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Costs and productivity in regional bus transportation in Belgium.

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Abstract

In this paper we develop productivity indices for regional bus transportation in Belgium on the basis of an estimated variable cost function. Two different but closely related indices are calculated as suggested in the literature. Both productivity measures correctly account for input substitution over time and the methodology used doesn't impose a priori restrictions on technology.

The results indicate important differences between our proposed measures and a less sophisticated productivity index defined as output per unit labor input. A completely different pattern of productivity growth over time emerges from our estimates.

1. Productivity measurement in the transportation industries

Productivity measurement is not an easy matter. This is especially true for the transportation industries where the heterogeneity of services provided even causes substantial difficulties in properly defining output. Moreover, factor substitution over time makes construction of indices of productivity growth a complicated task. It's not surprising, therefore, that only very recently acceptable methods have been developed to measure changes in total factor productivity (*).

Most early studies have used output for unit of input as a measure of factor productivity, see e.g. Abramovitz (1962). This type of productivity concept is at this moment still widely used by transportation firms in many countries. Many railroads e.g. use the number of passenger-miles per man-hour as an indicator of productivity in the production of passenger transport (**). Economists have been aware of the weakness of this approach for quite some time. Productivity growth was more formally defined as the shift in the production function over time as distinguished from movements along the production function (Solow (1957)). It measures the improvement over time in the efficiency with which a firm or industry transforms its factors of production into output. It's clear that indicators of the type "output per unit input" are poor approximations of productivity as they do not properly account for factor substitution over time. Indeed, many public transportation firms show substantial increases in passenger-miles for man-hour precisely because of capital-labor substitution. Use of the simplistic indicator of labor

(*) The difficulties involved in calculating the growth in productivity over time leads Meyer and Morton (1975, p. 497) to conclude: "At best, productivity measures incorporate an element of art as well as of science".

(**) This apparently simplistic productivity index is used by many firms responsible for passenger transportation in Belgium, including the firm we study in this paper.

productivity to measure total factor productivity leads to overestimating productivity growth.

A significant improvement was made by Jorgenson and Griliches (1967). They measured the shift in the production function over time and constructed productivity indices. However, they used the assumption of constant returns to scale prevailing in the industry under investigation. Moreover it was assumed that output was priced at marginal cost. There competitive conditions might be valid approximations for many industries but they cannot be accepted for the transport sector which is characterised by extensive regulation.

More recently, Meyer and Morton (1975) calculated productivity growth rates for the U.S. railroad industry using index procedures to represent aggregate output and input. Instead of measuring labor and capital productivity separately they used the input cost shares as weights to determine an aggregate input index. This was combined with an output index in order to calculate total factor productivity. However, the procedures they developed to characterise output and input implied very restrictive assumptions as to the nature of the underlying technology (*). Therefore, their proposed methods may not give very reliable results.

A powerful alternative was proposed recently by Caves, Christensen and Swanson (1980, 1981). Although their methodology is related to the work of Jorgenson and Griliches it provides substantial improvements by relaxing several restrictive assumptions. The productivity indices they propose are based upon the specification of a flexible functional form to describe the underlying production and cost structure. The authors choose the translog function as the appropriate description of technology.

(*) It has been shown that index numbers to represent input and output can be derived from particular aggregator functions describing the production' process (Samuelson and Swany (1974)). As a consequence a more satisfying approach would be to choose first a suitable function to describe the underlying technology and then to derive input, output and productivity indices.

This allows a very general production structure. Indeed it's well known that the translog cost function imposes no a priori restriction concerning the nature of returns to scale, the elasticity of substitution between inputs, separability between outputs and inputs, etc. Moreover productivity indices calculated from the translog cost function are compatible with nonneutral technological change. A further advantage of the proposed methodology is that it's possible to derive productivity indices both from the variable and the total cost function. In this paper, we will use the variable 'cost function' to develop productivity indices for regional bus transport in Belgium. This implies that we allow the bus company to be in disequilibrium with respect to its capital stock.

2. Productivity measurement: methodology

We now proceed to derive productivity indices for regional bus transportation in Belgium on the basis of a variable cost function to be estimated. The bus company is assumed to produce a single output (Y) and to use three distinct inputs in the production process, viz. labor (L), energy (E) and capital services (T). Under these circumstances the transformation' function can be written as

$$F(Y, L, E, T, t) = 0 \qquad (1)$$

where t is time. This variable is included to account for technological shifts in the production function and is an essential factor in calculating productivity growth. It's easy to show, by differentiating (1), that the following relation holds

$$F_{\bar{y}} d\bar{Y} + F_{\bar{L}} dL + F_{\bar{E}} d\bar{E} + F_{\bar{T}} d\bar{T} + F_t dt = 0 \qquad (2)$$

where F_X is the partial derivative of the transformation function with respect to X and $\bar{X} = \ln X$.

Following Caves, Christensen and Swanson (1981) we use two alternative definitions of productivity growth. The first one (P_1) is defined as the rate at which output can grow over time with all inputs held at a constant level. Using (2) it follows

$$P_1 = -\frac{F_t}{F_{\bar{Y}}} \qquad (3)$$

An alternative, though less natural, definition would be the common rate at which all inputs can be reduced over time with output held at a fixed level (P_2). Applying the definition we find

$$P_2 = \frac{F_t}{F_{\bar{L}}+F_{\bar{E}}+F_{\bar{T}}}$$

There is no a priori reason to expect these two productivity measures to be equal. Indeed, it's easy to show that $P_1 = P_2$

only if the production process is characterised by constant returns to scale (*).

Previous research has indicated that the transportation industries in general and passenger transportation in particular are typically operating with considerable excess capacity (**). Therefore imposing the assumption of total cost minimization would be quite unrealistic. We allow the firm to be operating at a less than optimal level for its capital stock and assume that the firm's main objective is to minimize variable costs. Although this is still a strong assumption to make, we believe it is more reasonable than to assume the firm to be in equilibrium with respect to its capital stock. This approach has still another practical advantage. Given the non-uniform treatment of capital costs in the firm's annual accounts over time, it's much easier to obtain accurate variable cost figures than to find reliable indicators of total costs. We are convinced, therefore, that estimation of a variable cost function and using the resulting estimates to construct productivity indices will lead to more reliable results.

Taking into account previous remarks, we assume that the firm minimizes variable costs subject to a well behaved technology i.e. it's required that the production function be continuously differentiable and strictly quasi concave in inputs. Under these conditions a unique variable cost function exists

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(*) The degree of returns to scale (R) has been defined in the literature as the proportional growth in output due to a proportional increase in all inputs. Holding time fixed in order to distinguish between returns to scale and productivity growth we find, using (2),

$$R = -\left(\frac{F_{\bar{L}} + F_{\bar{E}} + F_{\bar{T}}}{F_{\bar{Y}}}\right)$$

It's obvious that $P_1 = RP_2$ so that $P_1 = P_2$ only if R=1.

(**) See e.g. Keeler (1974), Meyer, Kain and Wohl (1965), Viton (1980).

giving the minimum variable costs to produce a specified output level at observed factor prices for the variable inputs (*). If we assume the capital stock to be fixed in the short-run we can write the variable cost function

$$c_V(Y,\ p_L,\ p_E,\ T,\ t)$$

where p_L and p_E are the input prices of labor and energy, respectively. It's possible to relate the defined productivity growth measures p_1 and p_2 to the variable cost function. In order to do so we first consider the relationship between the cost function and the partial derivatives of the transformation function (1). Noting that C_V is the result of minimising $(p_L L + p_E E)$ subject to (1) and using the envelope theorem it's straightforward to prove the following relations (**)

$$\frac{\partial \ln C_V}{\partial \ln Y} = - \frac{F_{\bar{Y}}}{F_{\bar{L}}+F_{\bar{E}}} \qquad (5)$$

$$\frac{\partial \ln C_V}{\partial \ln T} = - \frac{F_{\bar{T}}}{F_{\bar{L}}+F_{\bar{E}}} \qquad (6)$$

(*) See e.g. Diewert (1974), Mc Fadden (1978).

(**) The first-order conditions imply $p_L = \lambda F_L$, $p_E = \lambda F_E$ where λ is the Lagrange mulitplier. Further we have

$$p_L L+p_E E = C_V = \lambda(F_L L+F_E E) = \lambda(F_{\bar{L}}+F_{\bar{E}})$$

From the envelope theorem we have $\frac{\partial C_V}{\partial Y} = - \lambda F_Y$. Combining this and the previous relations yields

$$\frac{\partial C_V}{\partial Y}\ \frac{Y}{C_V} = -\frac{-F_Y Y}{F_L L+F_E E} = - \frac{F_{\bar{Y}}}{F_{\bar{L}}+F_{\bar{E}}}$$

Analogous reasoning leads to the expressions for $\frac{\partial \ln C_V}{\partial \ln T}$ and $\frac{\partial \ln C_V}{\partial t}$

$$\frac{\partial \ln C_V}{\partial t} = - \frac{F_t}{F_{\bar{L}}+F_{\bar{E}}} \qquad (7)$$

These expressions allow us to write the productivity measures P_1 and P_2 in function of the cost elasticities defined in (5), (6) and (7). Manipulation of (3)-(7) yields the following results (*)

$$P_1 = \frac{-\dfrac{\partial \ln C_V}{\partial t}}{\dfrac{\partial \ln C_V}{\partial \ln Y}} \qquad (8)$$

$$P_2 = \frac{\dfrac{\partial \ln C_V}{\partial t}}{1 - \dfrac{\partial \ln C_V}{\partial \ln T}} \qquad (9)$$

These relations can be used to calculate productivity growth on the basis of an estimated variable cost function. It's clear that our indices would be different if calculated from a total cost function. Indeed, as shown by Caves, Christensen and Swanson (1981, p. 996), the two procedures only lead to the same result if the firm is actually minimizing total costs.

(*) The expression for P_1 is easily found by comparing (3), (5) and (7). To arrive at equation (9) start from (4):

$$\frac{F_t}{F_{\bar{L}}+F_{\bar{E}}+F_{\bar{T}}} = - \frac{\partial \ln C_V}{\partial t} \left(\frac{F_{\bar{L}}+F_{\bar{E}}}{F_{\bar{L}}+F_{\bar{E}}+F_{\bar{T}}} \right) \qquad \text{using (7)}$$

$$= - \frac{\partial \ln C_V}{\partial t} \left(\frac{1}{1 - \dfrac{\partial \ln C_V}{\partial \ln T}} \right) \qquad \text{using (6)}$$

Also note that analogous reasoning can be used to prove $R = \dfrac{1 - \dfrac{\partial \ln C_V}{\partial \ln T}}{\dfrac{\partial \ln C_V}{\partial \ln Y}}$

3. A variable cost function for regional bus transportation

In this section we present a variable cost function for regional bus transportation in Belgium. Regional bus transport services are provided by a single firm (*) the "Nationale Maatschappij voor Buurtspoorwegen" (NMVB). The firm is regulated in the sense that the government is largely responsible for the prices charged. Moreover, government is involved in determining which routes should be served at which frequency i.e. in the determination of the firms vehicle-kilometers. Demand factors determine the ultimate number of passenger-kilometers produced by the firm. It's clear from this description that output is exogenously determined. This implies that output is no choice variable for the company and that its only flexibility is to be found at the input side: the firm is assumed to provide the transportation services at the lowest possible cost. These remarks also justify estimation of the cost function rather than the production function (**) in order to derive information on the underlying technology.

A substantial amount of research on the cost structure in the transportation industries has been done in the past decades. Several functional forms were used to describe the tech-

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(*) Due to the social character of regional bus transport and the limited extent of the market competition was never allowed in this sector. The NMVB has always been strongly regulated by the government in order to fulfil its social policy goals.

(**) Estimation of the production will in general suffer from simultaneous equations bias due to the dependence of factor demands on output. A better procedure is, therefore, to focus on the cost function which contains only exogenous variables.

nology involved in the production of transport services (*). The early studies specified linear cost functions (e.g. Meyer et alii (1959)) whereas in later years the Cobb-Douglas cost function was probably the most widely used functional form (Keeler (1974), Pozdena and Merewitz (1978)). Recently, however, a large number of studies have used the translog cost function which provides a second-order approximation to an arbitrary cost function (**). This flexible form contains all necessary information concerning the production structure such as economics of scale, price elasticities of input demands, elasticities of substitution etc. A minor disadvantage of the translog cost function is that it cannot be derived, using duality principles from an explicit production function (***). Given the exogenous nature of output in this study, we do not consider this to be important.

The translog cost function has recently been used in several submarkets of the transport sector. Railroad cost functions have been estimated by e.g. Caves, Christensen and Tretheway (1980), Spady (1977), Harmatuck (1979) and Viton (1980). The cost structure in the trucking industry has been

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(*) For a more extensive survey of the literature see De Borger and Deloddere (1980).

(**) Let f be the true functional form and x a vector of n variables which are assumed to determine costs. The second-order Taylor expansion of the true cost function around a point $\bar{x}$ can be written as

$$c(\bar{x}) \simeq f(\bar{x}) + \sum_{i=1}^{n} \frac{\partial f(\bar{x})}{\partial x_i} (x_i - \bar{x}_i) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 f(\bar{x})}{\partial x_i \partial x_j} (x_i - \bar{x}_i)(x_j - \bar{x}_j)$$

The translog cost function can be obtained from this approximation by using the logarithms of output and input prices as elements of the vector x and by specifying $\alpha_0 = f(\bar{x})$, $\alpha_i = \frac{\partial f(\bar{x})}{\partial x_i}$, $\alpha_{ij} = \frac{\partial^2 f(\bar{x})}{\partial x_i \partial x_j}$. As a consequence, the parameter vector α contains estimates of the value, the gradient and the Hessian of the true cost function at $\bar{x}$.

(***) In particular, the translog cost function is not the dual of the translog production function. For an application of the translog approximation to the CES production function see D'Haeseleer and Goossens (1982).

analysed by Friedlaender (1978), Spady and Friedlaender (1978), Klem (1978), Harmatuck (1981) and Keaton (1978).

Using our assumption of a fixed capital stock in the short run we can write the following translog variable cost function:

$$\begin{aligned} \ln C_V = {} & \alpha_o+\alpha_y \ln Y+\tfrac{1}{2}\alpha_{yy}(\ln Y)^2+\alpha_L \ln p_L+\alpha_E \ln p_E \\ & + \tfrac{1}{2}\{\alpha_{LE}\ln p_L \ln p_E+\alpha_{LL}(\ln p_L)^2+\alpha_{EE}(\ln p_E)^2+\alpha_{EL}\ln p_E \ln p_L\} \\ & + \alpha_{YL}\ln Y \ln p_L + \alpha_{YE}\ln Y \ln p_E \\ & + \alpha_T \ln T + \tfrac{1}{2}\alpha_{TT}(\ln T)^2 + \alpha_{YT}\ln Y \ln T + \alpha_{TL}\ln T \ln p_L \\ & + \alpha_{TE}\ln T \ln p_E + \alpha_t t + \alpha_{tY} t \ln Y + \alpha_{tT} t \ln T \\ & + \alpha_{tL} t \ln p_L + \alpha_{tE} t \ln p_E + \mu \end{aligned}$$

where all variables are as defined before and μ is a stochastic error term. It can be seen from this specification that the time trend, added to account for technological shifts, is allowed to interact with output, input prices and capital stock in order to impose no a priori restrictions on the nature of productivity growth.

In order for this cost function to correspond to a well-behaved technology it should be linear homogenous in factor prices. This implies that the following set of restrictions on the parameters should be imposed: $\alpha_{LE}=\alpha_{EL}$, $\alpha_L+\alpha_E=1$, $\alpha_{YL}+\alpha_{YE}=0$, $\alpha_{TL}+\alpha_{TE}=0$, $\alpha_{tL}+\alpha_{tE}=0$, $\alpha_{LE}+\alpha_{LL}=\alpha_{EL}+\alpha_{EE}=\alpha_{LE}+\alpha_{LL}=\alpha_{EL}+\alpha_{EE}=0$. Moreover, as the translog form provides a second-order approximation of the true cost function at a point, we have to specify this point. Following current practice we took the sample means of the included variables as the relevant approximation point. Therefore, all variables, except the cost variable, were divided by their mean. Taking into account previous remarks the translog variable cost function may be rewritten as:

$$\ln c_V-\ln p_E^* = \alpha_o+\alpha_Y \ln Y^*+ \tfrac{1}{2}\alpha_{YY}(\ln Y^*)^2 + \alpha_L(\ln p_L^* - \ln p_E^*)$$

$$+ \frac{1}{2}\alpha_{LL}(\ln p_L^* - \ln p_E^*)^2 + \alpha_{YL}\ln Y^*(\ln p_L^* - \ln p_E^*)$$
$$+ \alpha_T \ln T^* + \frac{1}{2}\alpha_{TT}(\ln T^*)^2 + \alpha_{TL}\ln T^*(\ln p_L^* - \ln p_E^*)$$
$$+ \alpha_{YT}\ln Y^* \ln T^* + \alpha_t t^* + \alpha_{tY} t^* \ln Y^* + \alpha_{tT} t^* \ln T^*$$
$$+ \alpha_{tL} t^*(\ln p_L^* - \ln p_E^*) + \mu$$

where starred variables are divided by their sample mean, e.g. $Y^* = \frac{Y}{\bar{Y}}$ with $\bar{Y}$: sample mean.

To make estimation of the cost function feasible some minor adjustments have to be made. Without additional information it's likely to obtain inefficient parameter estimates due to the high correlation between many of the cross-products among explanatory variables. In order to reduce this problem and to use additional information without introducing new parameters we consider estimating the cost function jointly with the factor share equations. These are derived using Sheppards lemma:

$$\frac{\partial C_V}{p_L^*} \frac{p_L^*}{C_V} = s_L = \alpha_L + \alpha_{YL}\ln Y^* + \alpha_{LL}(\ln p_L^* - \ln p_E^*) + \alpha_{TL}\ln T^* + \alpha_{tL} t^*$$

$$\frac{\partial C_V}{\partial p_E^*} \frac{p_E^*}{C_V} = s_E = (1-\alpha_L) - \alpha_{YL}\ln Y^* - \alpha_{LL}(\ln p_L^* - \ln p_E^*) - \alpha_{TL}\ln T^* - \alpha_{tL} t^*$$

Since the factor shares sum to unity use of both shares would introduce linear dependency in the data so that the variance covariance matrix of the error terms would be singular. Moreover, it's quite probable that the error terms of the cost function and the share equations correlate due to the large number of common explanatory variables. To account for these problems a correct procedure is to use a modification of the Zellmer technique for seemingly unrelated regressions. The method assumes that the expected value of the error term of each relation is zero and that the errors of different relations in different years are uncorrelated. It allows, however, the contemperaneous covariance between error terms in different equations to be nonzero. In order to avoid the singularity of the variance-covariance matrix one of the factor share equations is deleted before carrying out the second stage of the Zellmer procedure. It has been shown that the resulting parameter estimates have the same asymptotic properties as maximum likelihood estimates (Berndt et alii (1974)). Moreover the results of the procedure are insensitive to which factor share equation is deleted prior to the second stage.

The economic properties of the translog cost function are easily reviewed. The partial elasticity of substitution between labor and energy is given by (*):

$$\sigma_{LE} = \frac{-\alpha_{LL} + s_E s_L}{s_E s_L}$$

The input price elasticities for energy and labor are (**):

$$k_E = \frac{\alpha_{LL} + s_E(s_E - 1)}{s_E}$$

$$k_L = \frac{\alpha_{LL} + s_L(s_L - 1)}{s_L}$$

Finally, estimation of the translog cost model allows us to test whether the cost structure satisfies certain economic properties. The underlying technology is homothetic if and only if the cost function is separable in output. Our results would be compatible

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(*) See e.g. Uzawa (1962). The general formula is $\sigma_{ij} = \frac{C\frac{\partial^2 C}{\partial p_i \partial p_j}}{\frac{\partial C}{\partial p_i}\frac{\partial C}{\partial p_j}}$

This leads for our particular cost function to the value σ_{LE} given in the text.

(**) Note that for our two variable input model $s_L + s_E = 1$ which implies a fixed relation between k_L and k_E.

with a homothetic technology only if $\alpha_{YL} = 0$. This would imply that the cost elasticity with respect to output is independent of factor prices. A more severe restriction is homogeneity; this corresponds to assuming a constant output cost elasticity. This property would imply $\alpha_{YL} = \alpha_{YY} = \alpha_{YT} = \alpha_{Yt} = 0$. It can further be tested whether the elasticity of substitution equals one; we would then have $\alpha_{LL} = 0$. Finally, if the Cobb-Douglas cost function were a reasonable description of technology we would observe all previous conditions together.

Before turning to the estimation results it's useful to consider several problems that might be associated with estimating cost functions. These might lead to biased parameters. A first source of possible bias results from the classical "regression fallacy", as noted by Borts (1960). If costs are based upon a planned output level and differences between actual and planned output occur in practice the firm might be unable to adapt completely to these unforeseen circumstances. The regression fallacy implies overestimated marginal costs. A second possible source of bias is due to measurement errors in the variables used in the estimation procedure. Although this is a problem in all applied econometric work it's especially important in the transport sector. It's well known e.g. that the output data published in transport firms annual reports are in many cases determined using very crude methods. As a consequence measurement errors are to be expected.

Another more important difficulty with estimating cost functions concerns the assumptions of the methodology involved. Although the firm is allowed to be in disequilibrium with respect to the fixed capital stock in the short-run it's assumed that minimisation of variable costs is its main objective. If this is not the appropriate goal of the firm's management inefficiencies might bias the choice of inputs away from the cost minimising mix and the firm might be operating off its cost curve. As a consequence the parameters and hence the technological characteristics estimated from econometric cost relations will be subject to specification bias due to misspecification of the behavioral model. Moreover, the classical duality principles are no longer generally valid. Under these circumstances the estimated cost relation will reflect the firm's behavior rather than its technology (*). If one considers the bias introduced by possible deviations from cost minimising input use to be especially harmful, an alternative and more realistic behavioral assumption as to the nature of the firm's objectives must be developed (**). This is, however, beyond the scope of this paper as we are mainly interested in the construction of productivity indices (***).

(*) The methodology for correcting this type of possible bias is in its infancy. In an admirable foray Fuss and Waverman (1978) suggest how to estimate cost functions for the Averch-Johnson type firm operating under rate of return regulation. However, as this behavioral assumption is more unrealistic for the firm under investigation in this paper than the cost minimisation hypothesis, their methods do not seem very useful for our purposes.

(**) It has been suggested that the government uses the transport sector as an indirect means to achieve general social and economic objectives. This will not only affect the rate levels and the rate structure of the firm's output, but it might also influence the firm's input behavior. If this is the case the "shadow prices" of inputs upon which the firm bases its decisions might be different from observed input prices. A plausible assumption would then be that the firm utilises optimal input levels for the exogenous shadow prices they face. The costs actually incurred would be determined by these input demands and observed factor prices. Although it would be interesting to develop a cost model along these lines, this matter was not pursued in this paper.

(***) Note that, if cost minimisation is not the appropriate behavioral assumption, we would expect changes in efficiency to be reflected in our measured productivity indices.

4. Estimation results

The variable cost function discussed previously was estimated using time series data on output, input prices for labor and energy, a measure for the fixed capital stock and a time trend. The sample covers the period 1950-1979. We first turn to a brief discussion of the data used.

It has become quite common in recent studies (see e.g. Caves, Christensen and Tretheway (1980)) to consider passenger transportation as yielding several distinct outputs, e.g. passenger-miles and average trip length. This approach has the advantage to distinguish between the cost effects of increases in passengers and changes in distance traveled. However, data limitations precluded this type of analysis in this paper. Indeed, the only reliable output measure available was the number of seat-kilometers provided (*). Although this is not a very common variable in transport studies it has the advantage of being a pure supply measure which suggests a pretty close relationship to costs. As a consequence, significant estimates of marginal costs may be expected. This is far less likely if we used e.g. passenger-kilometers which is determined by both supply and demand factors. Once the routes to be served and the departure schedules have been fixed, passenger-kilometers are almost exclusively determined by demand factors. As long as we are unable to distinguish between peak and off-peak travel in our cost analysis the overall marginal cost of additional passenger kilometers is likely to be very low. A strong relation between costs and this output measure is not to be

(*) The bus company publishes passenger-kilometers and the number of passengers in its annual reports. However, these figures include output produced by private companies to whom the NMVB subleases some of its services. Of course the costs of this transport do not enter the NMVB's accounts. This implies that using the published data on passenger-kilometers or the number of passengers would give misleading cost-output relations.

expected and accurate estimates of marginal costs are doubtful. A further advantage of using seat-kilometers is the fact that this variable is probably subject to less measurement errors than demand determined output measures. This reduces the problem of errors in variables discussed before. A disadvantage of our output measure is probably that it's not straightforward to use the resulting marginal costs in the analysis of pricing policies. Precisely because seat-kilometers is supply determined there's no obvious relation between this variable, the price paid by users of the bus system and the demand for bus transportation. Thus a price rise e.g. may be matched by no reduction in seat-kilometers and costs at all, which makes the analysis of pricing policies rather complicated. However, as the main emphasis of this study is on the development of productivity indices we do not consider our output measure as a major disadvantage.

The prices for energy and labor were derived from the NMVB annual reports by dividing total expenditures on these inputs by the quantities used (*). Our variable cost figures were taken from these reports as well. In order to obtain a cost variable consistent with the theoretical analysis previously described only labor and energy costs were included. All cost and price variables were deflated using the wholesale price index.

As a measure for the capital stock we used the size of the bus fleet. Contractual arrangements between the firm and the producer of the vehicles suggests that in the short-run the stock of buses can indeed be assumed to be fixed. The lags between order and delivery are often quite long; over the period 1950-1979 these lags were between 2 and 5 years. Although the bus company has the possibility of changing its fleet size even in the short-run by a flexible policy of selling and buying on

(*) By using aggregate inputs labor and energy we implicitly assume input separability. This implies that we are assuming e.g. that the marginal rate of substitution between "line-haul" labor and "line-haul" energy is the same as between "line-haul" labor and "maintenance" energy. Data limitations precluded the use of more disaggregated inputs or activities to relax these assumptions.

the second-hand market and by changing its depreciation behavior, this is not very common practice (*). Therefore, following Williams (1979), we assumed the fleet size to be the fixed factor in our variable cost model (**).

We applied the modified Zellner procedure discussed in the previous section. Our initial regression results still suffered from multicollinearity and autocorrelation in both the cost function and factor share equations so that the assumption underlying the procedures were not met. Therefore we also reestimated the variable cost function after taking the total differential of the original relation. This procedure doesn't change the set of parameters to be estimated except that α_0 disappears and α_t becomes constant in the transformed equation. As far as the estimated parameters are concerned the results of both procedures were surprisingly similar but the transformed relation showed somewhat smaller variances for the estimated coëfficiënts. The results for the transformed equation are given in tabel 1 (***).

We observe that most parameters exceed their standard error by a factor larger than two. The explanatory power of the translog function appears to be satisfactory as well. Several interesting features emerge from the results. First of all the hypothesis of homotheticity is not contradicted as α_{YL} is not significantly different from zero. However the assumption of a constant cost elasticity with respect to output appears to be unreasonable as both α_{YY} and α_{tY} are highly significant. These results imply that the output cost elasticity varies with changes in output but not with variations in factor prices. The variability of the cost elasticity will be discussed in greater detail below. It further follows from the

(*) For a discussion of this point see De Borger and Deloddere (1980), p. 5-6.

(**) It should be noted that even if this assumption is not entirely correct no bias is introduced in our estimates. Spady and Friedlaender (1978) have indicated that treating as fixed a factor which is actually variable involves no misspecification. Econometric results will imply in that case that the factor is used at its optimal level.

(***) The transformation solved both the multicollinearity and the autocorrelation problem.

Table 1: parameters of the variable cost function

parameters	estimate	standard error	t-value
α_Y	- 1.388	0.029	- 47.86
α_{YY}	- 3.463	1.231	- 2.82
α_L	0.892	0.074	12.04
α_{LL}	0.046	0.015	3.07
α_{YL}	- 0.004	0.007	- 0.54
α_T	- 0.769	0.343	- 2.24
α_{TT}	0.001	0.001	0.96
α_{TL}	0.168	0.083	2.04
α_{TY}	0.003	0.001	3.14
α_t	- 0.018	0.008	- 2.22
α_{tY}	- 0.170	0.053	3.22
α_{tL}	- 0.0005	0.0003	- 1.53
α_{tT}	- 0.007	0.02	0.32

R^2 cost relation = 0.95

R^2 factor share = 0.96

estimates that the hypothesis of unitary elasticity of substitution is not supported by the data i.e. $\alpha_{LL} \neq 0$. It's obvious that this rules out the use of the Cobb-Douglas cost function as a reasonable description of the cost behavior of the firm (*).

Apart from the previous observations the parameters of the cost function have no clear direct interpretation. Their importance lies in their being estimates of the gradients and the Hessian of the true underlying cost function. This allows us to derive a set of relevant implications concerning the economic structure of the firm. Table 2 summarizes the main results. It contains an annual evaluation of the elasticity of substitution, the price elasticities of the firm's demand for labor and energy, the estimated marginal and average variable costs and the parameter D which is an indicator of the degree of economics of density. This measure was defined by Christensen and Greene (1976) as

$$D = 1 - \frac{\partial \ln C_V}{\partial \ln Y} .$$

A positive value (or an elasticity of costs with respect to output below unity) indicates economics of density i.e. variable costs increase less than proportionately with increases in output, given a fixed capital stock. In this sense economics of density are the short-run analogue of scale economics (**).

The most striking result is certainly the volatile evolution of marginal costs and economics of density over time.

(*) Note that the previous discussion is based on the estimated parameters. No formal likelihood ratio tests, as described in Berndt et alii (1974), were attempted.

(**) It's not surprising, therefore, to find a relation between D and the measure R of returns to scale. It's easy to show

$$R = (1 - \frac{\partial \ln C_V}{\partial \ln T})/(1-D).$$

Table 2:

Year	Elasticity of substitution σ_{LE}	Price elasticity of labor demand k_L	Price elasticity of energy demand k_E	Marginal cost	Average variable cost	Economics of Density D
1950	0,694	- 0.128	- 0.567	0.223	0.188	- 0.187
51	0,696	- 0.129	- 0.567	0.197	0.156	- 0.262
52	0,701	- 0.133	- 0.568	0.224	0.174	- 0.291
53	0,703	- 0.135	- 0.568	0.225	0.176	- 0.447
54	0.700	- 0.133	- 0.568	0.218	0.164	- 0.332
55	0.686	- 0.122	- 0.564	0.146	0.150	0.024
56	0.679	- 0.117	- 0.561	0.061	0.138	0.556
57	0.680	- 0.118	- 0.561	0.046	0.138	0.666
58	0.660	- 0.107	- 0.554	0.031	0.141	0.779
59	0.654	- 0.104	- 0.551	0.051	0.137	0.625
1960	0.627	- 0.090	- 0.537	0.032	0.127	0.746
61	0.606	- 0.082	- 0.524	0.049	0.126	0.608
62	0.555	- 0.065	- 0.490	0.033	0.123	0.732
63	0.541	- 0.061	- 0.480	0.023	0.124	0.811
64	0.517	- 0.055	- 0.462	0.029	0.123	0.767
65	0.496	- 0.050	- 0.445	0.061	0.130	0.531
66	0.490	- 0.049	- 0.441	0.103	0.141	0.271
67	0.483	- 0.048	- 0.436	0.165	0.149	- 0.107
68	0.482	- 0.047	- 0.435	0.224	0.154	- 0.456
69	0.462	- 0.044	- 0.419	0.269	0.156	- 0.720
1970	0.420	- 0.036	- 0.385	0.308	0.163	- 0.892
71	0.413	- 0.035	- 0.378	0.400	0.196	- 1.046
72	0.351	- 0.027	- 0.324	0.451	0.209	- 1.156
73	0.326	- 0.024	- 0.302	0.473	0.207	- 1.281
74	0.401	- 0.034	- 0.368	0.499	0.210	- 1.378
75	0.420	- 0.036	- 0.384	0.602	0.247	- 1.441
76	0.374	- 0.030	- 0.344	0.612	0.243	- 1.517
77	0.373	- 0.029	- 0.343	0.779	0.286	- 1.725
78	0.316	- 0.023	- 0.293	0.870	0.314	- 1.771
79	0.409	- 0.035	- 0.374	0.929	0.319	- 1.916

However, taking into account the NMVB's policy over the period 1950-1979 these results are not at all implausible. First of all the firm reduced substantially the size of its bus fleet in the early fifties and seemed to be unable to increase output without incurring important additional costs. However, during the period 1954-1965 the firm gradually adopted the one-man-car (OMC) exploitation system. This resulted in excess labor which could be used to expand output without much additional cost. The firm exploited the extremely low marginal cost over this period and the resulting strong economics of density to increase output substantially. New routes were served and on a lot of existing routes frequencies were augmented.

In the late sixties the evolution towards OMC exploitation was completed. The resulting increase in marginal costs was reinforced by rising inflation in labor and especially energy input prices during the last decade. From table 2 we observe highly increasing marginal costs over this period with diseconomics of density growing very strong in most recent years. It's clear that total variable costs increase more than proportionately with output expansion. Also note from the table that marginal cost was below average variable cost over the complete period 1955-1966, but that it largely exceeded average variable cost during the last decade.

A second piece of useful information concerns the price elasticities of the firm's demand for labor and energy and the associated elasticity of substitution between these inputs. First note that all three series are slightly trended over time. This is due to a special feature of the translog function which causes these elasticities to depend on the factor shares of the relevant inputs. Over the period 1950-1979 a trendwise increase in the cost share of labor and a trendwise decrease in the cost share of energy were observed which results in the trend in the calculated elasticities. However, the figures in table 2 are plausible. The elasticity of substitution appears to be substantially below unity in all years, confirming the inappropriateneness of the Cobb-Douglas specification (*). The demand for energy and especially for labor is very inelastic. These findings are consistent with previous research (e.g. Viton (1980)) and may be explained by the firm's contractual obligations. The higher elasticity for energy demand probably reflects the firm's greater flexibility in negotiating its energy contracts. Whereas labor contracts are usually negotiated over periods of several years changes in energy contracts can be accomplished within a much shorter time period. A lower elasticity for the demand for labor is also suggested by the fact that downward changes in the labor force are usually achieved by not replacing retiring workers rather than by direct lay-offs. This creates adjustment lags in the demand for labor which may be reflected in low price elasticities (**).

(*) We used simple t-test and found the elasticity of substitution to be significantly different from one over the complete sample period. Assuming that the cost shares of the inputs are nonstochastic (i.e. using observed rather than predicted shares) the standard error of the estimated elasticity of substitution can be calculated to be $\frac{1}{s_L s_E}\ \sigma_{\alpha_{LL}}$, where $\sigma_{\alpha_{LL}}$ is the standard error of α_{LL}.

(**) It should be admitted that the estimated input price elasticities for labor and energy were not always significantly different from zero during the period 1970-79. We calculated the standard errors of the input price elasticities to be $\frac{\sigma_{\alpha_{LL}}}{s_j}$, j=L, E, where $\sigma_{\alpha_{LL}}$ is the standard error of the parameter α_{LL}. Using t-tests we found the estimated price elasticities to be different from zero for all years between 1950 and 1970, but not for all years during the last decade.

5. Productivity indices for regional bus transportation in België

In this section we present productivity indices for regional bus transportation in Belgium, calculated on the basis of the cost function analysed in the previous paragraph. For each year in our sample we evaluated the expressions (8) and (9). Then, using the values for 1950 as basis, we transformed these figures into indices. Let's first take a look, however, at the basic inputs for the calculations. Table 3 contains the elasticities of variable cost with respect to output and capital stock and the shift in the cost function $\frac{\partial \ln C_V}{\partial t}$. The latter is of considerable importance in its own right. It may be interpreted as the change in variable costs that cannot be attributed to variations in output, capital stock and the real price of the inputs labor and energy. As this interpretation comes close to the intuitive notion of productivity we presented the shift in the cost function in index form as well (*). It follows from table 3 that large shifts in variable costs occured over the period 1950-1979. However these shifts were much more pronounced during the fifties than in later decennia.

It may be observed that the cost elasticity with respect to capital stock is negative over the complete sample period. This is consistent with long-run substitution patterns between capital and the variable inputs labor and energy. Although negative values would be implied by cost minimising behavior of the firm, these findings are of course insufficient to accept minimisation of total costs as a behavioral hypothesis.

(*) The shift in variable costs would be a correct indicator of productivity under several special circumstances. First note that $-\frac{\partial \ln C_V}{\partial t} = P_1$ if bus transport were caracterised by constant returns to density. Moreover, the shift in variable costs multiplied by the fraction of total costs which are actually variable would be a correct indicator of productivity changes if the firm minimised total costs. Under this assumtpion it's easy to show $\frac{\partial \ln C_V}{\partial \ln T} = -\frac{P_T T}{C_V}$, where P_T is the price of capital so that $-\frac{\partial \ln C_V}{\partial t} \frac{C_V}{C_T} = P_2$.

Table 3:

Year	Variable cost elasticity with respect to output	Variable cost elasticity with respect to capital stock	Shift in variable costs : $\frac{\partial \ln C_V}{\partial t}$ (index
1950	1.187	- 0.924	100
51	1.262	- 0.909	107.5
52	1.291	- 0.910	114.8
53	1.447	- 0.898	121.8
54	1.332	- 0.907	128.5
55	0.976	- 0.898	134.2
56	0.444	- 0.890	138.4
57	0.334	- 0.893	141.4
58	0.221	- 0.870	143.7
59	0.375	- 0.863	145.6
1960	0.254	- 0.844	147.2
61	0.392	- 0.835	148.4
62	0.268	- 0.799	149.1
63	0.189	- 0.787	149.2
64	0.233	- 0.777	148.9
65	0.469	- 0.768	148.5
66	0.729	- 0.753	148.3
67	1.107	- 0.745	148.4
68	1.456	- 0.748	149.0
69	1.720	- 0.719	150.0
1970	1.891	- 0.705	151.1
71	2.046	- 0.695	152.2
72	2.156	- 0.664	153.2
73	2.281	- 0.656	154.0
74	2.378	- 0.713	154.8
75	2.441	- 0.722	155.2
76	2.517	- 0.711	155.5
77	2.725	- 0.702	155.6
78	2.771	- 0.682	155.7
79	2.916	- 0.730	155.6

The calculated productivity indices are presented in the first two columns of table 4. We mentionned before that the firm constructs its own productivity indicator in its annual reports. In order to compare our indices with the NMVB productivity index, this is presented in the third column as well. It's of the classical type output per unit of input as it's defined as the number of seat-kilometers provided per man-hour labor used. Comparison of our indices, which are based on economic concepts, with the index proposed by the firm will provide interesting information as to the bias involved in using more naive productivity indicators.

Let's first consider our calculated indices. Recall that these would have given the same results only if the firm had exhibited constant returns to scale over the whole sample period. It's clear from our analysis of the economics of density parameter D and the relation between the returns to scale measure R and D that this was not the case. As a consequence, large differences in absolute value between both indices shouldn't be surprising. However, as far as the evolution over time is concerned both series show grosso modo the same consistent pattern of productivity growth. They indicate a remarkable increase in productivity over the complete period 1950-1960 followed by a long period of very modest growth. The conclusion is obvious: nearly all productivity increases of the sample period 1950-1979 occured during the first decennium. During the sixties and seventies no substantial growth was achieved. Both indices even show periods of slight productivity decline.

The indicator used by the bus company implies quite a different time pattern. Although it indicates substantial productivity growth over the period 1950-1960, it shows large further increases during the next decennia except for most recent years, where the index slightly declines.

The differences in time pattern and absolute productivity growth as given by our three alternative indicators can more clearly be observed by considering the information summarized in table 5.

Table 4:

Year	Productivity index P_1	Productivity index P_2	Productivity index NMVB
1950	100.0	100.0	100.0
51	106.0	103.9	104.5
52	111.6	107.6	109.3
53	116.7	111.2	114.3
54	121.1	114.5	119.4
55	125.4	117.1	129.4
56	130.9	118.9	137.7
57	137.8	120.4	142.2
58	145.7	121.5	145.6
59	152.2	122.4	150.7
1960	156.6	123.1	161.2
61	159.9	123.7	162.3
62	161.2	123.9	164.4
63	160.5	123.9	172.9
64	158.6	123.6	179.5
65	157.5	123.5	182.3
66	157.4	123.4	185.1
67	157.7	123.7	188.0
68	158.4	124.1	193.3
69	159.1	124.7	196.8
1970	159.6	125.4	196.1
71	160.1	126.0	195.5
72	160.6	126.5	201.3
73	160.9	126.9	206.9
74	161.2	127.3	209.6
75	161.4	127.6	211.6
76	161.4	127.7	213.2
77	161.5	127.7	210.6
78	161.5	127.7	209.8
79	161.4	127.6	206.7

This table contains the annual growth rates implicit in the three alternative indices as well as the average growth rates for successive five year periods and for the complete period 1950-1979. Note that the overall growth rate as measured by the firm largely exceeds that implied by the calculated indices. The figures in table 5 confirm our main findings: our two alternative definitions of productivity growth lead to substantially different results in absolute terms (especially over the period 1955-65 characterised by strong economies of density), but they show approximately the same evolution over time. The index given by the NMVB indicates a completely different time path and substantially overestimated productivity growth over the period 1950-1975.

We now turn to the public policy implications of our findings. First of all, it appears that our results are not irrelevant in view of the importance of productivity growth in transport policy discussions. The argument of productivity increases is often used by transportation officials to shift the responsability of financial losses completely to the government. The latter has a major impact on the determination of the firm's output in terms of vehicle-miles. Therefore, it's often argued that given the transport services the government imposes the firm to offer to the public, productivity. Growth is an indication of efficient operation. Inreased productivity in the provision of transportation facilities might in turn weaken the public opposition against subsidy payments to compensate deficits. These remarks show that the development of reliable measures is not an irrelevant academic exercise. Moreover, in many cases productivity growth is a major issue in the wage negotiation process. The use of biased estimates of productivity shifts may affect the relative strength of the participating parties and maybe even influence the final outcome of the negotiations. Therefore, policy makers are likely to consider reliable productivity estimators as very useful.

The most important result for public policy is not the fact that the use of the NMVB index has led to remarkable overestimates of the growth in productivity over the past 30 years, nor that the time path implied by this index was substantially different from the evolution of our calculated indices. Indeed, policy makers are much more interested in obtaining accurate productivity measures in the future, rather than having information on how misleading the previously used index has been in the past. Therefore, we consider the development of reliable indicators on the basis of readily available statistical data as the main contribution of this paper. If we assume the parameters of the variable cost function to remain stable at least in the near future, the estimated function and the simple formulas derived in the text can be used to calculate better productivity growth rates than those presented in the firm's annual reports (*).

(*) Our procedure to derive productivity indices might seem rather cumbersome. The question arises whether there is no easier way to obtain acceptable approximations to our calculated productivity growth rates based upon the relation between our indices and the NMVB index. If such a relation leads to a reasonable approximation of our correct indices, policy analysts would probably prefer this procedure and consider our proposed method as being not worth the effort. In the appendix to this paper we investigate the relation between our index P_1 and the NMVB index more thoroughly using a set of appropriate statistical tools. The results indicate, however, that the relation between the growth rates implied in both indices is so weak that it's impossible to obtain acceptable productivity measures on the basis of the growth rate of the NMVB index.

Table 5 a)

Year	Annual Productivity growth according to index P_1	Annual Productivity growth according to index P_2	Annual Productivity growth according to index NMVB
1951	6.0	3.9	4.5
52	5.3	3.6	4.6
53	4.5	3.4	4.6
54	3.8	2.9	4.5
55	3.5	2.3	8.4
56	4.4	1.6	6.4
57	5.2	1.2	3.3
58	5.7	0.93	2.4
59	4.5	0.77	3.5
1960	2.9	0.60	7.0
61	2.1	0.42	0.7
62	0.76	0.20	1.3
63	- 0.44	- 0.05	5.2
64	- 1.14	- 0.17	3.8
65	- 0.72	- 0.14	1.6
66	- 0.08	- 0.02	1.5
67	0.25	0.18	1.6
68	0.39	0.37	2.8
69	0.41	0.48	1.8
1970	0.38	0.51	- 0.4
71	0.33	0.49	- 0.3
72	0.28	0.44	3.0
73	0.22	0.38	2.8
74	0.16	0.28	1.3
75	0.09	0.16	0.9
76	0.05	0.09	0.8
77	0.03	0.05	0.0
78	- 0.01	- 0.01	- 1.6
79	- 0.03	- 0.06	- 1.5

Table 5 b)

Period	Average growth rates according to		
	P_1	P_2	NMVB
1950-54	4.90	3.45	4.55
1955-59	4.66	1.13	4.80
1960-64	0.84	0.20	3.60
1965-69	0.05	0.17	1.86
1970-74	0.27	0.42	1.28
1975-79	0.03	0.05	- 0.28
1950-79	1.68	0.84	2.54

6. Summary and conclusions:

In this paper we applied the methodology recently developed by Caves, Christensen and Swanson (1981) to calculate productivity indices for regional bus transportation in Belgium. The procedure involved two steps: first we estimated a flexible variable cost function to describe the cost and production structure. In a second step the estimation results were used to calculate two different indicators of productivity growth over the period 1950-1979.

The translog variable cost function was the flexible functional form chosen in this paper. Our main findings concerning the cost structure were the volatile evolution of both the economies of density parameter and the associated marginal cost. These were explained in terms of the NMVB's policy towards one-man-car exploitation during the period 1950-1965 and the rapidly increasing factor prices for labor and energy in recent years. Our results further indicated the elasticity of substitution to be significantly below unity, ruling out the use of simpler functional forms such as Cobb-Douglas to describe the cost and production structure in regional bus transportation. The price elasticities for energy and especially labor were found to be low, reflecting contractual obligations of the bus company.

In the second part of the paper we used two alternative definitions of productivity growth to calculate indices based upon our estimated variable cost function. The resulting productivity indices showed substantial growth over the period 1950-1960. However, no significant productivity increases were observed in later periods i.e. the growth in productivity over the period 1950-79 is almost exclusively due to the increases in the first decennium. The calculated indices were compared with a more naive indicator of productivity used by the bus company. The latter implied average growth rates which largely exceeded those implicit in our proposed indices. Moreover, the NMVB index showed a completely different time pattern, indicating quite large productivity increases during the sixties and the early seventies.

Finally we discussed the importance of reliable productivity indicators for public policy. In view of the role of productivity growth rates in policy discussions we would strongly argue for the construction of indices according to the method described in this paper. These can be used as alternative to the rather loosely defined productivity index presented in the firm's annual report.

Appendix

In this appendix we carefully analyse the relationship between the index used by the firm and our calculated productivity index P_1 (*). Our motivation was the following: suppose a very close relationship can be established between our index P_1 and the NMVB index. In that case we can probably exploit this relationship in order to obtain acceptable approximations of the time growth in productivity without calculating the latter. Indeed, it would be sufficient to substitute the observed NMVB figure into the relationship to obtain predictions of the correct growth rate in P_1. As a consequence, application of the method presented in this paper would not be necessary. The purpose of this appendix is to investigate whether a relation between both measures exists that allows us to determine the growth rate of P_1 on the basis of the NMVB index.

To ease the exposition let's consider some graphical representations of the relation between both indices and between their respective growth rates. Figures A1 and A2 are based upon the absolute levels of both indices. In figure A1 the evolution in time is indicated, whereas figure A2 contains the scatter diagram with the absolute levels of both indices on the axes. Similar graphs are presented for the growth rates of both indices in figures A3 and R4. Two results emerge from these figures: first, both indices are very closely related as far as their absolute levels are concerned. Figure A2 suggests that a log-log or a semi-log relation is likely to give the best fit to the data. Second, the growth rates of both indices are weakly related.

(*) We focus our attention on the relation between P_1 and the firm's index for several reasons. Although both P_1 and P_2 are valid productivity indicators the definition used to obtain P_2 is quite artificial in the single output case. Therefore the definition behind P_1 seems easier to understand intuitively. Moreover, the index P_1 is clearly the most conservative indicator over the complete period 1950-79 and as such compares most favorable with the NMVB index.

Figure A4 suggests that reasonable predictions of the growth rates in P_1 on the basis of an estimated relation between the growth rates of both indices are not to be expected.

These intuitive observations are confirmed by a formal analysis on the basis of some appropriate statistical tools. First we estimated the relation between the absolute levels of both indices. The best regression results were obtained for the following semi-log relation (*).

$$P_1 = \underset{(25.21)}{-255.42} + \underset{(4.93)}{79.05} \log (NMVB) \qquad R_C^2 = 0.96$$

We used this regression to predict the index P_1 on the basis of the NMVB index. From the predicted P_1-series we calculated the implied growth rates.

We used a set of statistical measures to determine the ability of the estimated equation to predict the index P_1 and the observed growth rates of P_1. In table A1 we give the mean and the standard deviation of observed and predicted values, the correlation coefficient, the inequality coefficient and information on the mean squared error. Finally the table contains the classical division of the latter into unequal central tendency, unequal standard deviation and unequal covariance as proposed by Theil (1960). Part a) of the table gives these prediction statistics for the presented regression. Part b) gives the same statistics to test whether the growth rates from the predicted P_1 series accurately predict the observed growth rates of P_1.

(*) Standard errors are in parenthesis.

Table A1

Table A1 a)

$\bar{Y}_o$	= 148.13	$\bar{Y}_p$	= 148.57		
σ_o	= 19.36	σ_p	= 18.65		
r	= 0.99				
U	= 0.021				
MSE	= 37.37	RMSE	= 6.14	%RMSE	= 4.15 %
%UCT	= 0.50 %	%USD	= 1.34	%UCOV	= 98.15 %

Table A1 b)

$\bar{Y}_o$	= 1.68	$\bar{Y}_p$	= 1.48		
σ_o	= 2.27	σ_p	= 1.47		
r	= 0.66				
U	= 0.36				
MSE	= 2.98	RMSE	= 1.73	%RMSE	= 103 %
%UCT	= 1.34	%USD	= 21.48 %	%UCOV	= 77.18 %

<u>Definition of symbols</u>

$\bar{Y}_o$	: observed mean	$\bar{Y}_p$	: mean predicted series
σ_o	: observed standard error	σ_p	: standard error predicted series
r	: correlation coefficient		
U	: inequality coefficient		
MSE	: mean squared error	(%)RMSE	: (percentage) root mean squared error

%UCT, %USD, %UCOV: percentage unequal central tendency, unequal standard deviation, unequal covariance.

The results are easily interpreted. The absolute levels of P_1 are very well predicted by the regression equation. Correlation between observed and predicted values is extremely high and the percentage root mean squared error amounts to less than five percent. Moreover the errors are almost exclusively due to less than perfect correlation. However, the results in table A1 b) imply that prediction of the productivity growth rates in P_1 using the growth rates from the predicted P_1 series is completely unacceptable. Correlation is rather low and the percentage root mean squared error exceeds 100 %. This implies that in many cases the error involved in predicting true productivity growth is larger than the growth figure itself. Moreover, a substantial fraction of the errors can be attributed to unequal central tendency and standard deviation. Indeed, average and standard deviation are rather poorly predicted. The conclusion is clear: although the estimated relation between P_1 and the NMVB index gives very good predictions of the absolute level of our calculated index P_1, the relation is of very little use to predict accurately the productivity growth rates implied by P_1.

In a final exercise we tried to use an estimated relation between the growth rates of our index P_1 and the firm's index to predict the former. The best fit was obtained for the following linear relation.

$$\mathring{P}_1 = 0.34 + 0.52\,(\mathring{NMVB}) \qquad R_C^2 = 0.46$$
$$(0.51)\quad(0.37)$$

where o indicates growth rates. The regression was used to predict the growth rate in P_1 on the basis of observed growth rates in the NMVB index. The statistical information concerning the predictive quality of this regression is given in table A_2.

Table A2

$\bar{Y}_o$ = 1.68	$\bar{Y}$ = 1.68	
σ_o = 2.27	σ_p = 1.27	
r = 0.52		
U = 0.38		
MSE = 3.43	RMSE = 1.85	%RMSE = 110 %
%UCT = 0.01 %	%USD = 29.03 %	%UCOV = 70.96 %

As can be seen the ability of this relation to predict the growth rate in our "correct" productivity index P_1 is even less than in the previous case. The correlation is quite weak and the errors involved very substantial. Although the average is very precisely predicted, the variance of the predictions is much lower than the observed variance.

The general conclusion to be drawn from the previous analysis is obvious. There is no simple relation between our calculated index and the firm's productivity measure that allows us to predict the true growth in productivity without having to calculate it. As a consequence we strongly suggest the use of our slightly more complicated method to derive annual productivity growth rates.

Figure A.3.

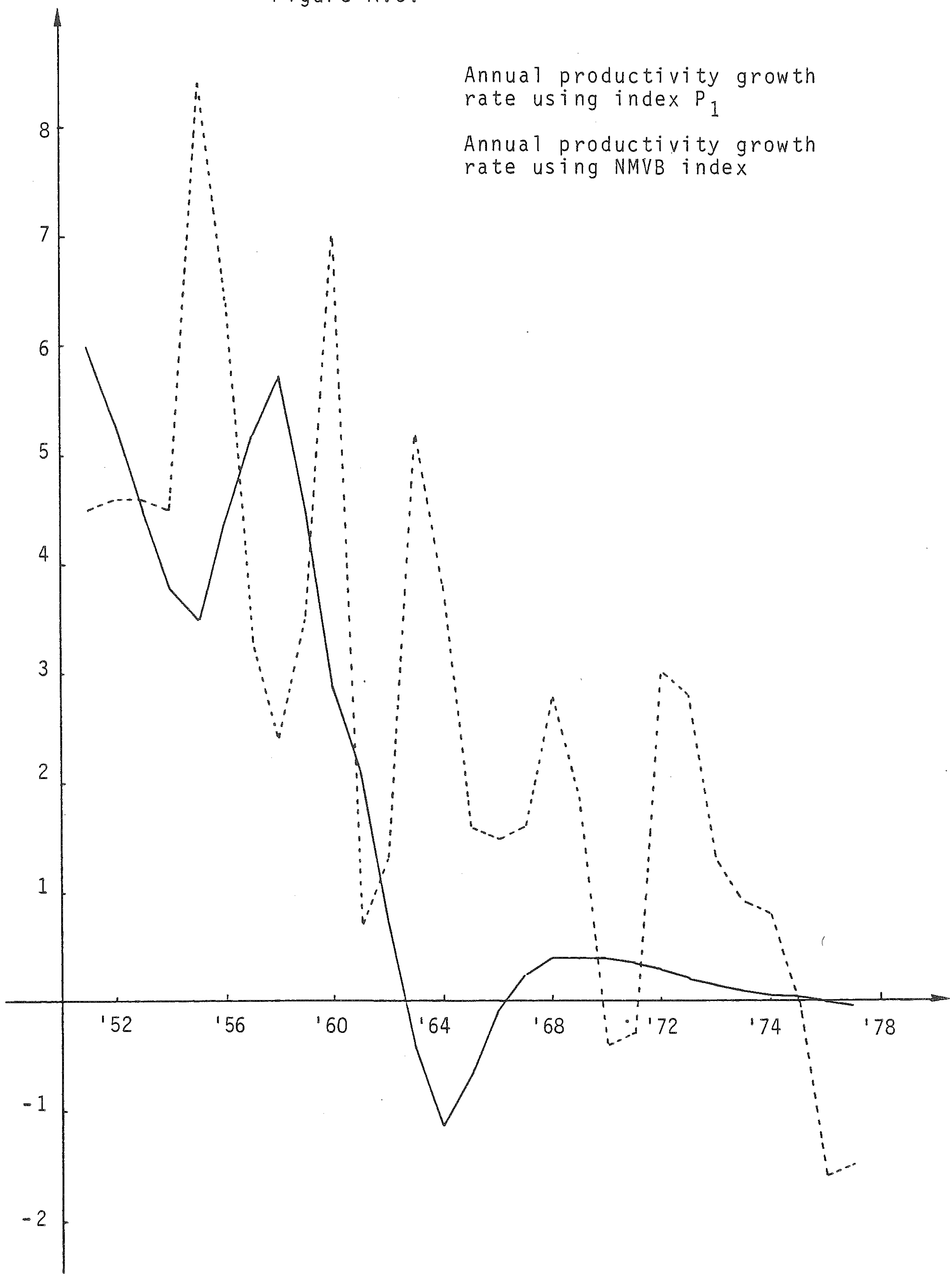

Annual productivity growth rate using index P_1
Annual productivity growth rate using NMVB index
8
7
6
5
4
3
2
1
-1
-2
'52
'56
'60
'64
'68
'72
'74
'78

Figure A.4.

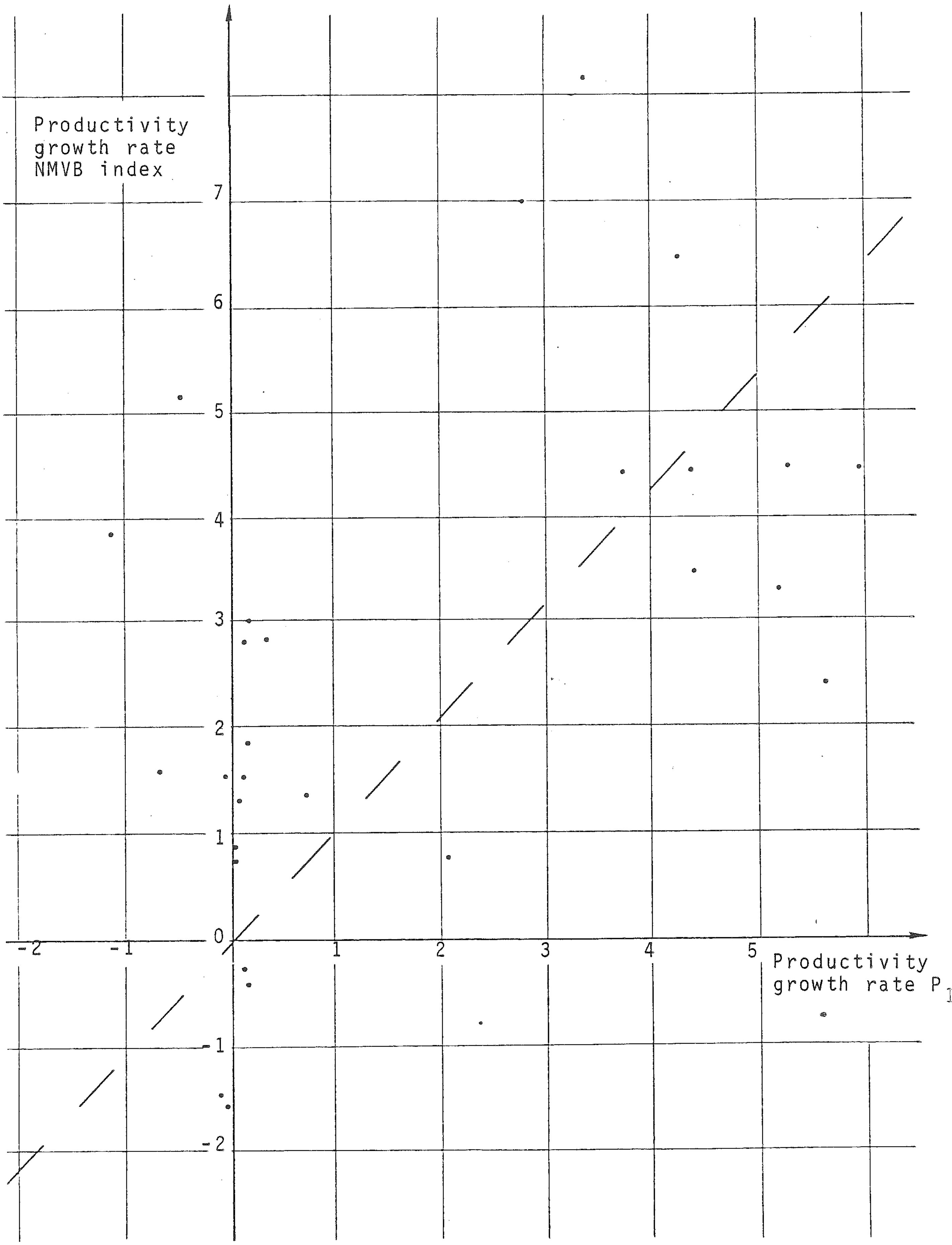
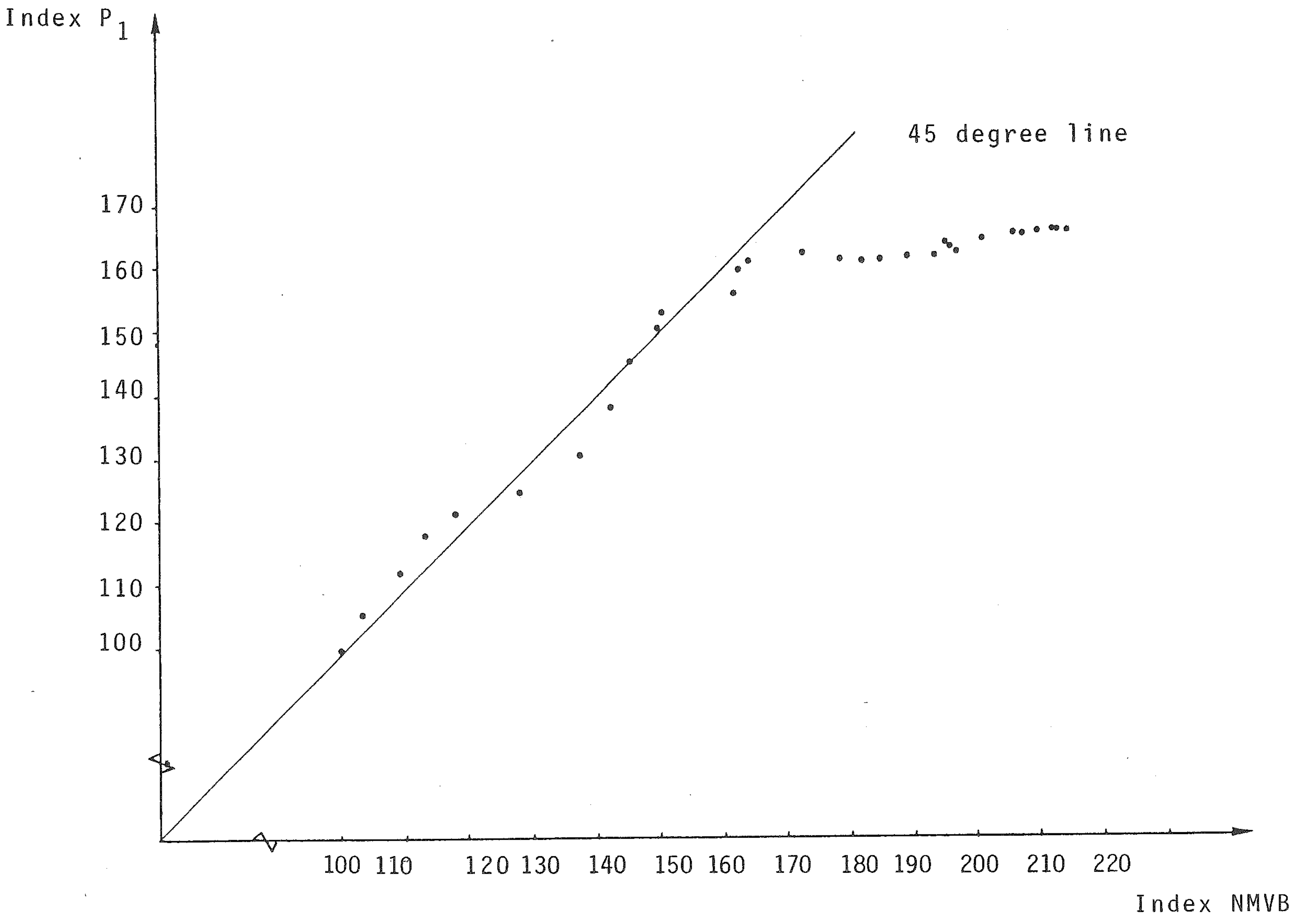
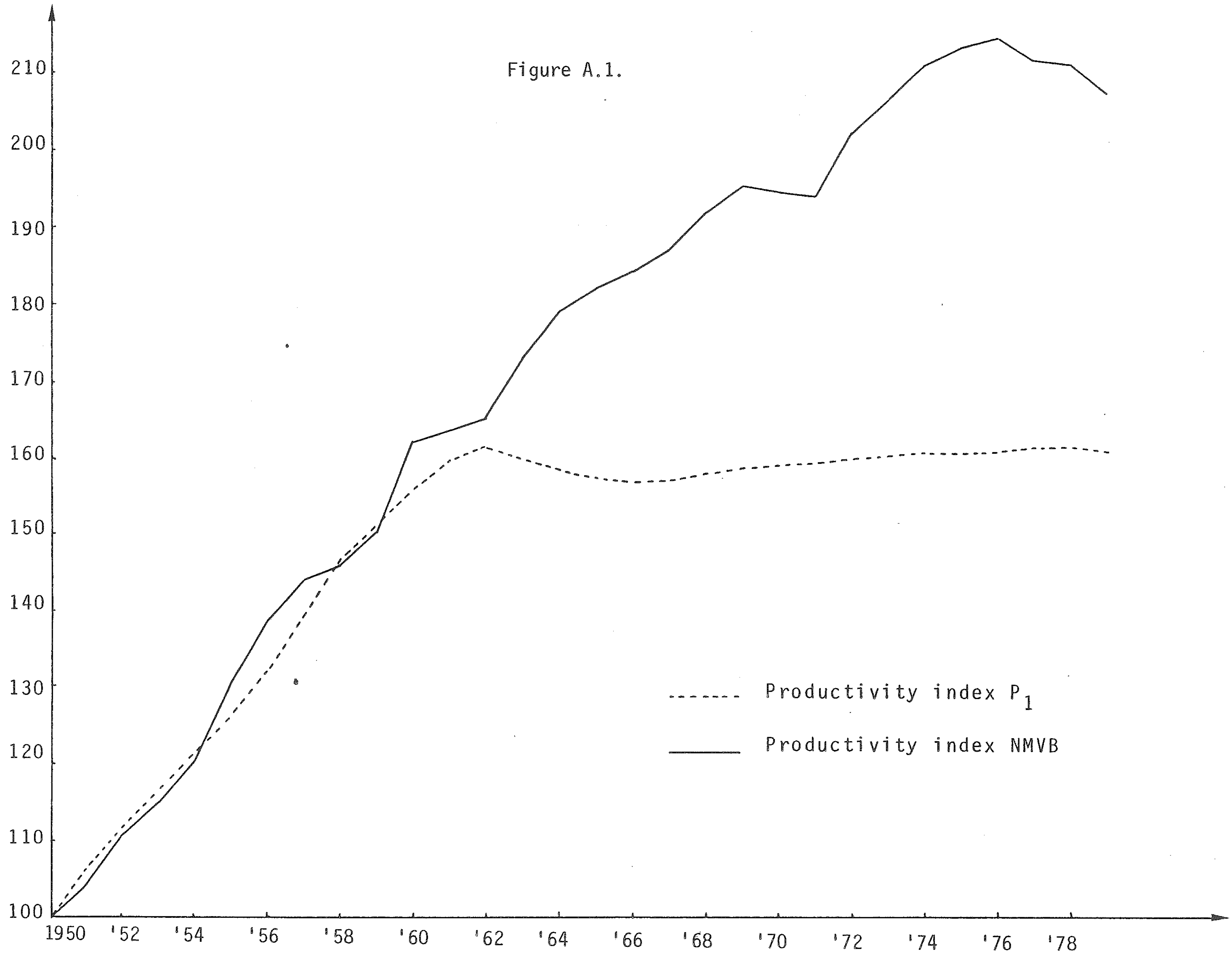

Productivity
growth rate
NMVB index
7
6
5
4
3
2
1
0
- 1
- 2
-2
-1
1
2
3
4
5
Productivity
growth rate P

Figure A.2.



Index P_1
45 degree line
170
160
150
140
130
120
110
100
100 110 120 130 140 150 160 170 180 190 200 210 220
Index NMVB

Figure A.1.


210
200
190
180
170
160
150
140
130
120
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