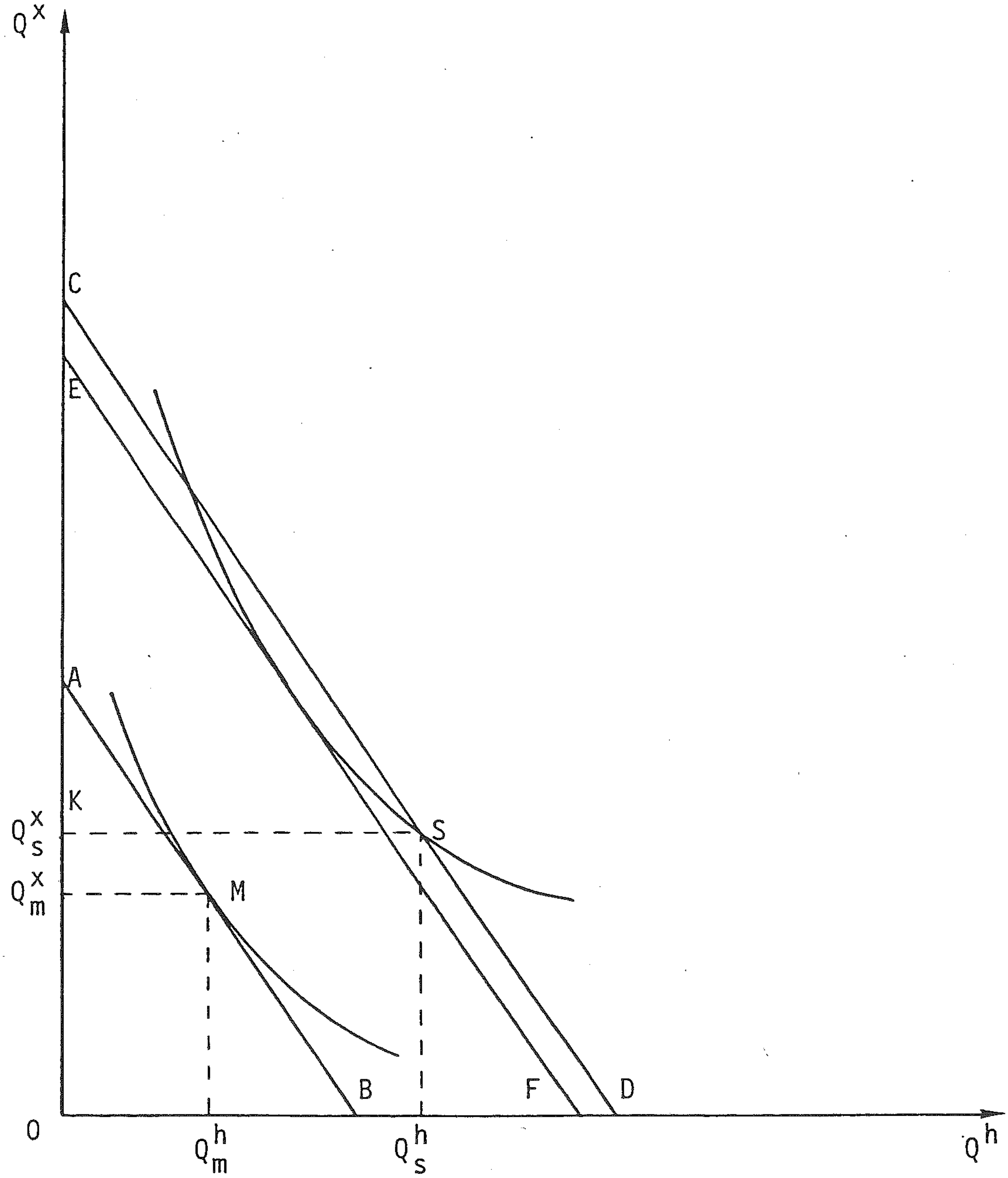
Using the relation $p_m^x\, Q_s^x = y - p_s^h\, Q_s^h$ we find, not surprisingly,

$$S^* = p_m^h\, Q_s^h - p_s^h\, Q_s^h$$

or, in other words, the subsidy equals the difference between the market value of the public housing unit and the rent charged by the Housing Authorities.

It's clear that the subsidy is not a very satisfactory measure of benefits. The subsidy would be exactly the same for all consumption bundles with the same market value, i.e. all bundles along CD, although it's obvious that people are not indifferent be-

Figure 1



where:	households income	OA
	subsidy	AC
	equivalent var.	AE
	charged rent public unit	AK
	market rent unit	CK

tween all such combinations. However, the subsidy serves as an upper bound to consumer benefits, because an in-kind transfer will be valued by its recipients at no more than an unrestricted cash transfer with the same market value (*).

A superior measure of benefits, usually referred to as an 'exact' measure of changes in welfare, is Hicks equivalent variation. It is defined as the amount we should give a household in the absence of the program in order to bring it on the same level of utility as achieved under the program (**). To represent the equivalent variation we shift the original budget constraint AB until it's just tangent to the indifference curve corresponding to utility level U^S under the program. It can easily be measured on the vertical axis as the distance AE.

Figure 1 illustrates all required information we need for the calculus of the equivalent variation. The ultimate location of E is clearly determined by the shape of the households indifference curves and the quantities housing and other goods consumed under the program. The latter is just the households income (OA) minus the rent charged for the public unit (AK), which are both observable quantities. However the remaining information is unobservable and must be estimated. Therefore, we need to make predictions of households housing consumption under the program and we should estimate the parameters of an a priori specified utility function.

(*) Note that the subsidy would be an acceptable measure of benefits if the point S corresponded to the utility maximizing bundle given the constraint CD. However in that case the subsidy would equal Hicks equivalent variation.

(**) There are, of course other 'exact' measures of welfare changes. We focus on the equivalent variation because it's probably the most widely used benefit concept in applied welfare economics and because it has a very appealing interpretation for policy makers. Moreover in the case of public housing programs it's not straightforward to calculate e.g. the compensating variation. Although both the equivalent and the compensating variation require information that is unobservable and must be predicted, the unknown inputs for the former are a lot easier to estimate than for the latter.

For purposes of comparison we will also calculate Marshallian benefits. Although it's well-known that this measure of consumer surplus is an unacceptable indicator of welfare changes unless strong restrictions are imposed on individual preferences this concept was the most widely used welfare tool until recently (*). Unless the income elasticity of the subsidised good equals zero, Marshallian benefit has no clear interpretation. Under the specified assumption, however, it is equal to the equivalent variation.

Marshallian benefit is usually represented on an ordinary price-quantity diagram. Let in figure 2 $D(Q^h)$ be the demand curve for housing services. The consumption level corresponding to the original utility U^m is a point on this demand curve viz. M'. The quantity consumed under the program is clearly a point S' off the Marshallian demand curve as the consumption bundle is not a utility-maximizing one (**). Marshallian consumer surplus MB is given by the area AM'CS'B. It can be calculated to be

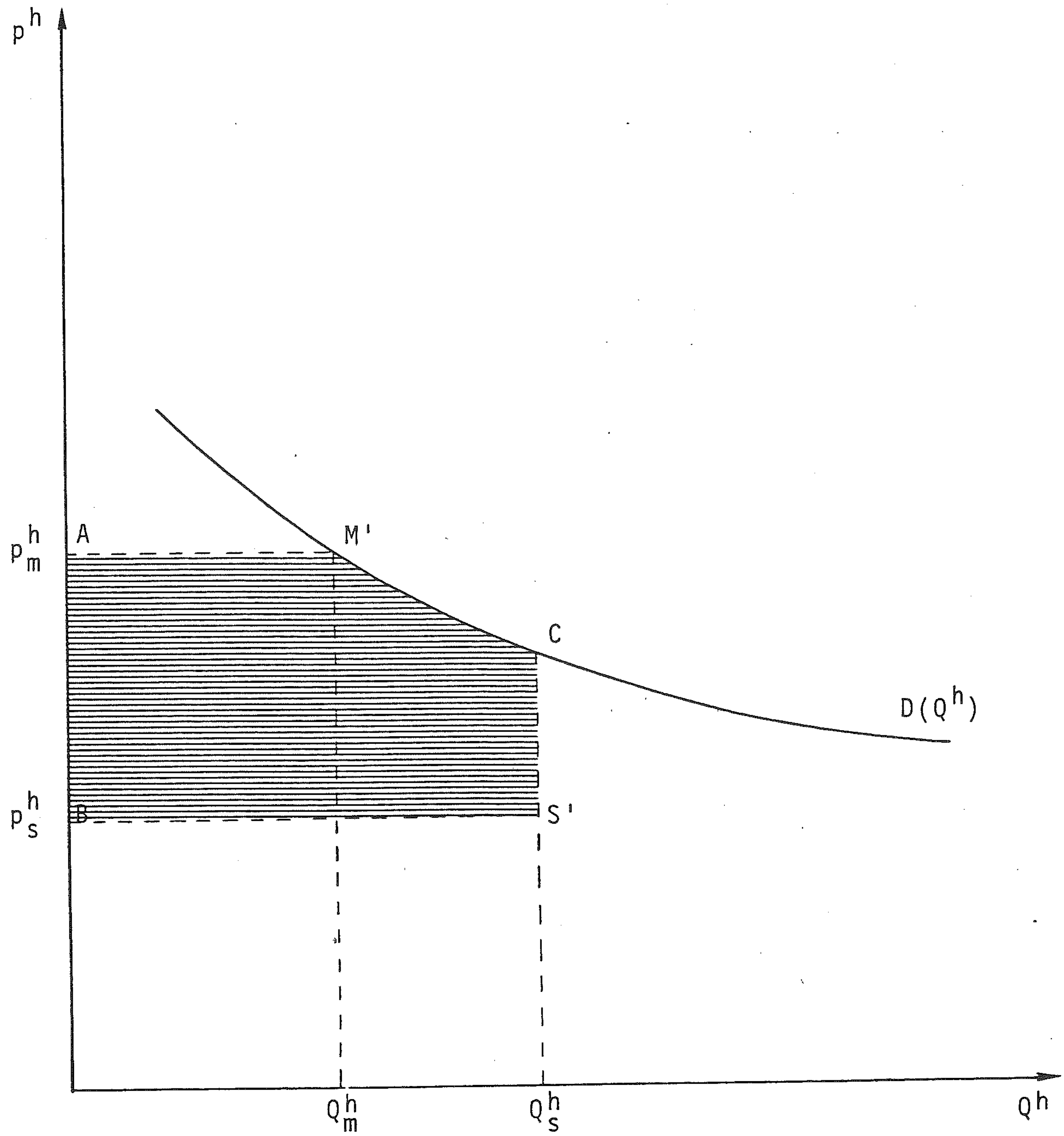
$$MB = p_m^h Q_m^h - p_s^h Q_s^h + \int_{Q_m^h}^{Q_s^h} D(Q^h)\, dQ^h$$

Evaluation of this expression requires of course the specification of a demand curve for housing services. Moreover, some additional and (in our case) unobserved information is needed. Indeed, we need to know the quantity of housing the household would have consumed without the program i.e. when facing market prices for all commodities. It will be suggested below how to obtain estimates for all required but unknown information, both for the equivalent variation and for Marshallian consumer surplus.

(*)There's a substantial litterature on the difficulties involved in using Marshallian benefits and the conditions under which it can be used as a valid welfare indicator, see e.g. McKenzie and Pearce (1976, 1982), Willig (1976), Hause (1979) Chipman and Moore (1980).

(**) An important implication is that a basic welfare theorem which can be shown to hold for pure price subsidies - namely that Marshallian benefit is always less than or equal to the equivalent variation, provided that the subsidized good is normal -, no longer holds in the case of quantity constrained price subsidies such as public housing. We do not show this statement in this paper, however, in order to save space.

Figure 2



3. Derivation of benefits

In recent years substantial progress has been made with respect to calculating exact benefit measures on the basis of observed information. It was pointed out that Hicks welfare measures could be calculated directly from market information, although they are based upon the unobservable utility concept. This resulted in two new approaches to benefit estimation based on duality theory, see Vartia (1978) and Hausman (1981)(*). Although these techniques may prove to be very useful in much applied welfare analysis they are not applicable for our purposes. This is due to the fact that the price of housing, unlike the price of most other commodities, is unobservable. We observe market rents (price times quantity of housing services per time period) or market values (a measure of the stock value) rather than the unit price of housing services. This peculiarity of the housing market implies that no price variation can be observed which excludes a lot of reasonable demand functions from consideration, as well as application of Hausmans or Vartia's procedures.

Therefore, we use in this paper a direct method for the calculus of the equivalent variation based upon a specified direct utility function. Again many useful utility functions are excluded because of the lack of observable price variation. The Stone-Geary or displaced Cobb Douglas utility function is an attractive candidate for our purposes (see e.g. Cronin (1982), Olsen and Barton (1983), H. Rosen (1978)), as its parameters can be obtained without explicit price information, see below.

Apart from being convenient analytically - it's one of the very few utility functions yielding an exact analytical expression for both Marshallian and Hicksian benefit measures - the Stone-Geary utility function has some other significant advantages. These

(*) The first method uses the identity $h(p,\bar{u}) \equiv x(p,e(p,\bar{u})$ i.e. that Hicksian demand at optimal utility level $\bar{u}$ equals Marshallian demand for income $e(p,\bar{u})$, where $e(p,u)$ is the expenditure function. This implicit equation in p can be integrated, using numerical integration methods, to find exact measures of consumer surplus. The second approach solves Roy's identity, which is simply a linear differential equation in indirect utility, to get the indirect utility function and thus Hicksian benefit measures. Both methods only require information on the Marshallian demand curve which can be estimated directly form market data.

should be traded off against a few very restrictive features.

The Stone-Geary utility function belongs to a group of quasi-homothetic functions, sometimes referred to as Gorman polar forms (Gorman (1961)). These are utility functions whose corresponding expenditure function adds a fixed cost element to a homothetic expenditure function (*). They retain some of the convenient properties of homotheticity while relaxing the most restrictive assumption. Although Engel curves are straight lines they needn't go through the origin so that the income elasticity is not restricted to be unity. The budget shares for any commodity i can be written as a weighed average of the expenditures of poor and rich people on this good, the weights being the percentages of 'subsistence' and 'above subsistence' expenditures as a fraction of total income. Moreover, the Stone-Geary expenditure function places convenient restrictions on substitution patterns, i.e. it's implied that richer people have more substitution possibilities than the poor. Finally, the displaced Cobb-Douglas function, unlike many other functional forms, automatically satisfies the theoretical restrictions of adding-up, homogeneity and symmetry.

The disadvantages of the Stone-Geary include the linearity of Engel curves, which implies that the marginal propensity to consume on a specific commodity is the same for all households. However, this restrictive assumption can easily be relaxed by making the parameters of the utility function dependent upon households characteristics. A further feature of Stone-Geary preferences is that inferior goods are excluded and that all commodities are gross substitutes. This clearly limits its use to very broadly defined composite commodities for which the implied assumptions are not too unreasonable (**). Given the purpose of this study we believe the Stone-Geary is a useful tool for analysing households preferences

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(*) More precisely, the expenditure function for a Gorman polar form can be written as $e(p,u) = u\, b(p) + a(p)$, where it's well-known that the first term on the right hand side is the general form of the expenditure function for homothetic utility functions. The fixed term $a(p)$ is usually interpreted as the cost of subsistence.

(**) Deaton and Muellbauer (1980, p. 151) conclude that, although the assumption of quasi-homotheticity remains somewhat implausible, it may not be an unreasonable assumption for broadly defined commodity groups.

with respect to housing and other goods (*).

We now proceed to derive the expressions for the equivalent variation and Marshallian benefit for the following Stone-Geary utility function

$$U = (Q^h - \beta^h)^{\gamma} \, (Q^x - \beta^x)^{1-\gamma} \qquad (1)$$

where γ is the marginal propensity to spend on housing. The β's are usually interpreted as subsistence parameters although this is by no means a theoretical necessity. Neither is there a theoretical reason to believe that these 'subsistence' parameters should both be positive.

The equivalent variation is easily derived as follows : first we derive the demand functions by maximizing utility subject to the constraint $p_m^h Q^h + p_m^x Q^x = y$. We find

$$Q^h = \frac{p_m^h \beta^h (1-\gamma) - \gamma p_m^x \beta^x + \gamma y}{p_m^h} \qquad (2)$$

$$Q^x = \frac{(1-\gamma) y + \gamma p_m^x \beta^x - p_m^h \beta^h (1-\gamma)}{p_m^x} \qquad (3)$$

Substitution of (2) and (3) into (1) yields the indirect utility function

$$v(p_m^x, p_m^h, y) = \frac{\gamma^{\gamma}(1-\gamma)^{1-\gamma} \, (y - p_m^h \beta^h - p_m^x \beta^x)}{(p_m^h)^{\gamma} (p_m^x)^{1-\gamma}} \qquad (4)$$

Inverting this expression and substituting for U^s in the resulting expenditure function $e(p_m^x, p_m^h, U)$ gives us the income a person should have in order to reach utility level U^s. Subtracting the observed income from this expression it's easy to show that the equivalent variation is given by

(*)Given the lack of observable price variation it's also the least restrictive utility function whose parameters can be estimated on the basis of our data.

$$EV = \left[\frac{Q_s^h p_m^h - p_m^h \beta^h}{\gamma}\right]^{\gamma} \left[\frac{Q_s^x p_m^x - \beta^x p_m^x}{1-\gamma}\right]^{1-\gamma} \tag{5}$$

$$+ p_m^h \beta^h + p_m^x \beta^x - y.$$

Note that evaluation of this expression requires information on the market rent of the public unit $Q_s^h p_m^h$, income y, the rent paid under the program (which determines $Q_s^x p_m^x = y -$ public rent) and the parameters of the utility function.

To derive Marshallian benefits we rewrite the demand function (2) as

$$p_m^h = \frac{\gamma(y - p_m^x \beta^x)}{Q^h - (1-\gamma)\beta^h}$$

substitute in the formula previously given

$$MB = p_m^h Q_m^h - p_s^h Q_s^h + \int_{Q_m^h}^{Q_s^h} \left[\frac{\gamma(y - p_m^x \beta^x)}{Q^h - (1-\gamma)\beta^h}\right] dQ^h$$

and evaluate the integral. The result can be written as :

$$MB = p_m^h Q_m^h - p_s^h Q_s^h + \gamma(y - p_m^x \beta^x) \left[\ln(p_m^h Q_s^h - (1-\gamma)\beta^h p_m^h)\right.$$

$$\left. - \ln(p_m^h Q_m^h - (1-\gamma) p_m^h \beta^h)\right] \tag{6}$$

As suggested before the calculus of Marshallian benefits requires additional information viz. the expenditures on housing in the absence of the program $p_m^h Q_m^h$.

We now turn to the problem of implementing the theoretical formulas derived in this section on the basis of a sample of individual households. It follows from the previous discussion and from the nature of our data that prediction methods should be developed to estimate three pieces of unobservable information. We need to estimate the parameters γ, β^h and β^x of the specified utility function, we need to predict the market rent of the public housing units and, for the calculus of Marshallian benefits, we need to estimate households expenditures on housing in the absence of the program (*). We discuss each problem in turn.

Rewriting (2) it follows that households with Stone-Geary preferences spend a fraction of their income on housing given by

$$\frac{p_m^h Q^h}{y} = \gamma + \frac{(1-\gamma) p_m^h \beta^h - \gamma p_m^x \beta^x}{y}$$

Adding a stochastic error term u to this equation which reflects both differences in taste and disequilibrium errors, direct estimation is not possible because prices are unobserved. Therefore, as suggested by Olsen and Barton (1983) we rewrite the relation as

$$\frac{p_m^h Q^h}{y} = \gamma + \alpha(\frac{1}{y}) + u, \qquad \text{where } \alpha = (1-\gamma) p_m^h \beta^h - \gamma p_m^x \beta^x$$

Although estimation of this equation doesn't allow all parameters of the utility function to be identified we will use a priori information on 'subsistence' housing expenditures to determine the remaining parameters β^x and γ. Moreover, in order to allow the parameters to be different for different types of households (**) we specify γ and α as a function of household characteristics z_j (***), so that the budget share equation now reads

$$\frac{p_m^h Q^h}{y} = \gamma_0 + \sum_{j=1}^{r} \gamma_j z_j + \alpha_0(\frac{1}{y}) + \sum_{j=1}^{r} \alpha_j z_j(\frac{1}{y}) + u, \qquad (7)$$

(*) Our data do not contain information with respect to public tenants expenditures on housing prior to entering the program. As a consequence, their expenditures on housing in the absence of the program must be predicted on the basis of the subsample of households in private housing.

(**) The estimated γ and α for households with given characteristics should be interpreted as the mean value of these parameters for the household type under consideration. Indeed, taste differences and households failures to maximize will cause random deviations.

(***) There is no reason why the relations between γ, α and household characteristics should be linear.

where u is the random error term assumed to be normally distributed with mean zero and constant variance. Estimating this equation and using extraneous information on subsistence housing expenditures we obtain $p_m^x \beta^x$ for each household from (*) :

$$p_m^x \beta^x = \frac{\left[1 - (\gamma_0 + \sum_{j=1}^{r} \gamma_j z_j)\right] p^h \beta^h - (\alpha_0 + \sum_{j=1}^{r} \alpha_j z_j)}{\gamma_0 + \sum_{j=1}^{r} \gamma_j z_j}$$

Knowledge of $\gamma, p_m^x \beta^x$ and $p_m^h \beta^h$ for each family provides sufficient information concerning the utility function for the calculus of benefits.

As public housing tenants are not characterized by utility-maximizing behavior we will estimate the parameters of the utility function on the basis of a sample of households living in private housing. Using these estimates for the calculation of benefits of public housing programs may lead to benefits which suffer from selection bias. The reason is simply that public tenants may have systematically different tastes than households buying all goods on the private market. A classical argument goes as follows : as public housing programs have distortive effects towards more housing consumption (**) and households with the largest expected benefits will participate it follows that public tenants have a higher taste for housing - i.e. a higher marginal propensity to spend on housing γ - than similar households in private housing. As a consequence benefits would be underestimated.

Although the possibility of selection bias cannot be denied the preceding argument is not entirely convincing because it ignores several special features of housing programs in Belgium (and in many other countries). Most programs have income limits for eligibility implying that not all households willing to participate are eligible. Moreover, they are no full entitlement programs in the sense that only a fraction of all eligible families willing to

(*) The equation follows from the definition of α.
(**) We indicated before that this is not necessarily the case. However, our empirical results discussed in the next section suggest that most public tenants consume indeed more housing than they would have done in the absence of the program.

participate is selected by the Housing Authorities. This is obvious from the long waiting lists observed in practice. Under these program rules it's easy to show that not only the magnitude but also the direction of the bias is unclear, i.e. it may be in either direction (*).

Econometric procedures for correcting selection bias are available in the literature, see e.g. Heckman (1979). However the data requirements for his method are nontrivial. Our data clearly lack some of the information necessary for its application.

Another possible source of bias in mean benefit is due to using the same indifference map for all households with the same observed characteristics although it's obvious that even within these groups taste variations will exist. The Stone-Geary, like most other utility functions, doesn't allow us to make individualized estimates of indifference map parameters, so that aggregation bias is likely, given the nonlinearity of the benefit formulas (**). Aggregation bias is a potential problem for any policy study based upon estimated indifference maps. Simulation techniques may be useful to discover the importance of the bias.

We now turn to the problem of predicting the market rent of public units. Under some simplifying assumptions this prediction can be obtained from hedonic regression techniques. Suppose we regress rent on observed housing characteristics for a sample of units rented on the private market. Then if we assume that the conditional distribution of rent given the observed characteristics would be the same for public and private units we can obtain unbiased predictions of the market rent of public units by inserting their observed attributes into the equation. The implicit assumption in this procedure is quite strong especially in view of the existence of unobserved and thus excluded housing charac-

(*) See Olsen and Bierman (1982) for a simple example. The authors further correct for selection bias and conclude that for their data the bias in mean benefit was less than five percent. This result can of course not be generalized to other data sets.
(**) The nonlinearity of the benefit measures suggests that the biases for different households will not cancel out in the determination of mean benefit.

teristics (*). If public housing units are systematically different with regard to unobserved characteristics the assumption is violated and biased predictions will result. Suppose e.g. that public units are located in areas having systematically better accessability to public services and no variables accounting for accessibility were included in the hedonic regression. In that case we would underestimate the the market rent of public units. The same can happen if there are systematic differences between public and private housing with respect to environmental quality or neighborhood quality and these variables are not included in the hedonic. However, it's not clear why these systematic differences would exist.

Finally, the prediction of household expenditures of public tenants in the absence of the housing program can be done using the estimated Stone-Geary budget share equation for housing.

(*) The existence of excluded characteristics implies biased estimates of the implicit prices of characteristics. Suppose the correct equation is

$R = H\alpha + Z\gamma + \varepsilon$ where
- R : rent
- H : matrix observed characteristics
- Z : matrix unobserved characteristics
- α, γ : parameter vectors
- ε : error term.

Now suppose we estimate $R = H\beta + \mu$ then a well known result is that $E(\hat{\beta}) = \alpha + T\gamma$ where $T = (H'H)^{-1}H'Z$, i.e. the matrix contains regression coefficients of the excluded variables on the complete set of included variables. Consequently unless $T = 0$ (no relation between included and excluded variables) or $\gamma = 0$ (the excluded variables have zero coefficients in the true hedonic) the estimated implicit prices are biased. However, even if biased coefficients result, it's easy to see that unbiased predictions of the market rent of public units can be obtained if the relation between included and excluded variables, i.e. the matrix T, is the same for public and private units.

III. Empirical implementation

In this chapter we report on the empirical results we obtained when applying the methodology discussed in the previous part of the paper. The data we used were derived from a small sample of households living in the town of Liège, Belgium. The survey, conducted in the early seventies, collected information on a set of household characteristics and some crude data on families housing situation. The survey included a question as to whether a household was living in a public housing unit so that we are able to identify public housing tenants in the sample. For reasons explained in appendix 1 - which also contains a description of the most important variables that will be used in this study - we decided to use only the subsample on rental housing. Our final data set contains 326 observations, of which 261 reported to live in private housing whereas 65 households reported to be public housing tenants.

This chapter is organized as follows : in a first section we describe the results of hedonic price regressions that were used to predict the market rent of public housing units. Next we analyse the estimation procedure and the empirical results with regard to the parameters of a specified Stone-Geary utility function. In a third section we report on the estimated benefits public housing tenants derived from the program. Both Marshallian consumer surplus and Hicks equivalent variation are calculated for the subsample of public housing beneficiaries. We also indicate the effects of replacing the program with a direct unrestricted cash grant with the same market value. Section four contains a detailed analysis of the sensitivity of the calculated benefits with respect to several predicted inputs to the benefit formula i.e. how sensitive are the results to prediction errors in market rent, the utility function parameters etc.. Finally, section five reports on the distributional effects of the public housing programs analysed i.e. how do benefits vary with a set of relevant household characteristics.

1. The market rent of public units

In this section we apply the procedures previously discussed in order to predict the market rent of public housing units. If we assume that the conditional distribution of rent given observed housing characteristics would be the same for public and private housing then unbiased predictions of market rent can be obtained by inserting observed attributes of public units into estimated hedonic rent equations. We should proceed, therefore, by specifying and estimating a relation between rent and a set of housing attributes using the sample on private rental units.

A potential difficulty arises because our data contain observations on both appartments and single-family units. It's not unrealistic to assume that there exist distinct markets for both housing types which would suggest that the estimated implicit prices of characteristics needn't be the same on the two markets. First of all, it's quite probable that the production and cost functions underlying the construction of appartments and single-family units are different as they require a different technology. Economics of scale - both with respect to housing units and the production of characteristics such as space, structural quality, etc. - may be quite different for both housing types. Moreover the effect of variations in input prices on costs will not be the same due to different input requirements.

On the demand side of the market it's not impossible that households living in appartments have substantially different preferences - and thus demand functions - with respect to housing attributes as compared to families in single-family units with the same observed characteristics. It may e.g. be the case that households in single-family housing have a systematically higher taste for space, environmental quality, privacy etc.

The previous remarks suggest that it should come as no surprise that the implicit prices resulting from demand-supply interaction may be quite different for appartments and single-family housing. This is even more acceptable if we take into account that not only cost and demand functions needn't be the same, but that even the adjustment processes towards equilibrium may be expected to differ. This is due to the fact that the processes are induced by different market forces. This is especially clear if deviations from equilibrium are corrected for through new construction of housing units and thus housing characteristics. Indeed, considering the market for new construction it's generally assumed that construction activity for multi-family dwellings is primarily supply-induced whereas new single-family units are largely generated by the demand of households. As a consequence quite different market forces operate on both markets which suggests seperate models should be developed (Jaffee and Rosen (1979)).

It seems feasible, therefore, to estimate seperate hedonic regressions for the subsamples of appartments and single-family housing and to test whether the vectors of coefficients of the variables included in both equations are significantly different (*).

It should be noted that the variables describing housing consumption - they are described more thoroughly in appendix 1 - were in most cases not directly useful for the analysis. Many variables were transformed before they could be entered into the regressions. The variable SPACE was transformed into a continuous indicator. Information with respect to the dwellings construction date was summarized into three dummy variables relating to the periods 1900 - 1918, 1919 - 1945 and 1946 - 1970. Dummies were also constructed for the story on which appartments were situated and whether the building had an elevator. The variables

(*) Still another reason to expect that this will be the case is that unobserved housing attributes may have a different impact on the estimated parameters in both relations. This is a reasonable assumption, especially in view of the data set used for this study. We do not have information on total lot size which is an important determinant of single-family house prices. The effect of deleting this variable together with other unobserved variables is likely to be different in both subsamples.

with respect to the heating system and the yard were used in their original form, i.e. as 'quality' indicators (*).

A set of alternative specifications were tried for the hedonic rent equations. For each subsample we chose the relation yielding the highest correlation between observed rent and the rent predicted by the equation (**). This criterion lead to the same specification being chosen for appartments and single-family housing (***). The estimated coefficients are presented in table 1. A sample of alternative regressions is reported in appendix 2. Several of these regressions were almost equivalent with respect to predictive power and significance of the estimated coefficients. This is neither unusual nor surprising. As noted by e.g. Dhrymes (71) the bundling of characteristics into housing units and the relatively limited range of observed quantities of characteristics on the market imply that only a small fraction of the complete price structure is covered by the sample. As a consequence many researchers in the past have found that different functional forms fit the data almost equally well (see e.g. Kain and Quigley (1970), Grebler and Mieszkowski (1974), Schnare and Struyk (1976)). This was certainly to be expected for our data set as many of the included variables are dummies. These implicitly capture some nonlinearities and restrict variability in the explanatory variables so that the marginal cost of searching for better functional forms quickly exceeds the marginal benefit in terms of higher explanatory power.

The appartment equation is very satisfactory, taking into account the lack of neighborhood and locational variables.

(*) Other information available in our data proved not to be useful at this stage of the analysis. Variables describing the 'quality' of the kitchen and restrooms and those indicating the availability of additional space (basement, depository, etc.) were completely insignificant and didn't increase the explanatory power of the regressions, if approppriately included in the equations.

(**) Direct comparison of alternative specifications on the basis of R^2 or the sum of squared residuals was not possible because the dependent variable was not always the same.

(***) Another reason for preferring the chosen specifications was that some other equations, having rent as the dependent variable, indicated slight heteroxedasticity of the residuals.

Table 1 a) Regression results appartments

Dependent variable : rent/room

Independent Variables	Description	Coefficient	t-value
Constant		- 1.379	-2.792
1/ROOMS	inverse number of rooms	15.151	5.514
BEDR/ROOMS	proportion of rooms that are bedrooms	4.512	3.609
SPACE	space indicator	0.028	1.945
WATER2	1 if hot water available; 0 otherwise	0.591	1.846
BATHR	1 if bathroom available; 0 otherwise	2.983	4.025
CH	central heating quality indicator	1.127	2.428
CD00	1 if constructed in 1900-1918; 0 otherwise	1.502	1.430
CD18	1 if constructed in 1919-1945; 0 otherwise	0.359	0.353
CD45	1 if constructed after 1945; 0 otherwise	3.459	3.081
STORY12	1 if unit on stories 1 or 2; 0 otherwise	-0.755	-1.083
STORY34	1 if unit on stories 3 or 4; 0 otherwise	-2.831	-2.268
STORY5+	1 if unit on story 5 or above; 0 otherwise	5.244	4.486
ELEV*STORY34		5.065	2.989

$R^2 = 0.686$

SSR = 1659.438

162 observations

Table 1 b) Regression results single-family housing

Dependent variable: rent/room

Independent	Coefficient	t-value
Constant	- 5.913	-3.144
1/ROOMS	+ 12.609	4.247
BEDR/ROOMS	1.673	1.732
SPACE	+ 0.01	0.61
WATER2	2.249	2.338
BATHR	2.487	1.986
YARD	1.263	2.21
CH	1.429	4.384
CD00	1.973	1.193
CD18	2.876	2.283
CD45	3.296	2.156

$R^2 = 0.515$
SSR = 1423.184
99 observations

(*) for description of variables see table 1a).

Many variables are significant at the 5 % level and the equation implies a correlation between observed and predicted rent of more than 0.85. Rent per room seems to be strongly affected by the space variables ROOMS, BEDR and SPACE. The availability of a bathroom and the quality of the heating system have a significant effect as well. We further find that a large and significant increase in rent per room is associated with recently constructed units as compared to units build before 1900. The coefficients of the dummies relating to the periods 1900-1919 and 1919-1945 are both small and unreliable.

There is some slight evidence that appartments located on the first or second story rent for somewhat less than those on ground level. Units located on higher stories clearly rent for more if the building has an elevator. If this is not the case rent is, not surprisingly, lower (*).

The equation for single-family housing is less satisfactory, though still acceptable. The main reason is undoubtedly the unavailability of lot size and reliable locational and environmental variables. Especially the former is clearly more important in determining market rents for single-family houses than for appartments. Despite the absence of unobserved variables the equation produces a reasonable fit to the data. The estimates confirm the importance of the number of rooms, the availability of hot water and a bathroom, and the quality of the heating system. The per room rent difference between recently build units and houses constructed before 1900 is comparable to the estimated effect for appartments. The rent difference between units constructed in the periods 1918-1945 and 1945-1970 is much smaller for houses than for multi-family units, which may simply reflect differences in maintenance and physical depreciation.

(*) Note that we didn't include an interaction term ELEV*STORY5+ into the equation. This merely reflects the fact that all units located at the fifth story or higher had an elevator available.

We tested the hypothesis that the vectors of coefficients corresponding to the variables appearing in both equations were equal (*). The test procedure is neatly described by Fischer (1970). It basically requires the estimation of the model with the restriction of equal coefficients imposed and compares the residual sum of squares from restricted and unrestricted models (**). The appropriate test statistic is

$$F = \frac{(u'u - u^{*'}u^{*})/(j)}{(u^{*'}u^{*})/(N_a+N_s-2j-k)}$$

where $u'u$: residual sum of squares restricted model

$u^{*'}u^{*}$: residual sum of squares unrestricted model

j : number of coefficients restricted to be equal

N_a, N_s : number of observations in the subsamples for appartments and single-family housing, respectively

k : total number of unrestricted parameters.

The statistic is F distributed with j and (N_a+N_s-2j-k) degrees of freedom. We calculated:

(*) The test vector includes the constant and the coefficients of 1/ROOMS, BEDR/ROOMS, SPACE, WATER, BATHR, CH, CD00, CD18, CD45.

(**) The unrestricted model refers to the regression equations reported upon in table 1. Assuming the following notation (s=single family housing, a=appartments):

Y_a, Y_s: vectors of observations on the dependent variable

X_a, X_s: matrices of observations on those independent variables for which the equality of the coefficients is to be tested

V_a : matrix of observations on those independent variables only appearing in the equation for appartments

W_s : matrix of observations on those independent variables only appearing in the equation for houses.

it can be written as

$$\begin{Bmatrix} Y_a \\ Y_s \end{Bmatrix} = \begin{Bmatrix} X_a & o & V_a & o \\ o & X_s & o & W_s \end{Bmatrix} \begin{Bmatrix} \alpha_a \\ \alpha_s \\ \beta_a \\ \gamma_s \end{Bmatrix} + \begin{Bmatrix} \varepsilon_a \\ \varepsilon_s \end{Bmatrix}$$

The restricted model clearly reads

$$\begin{Bmatrix} Y_a \\ Y_s \end{Bmatrix} = \begin{Bmatrix} X_a & V_a & o \\ X_s & o & W_s \end{Bmatrix} \begin{Bmatrix} \alpha \\ \beta_a \\ \gamma_s \end{Bmatrix} + \begin{Bmatrix} \varepsilon'_a \\ \varepsilon'_s \end{Bmatrix}$$

Estimation of the restricted model yielded an R^2 of 0.5609. The residual sum of squares was RSS = 3609.02. The coefficients of this model are not presented here, as they do not contain any interesting information above that already given.

$$F(10,236) = \frac{(3609.2-3082.62)/10}{(3082.62)/236} = 4.03$$

which is significant at the 5 % level. The hypothesis of equal coefficients can be rejected, implying that separate hedonic regressions for each subsample are to be preferred. This justifies the approach taken in this section.

2. The parameters of the utility function

We now turn to the estimation results for the parameters of the specified Stone-Geary utility function. As previously explained the procedure requires the estimation of relation (7) between the rent/income ratio and household characteristics. Given estimates of the coefficients of this equation we need a priori information on 'subsistence' housing expenditures $p_m^h\beta^h$ in order to derive all required inputs for the calculus of benefits. The main determinant of 'subsistence' housing expenditures is undoubtedly family size. We assumed that family size was the only relevant characteristic explaining differences in 'subsistence' housing and used for each group of households of a given size the observed sample minimum as an estimator of $p_m^h\beta^h$(*). This estimator is by definition upward biased. However, if the population of households of each size living in Liège contains at least one family at 'subsistence' and all housing expenditure data are reported without error, then the sample minimum is consistent in the sense that the bias declines to zero as the sample size approaches the population size. This fact together with the finding that the pattern of the sample minima is compatible with a priori expectations - viz. increasing with family size - are the main reasons why we used the observed minima to approximate $p_m^h\beta^h$. However, it's clear that this procedure may have undesirable implications for the calculus of benefits. Therefore it's important to analyse the sensitivity of our results with respect to the chosen 'subsistence' level (**).

In general a wide variety of household characteristics can be expected to affect preferences. However, the definition of our utility function in terms of the composite commodities housing and other goods is likely to reduce the list of potential taste determinants in a substantial way. Although household attributes such as educational level, professional status, single

(*) The sample minima were 500, 700, 800, 800, 900 Belgian francs for a household size of 2,3,4,5,6 or more, respectively.

(**) The sensitivity analysis is presented below, see section 4.

or double working couple etc. will almost certainly lead to variations in taste with respect to the composition of the housing bundle, there is no strong a priori reason to believe that differences in preferences will result towards housing and other goods as such. Whereas e.g. better educated households may exhibit a stronger taste for space or environmental quality, this doesn't imply a stronger preference for the housing commodity in general. It may be hypothesized that preferences with respect to housing and other goods will be largely determined by family size and by variables describing the position in the life-cycle at which the head of the family could be situated. Large families are likely to spend a higher fraction of a given income on housing: although their need for more space may to some extent be compensated through substitution of space for quality and location, it's unlikely that this will result in a lower rent-income ratio. On the other hand we would expect a households marginal propensity to spend on housing to decrease with later periods in the life-cycle and to depend upon the age of the children.

The estimated relationship between the rent/income ratio and family characteristics is presented in table 2. The results are consistent with the suggested hypothesis: many of the included variables pass the test of significance at the 5 % level and the explanatory power of the equation is reasonable, taking into account the inherent difficulties in explaining budget shares (*). The estimates imply that the marginal propensity to spend on housing increases in a nonlinear way with family size and decreases with age. There's also some evidence that it's somewhat higher for recently married couples and for families with small children, ceteris paribus.

Using a priori information on $p_m^h \beta^h$ we calculated 'subsistence' expenditures on other goods $p_m^x \beta^x$. In table 3 we present

(*) A set of additional variables with respect to professional status, education etc. proved to be irrelevant for the explanation of the rent income ratio. However, as the 'exact' set of explanatory variables could not be determined a priori on purely theoretical grounds some 'regression fishing' was unavoidable before the final results were obtained. This implies that the t-values reported in table 2 might be upward biased, see e.g. Lovell (1983).

Dependent variable: Rent/Income

Independent variables	Description	Coefficient	t-value
Constant	-	0.16292	(4.17)
FS3	dummy=1 if family size=3	0.01118	(0.68)
FS4	dummy=1 if family size=4	0.03998	(2.31)
FS5	dummy=1 if family size=5	0.06316	(4.20)
FS6+	dummy=1 if family size=6 or more	0.07948	(2.51)
MARR	dummy=1 if couple married since more than 6 years	-0.02458	(-1.40)
AGEYC	dummy=1 if age of youngest child is less than 3 years	0.05238	(2.73)
AGEM	age head of household	-0.0025	(-2.95)
1/INCOME	-	-4.42949	(-3.61)
FS3/INCOME	-	0.73159	(0.78)
FS4/INCOME	-	-3.8719	(-3.71)
FS5/INCOME	-	-6.7755	(-1.71)
FS6+/INCOME	-	-8.0134	(-2.85)
MARR/INCOME	-	-0.8719	(-0.86)
AGEYC/INCOME	-	-9.66943	(-2.96)
AGEM/INCOME	-	0.43828	(2.16)

$R^2 = 0.4883$

SSR = 0.478

T = 258 (*)

table 2

(*) Three observations were deleted because of inconsistencies in the data.

some summary information with respect to the estimated parameters γ and α and the calculated result for $p_m^x\beta^x$. The table contains the mean subsistence levels for other goods, the mean value for the estimated α and the minimum, maximum and mean of the marginal propensity to spend on housing for households of different size. Finally we give some information concerning the estimated standard errors of the parameters γ and α. However, it's clear that not only the estimate for γ and α depends on family characteristics but that also the estimated standard error of the parameters is different for households with different observed characteristics (*). Therefore, the standard errors in table 3 are <u>mean</u> standard errors for households within each group. Note that on average both γ and α exceed their standard error by a factor larger than two for most groups.

It's important to note that γ was always positive and varied between 0.034 and 0.240. The mean value over the sample was found to be 0.124. The results in the table confirm the positive relationship between family size and the marginal propensity to spend on housing, although the effects of the variables MARR, AGEM and AGEYC cause a substantial variation in γ for households of the same size. The most striking result in table 3 is, however, the fact that the estimated 'subsistence' expenditures for 'other goods' are on average negative. Though positive for large families, $p_m^x\beta^x$ tend to be strongly negative for families

---

(*) Consider a vector of random variables $Y = (y_1,\dots y_n)$. If Y has the multivariate normal distribution with mean u and variance-covariance matrix V and if p is a vector of constants then it follows that the random variable k defined by

$$k = \sum_{i=1}^{n} p_i y_i$$

has the univariate normal distribution with mean $(\sum_{i=1}^{n} p_i u_i)$ and variance $(\sum_{i=1}^{n} p_i^2\sigma_i^2 + \sum_{\substack{i=1 \\ i\neq j}}^{n}\sum_{j=1}^{n} p_i p_j \sigma_{ij})$, where u_i is the mean of y_i and σ_i and σ_{ij} are the standard error of y_i and the covariance between y_i and y_j, respectively. This theorem (see e.g. Mood and Graybill (1963), p. 211) allows us to calculate the standard error of γ and α for each household in the sample using their observed characteristics and the variance-covariance matrix of the estimated coefficiënts.

	γ_{min}	γ_{max}	γ_{mean}	σ_γ	α_{mean}	σ_α	$\beta^x p_m^x$
FS 2	0.034	0.117	0.087	0.021	8.07	2.76	-64.649
FS 3	0.045	0.153	0.100	0.053	8.37	4.25	-58.591
FS 4	0.099	0.157	0.132	0.042	3.32	1.19	11.557
FS 5	0.110	0.193	0.148	0.048	1.27	0.63	22.821
FS 6+	0.139	0.240	0.172	0.051	-1.55	0.64	45.422
All families	0.034	0.240	0.124	0.044	4.35	1.98	-12.684

table 3: summary of estimated utility function parameters

	Mean income elasticity	Within group variance	Mean price elasticity	Within group variance
FS 2	0.652	0.151	-0.842	0.039
FS 3	0.657	0.167	-0.852	0.037
FS 4	0.862	0.102	-0.785	0.071
FS 5	0.955	0.175	-0.787	0.083
FS 6+	1.033	0.143	-0.756	0.093
All families	0.816	0.209	-0.808	0.074

table 4: income and price elasticities of housing demand.

with less than two children (*).

This surprising result may seem somewhat implausible, although it is typical of studies based upon the linear expenditure system, especially those that define the utility function on two composite commodities, see e.g. Rosen (1978), Cronin (1982), Hammond (1982), Olsen and Barton (1983). Several reasons may be given to explain negative subsitence expenditures. First, we have measured expenditures on other goods as income minus rent

(*) Note that different results would have been obtained had we used a priori information on $p_m^x \beta^x$ to calculate $p_m^h \beta^h$ rather than vice versa. The impact of this procedure on estimated benefits may be analysed by testing the sensitivity of benefits with respect to the subsistence coefficients.

which implies that there is no saving or dissaving. If dissaving does occur (e.g. in the case of recently married couples without children (*)) then it's clear that the displacement parameter β^x can easily become negative. The latter merely indicates subsistence expenditures out of current income. Redefining the problem in an intertemporal framework that allows for saving and dissaving it's easy to show that β^x may be negative.

Two other possible reasons for negative β^x may be suggested. The first one concerns the fact that the Stone-Geary utility function implies a linear income-consumption path which passes through the displaced origin (β^x,β^h). If our data contained insufficient observations close to 'subsistence' levels and nonlinear effects occur in this region then it's possible that linearization of the income-consumption path produces a negative β^x. Finally, it may be the case that an inappropriate choice of subsistence expenditures on housing has caused the negativity of β^x. However, simulation experiments showed that negative β^x occurred for a wide range of a priori values for $p_m^h\beta^h$. Therefore, taking into account all previous remarks, we think it's useful to consider the displacement parameters as allowing for a more general indifference map without attaching too much value to a possible interpretation in terms of subsistence expenditures. Note, though, that the estimated pattern of β^x is not entirely inconsistent with such an interpretation. The estimates for β^x are clearly increasing with family size which would be compatible with an explanation in terms of subsistence levels (out of current income).

The estimated rent/income equation implicitly yields values for the price and income elasticity of the demand for housing. It's straightforward to show that the income and price elasticities $(\varepsilon_y,\varepsilon_p)$ are given by

$$\varepsilon_y = \frac{\gamma y}{\gamma(y-\beta^x)+p_m^h\beta^h(1-\gamma)}$$

(*) Dissaving would also be expected for retired people. However, the survey conducted in liège focused on relatively young families so that only in a few cases the age of the household head was over 50 years.

$$\varepsilon_p = \frac{-\gamma(y-\beta^x)}{\gamma(y-\beta^x)+p_m^h\beta^h(1-\gamma)}$$

Mean elasticities for the whole sample private rental housing are given in table 4 together with the variation over groups of households of different size. The income elasticity is estimated to be 0.816 on average and its value appears to increase with family size. Price elasticities do not vary very much around the estimated mean of -0.808. Although those values seem plausible and are within the range of acceptable estimates derived in the literature (see J. Quigley (79) for a review) they should carefully be interpreted. First of all, as indicated by S. Mayo (1981), both elasticities increase with income and decrease with price increases in absolute value. A major restriction of the Stone-Geary demand function is that both elasticities monotonically approach unity with changes in price and income. Moreover, the estimation procedure has used data on current rather than permanent income. It's well-known that this may have 'biased' the estimated income elasticity although the magnitude of the bias is unclear (*). Finally the estimated elasticities also depend upon the chosen a priori values for $p^h\beta^h$. If these subsistence levels are upward biased then this will result in a downward biased income elasticity. The price elasticity would similarly be upward biased in absolute value.

We should finally report on two further tests that were carried out. The first one considers the hypothesis that people living in appartments have systematically different preferences defined on housing and other goods than households in single-

(*)Use of the word 'bias' is somewhat misleading. The estimated income elasticity is an unbiased estimator of the true elasticity with respect to observed income. It does not estimate properly the elasticity with respect to permanent income, however. Consequently, the figures in table 4 are not biased in a statistical sense. They are simply estimates of a different elasticity than the one we'd like to have, i.e. the permanent income elasticity. Note, however, that even inclusion of a permanent income variable may lead to specification bias because it neglects the importance of transitory components for housing demand, see Goodman and Kawai (1982). They suggest using both permanent and transitory income in housing demand functions.

family units. Given the specification of the utility function there aro no theoretical reasons to believe that this will be the case. The test procedure reveals that our expectation is correct (*), yielding

$$F(16,226) = \frac{(0.478-0.447)/16}{0.447 \quad /226} = 0.98$$

(*) The procedure is to estimate the rent-income equation for both subsamples separately and to test whether the vectors of coefficients differ significantly. Using the following notation:

u'u: residual sum of squares pooled equation

$u^{*'}u^{*}$: residual sum of squares in the two separate regressions together

k: number of explanatory variables

T_{α}, T_s: number of observations in the subsamples appartment houses, respectively

the appropriate test statistic is

$$F = \frac{(u'u-u^{*'}u^{*})/k}{u^{'*}u^{*}/(T_a+T_s-2k)}$$

. It has the F-distribution with k and (T_a+T_s-2k) degrees of freedom. The following regression results were obtained for the subsamples on appartments and houses, respectively (t-statistics in parentheses):

```
appartments  RENT/INCOME=0.156+0.014(FS3)+0.051(FS4)+0.061(FS5)+0.069(FS6+)
                              (2.58)(0.78)      (3.15)      (3.29)      (2.83)
R2 = 0.492                     -0.009(MARR)+0.044(AGEYC)-0.003(AGEM)-7.182(1/INCOME)
RSS = 0.255                   (-0.92)      (1.89)       (-2.81)      (-3.06)
                               -0.124(FS3/INCOME)-5.631(FS4/INCOME)-10.213(FS5/INCOME)
                              (-0.12)            (-2.18)             (-3.48)
                               -7.185(FS6+/INCOME)+2.123(MARR/INCOME)-6.284(AGEY/INCOME)
                              (-1.17)              (0.92)               (-2.77)
                               +0.462(AGEM/INCOME)
                               (1.49)
houses  RENT/INCOME =           0.162+0.008(FS3)+0.073(FS4)+0.066(FS5)+0.081(FS6+)
                               (2.64)(0.44)      (2.44)      (1.74)      (4.18)
R2 = 0.513                     -0.052(MARR)+0.082(AGEYC)-0.0023(AGEM)-4.123(1/INCOME)
RSS = 0.192                   (-0.62)      (2.48)       (-4.22)       (-2.76)
                               +2.258(FS3/INCOME)-2.381(FS4/INCOME)-5.882(FS5/INCOME)
                               (1.29)            (-1.97)            (-0.98)
                              -11.183(FS6/INCOME)-1.182(MARR/INCOME)-13.821(AGEYC/INCOME)
                              (-3.61)            (-1.32)               (-3.48)
                               +0.0312(AGEM/INCOME)
                               (2.59)
```

This result is clearly insignificant at the 5% level. Consequently the estimation of a single indifference map appears to be justified.

A second test is concerned with the effect of constraining $\beta^x=\beta^h=0$ in which case the utility function reduces to the Cobb-Douglas. In view of the difficulties encountered when trying to identify all parameters of the Stone-Geary the question arises whether the hypothesis of a Cobb-Douglas type utility function can be rejected. The procedure is to test whether the coefficient vector α in the estimated rent-income relation differs significantly from zero (*). Performing the regression for the Cobb-Douglas rent-income ratio we obtained the following result (t-statistics in parentheses):

$$\begin{aligned} \text{RENT/INCOME} = {} & \underset{(3.25)}{0.122} + \underset{(1.843)}{0.021}(\text{FS }3) + \underset{(1.647)}{0.018}(\text{FS }4) + \underset{(2.159)}{0.026}(\text{FS }5) + \underset{(2.813)}{0.039}(\text{FS}6+) \\ & \underset{(-2.198)}{-0.023}(\text{MARR}) + \underset{(1.412)}{0.012}(\text{AGEYC}) + \underset{(0.884)}{0.00075}(\text{AGEM}) \end{aligned}$$

$$R^2 = 0.244$$
$$RSS = 0.706$$
$$T = 258$$

We calculated $F(8,242) = \dfrac{(0.706-0.478)/8}{0.478/242} = 14.44$, which is strongly significant. Imposing the restriction $\alpha=0$ clearly leads to inferior results. In other words, allowing the subsistence parameters β^x and β^h to be different from zero was worthwile, because the hypothesis that the indifference map is Cobb-Douglas cannot be accepted.

(*) The appropriate F-statistic is $F = \dfrac{(u'u-u^{*\prime}u^*)/j}{u^{*\prime}u^*/(T-k-j)}$

where
- $u'u$: residual sum of squares of the restricted ($\alpha=0$) rent-income equation
- $u^{*\prime}u^*$: residual sum of squares of the unrestricted rent-income equation
- T : number of observations
- k : number of explanatory variables in restricted model
- j : number of variables corresponding to the vector α.

3. Aggregate results: benefits and consumption effects.

Having estimated market rents of public units and the parameters of the utility function we are now in a position to calculate benefits and to review the main aggregate effects of the programs as derived from our small sample. Table 5 summarizes our most important results. The equivalent variation on average amounts to 775 Belgian francs per month. Mean Marshallian benefit is slightly lower, approximately 720 Belgian francs (*). It must be admitted that these estimates are relatively small when compared to a mean income of almost 14.500 Belgian francs. On average benefits were only a small fraction, some 6.5 %, of household income.

The calculated benefits must be interpreted in view of the data set used and some practical rules inherent in the operation of the programs. Our data did not contain information with respect to the year in which the participants had entered the program. It's clear, however, that the benefits derived from participation may drastically change over time. Although households are allowed to stay in their units even if their incomes exceed the limit for eligibility to the program rent increases are applied at regular intervals of time to adjust rents to income changes, if necessary. Moreover, preferences may change over time for example due to changes in household charcteristics. It's important, therefore, not to confuse the calculated benefits with the benefits of new entrants to the program. The latter may be quite different, and are probably higher, than the figures reported in this paper (**). The reason is just that benefits will vary with changes in income and preferences over time.

(*) Marshallian consumer's surplus exceeded the equivalent variation in approximately 20 % of the cases. As indicated before this is not inconsistent with basic welfare propositions as these are based upon pure price subsidies.

(**)It should also be noted that households will probably not even leave their units immediately if their benefits become negative over time. The reason is the existence of important monetary and psychological moving cost. This argument also suggests that the benefits for new entrants may be substantially higher than the figures calculated in the text.

Although the benefits appear to be relatively small, the consumption effects of the programs are substantial. First consider the percentage increase in housing consumption and other goods. These can be calculated as (*):

$$\frac{100\ (p_m^h Q_s^h - p_m^h Q_m^h)}{p_m^h\ Q_m^h}$$

and

$$\frac{100\left[(y-p_s^h Q_s^h)-(y-p_m^h Q_m^h)\right]}{(y-p_m^h Q_m^h)}$$

respectively. Expenditures on housing ($p_m^h Q_m^h$) and other goods in the absence of the program were predicted using the estimated demand functions derived from the Stone-Geary utility function. The results presented in table 5 indicate that public housing tenants consume, on average, approximately 34 % more housing than they would have done in the absence of the program. The increase in predicted expenditures on other goods is very small, about 3.5 % on average (**).

The large effects on housing consumption are partly due to the substantial excess of market rents over the rents charged by the housing authorities. The percentage reduction in the price of housing was calculated as

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(*) Basically we compare the market value of the public unit with the expenditures on housing in the absence of the program to arrive at the change in housing consumption. Note that the expression 'housing consumption' is used to refer to the market value of housing units. This is somewhat unsatisfactory and should be kept in mind when reading the remainder of this section.

(**) Note the huge variation in housing concumption increases and in the increase in expenditures on other goods. It's clear that several households consumed less of one of the two composite commodities than they would have done without the programs. This is not contrary to the theory of consumer behavior, however.

$$- \frac{100(p_s Q_s^h - p_m Q_s^h)}{p_m^h \; Q_s^h}$$

and found to be over 36 % on average. This implies that the program results in a large price subsidy.

The previous results, and especially the important effect of the program on housing consumption, are reassuring because one of the main arguments in favor of housing programs is precisely to provide the participants 'better' housing than they could afford in the absence of the programs. The results obtained thus far are consistent with this viewpoint; public tenants occupy housing units with a market rent far in excess of their housing expenditures without the program. However, both the Marshallian and Hicksian benefit measures are significantly below the 'subsidy'. This suggests the existence of nontrivial distortive effects on consumption patterns in the sense that observed consumption would be quite different if households were given a direct cash transfer with the same market value as the subsidy. For this reason many economists are opposed to in-kind subsidies even if they generate desirable increases in the consumption of the good in question. They argue that a cash grant with the same market value would yield the same benefit for the beneficiaries without distorting consumption patterns: households would just maximize utility subject to their new budget constraint. The argument continues that as a consequence a cash grant system is to be preferred to in-kind subsidies unless substantial externalities are associated with the subsidized commodity.

Although there is some evidence that external effects exist in the case of better housing for program participants the available studies do not allow to give a definite answer to the externality question so that the relevance of the previous argument is hard to evaluate in practice (*).

(*) For a review of some of these studies on the externalities of better housing see e.g. J. Weicher (1979).

	Mean	Standard Deviation (*)
Income	144.668	50.960
Market rent public unit	28.584	10.798
Public rent	17.554	7.148
Subsidy	11.030	5.081
% reduction in price of housing	36.132 %	19.842
Expenditures on other goods under the program	125.073	41.718
Expenditures on housing without the program	21.914	6.945
Expenditures on other goods without the program	120.713	40.832
% increase in consumption of housing	33.951 %	52.162
% increase in consumption of other goods	3.523 %	6.536
Hicks equivalent variation	7.755	3.606
Marshallian benefit	7.213	4.241

Table 5: aggregate effects of public housing programs. Monetary variables are in 10^2 Belgian francs per month.

(*) The standard deviations refer to observed variation over the sample, not to estimated standard errors of random variables. We did not attempt to estimate standard errors for the benefits calculated. Though it is theoretically possible to approximate the standard error the result would depend on the variances and covariances of all random variables in the benefit formula. Some of these covariances are unknown as different inputs to the formula were derived using different predicting equations.

Observing the long history of housing programs in many countries it's clear that policy makers do prefer in-kind subsidies over direct cash grants, however. It's probably true that their behavior is inspired by the belief that substantial positive externalities are associated with public housing programs and that the provision of decent housing for the poor should have a high priority. Without going into detail into the ethical arguments behind observed policies it must be noted that their belief will only justify housing programs if indeed public tenants consume more housing and less other goods than they would do when given an unrestricted cash grant with the same market value as the subsidy.

Using the information derived from our sample we can easily test whether the programs investigated in this paper have the desired effects. The expenditures on housing and other goods when a household would receive a grant δ equal to the subsidy are given by

$$\gamma(y+\delta) + (1-\gamma)p_m^h\beta^h - \gamma p_m^x\beta^x.$$

and

$$(y+\delta)(1-\gamma)-(1-\gamma)p_m^h\beta^h+\gamma p_m^x\beta^x.$$

respectively. Predicted consumption effects of a direct cash grant as compared to the actual housing program are summarized in table 6. On average this system of grants would reduce housing consumption by almost 7 % and increase expenditures on other goods by over 5 %. Note however the wild variations in consumption changes. Again it's clear that some families would increase housing consumption and reduce expenditures on other goods as the direction of the changes is largely determined by the location of the bundle consumed under the program relative to the income consumption path, making abstraction of prediction errors. On average, however, our results provide some justification for the current system of housing programs if one believes the externality argument to be relevant.

	Mean	Standard Deviation
Total income, including cash grant	155.698	52.354
Expenditures on housing with a cash grant	23.777	6.395
Expenditures on other goods with a cash grant	131.921	45.412
% increase in housing consumption with a cash grant as compared to the housing program	-6.77 %	23.833
% increase in consumption of other goods with a cash grant as compared to the housing program	5.39 %	9.578

Table 6: effect of replacing existing program by a direct cash grant with the same market value as the subsidy. Monetary variables are in 10^2 Belgian francs per month.

4. Some sensitivity results

Every policy-oriented study is sensitive to the assumptions underlying the analysis. The results derived in this paper must carefully be interpreted in view of a set of restrictive assumptions we had to introduce. First of all, the benefits we have calculated were based upon estimates of the market rent of public units and the parameters of a specified utility function. Moreover, we have assumed throughout the analysis that the available information with respect to household income and the rent paid under the housing program has accurately been reported by all households. Although the possibility of biased results due to misreported data or errors in the prediction of unobservable variables exists for all empirical work, the consequences of imperfections in the data or the predicted values may not be trivial for the results of this study. More importantly, in order to derive all required information for the calculus of benefits we have used extraneous values concerning subsistence expenditures on housing. The question arises how sensitive our results are with respect to the chosen level of subsistence housing. The purpose of this section is to provide some insight into the effects of using incorrect inputs to the benefit formula due to biased estimates of subsistence expenditures, prediction errors or data reporting and coding errors. As we will see below, it will be relatively straightforward to indicate the bias in benefit for an individual household and to suggest the magnitude of the bias in mean benefit in case of systematic errors. Unfortunately, the consequences of random errors for mean benefit are much harder to predict and require simulation procedures that are outside the scope of this paper.

To facilitate the exposition we rewrite the expression for the equivalent variation (5) in a somewhat simpler notation as follows (*):

(*) Although analogous procedures could be developed for Marshallian benefit we concentrate on the equivalent variation.

$$EV = \left(\frac{MR-S_h}{\gamma}\right)^{\gamma}\left(\frac{y-PR-S_o}{1-\gamma}\right)^{1-\gamma}+S_k+S_o-y$$

where MR : predicted market rent of the public unit

$S_h S_o$: 'subsistence' expenditures on housing and other goods respectively

PR : rent charged under the program

γ : marginal propensity to spend on housing

y : income

Consider the impact of upward biased a priori values for S_h on the calculated benefit of a particular household. Differentiating with respect to S_h and taking into account the relation $S_o = \frac{(1-\gamma)S_h-\alpha}{\gamma}$ we find

$$\frac{\partial EV}{\partial S_k} = \frac{1}{\gamma} - A^{\gamma-1} - \left(\frac{1-\gamma}{\gamma}\right)A^{\gamma}$$

where $A = \left(\frac{MR-S_h}{\gamma}\right)\left(\frac{1-\gamma}{y-PR-S_o}\right)$.

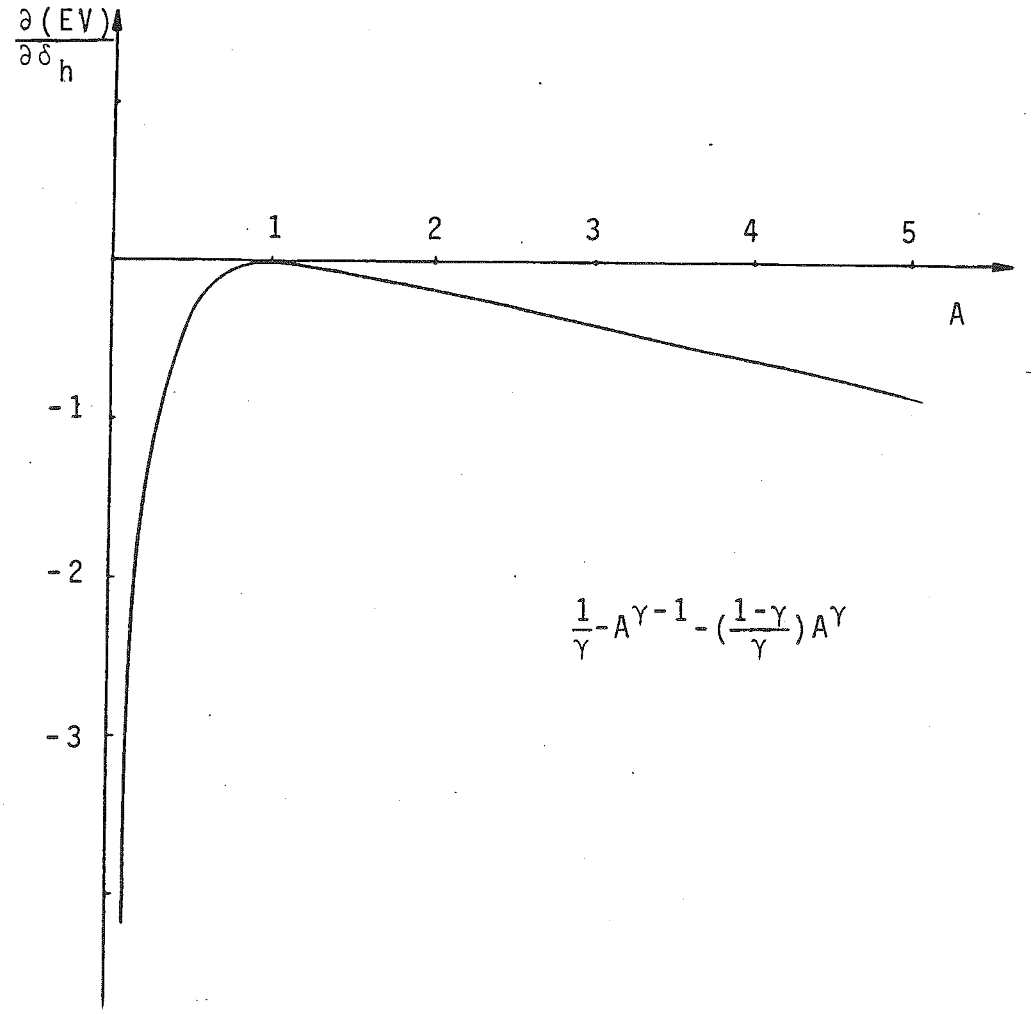
This derivative is graphically represented on figure 3a). A maximum is reached for A=1 in which case the effect of a small bias in δ_k on benefit is zero. It's clear that upward biased subsistence expenditures lead to a downward bias in benefit. However the effect will be small unless A is close to zero or extremely large.

It's instructive at this moment to analyse under what conditions A will be close to one, the case where erroneous subsistence expenditures have no biasing effects. It's straightforward to show that A=1 corresponds to the situation where the consumption bundle under the program is a utility-maximizing combination at income (y+MR-PR) i.e. it's a bundle on the income-consumption path. To see this remember the Stone-Geary demand function (2).

$$Q^h = \frac{p_m^h \beta^h (1-\gamma) - \gamma\ p_m^x \beta^x + \gamma y}{p_m^h}$$

Substituting $y = p_m^h Q^h + p_m^x Q^x$ and rearranging it immediately follows

Figure 3a) Effect of upward biased subsistence expenditures on housing on a households calculated benefit.



that

$$\left(\frac{p_m^h Q^h - p_m^h \beta^h}{p_m^x Q^x - p_m^x \beta^x}\right)\left(\frac{1-\gamma}{\gamma}\right) = 1.$$

Consequently if a public housing tenant happens to consume an optimal combination of housing and other goods under the program then the previous equality holds. Noting that $MR = p_m^h Q^h$, $y-PR = p_m^x Q^x$, $S_h = p_m^h \beta^h$ and $S_o = p_m^x \beta^x$ it's clear that the left-hand side of the expression is precisely the definition of A so that under the described conditions A=1. It's also obvious that in this case the equivalent variation exactly equals the subsidy.

The reason for the previous result follows from the fact that the income-consumption path passes through the displaced origin (β^h, β^x). Consequently shifts in the subsistence parameters have no effect on benefit for bundles on the income-consumption path. However for A close to zero (implying housing consumption close to subsistence or expenditure on other goods exceeding subsistence expenditures by a very large amount) or for A very large (because housing consumption under the program substantially exceeds subsistence levels or because expenditures on other goods are very close to subsistence) the bias of incorrect subsistence housing values may be large as shifts in β^h cause important changes in the indifference map estimated and thus in benefits (*).

We have constructed subsistence housing expenditures by taking the sample minimum for different groups of households. Consequently it's not unreasonable to assume that possible biases will be upward for all households. Some insight into the effects of this systematic type of bias on benefits may be gained by evaluating $\frac{\partial(EV)}{\partial S_h}$ over the sample of public housing tenants.

(*) Note that A < 1 corresponds to bundles to the left of the income-consumption path-provided we indicate housing on the horizontal and other goods on the vertical axis- whereas A>1 corresponds to bundles to the right.

We found $\frac{\partial(EV)}{\partial S_h}$ to be situated between -0.97 and -0.01 with a mean value of -0.17 (*). These figures suggest that in most cases the downward effect on households benefits would be small. A systematic bias of one unit in S_h would only result in an increase of mean benefit of less than 0.2 (**).

Analogous procedures can be developed to investigate the impact of prediction errors in market rent or in the budget share parameters γ and α on the estimated benefits of individual households. The effect of reporting errors in household income and public rent can be analysed along the same lines. The following expressions are straightforward to derive:

$$\frac{\partial(EV)}{\partial MR} = A^{\gamma-1}$$

$$\frac{\partial(EV)}{\partial \gamma} = +\left(\frac{S_o+S_h}{\gamma}\right)(A^{\gamma}-1) \qquad (***)$$

$$\frac{\partial(EV)}{\partial \alpha} = \frac{1}{\gamma}(A^{\gamma}-1)$$

$$\frac{\partial(EV)}{\partial y} = A^{\gamma}-1$$

$$\frac{\partial(EV)}{\partial PR} = -A^{\gamma}$$

where A is defined as before. Note, by simple substitution, that in case A=1, we have $\frac{\partial(EV)}{\partial \gamma} = \frac{\partial(EV)}{\partial \alpha} = \frac{\partial(EV)}{\partial y} = 0$, $\frac{\partial(EV)}{\partial MR} = 1$ and $\frac{\partial(EV)}{\partial PR} = -1$. Graphical presentations of all these expressions are given on figure 3b) through f) for some reasonable parameter values. One should be careful in interpreting these graphs, however, because A itself is not independent of the parameters γ, S_o and S_h in practice.

Evaluation of the derivatives over the sample again provides some guidance as to the effect of errors in the inputs to the

(*) The mean value for A was 1.46, confirming the difference between the equivalent variation and the subsidy.

(**) Simulation experiments with various different values for housing subsistence expenditures confirmed the relatively small effect of possible bias on mean benefit.

(***) To arrive at this result first obtain $\frac{\partial(EV)}{\partial \gamma} = -\left(\frac{MR-S_h}{\gamma}\right)A^{\gamma-1} + \left(\frac{S_h-\alpha}{\gamma^2} + \frac{y-PR-S_o}{1-\gamma}\right)A^{\gamma} + \left(\frac{\alpha-S_h}{\gamma^2}\right)$. Using the definitions of A and α the result presented in the text immediately follows.

benefit formula for individual households and to the consequences of systematic errors on mean benefit. Summarizing results are shown in table 7. Overestimated market rents may cause substantial upward bias in benefits. It's important to note that systematic prediction errors - these may be due to public units being systematically better or worse with respect to unobserved housing characteristics - will have nontrivial effects on benefits: if we assume e.g. that all market rents have been overestimated by 100 B.fr. then this would imply that we have overestimated mean benefit by approximately the same amount. Table 7 also shows the substantial effect of reporting errors in public rent. Errors in reported income have relatively small effects on benefits. This somewhat counterintuitive result is mainly due to the fact that we have constructed expenditures on other goods as income minus rent. If data on other goods expenditure had been available then it's clear that $\frac{\partial(EV)}{\partial y} = -1$ no matter the value of A. Finally, it should be noted that prediction errors in households marginal propensity to spend on housing of, say, 0.01 may lead to significant changes in a households estimated benefit. However, given the specification of the utility function and the procedures used to calculate benefits, the direction of the effect is not necessarily positive as it depends on the magnitude of A, γ, S_o and S_k.

Derivative of benefit with respect to	Min	Max	Mean
MR	0.32	3.11	0.95
PR	-1.25	-0.79	-1.03
γ	-69.12	94.76	12.67
α	-1.24	1.71	0.32
y	-0.22	0.31	0.09

Table 7

This provides some additional evidence that the simple selection bias argument - stating that we have probably underestimated γ for public tenants and thus their benefits - needn't be correct. Indeed, the direction of overestimating γ on benefits is not a priori known.

Figure 3b) Effect of prediction errors in market rent on a households calculated benefit.

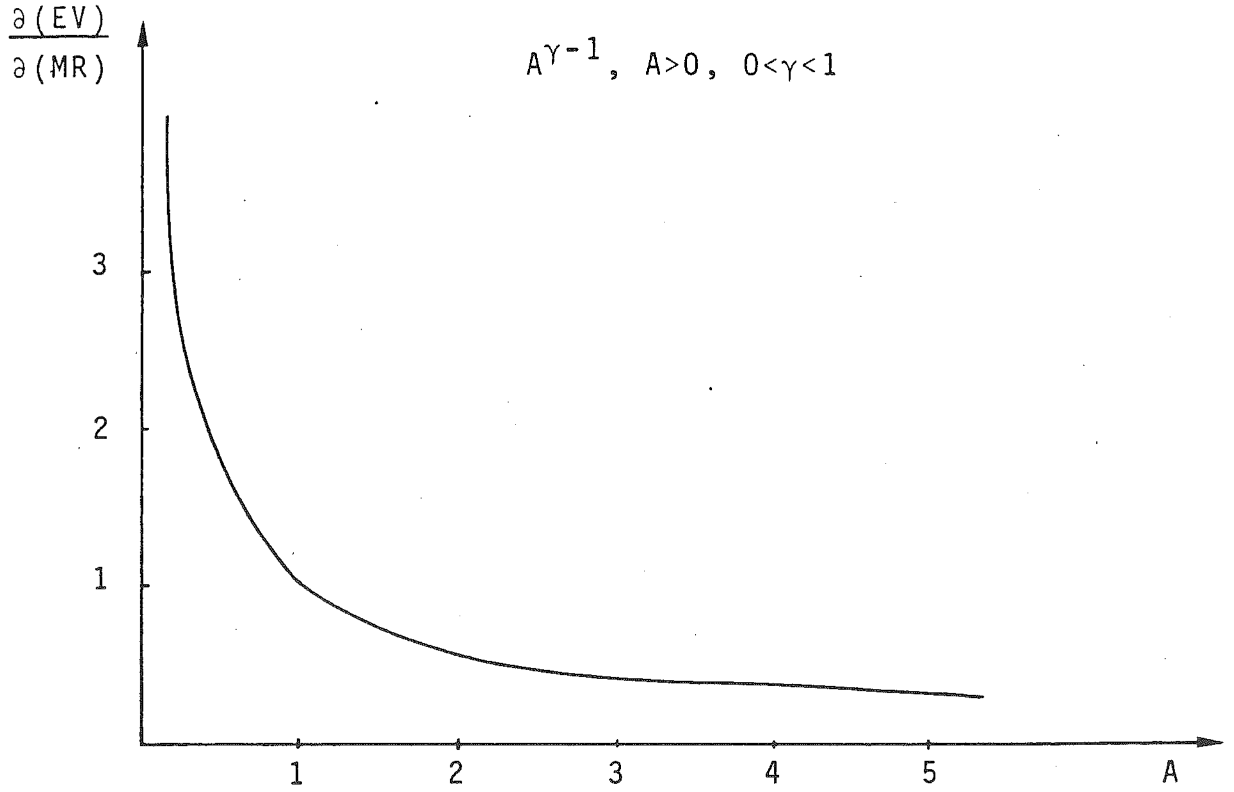


Figure 3c) Effect of reporting errors in income on a households calculated benefit.

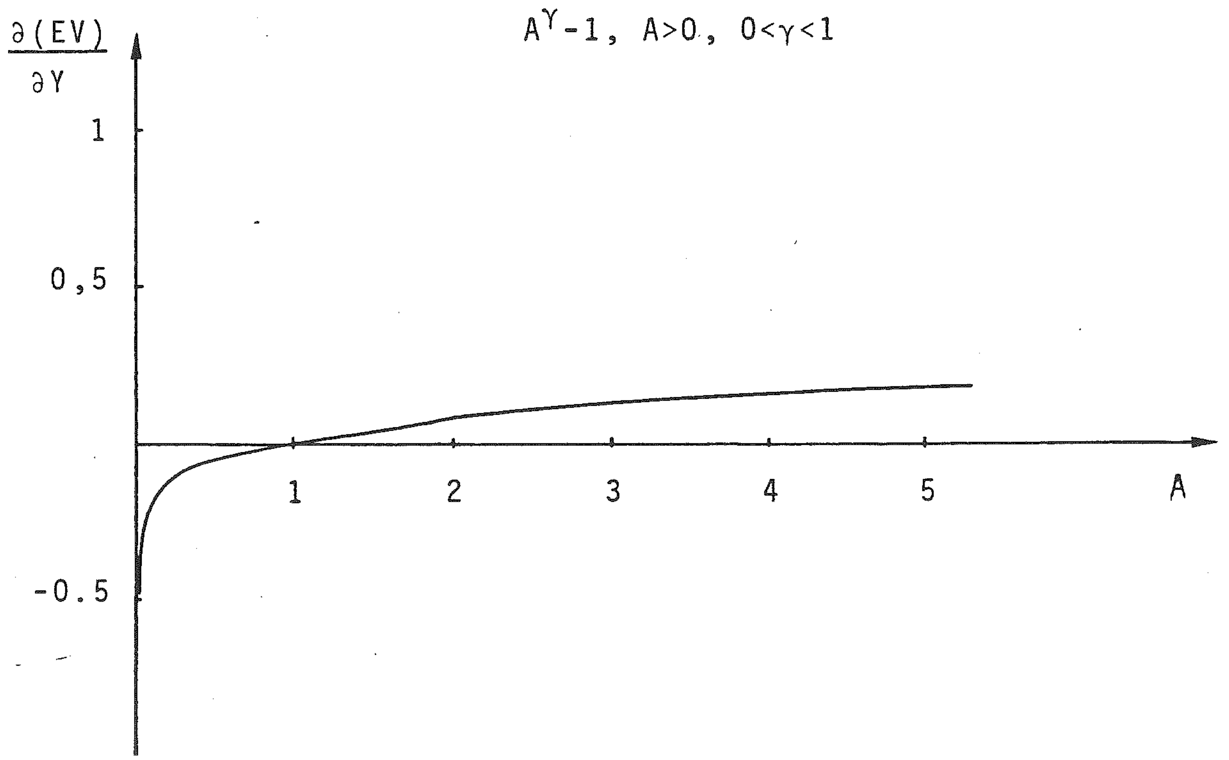


Figure 3d) Effect of reporting errors in public rent on a households calculated benefit.

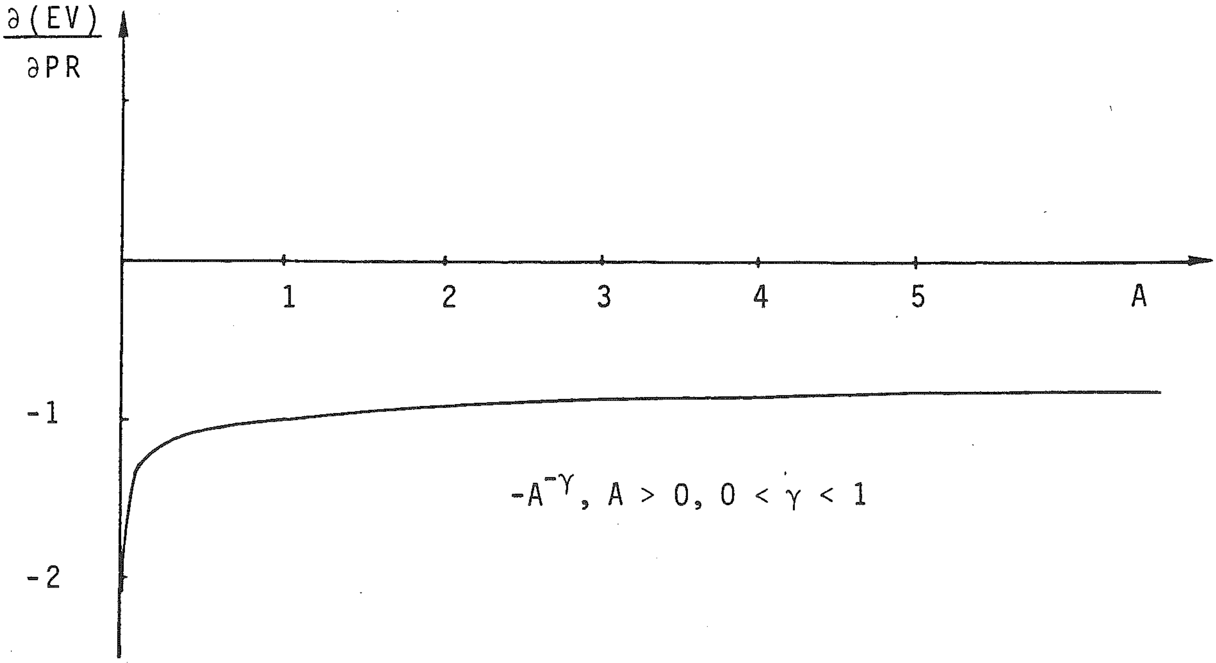


Figure 3e) Effect of prediction errors in α on a households calculated benefit.

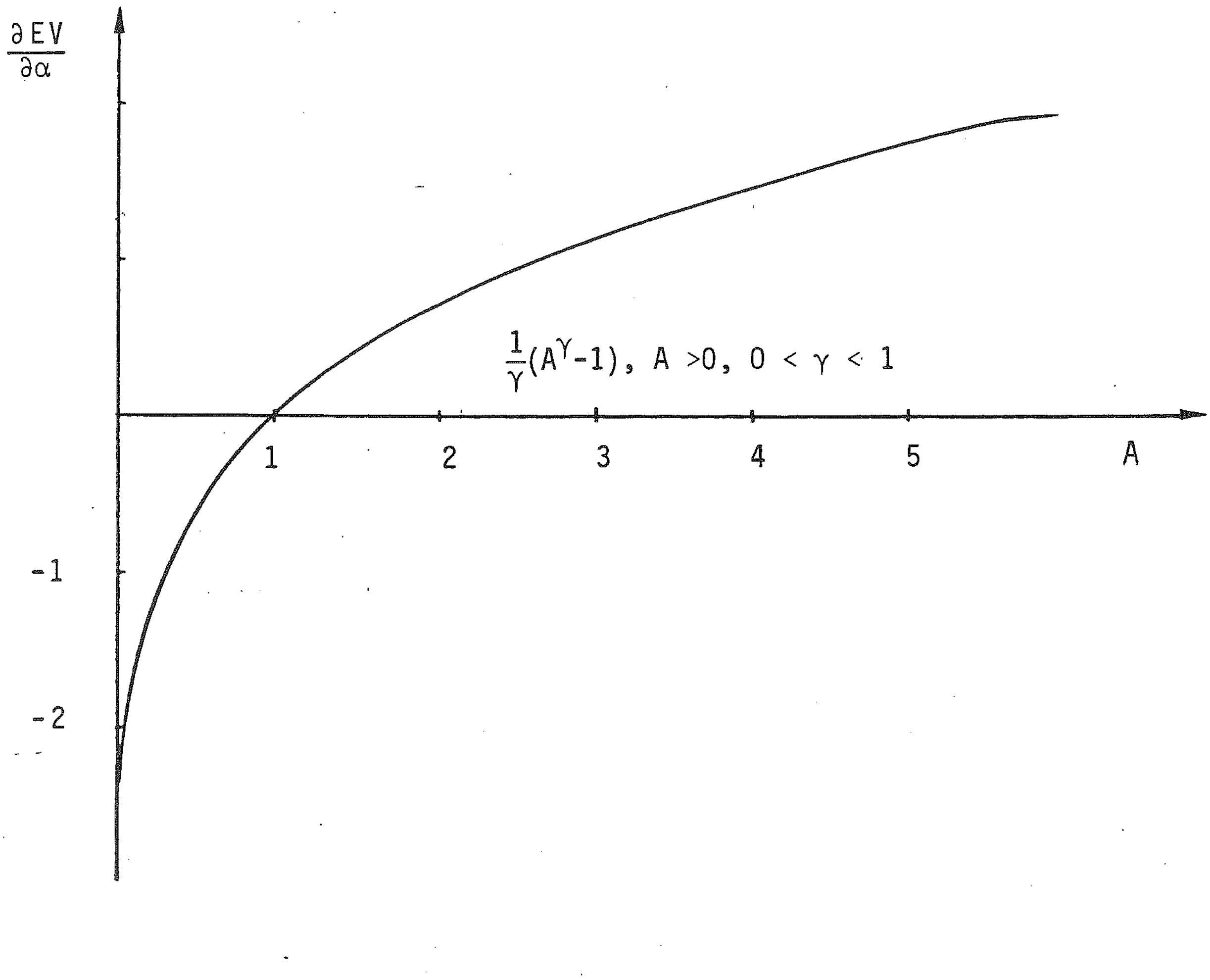
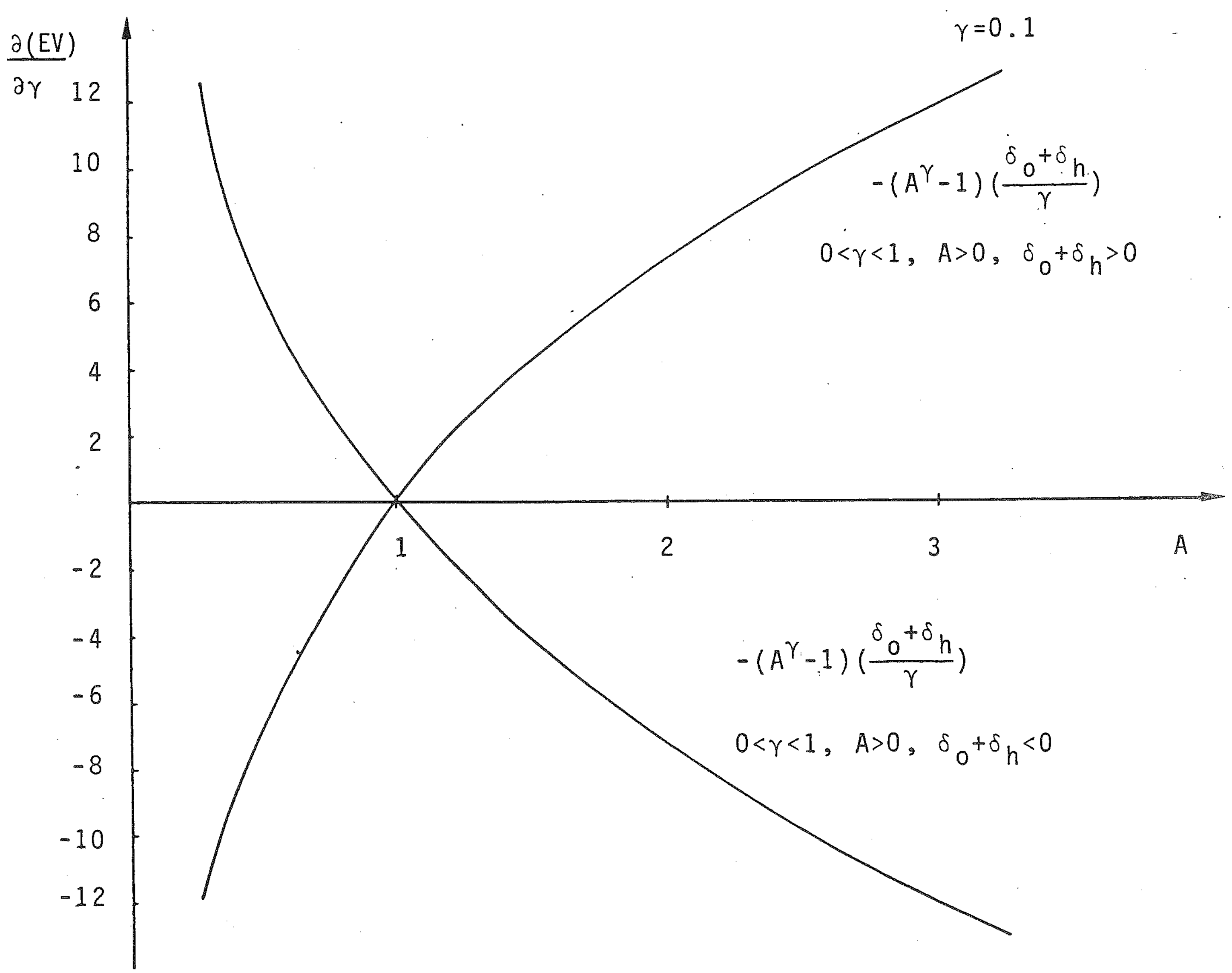


Figure 3f) Effect of prediction errors in marginal propensity to spend on housing on calculated benefit for an individual household.



The results presented in this section are useful to understand the effects of errors in the inputs of the benefit formula on households benefits. They do not provide, however, sufficient insight into the impact of prediction errors in general for the calculated value of mean benefit. Strictly speaking the mean derivatives reported in table 7 are only valid indicators of the impact of a change in the parameters on mean benefit if the change considered is equal for all households. Although such idealised systematic prediction errors are theoretically possible it's more likely that we have overestimated a particular parameter for some households while underestimating it for others. By estimating and using a mean marginal propensity to spend on housing for all households with the same observed characteristics we have undoubtedly used a value too low for some families and too high for others. The table and graphs give almost no guidance in the evaluation of this random type of error. In general, due to the nonlinearity of the benefit formula - reflected in the nonconstancy of the effects of errors on individual benefit - the effects of random errors will not cancel out in the calculus of mean benefit, even if the errors are normally distributed with mean zero. Thus, random errors in predicted market rent will not have a zero effect on mean benefit, nor will the impact of using the same γ for households of a particular type disappear in the determination of mean benefit (aggregation bias).

To evaluate random errors similation techniques might be very useful. However, to test the sensitivity of mean benefit to this type of errors in a satisfying way requires explicit modeling of the covariance structure of different prediction errors. Assumptions must be made regarding the correlation of prediction and reporting errors of different parameters and variables (*). In this paper, we did not attempt a formal simulation exercise to further analyse the sensitivity of mean benefit.

The sensitivity of our results both with respect to systematic and random errors in predicted and reported variables is

(*) The assumption of zero correlations might not be appropriate if e.g. people with a stronger than average taste for housing are more likely to underreport income.

typical for many public policy studies. The possibility of biases is inherent in the topics analysed and the methodology used. Rather than questionning the validity of the approach, we think this sensitivity stresses the importance of predicting the required inputs as accurately as possible. The development of better prediction procedures but especially the design of specialized surveys - yielding more and better information on the housing situation of both public and private housing tenants - might help to partially overcome the problems associated with sensitive empirical results.

5. The distribution of benefits

Apart from promoting homeownership in general one of the major goals of Belgian housing policy is to provide decent housing for those groups in society that cannot afford such housing without government support. Many programs and institutions emphasize the importance of better housing for relatively poor and large families, the elderly and handicapped people (*). The objective of the programs is to increase the opportunities to these households for the satisfaction of their housing needs, thereby increasing their well-being. The subsidisation of rents, by charging public rents well below the market rent, may be viewed as an indirect means to redistribute welfare towards the beneficiaries of the program. Given the importance of focusing on specific groups of households withing society the natural next step in the analysis is to investigate the distribution of benefits over the sample. How do benefits vary with household caracerististics? Are the largest benefits indeed obtained by the households for which the programs were initially developed? The answer to these questions at least yields some insight into possible inconsistencies between the basic goals of the programs and the outcome of their actual operation.

However, not too much information with respect to the distributional pattern and the implied equity aspects of benefits may be derived by a simple regression of calculated benefits on observed household characteristics. Although the sign, the magnitude and the standard errors of the estimated coefficients provide some information with respect to the distribution of benefits over the sample (and the R^2 can be considered as indicating the extent to which households with the same observed charcteristics receive equal benefits from the program), results of this type are insufficient to judge whether the program has desirable distributive

(*) For more detailed and qualified information on the objectives of housing policies in Belgium we refer to V. De Ridder and P. Minon (1980, p. 15-30), M. Durez-Demal (1982, chapter 1) and L. Goossens (1979).

effects. Several reasons account for this fact: first of all, different people may arrive at different conclusions with respect to the desirability of the programs on the basis of the same objective relation between benefits and household traits. Interpretation of the latter relation is largely a matter of taste and depends upon the individual's attitude towards equity and justice. Most people would object to programs that yield significantly higher benefits to high income households or that yield systematically lower benefits for elderly people. However, in some cases disagreement may occur. It's e.g. conceivable that not everyone would agree that larger families should necessarily receive larger benefits, because family size is to some extent a variable subject to individual choice. This leads us to a second problem: whereas most people would favor the statement that equal opportunities should be provided to equally situated families they find it hard to define 'equally situated' in terms of a few observable variables such as income, family size etc. They usually think of 'equal' in terms of unobservable variables (such as talent, ability, effort) or variables such as age and and sex that are not subject to individual choice (*). Consequently insofar as the concept 'equally situated families' cannot be translated in terms of the observables income, family size etc. many people probably don't even want a very close relationship between these variables and benefit. A strong relation is no guarantee of desirable distributive effects. On the other hand the absence of a close relationship is insufficient evidence to conclude that the program has inequitable effects. This is even more the case if we take into account that taste differences among households with the same observed characteristics imply that these families would by definition get different benefits, even if they obtained similar housing units and paid the same rent (**).

We believe, therefore, that a regression of benefits on observed household traits provides us with the objective distribu-

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(*) I am indebted to E.O. Olsen for this important remark.

(**) In this case benefits refer to the 'true' benefits. Estimated benefits would clearly be the same for all such households.

tion of benefits over the sample. It does not yield, however, a final answer to the question: does the program have desirable distributional effects? The answer will depend upon the preferences of the individual policy-maker asking the question.

In table 8 we finally present the regression results of the equations explaining three popular benefit measures in terms of a few household characteristics. The results are completely unsatisfactory from a statistical perspective (*): most variables are not significantly different from zero and the explanatory power is extremely low (**). The estimated relations are utterly disappointing: there is no evidence that higher income households receive lower benefits nor that larger families get higher benefits (the income coefficients have even the unexpected positive sign in two equations).

Although the magnitude of the sample public housing tenants doesn't allow us to conclude that this distributional pattern - or rather the absence of a pattern - is typical for rental housing programs in Belgium, we are convinced that the practical operation of the program may be largely responsible for the results obtained. Remember that for a given household benefit depend upon its income, the market rent of the unit occupied and the rent charged by the housing authorities. The analysis of the previous section shows that the effect of an increase in income on benefit may be positive or negative and that, for reasonable values of the parameters, the effect will be small. This implies that in order to guarantee a negative total effect of income on benefits a sufficiently large increase in public rent should accompany income changes.

The operation of the by far largest program for public rental housing in Belgium (***) strongly suggests that a close

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(*) Several other explanatory variables, if included in the equations, only contributed to producing results that were even worse than those reported in table 8.

(**) Note, by the way, that the hypothesis of all slopes equal to zero cannot be rejected at the usual significance levels.

(***) The program is operated by the 'Nationale Maatschappij voor de huisvesting (NMH)'.

Dependent variable Independent variables	EQUIVALENT VARIATION	MARSHALLIAN BENEFIT	SUBSIDY
CONSTANT	-3.233 (-2.55)	-2.913 (-1.832)	-1.689 (-1.991)
FS3	6.306 (1.727)	5.424 (1.813)	6.132 (1.608)
FS4	7.113 (2.243)	5.662 (2.829)	7.412 (1.867)
FS5	4.094 (1.067)	3.223 (0.955)	4.952 (1.238)
FS6+	6.659 (1.583)	7.819 (1.672)	6.486 (1.437)
INCOME	0.0057 (0.279)	-0.0032 (0.844)	0.0058(0.284)
AGEM	0.151 (0.886)	0.201 (1.246)	0.199 (1.123)
R^2	0.091	0.113	0.089
F (*)	1.18	1.50	1.15

Table 8: regression results for the distribution of benefits (t-statistics in parentheses)

(*) F-statistic for testing whether all slopes of the regression are equal.

relationship between income increases and adjustments in public rent doesn't exist. Although in practice the calculus of rent is a complicated matter subject to a multitude of exceptions to the general rule, the basis for the calculations is the construction date of the unit (especially units build after World War II are significantly more expensive than units of earlier periods) and the cost of certain additional attributes (such as central heating, elevators etc.) that are borne by the renter (*). Reduced rents are possible for large families and very low-income households. In general, however, the relation between rent and income is not very strong: rent is lower for households with an income below a given level, whereas rents are significantly increased if household income exceeds the limit for eligibility to the program. Moreover, the adjustment of rent is usually lagging behind because income controls are only done with broad time intervals.

The described general program rules suggests a rather weak relation between rent and income. To illustrate the argument we regressed public rent on income, family size and a few housing characteristics that were suggested to influence rents as calculated by the housing administration. The following equation was obtained (**):

$$\begin{array}{l} \text{PUBLIC RENT} = \underset{(2.356)}{10.299} - \underset{(-3.556)}{6.176}\ \text{CHILD} + \underset{(2.942)}{1.149}\ (\text{CHILD})^2 + \underset{(2.352)}{0.027}\ \text{INCOME} \\ \qquad + \underset{(2.194)}{2.815}\ \text{CD45} + \underset{(2.456)}{3.466}\ \text{ELEV} + \underset{(3.826)}{4.016}\ \text{CH} \qquad R^2 = 0.65 \end{array}$$

where CHILD is the number of children and the figures between brackets are t-values. This result shows that a large proportion of the variation in public rent can be explained by a few variables. Rent appears to decrease at a decreasing rate with the number of children and to be positively related to income. It should be

(*) The detailed program rules are almost continuously subject to change. For some of the recent developments see M. Durez-Demal (1982, p. 24-27).

(**) Additional explanatory variables such as space indicators were, somewhat surprisingly, totally irrelevant.

noted, however, that there are only large differences in rent between households without and those with children. The variations in rent between families with different numbers of children are relatively small. Furthermore, it follows from the equation that the difference in rent between two households with the same values for all included variables but with a difference in income of 1000 B.fr. will be less than 30 B.fr. on average. Although it cannot be denied that a significant relation between rent and income exists there is clearly some evidence that many changes in income in practice are accompanied by small or zero rent adjustments. As previously explained only a large increase in rent corresponding to increased earnings - which would lead us to observe significant rent variations for households with the same housing units and observed characteristics but with different incomes - can guarantee a negative relation between benefits and income. We believe the absence of such a close relationship is largely responsible for the disappointing results of this section (*).

(*) If the results of this section would be reproduced for larger data sets and if one believes a negative relation between benefits and income is absolutely crucial for equitable programs, then this would indicate that changes in program rules are highly desirable. Simulation techniques would be extremely useful to evaluate alternative sets of program rules.

IV. Summary and conclusions

In this paper we have analysed some of the economic effects of housing programs for rental housing in Belgium on the basis of a small sample of individual households. Although a more detailed study of the programs is desirable - e.g. in terms of the housing characteristics contained in the public units - we have used the classical composite commodity approach for the estimation of the benefits and the consumption effects that result from subsidized rents. In a theoretical part of the paper we have discussed the methodology implemented to calculate Hicks equivalent variation and Marshallian benefits for each household in the subsample of public housing tenants. To accomplish this task we specified a Stone-Geary utility function to represent household preferences and estimated its parameters. Furthermore, we had to predict the market rent of the housing unit occupied by each family. Given these estimates the calculus of benefits was shown to be a straightforward exercise. We also investigated the distortive effects of the housing subsidies on consumption patterns by comparing the consequences of the program with the results that would have been obtained if the participants had been given a simple monetary grant with the same market value as the subsidy under the program.

Two further issues were dealt with. We presented some evidence with respect to the sensitivity of our results to changes in the most important determinants of benefits. This provided some insight into the effects of possible prediction errors or reporting errors on the calculated benefits for individual households. However, insufficient information was available to arrive at strong conclusions regarding the sensitivity of calculated mean benefit to random errors in the parameters of the benefit formula. Finally, we also investigated the relation between benefits and household characteristics in order to describe the distributional pattern of benefits over the sample.

In our view useful information results from the empirical work reported in this paper. We found the marginal propensity to spend on housing to be 0.127 on average over the sample. It fluctuated strongly, however, with family characteristics such as family size, age of the head of the household and of the youngest child. The price and income elasticities derived from the Stone-Geary demand functions were within the range of accepted values in the urban economics literature. The income elasticity was estimated to be 0.816 on average and found to be slightly increasing with family size. Price elasticities were in the range from -0.75 to -0.85.

Participants in the programs occupy housing units with a substantially higher market rent than the amount charged by the housing authorities. On average the programs correspond to a subsidy of 36 % in the price of housing. The over-all consumption effects were also found to be large: most households consumed significantly more housing than they would have done without the program, 34 % on average. Moreover, many families also consumed more of other goods (3.5 % on average). There were, of course, households that consumed actually less of one of both composite commodities in the sample as well.

The calculated benefits from the program were relatively small amounting to approximately 750 Belgian francs per month on average. This corresponds to some 6.5 % of monthly income. Although substantial differences in benefit were found between the Hicksian and Marshallian measures for individual households mean benefits from both concepts proved to be quite similar. In each case mean benefit was significantly below the subsidy which suggests important distortions in consumption patterns caused by the programs. We calculated that replacing the programs with a system of direct cash grants with the same market value as the subsidy would reduce average housing consumption by almost 7 % while increasing the consumption of other goods by some 5 %. Although a wide variation existed over the sample these results give at least a partial justification for the housing programs analyzed if one believes that important externalities are associated with the fact that the program participants consume more housing.

We did not find strong evidence that benefits are closely related to household traits such as income, family size etc. Although this is somewhat disappointing we have stressed the fact that the absence of this relation doesn't allow any conclusions with respect to the desirability of the program. Individual preferences and personal attitude towards equity and justice will lead to widely diverging interpretations of any objective distributional pattern of benefits over households with different characteristics. Moreover, it's not sure whether the weak distributional results obtained in this paper are typical of all housing programs for rental housing in Belgium. We did suggest, however, that the operation of these programs might lead to a particular, insignificant relation between benefits, income and family size. Indeed, several specific features of the programs - such as the basic calculation procedure to determine rent and the lack of strong adjustments of rents for changes in income - are consistent with the results obtained.

Appendix 1: a summary description of the raw data

The data we will use in this study of public housing programs in Belgium come from a survey conducted in the city of Liège in the early seventies. The original sample contains 522 observations and a wide variety of variables relating to the socio-economic characteristics of the surveyed households.

Many observations were not useful for the purposes of this paper, however. As the analysis concentrates on the effects of public housing programs and only a few owner-occupiers were living in public units the subsample on owner-occupied housing didn't seem to be very informative. Moreover, insufficient information was available on the financial conditions of the house sales so that it would be very difficult to get an accurate picture of the price of housing services for owner occupiers. It was decided to concentrate the analysis on the sample on rental housing. The latter consisted of 338 observations (*).

A few more observations turned out to be useless. Some households didn't answer the question whether they were living in a public housing unit, whereas others didn't report the rent they were paying. As both these variables are crucial for the analysis it was decided to delete the corresponding households from the sample, rather than trying to predict the missing observations. Consequently, we ended up with a sample of 326 households living in rental housing. The subsample on public tenants contained 65 observations, whereas 261 households were living in private housing.

In this appendix we briefly describe the general characteristics of our data and indicate how some missing observations were predicted. We can classify the available information into two groups that will be discussed consecutively: data with respect

(*) Working with the renter data only has a significant advantage. Whereas a wide variety of government programs exist for owners - each of which requires a somewhat different approach - public programs for rental housing are quite uniform. As a consequence our results provide some preliminary information on the general effects of government programs for rental housing.

to housing conditions and data describing household characteristics. Although in both cases the information is far less than desirable it is comparable to data sets that have been used in previous studies of public housing programs.

Let's first consider the data on housing characteristics. In table A1 we give a summary description of all important variables together with their possible numerical values, i.e. whether they are dummies, continuous variables or whether they are given on a quality scale. The table shows that we have, apart from monthly rent, the number of rooms and bedrooms and total living space. These variables should give a reasonable description of the first aspect of housing consumption viz. space. Further we have a set of variables that may be assumed to represent the quality dimension of housing. As can be seen from table 1 most of these variables have to be transformed before they can be used for hedonic estimation purposes. We have a rather crude classification of the construction date of the unit, we know whether or not a kitchen is available and we have some information on the type of kitchen. We know whether the housing unit has hot water in the kitchen, whether a bathroom is available and there's some information relating to the heating system. Other variables describe the availability and 'quality' of additional spaces such as e.g. a basement, a yard etc.

Finally two other variables concerning housing consumption are a dummy variable for the housing type, i.e. whether the unit is an appartment or a single family house, and (in the case of appartments) the story on which the unit is located. Of course we also know which houses or appartments are public unit.

Unfortunately we have almost no information on environmental quality and on the accessibility of the unit with respect to public and recreational facilities. There's also insufficient data on the location of the housing unit, although we do have some crude variables describing the relative location of the unit with respect to the workplaces of the working members of the household. This will be indicated below.

A particular problem arises with the variables OTHSPA and ATTR1. The way these are constructed it's not always possible to determine which individual attributes are available in each unit and which are not. The reason is that they are defined on a scale and that it's not possible to identify each number on this scale with a particular attribute (*). As a consequence some probably useful information is lost.

Table A1: data on housing

Variable	Description	Type-Range
RENT	Monthly rent, excluding heating and electricity.	Continuous
TYPRH	Type of housing i.e. appartment or single-family house	Dummy
CONSTRDAT	Construction date of unit; classified in four periods.	1: before 1900 2: 1900 - 1918 3: 1919 - 1945 4: after 1945
STORY	Story on which the unit is located	0 - 5 (5=5 or higher)
ATTR 1	Availability of certain attributes such as an elevator, etc.	Scale 0-8, each number referring to one or more specific attributes
ROOMS	Number of rooms	0 - 9
BEDR	Number of bedrooms	0 - 9
SPACE	Living area	Continuous
HEIGHT	Height of the rooms; classified in four categories	1: < 2.4 meter 2: 2.4 - 2.7 meter 3.: 2.7 - 3.5 meter 4: > 3.5 meter
KITCHEN	Availability and 'quality' of kitchen	1: no kitchen 2 - 5: quality scale
WATER 1	Whether or not running water available	Dummy
WATER 2	Whether or not hot water available	Dummy
BATHR	Whether or not the unit has a bathroom	Dummy
WC	Quality of restrooms	Quality scale 1-6

(*) E.g. a particular number on the scale indicates that at least 2 out of 4 attributes are available without specifying which ones.

table A1 continued

Variable		
OTHSPA	Whether the unit has such additional spaces as a basement, depositories, a terrace etc.	Scale 1-7, each number referring to one or more specific item.
YARD	Availability and 'quality' of yard	1: no yard 2-4: quality scale
CH	Information on the heating system	3: individual central heating 2: collective central heating 1: no central heating
LIGHT	Power of the electric light installations	Continuous

For some households not all the specified variables were available due to their not answering all questions or due to coding errors. The missing observations were predicted using standard regression techniques. The procedure looks at the relation between each variable for which some observations are missing and a set of independent variables specified by the investigator. Stepwise regression is used until all variables that meet the F-to enter test are included in the equation (*). In table A2 we give an overview of the variables for which some observations were predicted: in each case we indicate the percentage of missing data for the subsamples on public and private housing together with a list of the variables that appeared in the final prediction equations (**). The R^2 of the prediction relations ranged between 0.25 and 0.489. Although our procedure may not be entirely satisfactory we are convinced that prediction was a superior solution to the problem of missing observations than deleting the corresponding households from the sample.

(*) The square of the ratio of a variables regression coefficient to its standard error is F distributed. We required that variables were entered into the equation if their F-value exceeded 4.

(**) Note that we did not use RENT in predicting missing observations. Though this variable correlated very well with some of the variables to be predicted this procedure might artificially inflate the explanatory power of hedonic regressions to be estimated later.

Variable	% Missing Private Housing	% Missing Public Housing	List of explanatory variables
CONSTRDAT	7.3	1.5	ROOMS,BATHR,HEIGHT,LIGHT
BEDR	0.4	1.5	ROOMS,WATER2
SPACE	8.0	7.3	ROOMS,BEDR,WEIGHT,LIGHT
HEIGHT	3.1	0.0	CONSTRDAT,ROOMS
KITCHEN	2.7	1.5	BATHR,LIGHT,CONSTRDAT
WATER2	0.8	0.0	BATHR,LIGHT
BATHR	3.8	0.8	CONSTRDAT,WATER2,LIGHT
YARD	0.8	0.8	STORY,ROOMS,HEIGHT
LIGHT	2.7	0.0	KITCHEN,BATHR,STORY

Table A2: Missing observations

Next consider our data on household characteristics. Table A3 presents a summary overview of the most important variables. We have information on wages, family size, the ages of husband, wife and youngest child and some professional information. Very crude information is also available concerning the workplaces of the households working members. Indeed we know the monthly costs of the journey to work as well as the daily commuting times. Unfortunately no useful information is available on the location of the workplaces and the families house itself. Other data, which we think will be not very useful for the purposes of this study include the birthplace of husband and wife, whether the latter intends to stop working in the near future etc.

The data on household attributes were fairly complete. We only predicted a few missing observations for the transport cost variables using the stepwise regression techniques previously discussed.

It should finally be noted that not all variables will be used in the empirical analysis of this paper.

Table A3: description of household characteristics

Variable	Description	Type-Range
CHILD	Number of children	0 - 4 (4=4 and above)
MARR	Number of years married	1: 1-6 years 2: 6-11 years 3: > 11 years
BIRPLM	Birthplace husband	
BIRPLW	Birthplace wife	
AGEM	Age husband	8 age categories
AGEW	Age wife	8 age categories
AGEYC	Age youngest child	4 age categories
PROCLM	Whether husband is a laborer or an employee	dummy
PROCLW	Idem - wife	dummy
PROSEM	Whether husband is working in public or private sector	dummy
PROSEW	Idem - wife	dummy
WRKPLM	Workplace husband	Central Liège, SMSA Liège, other
WRKPLW	Workplace wife	
WAGEM	Wage husband	Continuous
WAGEW	Wage wife	"
EDM	Educational level	Scale from elementary school to university degree
PRACTW	Whether the wife has some professional activity	Dummy
POFTW	Whether the wife works part- or full-time	Dummy
STOPW	Whether the wife intends to stop working	Dummy
RMOVED	Whether the household has recently moved	Dummy
TRTM	Commuting time husband	6 categories
TRTW	Commuting time wife	"
TRCM	Commuting costs husband	8 categories
TRCW	Commuting costs wife	"

Appendix 2: regression results hedonic equations

Here we report on some alternative hedonic models that were close to the chosen specifications as far as predictive power is concerned. The results confirm the statement in the text that a lot of different specifications produced quite similar results.

Dependent / Independent	RENT		RENT		RENT		RENT/ROOMS	
	Coefficient	t-value	coefficient	t-value	coefficient	t-value	coefficient	t-value
CONSTANT	9.921	(4.286)	11.458	(4.123)	18.2561	(3.987)	12.939	(2.881)
ROOMS	1.584	(1.432)	1.428	(1.312			-2.954	(-5.960)
BEDR/ROOMS	7.369	(2.561)	6.425	(2.259)	7.332	(2.672)	3.632	(2.819)
SPACE	0.083	(2.622)	0.096	(3.036)	0.081	(2.563)	0.031	(2.201)
WATER	1.936	(1.858)	1.996	(1.307)	1.958	(1.279)	0.697	(1.011)
BATHR	6.219	(3.796)	5.814	(3.593)	6.068	(3.731)	2.671	(3.639)
CH	2.819	(2.751)	2.797	(2.788)	2.788	(2.736)	1.206	(2.624)
CD00	3.089	(1.336)	2.959	(1.296)	3.090	(1.341)	1.459	(1.409)
CD18	1.655	(0.745)	1.377	(0.629)	1.892	(0.849)	0.605	(0.608)
CD45	6.613	(2.661)	5.326	(2.134)	6.559	(2.661)	3.163	(2.842)
STORY12	-1.580	(-1.028)	-2.309	(-1.440)	-1.378	(-0.912)	-0.406	(-0.59)
STORY34	-3.806	(-1.419)	-3.267	(-1.331)	-3.0964	(-1.131)	-1.922	(-1.598)
STORY5+	14.589	(5.646)	7.691	(2.444)	14.840	(5.786)	5.756	(4.972)
ELEV			7.596	(3.346)				
ELEV*STORY34	8.426	(2.267)			8.155	(2.193)	4.889	(2.936)
1/ROOMS					-10.237	(-2.698)		
R^2	0.7055		0.6927		0.6725		0.6812	

appartments

Dependent / Independent	LN(RENT) Coefficient	t-value	(RENT) coefficient	t-value	LN(RENT) (*) coefficient	t-value
CONSTANT	2.175	(2.972)	2.439	(4.172)	2.538	(4.123)
LN(ROOMS)	0.239	(2.171)	5.488	(1.846)	0.065	(1.587)
LN(BEDR)	0.167	(2.365)	4.932	(2.588)	0.242	(2.262)
LN(SPACE)	0.183	(2.31)	4.689	(2.187)	0.003	(2.634)
WATER	0.071	(1.245)	1.940	(1.261)	0.073	(1.279)
BATHR	0.292	(4.820)	5.979	(4.652)	0.301	(5.928)
LN(CH)	+0.101	(2.672)	+2.796	(2.727)	0.104	(2.826)
CD00	0.134	(1.568)	3.199	(1.382)	0.129	(1.505)
CD18	0.081	(0.983)	1.893	(0.85)	0.069	(0.843)
CD45	0.277	(3.017)	6.656	(2.681)	0.271	(2.926)
STORY12	-0.077	(-1.352)	-1.596	(-1.370)	-0.076	(-1.332)
STORY34	-0.167	(-1.670)	-3.262	(-1.205)	-0.192	(-1.918)
STORY5+	0.332	(4.484)	14.754	(5.724)	0.328	(3.411)
ELEV*STORY34	0.269	(2.154)	8.272	(2.220)	0.283	(2.044)
R^2	0.679		0.6922		0.6751	

appartments

(*) Independent variables are untransformed i.e. this columns reports the results of semi-log specification.

Dependent	RENT		RENT/ROOMS		RENT/ROOMS	
Independent						
	Coefficient	t-value	coefficient	t-value	coefficients	t-value
CONSTANT	10.661	(3 ?21)	5.325	(9.998)	-5.507	-(4.762)
ROOMS	2.538	(2.936)	-1.979	(-4.141)	2.384	(2.942)
BEDR/ROOMS	2.258	(1.348)	0.766	(0.326)		
SPACE	0.003	(0.389)	0.001	(0.41)	0.009	(1.126)
WATER	7.729	(2.915)	1.975	(2.044)	7.693	(2.918)
BATHR	5.480	(1.623)	1.883	(1.530)	5.603	(1.677)
YARD	3.119	(1.959)	1.104	(1.902)	3.046	(1.941)
CH	4.534	(2.734)	1.799	(2.977)	4.468	(2.726)
CD00	2.259	(0.581)	1.449	(1.021)	2.224	(0.575)
CD18	4.504	(1.289)	2.361	(1.857)	4.404	(1.271)
CD45	5.173	(1.831)	2.418	(2.578)	5.199	(1.942)
R^2	0.515		0.482		0.483	

Single-family housing

Dependent	LN(RENT)		RENT		LN(RENT) (*)	
Independent						
	Coefficient	t-value	coefficient	t-value	coefficient	t-value
CONSTANT	2.063	(3.182)	-4.552	(4.183)	1.814	(2.256)
LN(ROOMS)	0.237	(1.421)	4.735	(1.951)	0.085	(2.591)
LN(BEDROOMS)	0.1145	(1.823)	1.923	(1.422)	0.054	(0.791)
LN(SPACE)	0.005	(0.119)	1.416	(0.281)	0.001	(0.532)
WATER	0.235	(2.642)	7.479	(2.826)	0.251	(2.788)
BATHR	0.226	(1.956)	4.770	(1.386)	0.276	(2.408)
LN(CH)	0.308	(3.064)	9.067	(3.03)	0.157	(2.795)
YARD	0.105	(1.975)	2.946	(1.867)	0.107	(1.997)
CD00	0.189	(1.553)	1.827	(1.471)	0.219	(1.659)
CD18	0.282	(2.415)	4.053	(1.165)	0.302	(2.540)
CD45	0.345	(2.456)	4.494	(2.076)	0.373	(2.615)
R^2	0.572		0.512			0.564

Single family housing

(*) Independent variables are untransformed i.e. this column reports results of semi-log specification.

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