STUDIECENTRUM VOOR ECONOMISCH EN SOCIAAL ONDERZOEK

SHIPPING LOGISTICS ;

A Revisitation of applications of Linear programming

Evrard CLAESSENS *(&)*

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UNIVERSITAIRE FACULTEITEN SINT-IGNATIUS
Prinsstraat 13 - 2000 Antwerpen
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a b s t r a c t

The note deals with LP applications of establishing an optimum least-cost shipping sailing list on a number of alternative routes with a given number of ships of one or more types (classes). The discussion analyses the basic optimum solution with the dual optimum values. Afterwards specific real life situations are addressed, such as the backloads, lay-up costs and the possibility of entering the time-charter market.

table of contents

1. Introduction

Since Dantzig's benchmarking initiatives on the matter of programming techniques now over thirty years ago, linear programming procedures have achieved vast strides forward, both in terms of reaching an ever wider span of applications as with respect to the methodological elaborations and refinements. As often happens in these matters there is a growing gap threatening between on the one hand the possibilities of hard- and software and the methodlogical proficiency of the "happy few", and on the other hand the actual accessibility of existing knowledge and software to the larger public. Among those, many are involved in daily management without much spare time to endeavour in keeping up with the technical literature.

That threat is not likely to soften; especially noteworthy is that the more recent new endeavours are progressively restricted to the so-called "commercial software literature", i.e. without the larger access of the "academic literature". Such observations contrast fairly sharply with the recent proliferation of personal and mini-computing system, offering standard packages to cope with traditional problem statements and covering an increasingly wider calculating capacity. There is thus an urgent need to give new impetus toward a wider application range of existing software.

The present note aims to address a number of well-known problems in shipping logistics by means of the "assignment-problem" case in linear programming. The case study is elaborated with realistic cost-figures and operations data, only occasionally "adapted" in order to check instructively the sensitivity of the formulation.

A first section presents a basic "assignment problem" in the shipping industry, such as when one owns a certain number of ships and the point is to achieve an optimal sailing pattern to match demand on a number of routes. This section also introduces the Linear Programming (LP) procedure, for those not (anymore or yet) familiar with the basic output of current computer packages.

Afterwards a number of refinements are successively intro-troduced, such as the economic interpretation of the dual variables, the organization of the "backload problem" and the vital choice between alternatives such as time-charters versus operations on own account, or operating-at-a-loss versus involving lay-up costs for the idle fleet.

With those introductory elaborations the hope is substantiated that even in the shipping industry where the traditional virtues of skill, judgment and care are only appatently counterparts of technical once-for-all solutions, some technical background is instrumental in building up further proficiency in tackling daily management decisions.

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2. : Procedures of linear programming

Consider you have to reorganize the operations logistics of a container line which currently owns 23 ships of which:

- 5 "second generation" vessels of a rated 1500 TEU capacity,
- 8 " first generation" vessels of a rated 850 TEU capacity and
- 10 feeders with a 500 TEU capacity.

Your company serves four routes from one port to four destinations (viz.origins). Your agents in each of those ports estimated market potential demand at 3000, 6000, 2500 and 3500 TEU's respectively. Further information is available on:

- the maximum number of monthly sailings per route and by ship's class *(which is derived from average sailing speeds and corrected for port calls, i.e. waiting and handling times)*,
- the average operating costs for round trips on each vessel/ route combination.

The above information is schematically presented in Table 1. The question now is to establish a "least cost" logistical operations schedule. With "costs" we mean "systems costs", i.e. the operating costs of seagoing cargo but also the so-called "penalties" for each forgone opportunity of loading cargo. Those penalties are expressed in terms of net-profit per TEU and are evaluated for each route at 40,- 50,- 40,- and 70,- respectively.

Table 1 : Data input for the shipping logistics exercise

ship characteristics: type	capacity	number	maximum monthly trips on route: 1.	2.	3.	4.
1.	1500 TEU	5	3	2	2	1
2.	850 TEU	8	4	3	3	2
3.	500 TEU	10	5	5	4	2
1.	operating costs per vessel and by route;		10,000	11,000	12,000	15,000
2.			8,000	9,000	10,000	10,000
3.			6,000	8,000	8,000	9,000
penalty per TEU			40,-	50,-	40,-	70,-
TEU-demand per month by route: (in TEU's)	case 1. :		3,000	6,000	2,500	3,500
	2. :		6,000	9,000	5,000	5,000
	3. :		6,000	10,000	5,000	10,000
	4. :		6,000	20,000	5,000	10,000
	5. :		6,000	20,000	5,000	20,000
	6. :		10,000	20,000	5,000	20,000

Table 2. :

Operating cost per TEU by ship's type and route:

1.	6.67	7.33	8.00	10.00
2.	9,41	10.59	11.76	11.76
difference in %	*+ 41%*	*+ 44%*	*+ 47%*	*+ 18%*
3.	12.00	16.00	16.00	18.00
difference to 1.	*+ 80%*	*+118%*	*+100%*	*+ 80%*
difference to 2.	*+ 28%*	*+ 51%*	*+ 36%*	*+ 53%*

2.1. . : Technical description

The technique of linear programming enables the user to optimize a so-called "objective function" under the restriction that a number of conditions (constraints) are not violated. The method features the advantage that it quickly resolves those optimization problems without having to simulate all possible combinations between the decision variables. This does not mean that all trial-and-error becomes an unnecessary game; at least it becomes redundant for the purely technical calculus.

The LP-technique is backed by a copious body of mathematical theory and mechanical interpretation, for which we refer to the specialized textbooks (1). Rather we focuss on the major possibilities and warnings in applying the method, as far as the herewith discussed applications to the shipping industry are concerned. An extensive discussion of a typical "case study" is of vital importance, the more since modern micro-computers allow easy and cheap applications of LP, without a preliminary proficiency in mathematical calculus. Therefore we ommit the basic calculus, except for the formulation of the input format and some considerations of organizational nature.

The input format of an LP problem requires firstly that each equation (*i.e. the objective function and the constraints*) is expressed in a common dimension, such that each item which conditions the optimization process is formulated in a consistent way. Those dimensions; however, may vary accross the equations. Thus the objective function may take a monetary dimension (*costs, revenues, profits or opportunity costs*) whereas the constraints express operating frequencies, demand units or a variety of operating conditions.

Second, all variables introduced in the constraints, must also feature in the objective function. Otherwise, the optimization could not be reached in a comprehensive way *(i.e. taking into account the potential activation of constraints).*

Third, constraints are either:

a/ inequalities, which express the necessity that a number of quantified items should not exceed a given ceiling, or violate a minimum value. For each inequality constraint, the program itself calculates the extent to which the inequality applies through the introduction of a so-called "slack-variable". *For example, 7 is smaller than 8, but 5 too. If the constraint sais "less than or equal to 8", then the slacks are respectively -1 and -3.* The program thus calculates those slacks for each ineaquality, of which the economic meaning denotes "idle capacity". In the present application idle capacity means "the average number of ships laying up during a month".

b/ equalities, which state that a number of items have to sum up to a given amount. Generally, those equalities follow a concept, similar to the inequalities, except for the slacks which are replaced by the explicit introduction of a "penalty". Thus, equalities may be expressed in two alternative formats:
- strict equalities, eg $X_1 = X_{17}$ or $X_1 - X_{17} = 0$ (cfr.infra 4.2.4.2)
- compound equalities, which include a penalty, eg $X + p$ = a given number.

In the initial case study, we will use compound equalities which denote the number of TEU not shipped because of capacity shortage. In a later elaborate version, dealing with the backhaul, strict equalities are suggested to impose the condition that the number of monthly backhauls should equal the number of outbound departures (except of course for tragedies at sea).

Fourth, both the objective function and the set of constraints ought to be a linear combination of the introduced variables. Otherwise, a different approach (non-linear programming) is required. It should nevertheless be mentionned that a substantial number of so-called "non-linearities" can be "linearized" through either : - adjustment procedures (stepped functions),

or - the organization of the LP-format itself.

The latter option is followed in the present case-study as far as the operating costs (which may not vary proportionally with the distance) and the turn-around times (which vary not fully proportionally with the speed and the distance) are concerned.

The formulation of an LP-problem is furthermore simplified by virtue of the possibility to classify LP-cases in a number of "typical problems". The present case on shipping logistics resorts to the class of LP-applications, piled together in what has become to be known as the "assignment problem" (2).

2.2. . : The basic format

The objective which comprehensively spans all items coming up for joint optimization is that of cost-minimization. With costs we understand "system costs", including:

a/ operating costs, *which sum up all voyage costs by type-of-ship and route. For a particular ship/route combination a roundtrip cost equals the standard cost for a roundtrip times the number of roundtrips* (R_{ij}) *for the vessel of class "i" on route "j".*

b/ opportunity costs, *which cover the penalties (lost profit) for cargo (TEU) not shipped because of capacity shortage. Those penalties* "P_j" *are specified by route "j" only, since indeed any type of ship is a candidate to perform the shipment.*

Through the action of such compound objective function, we include a minimization (*costs*) problem as well as a reciprocal maximization *(of profits through the minimization of identified lost profits).*

In mathematical language the objective function is translated:

$$
\begin{array}{lllll}
10{,}000\ R_{11} + & 11{,}000\ R_{12} + & 12{,}000\ R_{13} + & 15{,}000\ R_{14} + & \textit{(1500 TEU ships)} \\
8{,}000\ R_{21} + & 9{,}000\ R_{22} + & 10{,}000\ R_{23} + & 10{,}000\ R_{24} + & \textit{(850 TEU ships)} \\
6{,}000\ R_{31} + & 8{,}000\ R_{32} + & 8{,}000\ R_{33} + & 9{,}000\ R_{34} + & \textit{(500 TEU ships)} \\
40\ P_{1} + & 50\ P_{2} + & 40\ P_{3} + & 70\ P_{4} & \textit{(penalties)}
\end{array}
$$

where, for example, "R_{23}" stands for the number of return trips of a 850 TEU vessel on the third route, which after multiplication by 10,000 results in the total operating costs of all ships of class 2 on the third route. Finally, as stated in the problem input, the penalties are summed up by route, since it is irrelevant to allocate them at a particular class of vessel.

The whole objective function was expressed in one common "monetary" dimension, i.e. opportunity costs *(approximately in 1,000 BF)*. Similarly, the constraints ought to feature a common dimension, constraint by constraint. The problem-statement suggests two classes of constraints which consequently feature two different dimensions:

a/ Fleet-capacity constraints relate to the availability of ships by class. The constraint expresses the condition that one cannot perform more monthly roundtrips than indicated by the number of ships available and by their speed. Speed does not act in the objective function but is expressed by the maximum number of monthly sailings. At the same time this procedure allows to introduce non-linearities *(since roundtrip times may not vary proportionally to the distance)*. We obtain:

$$
\begin{aligned}
i=1;\ & 0.33\ R_{11} + 0.50\ R_{12} + 0.50\ R_{13} + 1.00\ R_{14} \leq 5\\
i=2;\ & 0.25\ R_{21} + 0.33\ R_{22} + 0.33\ R_{23} + 0.50\ R_{24} \leq 8\\
i=3;\ & 0.20\ R_{31} + 0.20\ R_{32} + 0.25\ R_{33} + 0.50\ R_{34} \leq 10
\end{aligned}
$$

On the right-hand side, we formulate indeed that the "reciprocal" of commercial speed (*i.e. slowness*) and the number of trips (R_{ij}) actually occupy the available ships' "shuttle-capacity". For example, the coefficient of R_{32} states that each small 500 TEU ship of class 3 performs a maximum number of 5 monthly sailings. Therefore, each trip takes 20 % of the monthly trip-performance and "0.20" is the coefficient of R_{32}.

b/ Traffic-demand requirements state that all three classes of vessels, operating on a particular route, should meet the demand on that route. Moreover, each TEU which cannot be shipped because of capacity shortage, activates a penalty. Those penalties sum up to "P_j" by route, and are simultaneously translated into opportunity-costs through the associated coefficient in the objective function.

$$
\begin{array}{llllllllll}
j=1 & 1500\ R_{11} & + & 800\ R_{21} & + & 500\ R_{31} & + & P_1 & = & 3,000 \\
j=2 & 1500\ R_{12} & + & 800\ R_{22} & + & 500\ R_{32} & + & P_2 & = & 6,000 \\
j=3 & 1500\ R_{13} & + & 800\ R_{23} & + & 500\ R_{33} & + & P_3 & = & 2,500 \\
J=4 & 1500\ R_{14} & + & 800\ R_{24} & + & 500\ R_{34} & + & P_4 & = & 3,500
\end{array}
$$

Those constraints feature the effective market demand at the right hand side (RHS). For each separate route is stated at the left hand side (LHS) that the demand:

a/ is either shipped by any class (i) of ships. Therefore, the number of roundtrips (R_{ij}) is multiplied by the ships' capacity (in TEU) and added over ship classes by route. This number eventually expresses the "effective TEU roundtrip capacity for all vessels in operation by route".

b/ or not shipped. In this case a penalty is activated (P_j) which expressed the number of TEU not shipped by the monthly service.

Thus, whereas fleet-capacity constraints had been expressed by class of ship, traffic demand constraints are introduced by route.

The general formula of such shipping logistics problem becomes:

Minimize $\Sigma_i \Sigma_j \; c_{ij} . R_{ij} \; + \; \Sigma_i \; d_j . P_j$

subject to:

a/fleet capacity constraints:

$$\Sigma_j \; a_{ij} . R_{ij} \leqq S_i$$

b/traffic-demand constraints:

$$\Sigma_i \; b_i \; . R_{ij} + 1. \; P_j = T_j$$

in which we distinguish between parameters:

c_{ij} : operating costs by ship's class and route,

d_j : penalty value per TEU by route,

a_{ij} : monthly turnaround times *(expressed in fractions of months)*, or the reciprocal value of the maximum number of monthly roundtrips,

b_i or b_{ij} : rated TEU capacity by ship class. Since this "rated" capacity may give rise to different "effective" capacity standards, according to the "Plimson mark" information, the "b_i" parameter may also be specified by route. See further section 4.2/2/c (including note 10).

S_i : total number of ships available by class,

T_j : total freight demand by route (in TEU),

and variables:

R_{ij} : number of monthly roundtrips by class and route,

P_j : number of TEU's not shipped by the monthly shuttles.

3. Optimality Analysis

Current LP computer packages require the input data by a one dimensional indexation. Thus our previous double dimension by ship's class (i) and route (j) should be adapted, and also the different symbols of variables (R_{ij}) and P_j require a common "name" (X_i):

Table 3 : Conversion of problem data in a LP-format

route(j) / ship class (i)	1	2	3	4
1.	$X_1 = R_{11}$	$X_2 = R_{12}$	$X_3 = R_{13}$	$X_4 = R_{14}$
2.	$X_5 = R_{21}$	$X_6 = R_{22}$	$X_7 = R_{23}$	$X_8 = R_{24}$
3.	$X_9 = R_{31}$	$X_{10} = R_{32}$	$X_{11} = R_{33}$	$X_{12} = R_{34}$
penalties	$X_{13} = P_1$	$X_{14} = P_2$	$X_{15} = P_3$	$X_{16} = P_4$

The schematic overview of earlier reported objective function and constraints is repeated in Table 4. using the standard X-variables. The presentation of the optimality procedure follows four main steps. First, the general optimum results are presented in section 3.1. together with a graphical analysis of a suggestive fraction of the assignment problem. Third, one tackles the information given by the dual optimum values, and in section 4. some more elaborate amendments to the basic LP-problem cover the issues of backloads and costs of idle-time.

Table 4: General scheme of the LP "assignment problem"

Minimize:

$$10000X_1+11000X_2+12000X_3+15000X_4+8000X_5+9000X_6+10000X_7+10000X_8+6000X_9+8000X_{10}+8000X_{11}+9000X_{12}+40X_{13}+50X_{14}+40X_{15}+70X_{16}$$

subject to:

a: fleet-capacity constraints

$$0.333X_1+0.500X_2+0.500X_3+1.000X_4 \leqq 5$$

$$0.25X_5+0.33X_6+0.333X_7+0.500X_8 \leqq 8$$

$$0.20X_9+0.20X_{10}+0.25X_{11}+0.50X_{12} \leqq 10$$

b: traffic-demand constraints

$$1500X_1 + 850X_5 + 500X_9 + 1X_{13} = 3000$$

$$1500X_2 + 850X_6 + 500X_{10} + 1X_{14} = 6000$$

$$1500X_3 + 850X_7 + 500X_{11} + 1X_{15} = 2500$$

$$1500X_4 + 850X_8 + 500X_{12} + 1X_{16} = 3500$$

1500 TEU units	850 TEU units	500 TEU units	penalties

3.1. : Optimal solutions at various demand-levels

A standard LP-program has been used to generate the optimal ship-assignment to each of the four routes for the six exemplative cargo-demand cases of Table 4.1. Those solutions are exibited in Table 5.

In case "1", all "1500 TEU"-vessels are assigned (slack = 0), whereas none of the small 500 TEU units is used (slack = 10). This simply means that the larger vessels of type "1" feature lower costs per rated TEU-capacity, and consequently a less efficient unit (in this case a 800 TEU unit of type "2"), is only partly used:
- when the most efficient capacity (type 1) is in full operation (slack = 0.),
- on those routes where the efficiency gap between the two types of vessels is least influencing the objective function.

Thus, in the first case, a 800 TEU vessel executes 1.5 sailings per month on the fourth route since:
- all 1500 TEU units are taken, and
- the operating costs per TEU for type "2" compares relatively least costly on the fourth route. Since indeed, The operating cost of one TEU by a 850 TEU vessel is only 18 % higher than by a 1500 vessel on the fourth route. For all other routes the difference is more than 40 %. Thus, the operation of one 850 TEU "first generation" ship on the fourth route will give the lowest increase in the value of the objective function (i.e. the lowest over-all cost-increase).

A second point to be noticed is the possibility of the ships' frequencies to obtain fractions of trips too. Thus 1.5 trips can be interpreted as 3 sailings over 2 months. One might advance the additional constraint that ships follow a strict schedule such that the number of monthly sailings ought to be an INTEGER. If such conditions are requested the optimal solution is found by "integer programming" (cfr.infra, conclusions & 14).

Table 5.: Solutions of the ships assignment exercise

routes :		1.	2.	3.	4.	slacks
case 1 : demand		3,000	6,000	2,500	3,500	
assigned ships	5:	2	4	1.7	1.5	0
to sailings:	8:	-	-	-	1.5	7.3
	10:	-	-	-	-	10.
case 2 : demand		6,000	10,000	5,000	5,000	
assigned ships	5:	4	4	3.3	-	0
to sailings:	8:	-	4.7	-	5.9	3.5
	10:	-	-	-	-	10.
case 3 : demand		6,000	10,000	5,000	10,000	
assigned ships	5:	4	4	3.3	-	0
to sailings:	8:	-	4.7	-	11.8	0.6
	10:	-	-	-	-	10.
case 4 : demand		6,000	20,000	5,000	10,000	
assigned ships	5:	4	4	3.3	-	0
to sailings:	8:	-	16.5	-	5.0	0
	10:	-	-	-	11.4	4.3
case 5 : demand		6,000	20,000	5,000	20,000	
assigned ships	5:	4	-	3.3	2	0
to sailings:	8:	-	-	-	16	0
	10:	-	40	-	4	0
active penalties :		-	-	-	1398 TEU	
case 6 : demand		10,000	20,000	5,000	20,000	
assigned ships	5:	6.7	-	3.3	1.1	0
to sailings:	8:	-	-	-	16	0
	10:	-	40	-	4	0
active penalties :		-	-	-	2730 TEU	

Further clarification of the optimality procedure is now executed graphically with respect to the allocation of ten *"500 TEU capacity"* vessels of class 3. on routes 2 & 4 (see Figure 6). There we observe that the more efficient units of classes 1. and 2. are already assigned, such that only two "variables" come up for discussion, i.e.:

a/ the number of trips of class 3. on the second route, (variable X_{10} or $R_{3,2}$ in the initial notation),

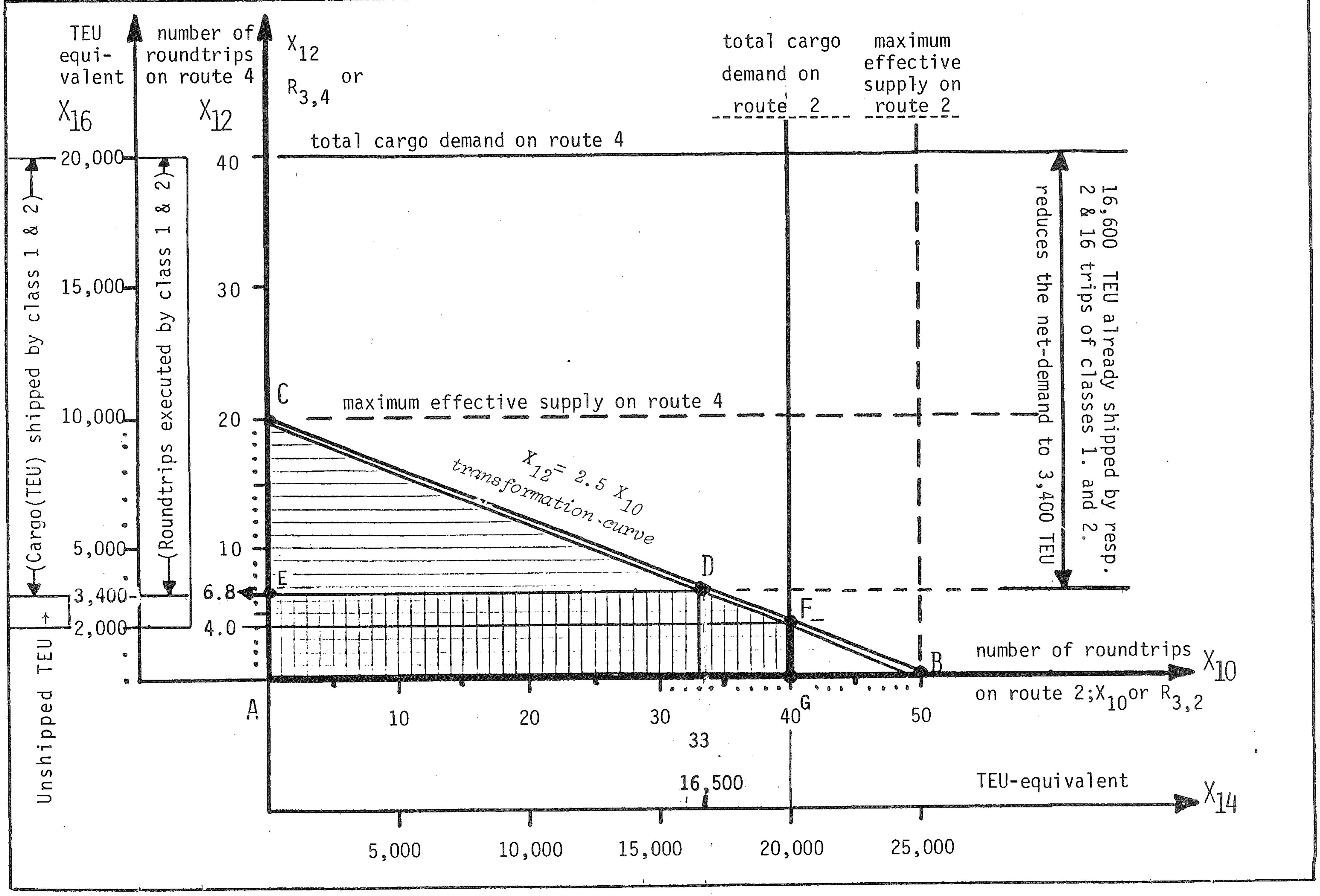
b/ the number of trips of class 3. on the fourth route, (variable X_{12} or $R_{3,4}$ in the initial notation).

Both variables can also feature in TEU equivalents since each trip of a class 3. vessel rates a 500 TEU capacity. This alternative dimension is put on two parallel axes. They are especially useful in calculating the penalties, which are indeed expressed in TEU. Therefore, those axes exibit the penalty-variables X_{14} and X_{16} for the second and fourth route respectively.

Two vertical lines indicate the maximum effective capacity on route 2 (dotted line) and the total demand (maximum level of effective traffic). Equivalent horizontal lines suggest those characteristics regarding the fourth route. Thus, on both relations there is an equal demand of 20,000 TEU containers, which is the equivalent of 40 roundtrips with our *500 TEU* vessels of class 3. The latter traffic demand ceiling is the actual boundary of a square (see full lines) representing the potential market (ie money making) opportunities. Any supply beyond (top-right) that region will cause idle capacity *(in which case slacks would become activated)*.

On the supply side (Table 1.) vessels belonging to class 3 can achieve a maximum turn-around of 2 trips per month on the fourth route OR 5 journeys on the second. With 10 vessels this part of the fleet is thus able to execute either 20 or 50 roundtrips on those routes (X_{12} or $R_{3,4}$ & X_{10} or $R_{3,2}$). Those boundaries of maximum supply appear on the graph by means of dotted lines.

Figure 6. : Graphical overview of the assignment of the ten 500 TEU units in case 5



Those (dotted) boundaries however are not realistic since they both assume that all 10 *"500 TEU"* are exclusively operating on one of the two routes. Therefore we construct a "transformation curve" between the two extreme points (C) and (B). This transformation curve gives the actual possibilities of operating on both routes with the fleet of 500 TEU vessels according to the next alternatives:

X_{12}	X_{10}
20	0
16	10
12	20
8	30
4	40
0	50

This technical substitution between X_{12} and X_{10} can also be expressed algebraically by the next transformation :

$$X_{12} = 20 - 0.4\, X_{10}$$

or $$\Delta X_{12} = -2.5\, \Delta X_{10}$$

or $$\Delta X_{10} = -0.4\, \Delta X_{12}$$

That transformation curve thus becomes the actual boundary of the FEASIBLE SUPPLY REGION (triangle A-B-C) in which our solution must be located. The whole area at the top-right of the transformation curve B-C is called "unfeasible" since there are not enough 500 TEU units available to perform the equivalent supply.

Out of Table 5 (case 5) however, we are informed that on the fourth route, more efficient vessels of the first and second generation are already operating with:

- 16 sailings of class 2., carrying 16 x 850 = 13,600 TEU,
- 2 sailings of class 1., carrying 2 x 1500 = 3,000 TEU.

with a total effective supply of: 16,600 TEU,
thus leaving a net-demand of only (20,000 - 16,600) = 3,400 TEU on that fourth route.

The feasible area is consequently reduced by lowering the horizontal DEMAND line to the level of:

$$X_{12} \text{ (trips)} = 6.8 \quad \text{or} \quad X_{16} \text{ (TEU)} = 3{,}400$$

Our optimum solution must now be found in that confined feasible area, since it would not pay off to supply any capacity in the "C-D-E" area (market already served). A similar redundancy had been found with respect to the right-hand "B-F-G" area where the maximum effective supply exceeds the potential demand.

Thus our "technically feasible area" (constituted by the transformation curve) has been reduced two times by the actual market opportunity-lines "E-D" and "F-G". The area left for optimization thus becomes:

A - E - D - F - G

Furthermore, the objective is the minimize operating and penalty costs, of which we now only consider:

$$\min.\ 8{,}000\, X_{10} + 50\, X_{14} + 9{,}000\, X_{12} + 70\, X_{16} + \text{others}$$

operating & penalty-costs *operating & penalty-costs*

on the

second route *fourth route*

For the sake of clarification operating costs can be expressed per TEU-capacity, in order to compare with penalty-costs:

(cfr; Table 2.	operating costs per TEU when shipping	penalty costs per TEU when not schipping
route 2	16	50
route 4	18	70

Consequently in both cases, one will always sail up to the full capacity use in order to avoid even higher penalty-costs. Therefore, out of the E-D-F-G boundary, only D-F is relevant.

Finding a cost minimum along the D-F section is executed by inspecting the cost balance of the extremes.

In "F", the 500 TEU vessels perform:

- 40 trips on route 2 at 8,000	=	320,000
- 4 " " " 4 at 9,000	=	36,000
- and leave 1400 TEU unshipped at a penalty of 70.- each	=	98,000
		454,000

In "D" the company first concentrates on the 3,400 TEU on the fourth route which were not served by vessels of class 1. & 2. They require 6.8 trips by units of class 3. à 9,000 (X_{12}) = 61,200

But because of those 6.8 trips on the fourth route, we can only perform (3):

$$50 - (6.8 \times 2.5) = 33. \quad (X_{10})$$

trips on the second route à 8,000	=	264,000
With those operations, we handle 33. x 500 = 16,500 TEU out of the market demand of 20,000. There, 3,500 are left at a penalty of 50.-	=	175,000
		500,200

which results in a higher systems cost. Because of the linear D-F section, this cost-increase builds up progressively. Therefore, "F" is the cost-minimum.

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(3): *earlier the transformation curve was expressed as $X_{12} = 2.5\ X_{10}$, which states that one additional X_{12} trip on the fourth route requires 2.5 trips to be dropped from the second route. Or 6.8 trips more on route 4. gives 6.8 x 2.5 = 17 less on route 2.*

At this point of the discussion, it is interesting to endeavour in the speculative reflection, why in the latter example the optimum solution did concentrate on all cargo on the second route (X_{10} *up to total cargo demand for 40 trips*), whereas the resulting penalties on the fourth route are higher (X_{16}=70.-) than those on the second route(X_{14} à 50.-).

The reason is that one cannot simply compare operating costs (per TEU) and penalties on their straightforward numerical value. In fact, in order to save a penalty of 70.- on the fourth route, the company ought to perform 2.5 times more roundtrips than needed for saving a penalty on the second route *(following the transformation curve* $X_{12}=2,5X_{10}$*)*.

Thus, when on the second route, unitary operating costs per TEU of "16" compare to a penalty of "50", which is 32 %, on the fourth route we have to adjust those unitary operating cost by the longer trip time, i.e. 18 x 2.5 = 45.-, in order to make it comparable to the penalty of 70. Then operating costs take 44 % of the penalty.

Therefore, on the fourth route, the effective operating costs per TEU are relatively more expensive in relation to the (avoided) penalty, as compared to the second route. In other words, on the second route the 50.- penalty cost penalizes the loss of a TEU shipment relatively more in comparison with the operating costs.

Such example once again shows the underlying cost-logistical straightforwardness of LP-poogramming, in cases where simple accounting rules-of-thumb might have given erroneous results or misleading interpretation.

3.2. The dual problem

A standard LP package also provides information on the optimum values of the so-called "Dual problem". From the technical standpoint of the program mechanics, the dual constitutes the reciprocal of the original (i.e. primal) problem.

In standard notation of table 4.4 the primal can be expressed synthetically by:

$$\text{Minimize } \Sigma_i \, c_i \, . \, X_i \qquad (i= 1 \ldots 16)$$

subject to (4):

$$\Sigma_i a_{ij} \, . \, X_i \leqq u_j \qquad (j= 1,2,3 \; : \; i=1 \ldots 16)$$
$$\Sigma_i a_{ij} \, . \, X_i = u_j \qquad (j=4,5,6,7; \; i=1 \ldots 16)$$

of which all X_i which did not feature in the problem statement of Table 4.4. obtain a zero coefficient (a_{ij}=0).

The dual then becomes:

$$\text{Maximize } \Sigma_j \, u_j \, . \, Y_j \qquad (j= 1 \ldots 7)$$

subject to:

$$\Sigma_j a_{ji} \, . \, Y_i \geqq c_j$$

When, as in the present case, the primal is a cost minimization problem, the optimal dual variables Y_j (j=1,7) express the decrease in the optimum value (costs) when the primal constraints (u_j) are relaxed by one unit. They also give the equivalent INCREASE in the optimum (cost)value when the original constraints are INCREASED by one unit.

---

(4) *conform to our initial notation* $u_1=S_1$; $u_2=S_2$; $u_3=S_3$ $u_4=T_1$; $u_5=T_2$; $u_6=T_3$ *and* $u_7=T_4$.

The most interesting application of the dual problem output is the interpretation of the dual optimum values in terms of "marginal opportunity costs"(the so-called shadow-costs or shadow-benefits in the case of a maximization resp.minimization primal).

In the present case there are seven dual values, as many as the number of constraints in the primal. When applied to the cost-minimization primal, the dual values inform on:

" the increase in system-costs (operating
" and/or penalty-costs) when one of the
" primal constraints is increased by 1 unit.
" i.e. - one additional ship (u_1 to u_3), or
"   - one additional TEU demand (u_4 to u_7)

The dual values of the six successive demand cases are listed in Table 4.7, together with the total systems costs in the first column. They are grouped according to the primal constraints they belong to.

The duals in columns S_1 to S_3 (Y_1 to Y_3) give a negative cost increase, ie the cost-decrease of the system when one additional vessel of respectively class 1, 2 and 3 enters in operation. The four right-hand columns T_1 to T_4 (ie Y_4 to Y_7) give the cost increase of either operating one additional TEU container, or of paying the penalty when shipment is not possible.

Table 7: Dual values (optimal solution) with post-optimality exercises

a/ dual variables as computed by the LP-optimality procedure

case	costs	S_1	S_2	S_3	T_1	T_2	T_3	T_4
1.	121,204	- 2,647	-	-	7.25	8.22	8.88	11.76
2.	225,165	- 9,765	-	-	8.83	10.59	11.25	11.76
3.	283,987	- 9,765	-	-	8.83	10.59	11.25	11.76
4.	425,544	-22,223	-10,600	-	11,60	14,74	15.41	18.00
5.	723,880	-90,000	-99,000	-52,000	26.65	36.80	38.00	70.00
6.	830 466	-90,000	-99,000	-52,000	26.65	36.80	38.00	70.00
maximum dual value for T-constraints					40.--	50.--	40.--	70.--

b/ dual variables as computed out of the sensitivity analysis *(&)*

case		S_1	S_2	S_3
1.		*- 2,206*		
2.		- 9,765		
3.		- 9,765		
4.		-22,223	-10,600	
5.		*-89,456*	*-97,389*	-52,000
6.		-90,000	-99,000	-52,000

(&) use of "*light italic*" when figures of b/ differ from a/

3.3. : dual values of the traffic-demand constraints

The four right hand side columns exibit the net average increase in system costs, when on one of the four routes (T_1 to T_4 or u_4 to u_7) there happens to be a market traffic-increase of one TEU waiting for shipment. Evidently, that TEU gives either rise to an increase :

- in operating costs when it is shipped, or
- in penalty costs when it cannot be shipped.

The latter point is easily verified in the fifth and sixth demand case where out of table 5 (bottom right) one observes that all shipping capacity is taken *(slacks=0)*, and there is some cargo left *(penalties resp. 1398 & 2730)*. Any additional TEU will simply rise the penalty by one and the penalty costs by 70.- on the fourth route. The penalty costs consequently represent the maximum dual value in the extreme case when no additional sailings can be organized.

When the dual-values are less than the (maximum) penalty costs they express the increase in operating costs, necessary to carry one additional TEU on a particular route. For example, the four dual values are equal in the second and third demand case. In those two situations the optimum solution (Table 5.) still shows some of the first generation vessels (= class 2. of 850 TEU) laying up (slack > 0). They can perform (fractions) of additional trips to handle the cargo. Taking then the fourth route, where a 850 TEU ship may perform one additional trip à 10,000.-, the incremental cost of one TEU becomes: 10,000/850 = 11.76, which is the dual value of T_4.
On the same route, when demand increases to the level of "case 4". There is no 850 TEU unit left and additional TEUs require 500 TEU ships, which are already operating. Their roundtrips cost 9,000 each or 9000/500=18 per TEU, which is once again equal to the dual value.

Similarly to the latter example on route 4., roundtrips on route 2. cost 9,000 for 850 TEU vessels or 10.59 per TEU. That value consequently determines the dual in cases 2. and 3. where on the second route 850 TEU ships can still be added to the fleet operations. Thus far, route 2. and 4. allow easy calculations.

But on the first and third route there are only efficiency-units of 1500 TEU in operation for all levels of demand, and for all cases they are used up to their rated capacity. Therefore, we cannot calculate the dual value by simply dividing operating costs by the rated TEU capacity (cfr.Table .2.), neither for a class 1. vessel nor for one of class 2. Expanding on this issue we again look at the second and third demand case, for which the dual values are:

- 8.83 on the first route, and
-11.21 on the third route.

On the first route the average operating cost per rated TEU is:
- for 1500 TEU units 10,000/1,500 = 6.67, and for
- for 850 TEU units 8,000/850 = 9.41,

or respectively lower and higher than the dual value (8.83). Similarly on the third route, operating costs average per TEU in
- 1500 TEU units 12,000/1,500 = 8.0 , and in
- 850 TEU units 10,000/850 =11.76.

Once again the dual value (11.25) lays in between, since it calculates the total systems cost-increases of;

a/ handling the additional container by an efficiency unit of 1500 TEU on the first or third route, with an average cost of respectively 6.67 and 8.,

b/ rearranging the capacity shortage *(of one TEU on routes 1. and 3.)* along the other routes *(i.e. the second & fourth)* where apparently the use of less efficient 850 TEU vessels is less harmful in terms of cost-increases.

The latter example clearly complicates the calculus of dual values since it partly includes a marginal adaptation of the sailing frequencies of vessels along the routes. This perfectly follows the principle of "marginal opportunity costs" which represent the dual value. These are not only *the incremental costs of performing one additional job besides other jobs*, but obtain a far more comprehensive meaning of *the least-cost incremental use of resources needed to perform an additional job, under the condition that the whole system should work at lowest costs.*

Thus, in the last reported example, it is possible to allocate the difference between dual-values and average operating costs/TEU to comparative efficiencies between 1500 TEU and 850 TEU vessels. That calculus is quite tedious to do for such small differences. The calculus is however worth to be executed with respect to the fleet-capacity constraints.

3.4.: dual values of the fleet capacity constraints

Repeating the analysis for the dual variables of the fleet's capacity, let us consider the "- 9,765" value of "Y_1" in the second and third cases of market demand. It denotes the change in the optimum value of the objective function, when the constraint, associated with S_1, is relaxed by one unit. In the current example:

a/ the objective function is expressed in terms of costs, (including operating costs and penalties for lost cargo),

b/ the S_1-parameter stands for "5 ships" of a 1500 TEU capacity.

Thus "9,765" is the amount of

a/ the reduction in system-costs *(since it takes a negative sign)* of operating the whole fleet, when

b/ the company introduces a sixth unit of 1500 TEU capacity, and keeps others either in operation or as standby-use.

Repeating the assignment exercise of Table 5 with S_1=6 instead of S_1=5, we obtain the next sailings for the second and third demand-cases:

Table 8/a Optimal ships assignment with S_1=6

case 2/b; demand		6,000	10,000	5,000	5,000	slacks
sailings of	1.	4	6 (4)	3.3	--	0
	2.	-	1.2(4.7)	-	5.9	4.7
	3.	-	-	-	-	10.0
total costs: 215,399; costs with(S_1=5)225164 ;difference -9,765						
case 3/b; demand		**6,000**	**10,000**	**5,000**	**10,000**	**slacks**
sailings of	1.	4	6 (4)	3.3	-	0
	2.	-	1.2 (4.7)	-	11.8	1.7
	3.	-	-	-	-	10.0
total costs: 274,222; costs with(S_1=5) 283987;difference:-9,765						

figures between brackets refer to the standard allocation (if different) with S_1=5; S_2=8 and S_3=10 as given in Table 5.

The conclusions from our traffic engineering adjustment are clear. The new 1500 TEU ship has been assigned to the second route on which it can perform two round trips (cfr. Table .1). Therefore, the total number of round trips on the second route rises from 4 to 6. The additional costs of those two round trips is 22,000 (2 times 11000). Since the second route is now served by two large vessel we can withdraw a number of smaller 850 TEU ships of the first generation. In fact, we withdraw the capacity-equivalent of 1.2 ships *(which follows from comparing the slack values in Table* .5(3.5) *&* .*8/a* (4.7).
The number of sailings by 850 TEU ships drops from 4.7/month to 1.2 or minus 3.5. This causes a saving of operating costs of 31500 (3.5 times 9000). The total savings from the withdrawal exceed the additional costs of the large vessels by:

31,500 - 22,000 = 9,500. = *9,765.* (5)

Nevertheless, there is some danger in the straightforward interpretation of the dual variable as the ACTUAL amount of cost-savings resulting from a unit-increase in the S_i-constraints. In order to clarify this warning we now turn to the first demand-case, in which we expect a cost-saving of 2647.- when increasing the number of 1500 TEU ships from 5 to 6.

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(5): *Actually, with a 5 digit precision, the number of sailings drops from 4.7012 down to 1.1718 or minus 3.5294. Savings are evaluated à 9,000 per trip, or 31,765 instead of the above figure 31,500. Thus the net savings are 9,765 as in Tables 8/a (cases 2/b and 3/b).*

However, when we solve the original LP problem for $S_1=6$, the next optimal sailings are obtained:

Table 8 /b : Optimal ships assignment with $S_1=6$

case 1/b; demand	3,000	10,000	2,500	3,500	slacks
sailings of: 1.	2	4	1.7	2.3(1.5)	0.16(0)
2.	-	-	-	0 (1.5)	8(7.3)
3.	-	-	-	-	10

total costs 121,204; costs with ($S_1=5$) 121,204, difference 2,204 instead of 2,647(443)

The net cost savings are 2,204, i.e. 443.- lower than the figure suggested by the dual variable. Similarly, to the previous calculations, let us discover the meaning of our two different "shadow evaluations".

Firstly, all trips by 850 TEU vessels are skipped which now all lay up (slack = 8). Their operating costs are reduced by: 1.5 x 9,000 = 15,000. Additional sailings by the new 1500 TEU vessel increase from 1.5 to 2.3 or by 0.8 trips; therefrom results an operating cost-increase of 0.8 x 15,000 = 12,000 *(or 12,490 with a six digit precision)*

The net balance becomes	savings	15,000 = *14,694* (6)	
	- costs	12,000 = *12,490* (6)	
	balance	2,204	

which is indeed 443.- lower than the dual value of 2647 or 16 % (*2647 x 0.16 = 2204*). The reason is that our new unit of 1500 TEU is partly an overinvestment of which 16 % remains idle capacity (slack value = 0.16). The calculation of dual variables by an LP-program thus assumes that all such idle capacity comes up for substituting less efficient capacity, even when such substitution has only occurred in part.

---

(6) *skipped trips of 1.469413 à 10,000 or savings of* *14,694*
Additional 1500 TEU trips from 1.500667 to 2.33333 or 0.832666 additional trips à 15,000 equals *-12,490*
2,204

The three earlier examples all exibited substitution of vessel on the same sea route. In the fourth case, the higher demand level now also activates a substitution between ships and routes:

Table 8/c : Optimal ships assignment with S_1=6					
case 4/b; demand	6,000	20,000	5,000	10,000	slacks
sailings of: 1.	4	6(4)	3.3	0	0
sailings of: 2.	-	12.9(16.5)	-	7.38(5)	0
sailings of: 3.	-	-	-	7.47(11.4)	6.3(4.3)

The dual variable of 22,223 can be interpreted following:

a/ a substitution of 850 TEU units by 1500 TEU on route 2.

$\frac{(12.93648)}{(16.46589)} = 3.52941$ x 9,000 savings : - 31,765

$\frac{(6.002665)}{(4.002667)} = 2.$ x 11,000 costs : + 22,000

which results in a net saving of : 9,765

b/ a switch of those 850 TEU units to the fourth route where they replace 2 smaller units of 500 TEU (slack rises from 4.3 to 6.3):

$\frac{(7.446677)}{(11.44267)} = 3.99599$ x 9,000 savings - 35,964

$\frac{(7.384309)}{(5.033722)} = 2.350587$ x 10,000 costs + 23,506

which results in a net saving of: 12,458

The total gain of the logistical operation becomes: 22,223, from:
- four abandoned sailings on the fourth route by class 3. (35,964), which are replaced by:
- 2.35 additional trips by class 2. (23,506).
- The latter capacity is removed from the second route à 3.5 trips (31,765), where they are replaced
- by two addtional crossings of 1500 TEU vessels.

The highest dual values of the S_1 constraint are evidently found when market demand experiences strong capacity strains. This happens in the fifth and sixth case where respectively 1398 and 2730 TEU containers could not have been shipped at the present ($S_1=5;S_2=8;S_3=10$)-configuration. Now the dual calculations also involve the evaluation of penalties:

Table 8/d : Optimal ships assignment with $S_1=6$					
case 5/b; demand	6,000	20,000	5,000	20,000	slacks
sailings of: 1.	4	0.7(0)	3.3	2.7(2)	0
2.	-	-	-	16	0
3.	-	38(40)	-	4.8(4)	0
penalties (TEU)	-	-	-	0(1398)	
system costs 634,424 ($S_1=6$)					
723,880 ($S_1=5$)					
savings 89,456 ; dual = 90,000 ; difference 544.-					

Similarly to the first case, (see table 8/b) the dual variable exceeds the actual cost-savings, but upon a minor difference of 544. This small value is not sufficient to activate a slack up to the fourth right-hand digit.

Therefore, our fleet has almost obtained a perfect operational structure, given the demand at each route and the capacity available (6-8-10). There are no idle ships *(all slacks are zero)* and all 1398 TEU's are regularly shipped by the monthly sailings *(all penalties are zero too)*. The additional 1500 TEU ship has exclusively substituted 500 TEU vessels on routes 2. and 4., which now are able to take care of the 1398 TEU's. Ships of class 2. all remain on the second route.

Our suggestion about the "perfect operational structure" is only valid for the present constraints, and does not warrant an absolute cost-minimum *(or minimum minimorum)*. It is indeed possible still to obtain lower costs(without activated slacks or penalties) at another fleet-configuration (cfr. infra table 8/f).

The quite substantial savings now only result in part from an improved fleet with one additional 1500 TEU efficiency-unit but mainly from the fact that the 1398 containers could now be shipped and have increased our profits by 70 x 1398 or 97860.-. The difference (97860 - 89456 = 8404.-) follows from the higher operating costs of our enlarged fleet, with a correction for the improved efficiency.

A clarification of the latter point is exemplified in the sixth demand case, where the new 1500 TEU vessel is exclusively operating on the fourth route.

Table 8/e : Optimal ships assignment with $S_1=6$

case 6/b; demand		10,000	20,000	5,000	20,000	slacks
sailings of:	1.	6.7	-	3.3	2.1(1.1)	0
	2.	-	-	--	16	o
	3.	-	40	-	4	0
penalties (TEU)		-	-	-	1230(2730)	

system costs: 740,467 ($S_1=6$)
830,467 ($S_1=5$)
90,000 = dual' value

The additional 1500 TEU ship is assigned to the fourth route where it moves 1500 of the 2730 TEU; the 1230 remaining keep waiting for another company. The savings are now:

- increased profits ã 70 x 1500		105,000
- minus operating costs of one trip	-	15,000
		90,000

In a identical way, we are able to discuss the potential gains of one additional S_3 ship added to the existing fleet (S_1=5; S_2=8). As suggested already in Table 5 benefits are zero since the eleventh vessel of class 3. remains idle and would only increase the slack-value from 10. to 11. Only in the fifth and sixth case, the ship can be used to carry some of the 1398 TEU waiting on the quay. The 500 TEU unit can perform two roundtrips per month at 9,000.- operating costs. Therefore, the systems savings are:

- increased profits à 70 x 2 x 500		70,000
- minus operating costs 2 x 9000		18,000
	or:	52,000

for both the fifth and the sixth demand case. The only difference is that respectively 398 and 1730 TEU's will remain waiting for seaborne capacity, since the additional S_3 unit can only handle 1000 additional TEU's per month.

To conclude our discussion on the shipping S_i-constraints we finally turn to the duals associated with the S_2 constraint. Here we are really surprised by noticing that the S_2 duals in the 5*th* and 6*th* case are larger that those associated to the S_1 constraint (99,000 instead of 90,000). Once again the logic of the assignment procedure provides the right answer.

The program input stated that 850 TEU ships can perform two roundtrips on the second route, thus bringing the capacity to 1700 TEU's. A larger ship does carry 1500 TEU's but executes only one monthly sailing. Those 200,- additional containers are worth 70,- on the fourth route (where the capacity is needed). But two trips with the 850 TEU vessel cost 2 x 10,000 = 20,000, or 5,000 more than the 15,000 budget for a single shuttle with a 1500 TEU unit. Thus the comparison in systems costs balances:

- additional profits: 200 TEU's à 70 = 14,000,
- additional operating costs - 5,000.-,

or 9,000, being the difference between the S_1 and S_2 duals in the fifth and sixth market-demand cases.

Thus far the difference is explained between u_1 and u_2 dual values. But out of comparing Tables 8/a and b/, one obtains a warning that the latter duals slightly overestimate the actual cost savings. Again we list the optimal routing with $S_1=5$, $S_2=9$ (instead of 8) and $S_3=10$.

Table 8/f : Optimal ships assignment with $S_2=9$ ($S_1=5$; $S_3 = 10$)

case 5/c; demand		6,000	20,000	5,000	20,000	slacks
sailings of:	1.	4	2(0)	3.33	1(2)	0
	2.	-	-	-	18(16)	0
	3.	-	34(40)	-	6.4(4)	0
penalties (TEU)		-	-	-	0(1398)	0

systems costs 626,491 ($S_2=9$)
723,880 ($S_2=8$)

net savings 97,389 instead of the 99,000 dual value.

The dual value had indeed calculated excess benefits of attributing all newly available capacity of a S_2-vessel to carry containers on the fourth route. The basic dual calculation is easy to make, just as in case of the S_3 vessel:

- two additional sailings of a S_2 ship with a rated capacity of 850 TEU with two monthly sailings given 1700 TEU à a 70.- penalty saving = 119,000
- operating costs for two sailings à 10,000 per roundtrip 20,000

99,000

But in the fifth demand case there are only 1398 TEU waiting or only 82% of the rated monthly roundtrip-capacity of a vessel of class 2. (1700/1398 = 0.82).

Therefore, the benefits from reduced penalties are evaluated à 70 x 1398	= 97,000
and the additional operating costs of the last vessel are: 2 x 10,000 x 0.82	-16,447
The remaining idle capacity of the class 2 vessel (18%) has been optimally used to substitute some of the less efficient units of class 3, following the complex scheme of Table 8 /f. This results in an additional cost savings of:	16,836
wherefrom the actual systems-cost savings are:	97,389

Thus, once again, the dual variable, as calculated by the initial LP-program :

a/ concentrates on the most activated constraint(here the demand on the route with the activated penalty P_4),
b/ but extrapolates those benefits to the extent of the TOTAL ADDED CAPACITY (1700 TEU instead of 1398),
c/ and does not correct for the potential occurrence of idle capacity when relaxing the constraint.

In the last example (case 6) there are 2730 TEUs which clearly exceed the added S_2 capacity of 1700 TEU. Therefore the dual of S_2 correctly calculates the benefit of two additional crossings by a second class vessel being (850x2x70) - (2 x 10,000) = 99,000.

In all eleven cases of dual exercises, only two cases have been met where the shipping capacity was almost tailored down to the strict demand needs. There were no activated slacks (idle capacity) and no penalties (surplus demand).

Those "optimal operating" circumstances have occurred:

a/ in the last case (Table 4.8/f) where the system cost was 626,491 with $S_1=5$; $S_2=9$; $S_3=10$ and in

b/ an earlier case with $S_1=6$; $S_2=8$; $S_3=10$ where the system cost amounted to to: 634,424 (in Table 4.8/d).

Both cases feature a small difference between dual values and actual cost savings (respectively 1411 and 544) which cannot account for the difference in system costs. Both cases also had a similar demand-structure since they both concerned the "fifth case" with the T_j being 6,000 ; 20,000 ; 5,000 and 20,000 respectively.

Remarkably, the lower system-costs occur when one additional 850 TEU vessel was added to the fleet instead of a more efficient 1500 TEU unit. Thus the difference in system costs is to be attributed to an improved over-all configuration of the company's fleet over the routes. The example sufficiently shows that from the standpoint of "INTEGRATED LOGISTICS" the LP optimal solutions only give a "local optimum". This in the end may be improved by establishing a a more efficient logistical scheme, through an adaptation of the basic parameters (constraints).

4. Logistical Management amendments

The previous cases only deal with one-way flows taken as a whole (eg with empty backhauls). Such extremely simplified presentation allowed us to introduce the main programming concepts without excessive intracacies, and also to obtain some caution against erroneous interpretation of the results. Now, time has come to add real world features which improve the relevance of the used technique. Those amendments cover the issues of planning backload cargo, the provision for costs of laying up idle capacity, and the choice between time charters versus operations on own account.

Most of those amendments share a common feature in that their cost- and operating implications add further "non-linear" items *(shifts and kinks in the underlying functional relàtionschips)*. It is however possible to re-organize and adapt the constraints for those new evidences, such that the basic linear structure of the procedure is kept conform to the initial requirements (cfr. supra section 1). Those amendments allow the simulation of a number of decision-alternatives which specificallly relate to the compound cost-structure and operating schedule. This particular theoretical issue is first addressed by a short overview of the costs of associated production.

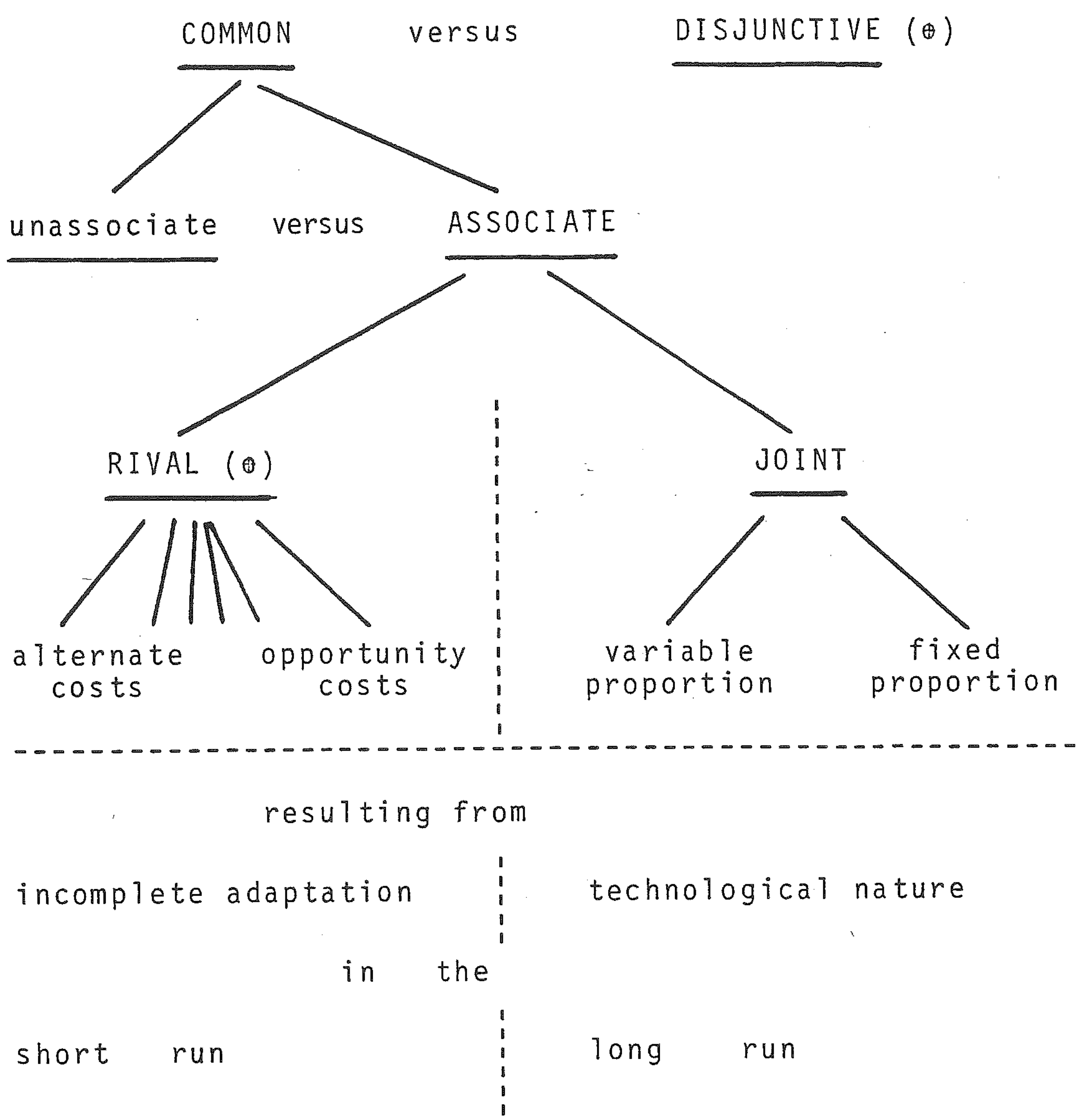
4.1. : Problems of "associate production"

In tackling the issue of backhaul trips and the evaluation of idle times, a preliminar insight is necessary on the concept of associate costs. The chief problem in this matter is that current procedures of cost accounting as applied to multiple outputs requiring a common fraction of resources. Economists and accountants have distinguished several cases of such associate production as related to various specific business situations. The main vocabulary used in this matter is suggestively reproduced in Figure 9.

In fact there are two pure cases, being:
- disjunctive costs in which the costs of two different services do not feature any common component (eg a roundtrip from Tangung Priok to Chittagong and one from Dar es Salaam to Port Said).
- Joint costs in fixed proportion which is for example the fuel cost of the empty backhaul when the outbound trip is executed fully loaded at the same route in the same weather and sea conditions.

In the marginal approach (in which we count opportunity costs or those which are AVOIDABLE by not executing a particular cost-generating action), disjunctive costs are fully traceable from the evidence of the operations, whereas joint costs in fixed proportion are not empirically traceable (to either the outbound trip or the empty backhaul). Thus, in the pure joint cost-case, costs are only traceable to compound demand units. A well known scheme linking those two extremes has been established by DEAN (7) of which a slightly adapted version is presented in Figure 9(8). Some notions are not self-evident and will remain contingent upon the accountant's judgment of the cost-structure from the "outside of the production process". For example, "unassociate-common" costs may refer to technically disjunctive technologies of which some costs are aggregated by several administrative practices..Under the heading of "rival costs", alternate and opportunity costs may also reveal some overlapping connotations.

Figure 9. : Scheme of multiple production and costs



sources: graph drawn on various discussions in:
DEAN,(1950, pp. 263-5; p. 270 & pp. 311-9)
EDGEWORTH (for ⊕-marked items; 1911,p.558, footnote 4)

The latter point indeed suggests that it is often possible to reveal the underlying production process in a finer detail than is possible with the costs of operating the process as an integrated system, where most decisions are taken "on the margin" of current operations.

In the present case study most issues of cost-indeterminacy can be solved by appropriately formulating the incremental costs (or cost-savings) as resulting from a marginal decision. Special care should be given to:

a/ consistency in applying the marginality concept, and

b/ cost-recovery of the system (ie including basic and marginal operations).

For example, in section 4.2 the backhauls will be charged the marginal costs of engaging in carrying backloads. This approach is only valid if the LP procedure warrants never to organize more backhaul than outbound trips. In such case it is only necessary to define the outbound section of a route as the direction with the dominant cargo flow. Otherwise, a number of backhauls will become charged at the margin, without the fixed (joint) costs of organizing the basic roundtrip *(i.e. the loaded outbound trip with empty return)* being covered and paid eventually.

4.2. Backhauls

At this stage of the discussion we had assumed that both parts of a roundtrip occurred along an identical route and at one global operating cost. The possibility to include explicit backhaul cargo requires a potential correction for additional port handling costs, turnaround times and penalties for lost cargo.

The basic difference between outbound and backhaul trips indeed refer to costs and demand conditions, such as:

a/ the traffic level. Some routes may show balanced flows whereas, others may feature ennoying surplus cargo in one direction.

b/ the penalties to be allocated. Routes with fierce competition will feature lower mark-ups and therefore lower penalties compared to routes on which the company ia able to activated some monopoly power. Further difference in the penalties refer to differences in the commodity-package, though the latter point becomes less observable in container transport.

The previous debate suggests that the major adaptations to be applied to the traffic-demand constraints, which are indeed specified by route. Nevertheless there are also:

c/ incremental costs incurred. Earlier operating costs had been specified for whole round trips. Since the backhaul is to a certain extent an intrensic part of the roundtrip the costs to be allocated to the backhaul cargo are likewise "joint costs". Thus incremental costs for organizing the shipment of return cargo are to be identified when:

1/ return cargo necessitates more complicated route-loops to be organized (e.g. additional port calls etc.),

2/ port costs increase to the extent that the loading of backhaul cargo requires additional time and/or costs paid to port authorities, agents and freight forwarders.

Some of those costs may behave proportionally to the amount of return cargo, others may take the form of a fixed mark-up (9). In our case we will assume a compromise and take a fixed mark-up per class of vessel, being 500,- for a 1500 TEU vessel, 400 for a 850 TEU unit and 350,- for the smaller feeder class.

Therefore, the problem statement of section . 2., as generalized in section 3. (Table 3 & 4) is to be adapted by the variables, the parameters and the constraints.

1. The variables

Each backhaul trip should obtain a specific variable. Thus for each R_{ij} now corresponds an additional R_{ji}. Or in the one-dimensional X-notation of Table we obtain:

Table 10: Trip sailings for outbound and backhaul

route / ship class	1.	2.	3.	4.	direction
1.	X_1	X_2	X_3	X_4	OUTBOUND (cfr. Table 3)
2.	X_5	X_6	X_7	X_8	
3.	X_9	X_{10}	X_{11}	X_{12}	
penalties	X_{13}	X_{14}	X_{15}	X_{16}	
ship class					
1.	X_{17}	X_{18}	X_{19}	X_{20}	BACKHAUL
2.	X_{21}	X_{22}	X_{23}	X_{24}	
3.	X_{25}	X_{26}	X_{27}	X_{28}	
penalties	X_{29}	X_{30}	X_{31}	X_{32}	

(9) *one observation to substantiate the relevance of the fixed mark-up is the SUEZ CANAL toll-defining practice that only difference is made between general cargo ships fully empty or fully loaded, in which 1 ton cargo is considered as a full load.*

2. The problem formulation

To the original cost-minimizing objective function of the systems roundtrips:

$$
\begin{aligned}
& 10{,}000\ X_1 + 11{,}000\ X_2 + 12{,}000\ X_3 + 15{,}000\ X_4 \\
+ & \ 8{,}000\ X_5 + 9{,}000\ X_6 + 10{,}000\ X_7 + 10{,}000\ X_8 \\
+ & \ 6{,}000\ X_9 + 8{,}000\ X_{10} + 8{,}000\ X_{11} + 9{,}000\ X_{12} \\
+ & \ 40\ X_{13} + 50\ X_{14} + 40\ X_{15} + 70\ X_{16}
\end{aligned}
$$

we now add the marginal backhaul costs:

$$
\begin{aligned}
& 500\ X_{17} + 500\ X_{18} + 500\ X_{19} + 500\ X_{20} \\
+ & \ 400\ X_{21} + 400\ X_{22} + 400\ X_{23} + 400\ X_{24} \\
+ & \ 350\ X_{25} + 350\ X_{26} + 350\ X_{27} + 350\ X_{28} \\
+ & \ 20\ X_{29} + 20\ X_{30} + 40\ X_{31} + 50\ X_{32}
\end{aligned}
$$

in which the last row gives the penalties (lost profit) of not taking the backhaul cargo (by TEU). Furthermore we adapt the constraints:

a/ fleet capacity constraints:

$$
\begin{aligned}
0.333\ X_1 + 0.50\ X_2 + 0.50\ X_3 + 1.0\ X_4 + A\ X_{17} + A\ X_{18} + A\ X_{19} + A\ X_{20} &\leq 5 \\
0.25\ X_5 + 0.33\ X_6 + 0.33\ X_7 + 0.5\ X_8 + B\ X_{21} + B\ X_{22} + B\ X_{23} + B\ X_{24} &\leq 8 \\
0.20\ X_9 + 0.20\ X_{10} + 0.25\ X_{11} + 0.5\ X_{12} + C\ X_{25} + C\ X_{26} + C\ X_{27} + C\ X_{28} &\leq 10
\end{aligned}
$$

in which "A", "B" and "C" stand for the additional time needed for loading and unloading the backhaul cargo; e.g. A = 0.04 for 1.2 days (0.04 months x 30 days); B=0.03 (for 0.9 days) and C=0.02 (for 0.6 days). In this section, we consider the backhaul-time costs as fully joint to the outbound trip with A=0, B=0 and C=0.

b/ outbound market demand constraints, remain unchanged with:

$$
\begin{aligned}
1500\ X_1 + 800\ X_5 + 500\ X_9 + 1\ X_{13} &= 10{,}000 \\
1500\ X_2 + 800\ X_6 + 500\ X_{10} + 1\ X_{14} &= 20{,}000 \\
1500\ X_3 + 800\ X_7 + 500\ X_{11} + 1\ X_{15} &= 5{,}000 \\
1500\ X_4 + 800\ X_8 + 500\ X_{12} + 1\ X_{16} &= 20{,}000
\end{aligned}
$$

as earlier reported for the "sixth demand case"(Table1.).

c/ backhaul market conditions (traffic-demand constraints):

$$
\begin{array}{llllllll|l}
1500\ X_{17} & ++ & 850\ X_{21} & + & 500\ X_{25} & +\ 1 & X_{29} & = 5{,}000 & 1. \\
1500\ X_{18} & + & 850\ X_{22} & + & 500\ X_{26} & +\ 1 & X_{30} & = 10{,}000 & 2. \\
1500\ X_{19} & + & 850\ X_{23} & + & 500\ X_{27} & +\ 1 & X_{31} & = 5{,}000 & 3. \\
1500\ X_{20} & + & 850\ X_{24} & + & 500\ X_{28} & +\ 1 & X_{32} & = 20{,}000 & 4. \\
\hline
\text{class 1} & & \text{class 2} & & \text{class 3.} & \text{penalties} & & \text{TEU market} & \text{route}
\end{array}
$$

in which the RHS figures express the number of TEU available for the backhaul shipments by route, and the Left Hand Side denotes the total available shipment capacity added over the three ship classes including the penalties (X_{29} .... X_{32}) for surplus-containers left for unavailable shipment capacity. As suggested before the used coeficients (1500, 850 and 500) are "rated" capacities. They may be different from those used on the outbound journey, a detail which is omitted in the present discussion (10).

The reported RHS figures (c/) are exemplative suggestions in additional to the "sixth demand case" of the outbound market. Thus with 5,000 and 20,000 TEUs in respectively the third and the fourth route, we assume that freight market on those routes are perfectly balanced in both directions. On the opposite routes 1. and 2. only provide backhaul cargo at 50 % of the outbound market demand (cfr. supra Table 1.). Also the penalties of the backhaul market are relatively higher on the third and fourth route.

In order to keep the problem statement realistic, it is essential to consider the same backhaul trip in the objective function and in the backhaul demand constraints.

---

(10) *For example a northbound journey toward the "NORD ATLANTIC" area may feature the rated 1500 TEU capacity with a ship used up to her load-line, Then the southbound backhaul will most certainly obtain a somewhat lower capacity because of corrections to be made for the weight of bunker fuel and changing salt-composition of the crossed seas. Since all this is a matter of WEIGHT of the cargo it is very arbitrary to translate those items in terms of NUMBER OF TEU*

Thus, when on the first route the outbound cargo (10,000) exceeds the backloads by 5,000 TEU the cost-coefficients of the backhauls in the objective function (X_{17},X_{21}& X_{25}) must express the marginal costs of loading a vessel on her backhaul instead of returning empty. The reverse, ie backloads exceeding outbound cargo, is not possible. Indeed one would have organized "phantom trips", i.e. some backhauls at marginal loading costs without a corresponding number of outbound journeys which bear the fully joint fraction of common round-trip costs. This point is systematically introduced by the:

d/ Backhaul trip-frequency constraints

which state that the number of (fully loaded) backhauls should not exceeds the number of outbound departure by route and ship-class:

$X_1 \geqq X_{17}$	or	$X_1 - X_{17} \geqq 0$	for class 1,	route 1
$X_2 \geqq X_{18}$		$X_2 - X_{18} \geqq 0$	1,	2
$X_3 \geqq X_{19}$		$X_3 - X_{19} \geqq 0$	1,	3
$X_4 \geqq X_{20}$		$X_4 - X_{20} \geqq 0$	1,	4
$X_5 \geqq X_{21}$		$X_5 - X_{21} \geqq 0$	2,	1
$X_6 \geqq X_{22}$		$X_6 - X_{22} \geqq 0$	2,	2
::::::::::		::::::::::		
:::::::::		::::::::::		
$X_{11} \geqq X_{27}$		$X_{11} - X_{27} \geqq 0$	3,	3
$X_{12} \geqq X_{28}$		$X_{12} - X_{28} \geqq 0$	3,	4

Those constraints simply follow hhe meaning of earlier program assumptions that:

a/ a ship is either sailing or laying up for a full month,
b/ as far as she sails, she can perform fractions of monthly trips, but ALWAYS FULLY LOADED,
c/ penalties can be activated for surplus cargo, but THE OPPOSITE is not possible, in that one cannot organize outbound journeys with a load factor of less than one.

The above inequalities must remain inequalities. If they would have been introduced as equalities (eg $X_1=X_{17}$) they would additionally require that the actual outbound cargo equals the return load which is not realistic with actual demand conditions.

Moreover, the backhaul inequalities should logically correspond to the formulation of roundtrip features in the cost- and demand-equaticns. Let us expand on this issue with the first route in mind. There, the outbound cargo (10,000) substantially exceeds the backloads (5,000 TEU). Consequently, the backhauls were evaluated in the objective function at the marginal costs (coefficients of X_{17} to X_{20}). Furthermore, the first four backhaul-frequency constraints express the condition that the loaded backhauls should never exceed the outbound departures and there ratio simply gives the load factor of the return voyage (eg X_{17}/X_1 , X_{18}/X_2 etc.). With this procedure,

a/ the backhaul is always defined as the "route-leg" with the lowest cargo demand, a condition which gurantees the consistency between the objective function and the cargo-demand constraints,

b/ the return trips will never exceed outbound departures since the latter are evaluated at incremental costs with respect to the outbound.

Applied to the sixth demand case, the optimum is represented in Table 4.11. The number of trips between brackets give the frequencies in the case of no backhaul cargo *(such as was exhibited in Table 5)*.

The only point of difference concern the third ship's class *(i.e. the smaller 500 TEU units)*. Now, the 40 roundtrips on the second route are reduced to 26.4 departures of which 20 monthly crossings find at a average a full return cargo. On that route 6,825 TEU are not shipped on the outbound leg. The reduction of (40-26.5=) 13.5 trips on the second route balances an increase of (9.5 - 4=) 5.5 additional trips on the fourth route, which is indeed compensated for the availability of full backloads in that area. Consequently the penalties have been allocated to the second route instead of the fourth .

Table 11.:Roundtrip optimum trip assignment with $S_1=5$; $S_2=8$; $S_3=10$

OUTBOUND	demand	10,000	20,000	5,000	20,000	slacks
	sailings of: 1.	6.7	-	3.3	1.1	0
	2.	-	-	-	16	0
	3.	-	26.4(40)	-	9.5(4)	0
	penalties TEU:	-	6,825(0)	-	0(2730)	
BACKHAUL	penalties TEU:	-	-	-	-	
	sailings of: 1.	3.35	-	3.3	1.1	0
	2.	-	-	-	16	0
	3.	-	20	-	9.5	0
	demand	5,000	10,000	5,000	20,000	
	backload load factors 1.	0.5	-	1	1	
	2.	-	-	-	1	
	3.	-	0.77	-	1	

OPTIMUM COST VALUE : 941,157

As compared to the operations-schedule of Table 5. , the system costs have increased from 830,466 up to 941,157, or by some 13 %. At the same time the number of TEU shipments is pushed from (55,000-2730=) 52,270 to (95,000 - 6825 =) 88,175 units or by 69 %.

This examplative story tells something about "scale-economies" rather resulting from a comprehensive approach of integrated operations logistics than from the only use of larger ships.

4.3. Laying-up costs

Untill now, "slacks" have been assigned to the average monthly number of vessels laying up because of shortage of cargo. Those slacks are computed simultaneously with the variables of the optimum solution, but did not enter in the objective function. In fact we could have put those slacks in the objective function too, but with a zero-coefficient such as to neutralize their influence, and change the associated inequalities by equalities (11). In such cases, one explicitly states that there is no specific cost associated to the time a ship is laying up either in some foreign port or a Norvegean Fjord. Apparently, this approach is correct if one assumes that the operating costs have been calculated incrementally to the costs of laying up.

Nevertheless, that assumption is only valid in part since lay-up costs dominantly contain "period costs" by class of ship, whereas operating costs also vary with route characteristics. Therefore, a more realistic formulation of lay-up features is obtained:

1/ by changing the "fleet-capacity constraints" from: inequalities: $RHS \leq 5$ to equalities: $RHS + X_{33} = 5$,

$RHS \leq 8$ $\quad$ $RHS + X_{34} = 8$,

$RHS \leq 10$ $\quad$ $RHS + X_{35} = 10$,

2/ and adding those variables to the cost-minimizing objective function:

Minimize $10{,}000\ X_1 + \ldots + 50\ X_{32} + a'\ X_{33} + b'\ X_{34} = c'\ X_{35}$.

(11): *this procedure is also called "artificial variables". Artificial variables are variables which feature in the constraints in order to change inequalities into equalities. This follows from two LP formats, being either in a STANDARD FORM (with only equalities) or the CANONICAL (in which also inequalities occur).*

The new coefficients in the objective function (a',b' &c') express the average lay-up cost per month by class of ship. It is not easy to attribute provisional exemplative values to those parameters, because they depend in reality upon:

a/ the actual place of laying up (ports, fjords etc.)

b/ the length-of-time of contineously laying up,

c/ the necessity of executing regular overhauls and periodic repairs of the operating fleet, which may consequently allow to turn the evil into a benefit (12).

Evidently, whatsoever their value those costs will not influence the optimum solution when slacks are zero and the all fleet is in permanent operation . If some idle capacity exists (and some slacks, ie variables X_{33} ... X_{35}, obtain positive values) the optimum will exhibit higher systems-costs to the extent of those slacks, each of them multiplied by the appropriate cost of laying up. But even in those cases the introduction of lay-up costs will not change the optimum sailing list bacause:

- slacks are only activated when all cargo is shipped,
- slacks only appear progressively from the less efficient (class 3) to the more efficient units (class 2).

Thus, in "normal" circumstances, the correction for idle time into lay-up costs only consolidates the optimum solution in the cases reported before.

The programming concept of lay-up costs may nevertheless introduce relevant management decisions in case of an occasional or structural crisis in world freight markets. It would indeed be vital information to know if it still worth performing roundtrips:

a/ when on certain routes the company operates at a loss

b/ which may be balanced in part or in full by saving lay-up costs.

Such problems can only be addressed by an integrated logistical approach since indeed, markets may slow down on some routes where the most efficient vessel happen to operate, and the company will certainly try to lay up the least efficient vessels which consequently have to be removed from routes with good market prospects. In order to simulate such circumstances, one only has to introduce NEGATIVE penalty values *(i.e. cost-savings)* to the freight-penalties of not carrying an available TEU.

Our scenario of the "freight market crisis" is established:

- by reducing all freight penalties by 50 % (profit reduction),
- and by taking negative values on the outbound sections of the second and foruth route.

Thus, in the objective function the new coefficients become:

variables :	X_{13}	X_{14}	X_{15}	X_{16}	X_{29}	X_{30}	X_{31}	X_{32}
crisis-values:	20	-25	-20	35	10	10	20	25
previously :	*40*	*50*	*40*	*70*	*20*	*20*	*40*	*50*
route :	1.	2.	3.	4.	1.	2.	3.	4.
SECTION :	o u t b o u n d				b a c k h a u l			

Furthermore, we assume fairly substantial lay-up costs of 05000.-/month for a 1500 TEU vessel and respectively 4500 and 4000 for the smaller ships of class 2. and 3.

Table 12 : Optimal ship assignment with $S_1=5;S_2=8;S_3=10$ given backloads at market-crisis prices and lay-up costs of 5000, 4500 and 4000/month.

route	1.	2.	3.	4.	ships laying up (slacks)	
demand:	10,000	20,000	5,000	20,000		
sailings 1:	6.6	0	0	2.8	0	OUTBOUND
sailings 2:	0	0	0	16	0	
sailings 3:	0	0	0	4.6	7.7	
penalties (TEU)	0	20,000	5,000	0		
penalties (TEU)	0	10,000	5,000	0		BACKHAUL
sailings: 1.	3.3	0	0	2.8		
sailings: 2:	0	0	0	16		
sailings: 3:	0	0	0	4.6		
demand:	5,000	10,000	5,000	20,000		

The crisis has indeed drastically changed the operating picture. The loss on the outbound sections of route 2. & 3. made the company cancelling all trips on those routes, even when the backloads still allowed some profit. At the same time large 1500 TEU units substitute for small 500 TEU units (compare sailings of Table 11). The latter vanish completely from route 2. and are substantially reduced on route 4. At the average month, 7.7 of those feeder ship now lay up permanently.

A third rather elaborate way to cope with costs of laying up is to reduce speed on current operations. Such circumstances are very realistic to-day and can be introduced by:

a/ raising the roundtrip times, and

b/ adapting operations costs for a decreased fuel consumption and a small increase in crew costs.

Such simulations require the basic reformulation of the initial problem statement, which goes beyond the scope of this introductory presentation.

A fourth final proposal to evade costs of laying up is trying to enter the charter market to the extent the company wishes to renumerate idle or less efficient shipping capacity. To a certain extent, the concept of this situation is similar to the crisis-market with lay-up costs, but the used coefficients are opposite. This case is separately discussed in the next section.

4.4. Managing time charters

The previous discussion on lay-up costs aimed to obtain proper cost-accounts of the whole operating system, but also addressed some managerial decision alternatives in times of freight-market crisis with explicit losses on some routes. It often occurs, however, that a market slowdown is less harmful in general, but for example especially affects the route-areas which are served by the company under consideration. Since the shipping market may be spatially articulated, other shipping areas may still exibit good opportunities. In other areas, companies may be temporarily in short capacity-supply. In the short term, before our company is able to engage itself on new spatial markets or organize pools, it may consider to bring some of its vessels in the time charter market. Our firm then faces the alternative decision of providing time charters instead of operating the whole fleet on its own account.

Such decision may be modeled by:

a/ keeping the freight penalties ($X_{13} \ldots, X_{16}$; $X_{29} \ldots X_{32}$) associated with positive values but at the reduced level ie + 20, 25, 20, and 25 for outbound routes, and + 10, 10, 20 and 25 for backload TEUs,

b/ attributing NEGATIVE values to the (slack)-penalties of idle fleet capacity (X_{33}, X_{34} & X_{35}).

The latter "charter terms" might obtain a value between the opportunity costs of saving lay-up time and the maximum money making capacity, that is the ship's capacity times the positive value of freight penalties. For 1500 TEU vessels the terms compare to the net profit potential by route:

(capacity	x penalty	x nr.of trips)-	(operating costs	x trips)	= maximum
1500	20	3	10,000	3	60,000
1500	25	2	11,000	2	53,000
1500	20	3	12,000	2	36,000
1500	35	1	15,000	1	37,500

Therefrom, we derive for the three ships' classes:

	1500 TEU	*850 TEU*	*500 TEU*	(Table)
- mean charter terms	50,000	45,000	40,000	(4.13)
- high charter terms	64,000	60,000	56,000	(4.14)

As compared to Table 11 the system cost decrease from 941,152 down to 728,775 which represents a cost-savings of almost 33 %, in case of the average level of charter terms (cfr. Table 13). The total package of shipped containers however only drops by less than 10 % (from 88,175 to 80,000) , and at the average 2.3 *500 TEU* ships are rented out on a month-to-month basis.

It is fairly easy to enumerate what happened:

1/ The company has especially reduced services on those markets where the backloads does not match outbound traffic. Consequently, routes 1. and 2. now feature an outbound surplus of 5,000 and 10,000 respectively for which the penalty costs are activated. All flows are featuring a load factor equal to 1.

2/ The lower effective demand allows a further specialization by the efficiency units of class 1. on routes 1., 2. and 3., whereas vessels of Class 2. keep sailing exclusively on the fourth route.

3/ Consequently, the small vessels of class 3 (500 TEU) are substantially withdrawn from the second route (where they are substituted by class 1 ships) and consolidate their operation on route 4, apart from

4/ 2.3 monthly units which earn money on a time-charter basis.

Table 13: Optimal ship assignment with $S_1=5$; $S_2=8$; $S_3=10$ given backloads and charter terms of respectively 50,000 (class 1); 45,000 (class 2.); 40,000 (class 3.)

		1.	2.	3.	4.	charters / month	
route demand:		10,000	20,000	5,000	20,000		OUTBOUND
sailings:	1.	3.3 (6.7)	4.4 (0)	3.3	0 (1.1)	0	OUTBOUND
	2.	0	0	0	16	0	OUTBOUND
	3.	0	6.7(26.4)	0	12.8(9.5)	2.3(0)	OUTBOUND
penalties (TEU)		5,000	10,000	0	0		OUTBOUND
penalties (TEU)		0	0	0	0		BACKHAUL
sailings	1.	3.3 (6.7)	4.4 (0)	3.3	0 (1.1)		BACKHAUL
	2.	0	0	0	16		BACKHAUL
	3.	0	6.7(20)	0	12.8(9.5)		BACKHAUL
demand:		5,000	10,000	5,000	20,000		BACKHAUL

OPTIMUM COST VALUE : 728,775

Table 14: Optimal ship assignment as in 4.13 with terms of 64,000 (class 1);60,000 (class 2);56,000 (class 3);

route		1.	2.	3.	4.	charters/month	
sailings	1.	3.3	4.4	3.3	0	0	OUTBOUND
	2.	0	0	0	16	0	OUTBOUND
	3.	0	0(6.7)	0	0(12.8)	10	OUTBOUND
penalties (TEU)		5,000	13330(10000)	0	6400(0)		OUTBOUND
penalties (TEU)		0	3330 (0)	0	6400(0)		BACKHAUL
sailings	1.	3.3	4.4	3.3	0		BACKHAUL
	2.	0	0	0	16		BACKHAUL
	3.	0	0(6.7)	0	0(12.8)		BACKHAUL

OPTIMUM COST VALUE : 584,753

In the previous case, chartered capacity was only removed from routes where outbound traffic demand exceeds backload-possibilities, and only to the extent of those surplus demand.(i.e. 5000 and 10000 on the first and second route).

Next our firm faces the same market volumes at the same profit potential (equal penalty values of freight penalties $X_{13} \cdots X_{16}$ & $X_{29} \cdots X_{32}$) but is able to raise the terms of the time-charters from a 50-45-40 combination up to a 64-60-56 level.

The optimality analysis discloses a complete removal of all small *1500 TEU capacity* vessels from the own operations and all of them are rented out. Class 2. remains on route 4 where it now becomes the exclusive operator and class 1. covers the first three routes. The operations thus show a very simple picture:

- 5 1500 TEU units on routes 1, 2 & 3,
- 8 850 TEU units on route 4,
- 10 500 TEU units in the time-charter market.

Cargo services are especially removed from the:
- first route to the extent of the outbound freight surplus (5000 TEU)
- second route to the amount of the outbound surplus (10000 TEU), in addition to 3,330 TEU on both outbound and backhaul trips,
- third route up to 6,400 TEU on both directions or 32 % of the local market potential.

Systems costs are furthermore reduced by 20 %, but the number of shipped containers drop by almost 25 %. This is the evident consequence of straightforward possibilities on the charter market.

5. Conclusions

Most LP application of assigning scarce resources to alternative users are dealing with the so-called "short term", in which production factors are contingent upon one or more operating constraints (13). This is the essence of the assignment problem, since without those constraints, the search for optimum operating conditions would become redundant.

Shipping companies do meet regularly such issues since their basic equipment is long lasting and market conditions uncertain and often volatile. The case studies, dealt with in the previous sections, are basic and exemplative exercises. Even the amendments of backload opportunities, market decline or chartering and lay-up costs are realistic refinements, but still cover fairly standard issues in operating a fleet of vessels (14).

Further refinements can be added to the basic format in the sense of a more systematic organization of the sensitivity-analysis. Examples are the incidence of port costs and delays, the simulation of more elaborate route-loops, the effect of traffic-pools and conferences, new vessel design and the hinterland-coverage of port calls (eg. with respect to items of seasonality and cyclicality). All of the latter problems require more basic changes in the initial model compared to simple one-by-one amendments, as was done before. As such, they are technically possible to be established on the model here discussed, but provisionally exceed the scope of the present note.

o 0 o

6. Notes and References

(1) Among a well known sample of standard textbooks we note: YAMANE T., Mathematics for Economists, 2nd edit., Prentice Hall, London, 1968, 714 p.;pp.553-55, HORNGREN Ch.I., Cost Accounting, a Managerial-Emphasis, 4th edition, Prentice Hall, Engl.Cliffs,N.J., 1977, 934 p.;pp.866-75, WAGNER H. M., Principles of operations Research with Applications to Managerial Decisions, 2nd ed., Prentice Hall, Engl.Cliffs, 1975, TAHA, H. A., Operations Research, New York,Macmillan, 1971, 603 p., HESKETT J. L., GLASKOWSKY N. A., IVIE R. M., Business Logistics, 2nd ed., Ronald Press, New York, 1973,pp.414-50, NAYLOR Th. H. & VERNON J. M., Micro-economics and Decision Models of the Firm, Harcourt, Brown & World, New York, 1969,pp. 213-45, STEENBRINK R., Optimization of Transport Networks,London,Wiley,1973p.,325p. SIMMONS D. M., Linear Programming for Operations Research, Holden Day, San Francisco, 1972, 288 p.

(2) Nevertheless, the "assignment problem" as applied to the present cases in shipping logistics features a different format as the assignment exercises which resort to the so-called "transportation problem", e.g.in plant-warehouse logistics and optimal routing algoriths; see for example, BOWERSOX P. J., Logistical Management, 2nd ed., MacMillan, New York 1978, 528 p.;pp.347-53 & pp. 395-410. WAGNER, o.c., pp. 671-76, HEGGIE I. G., Transport Engineering Economics, Mc Graw Hill, London, 1972, pp.215-19.

(3) Earlier the transformation curve has been expressed as $X_{12} = 2.5 \times X_{10}$, stating that one additional X_{12} trip on the fourth route requires 2.5 trips to be cancelled on the second route. Thus, 6.8 trips more on route 4. gives 6.8 times 2.5 = 17 trips less on route 2.

(4) conform to our initial notation $u_1=S_1$; $u_2=S_2$; $u_3=S_3$;$u_4=T_1$; $u_5=T_2$; $u_6=T_3$ and $u_7=T_4$.

(5) Actually, with a 5 digit precision, the number of sailings drops from 4.7012 down to 1.1718 or minus 3.5294. Savings are evaluated â 9,000 per trip, or 31765 instead of the reported 31,500. Thus the net savings are 9,765 as reported in Tables 8/a (cases 2/b and 3/b).

(6) cancelled trips of 1.469413 à 10000 or savings of 14,694,
additional 1500 TEU trips from 1.500667 up to
2.3333 or plus 0.832666 à 15,000 equal - 12,490
2,204

(7) see DEAN, Managerial Economics, Prentice Hall, Engl.Cliffs, 1951, 619 p.; and for some additional notions EDGEWORTH, Contributions to the theory of railway rates, the Economic Journal, Volume 24, December 1911, pp. 551-71.

(8) a more elaborate digression on the subject has been given in CLAESSENS E. M., Methods of applied railway economics, the case of the EEC, unpublished Ph.D. dissertation, Antwerp SESO, 1980, 1022 p.; pp. 246-56.

(9) one observation to substantiate the relevance of the fixed mark-up is the SUEZ canal toll-establishing practice in which only difference is made between empty and loaded vessels and 1 ton net cargo-load is considered as a full load.

(10) For example a northbound journey toward the "Nord Atlantic" area may feature the rated 1500 TEU capacity with a ship fully charged up to her NA-load line when entering the NA-area. She thus may be loaded above that line in tropical waters when the loss of bunkering weight allows the NA load line be reached in the NA area. However, the southbound backhaul will most certainly obtain a somewhat lower capacity because of the progressively increasing salt composition of the crossed seas. Since all of this is only a matter of net LOAD, it is fairly arbitrary to translate those items in terms of net NUMBER OF TEU.

(11) this procedure is also called the "artificial variables" method. Those variabees indeed feature in the constraints in order to change inequalities to equalities. This corresponds to LP formats being either in a STANDARD form (with only equalities) or the CANONICAL form (in which also inequalities may occur).

(12) The exact timing of ship repairs and overhauls is subject to some freedom. For example, the LLoyd's Register of Shipping allows the "annual & docking survey" within a period of 2 successive years. Special surveys are to succeed every 4 years correction made for one "year of grace". Furthermore, passenger vessels are subject to the possibility of "progressive" surveys, and, finally, "contineous surveys" may substitute the special surveys but only for motive power. see e.g. R. VLEUGELS, Vervoer ter Zee (sea navigation), Antwerpen, UFSIA, 1982,pp.56-57.

(13) see for example HOROWITZ H., Decision Making and the Theory of the Firm, New York, Holt, Rhinehart & Wilson, 1970, 468 p.;pp.105-11.

(14) A final amendment may be introduced by applying Integer programming in which the monthly sailing cannot anymore obtain fractions of trips. However, the drawback of the present "simplification" should not be carried too far either. One the one hand, the present exercises actually represent a "regular service" with backhauls immedeately following the outbound trips i.e. somehow in between the "liner" and "tramping" system. As repeatedly mentionned on the November 1983 Bremen-congress on liner shipping the above distinction progressively vanishes such that the "in between" situation may as well become more relevant. On the other hand it may indeed be unwise to tackle an assignment problem of this kind immedeately with integer restrictions without having crossed the initial exercise with fractions of trips on a month/to/month schedule.

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