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CORRECT AND INCORRECT MEASURES OF THE DEADWEIGHT LOSS OF TAXATION

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Abstract

When defining a measure of the deadweight loss of taxation, it is important to make a distinction between two different approaches. In one approach, the deadweight loss is defined as the gain that could have been realized if all taxes had been lump sum. The other approach defines the deadweight loss as the loss that would remain if all tax receipts would be returned, in a lump sum way, to the consumer. These two approaches yield different loss measures. Moreover, within each of the two approaches, two loss measures can be defined, one based on the equivalent variation, and the other based on the compensating variation. It will be shown that all the measures based on the compensating variation are unreliable.

In their well-known article of 1974 {1} Diamond and Mc Fadden proposed a precise definition of the deadweight loss of taxation. In 1980 Kay {2} and Pazner and Sadka {4} independently showed that the loss measure proposed by Diamond and McFadden is unreliable. They then defined an alternative (reliable) measure based on the equivalent variation. In 1982 Zabalza {5} proposed another measure based on the compensating variation.

The purpose of the present paper is to critically review the issues covered in this literature. It will be shown that when defining a loss measure, it is important to make a distinction between two different approaches. In one approach, the deadweight loss of taxation is defined as the gain that could have been realized if all taxes had been lump sum. The other approach defines the deadweight loss as the loss that would remain if all tax receipts would be returned, in a lump sum way, to the consumer. These two approaches yield different loss measures. Moreover, within each of the two approaches, two loss measures can be defined, one based on the equivalent variation, and the other based on the compensating variation. It will be shown that all the measures based on the compensating variation are unreliable.

In section I we review some important properties of the equivalent and the compensating variation. Section II analyzes the two approaches one can follow when defining loss measures. Finally, in section III, we give an example to show that the loss measures based on the compensating variation are unreliable. We assume throughout a single consumer economy, with fixed producer prices.

I. Some preliminary results on the compensating and equivalent variation

In this section we want to review some properties of the compensating and equivalent variation which will prove useful in the following section. They have been discussed more extensively in reference {3}. Let there be 1+n commodities, the quantities of which are represented by a vector $(z,x) \varepsilon R_+^{1+n}$, where R_+^{1+n} is the nonnegative orthant of R^{1+n}. The quantity z is the quantity of a numéraire commodity, the price of which is identically equal to one. Prices of the other n commodities are given by a vector p which is strictly positive. Let the consumer's preferences be represented by a direct utility function U (z,x). We will assume that U is twice differentiable and strictly quasi-concave in R_+^{1+n}, and that its first order partial derivatives are positive in R_+^{1+n}.

If I represents the consumer's given (unearned) income, then by maximizing U subject to z + px = I one obtains the Marshallian demand functions z=z(p,I) and x=x(p,I). The indirect utility function is then given by V(p,I)=U{z(p,I), x(p,I)}. By solving U = V (p,I), for I, one obtains the expenditure function E(p,U). This function gives the minimum amount of income required to attain a utility level U, given a price vector p. It is increasing in p and U.

Consider now a given initial state (state o) of the consumer, characterized by a vector (p°, I°), and a corresponding utility level $U^\circ = V(p^\circ, I^\circ)$. Consider also various alternative states (p^i, I^i), i=1,2..., with utility levels $U^i = V(p^i, I^i)$.

The <u>compensating variation</u> (CV) resulting from the transition from state 0 to state i is then defined as that decrease (positive or negative) in income CV^{oi} for which

$$V(p^i, I^i - CV^{oi}) = V(p^o, I^\circ)$$

As one must obviously have

$$V(p^i, E(p^i, U^o)) = V(p^o, I^o),$$

it follows that

(1) $$CV^{oi} = E(p^i, u^i) - E(p^i, U^o)$$
$$= I^i - I^o + E(p^o, U^o) - E(p^i, U^o)$$

Similarly, <u>the equivalent variation</u> (EV) resulting from the transition from state 0 to state i is defined as that increase (positive or negative) in income EV^{oi} for which

$$V(p^o, I^o + EV^{oi}) = V(p^i, I^i)$$

As one must have that

$$V(p^o, E(p^o, U^i)) = V(p^i, I^i)$$

it follows that

(2) $$EV^{oi} = E(p^o, U^i) - E(p^o, U^o)$$
$$= I^i - I^o + E(p^o, U^i) - E(p^i, U^i)$$

As E is an increasing function of U, it follows from (1) and (2) that

(3) $$U^i \gtrless U^o \iff CV^{oi} \gtrless 0 \iff EV^{oi} \gtrless 0$$

In other words, the sign of CV^{oi} and of EV^{oi} informs us about the desirability of a move from state o to state i.

Suppose now the consumer is in state o, and in considering a move to state i or to state j. Will then the following equivalence hold?

(4) $$CV^{oi} \gtrless CV^{oj} \iff U^i \gtrless U^j$$

The answer is no. In other words, when evaluating alternative states of the economy, starting from a given initial state 0, it may happen that the state for which the CV is greatest is not the most desirable one in terms of utility.

This is the basic message of reference {3}. This reference also gives an example which shows that (4) is false. Other examples in the context of the dead weight loss of taxation will be given in section III.

On the other hand, the equivalence

$$(5) \qquad EV^{\circ i} \gtrless EV^{\circ j} \iff U^i \gtrless U^j$$

does hold. Indeed,

$$EV^{\circ i} = E(p^{\circ},U^i)-E(p^{\circ},U^{\circ}) \gtrless EV^{\circ j}=E(p^{\circ},u^j)-E(p^{\circ},U^{\circ})$$

$$\iff E(p^{\circ},U^i) \gtrless E(p^{\circ},U^j) \iff U^i \gtrless U^j$$

Therefore, when evaluating alternative states of the economy, starting from a given initial state o, it will always be true that the state for which the EV is greatest is also the most desirable one in terms of utility.

II Alternative measures of the deadweight loss of taxation

Using the notation of the previous section, let us define state O of the consumer as the state where there are no taxes. Prices are given by p°, equal to producer prices which are assumed to be constant. There are no lump sum taxes, and income is given by I°. The utility level achieved in this state is $U^{\circ}=V(p^{\circ},I^{\circ})$.

We now introduce a tax structure. A tax structure is defined by a vector (t,T^L) where t is a vector of commodity taxes (positive or negative) paid per unit of the commodities in the vector x, and where T^L is a lump sum tax (positive or negative).

The problem of measuring the deadweight loss of such a tax structure can be approached in two different ways. In the first approach, one assumes that a given amount of total tax revenue $\bar{T}$ has to be raised. One can then define all tax structures (t,T^L) which yield the required total revenue $\bar{T}$. These tax structures result in corresponding states for the consumer. These states are then compared with the state where $\bar{T}$ is raised only by lump sum taxes. In the second approach, one assumes that the tax revenue resulting from a tax structure is fully returned to the consumer in a lump sum way. The resulting state of the consumer is then compared with the initial state 0.

Let us start with the first approach. Assume that, starting from state 0, a given amount of taxes $\bar{T}$ has to be raised. We can then consider all tax structures (t,T^L) which will yield a total revenue equal to $\bar{T}$. These are all the tax structures satisfying

(6) $$T^L + T^C = \bar{T}$$

where T^C are the commodity taxes given by

(7) $$T^C = tx(p^\circ+t, I^\circ-T^L)$$

The resulting states for the consumer are then given by

(8) $$(p^\circ+t, I^\circ-T^L), \text{ with utility } V(p^\circ+t, I^\circ-T^L)$$

A special case occurs when all taxes are lump sum. This gives a tax structure $(0,\bar{T})$, which results in a state

(9) $$(p^\circ, I^\circ-\bar{T}), \text{ with utility } V(p^\circ, I^\circ-\bar{T})$$

Let us call (9) state 1, with corresponding utility level $U^1=V(p^\circ,I^\circ-\bar{T})$. State 1 will be the basic reference state with which all other states of type (8), with (t,T^L) satisfying (6) and (7), will be compared. The deadweight loss of a tax structure (t,T^L) is then defined as the gain that could have been realized if all taxes had been lump sum. In other words, it is the gain that would occur from the transition from a state of type (8) to state 1 given by (9).

Let (t^A, T^{LA}) be a tax structure satisfying (6) and (7), resulting in a state A

$$(p^\circ + t^A, I^\circ - T^{LA}), \text{ with utility } U^A = V(p^\circ + t^A, I^\circ - T^{LA})$$

The dead weight loss of taxation can then be measured either as (the negative of) the EV or (the negative of) the CV, resulting from the transition from state 1 to state A. Applying (2) and (1), this leads to the follwing definitions.

$$(10) \qquad -EV^{1A} = E(p^\circ, U^1) - E(p^\circ, U^A)$$
$$= E(p^\circ + t^A, U^A) - E(p^\circ, U^A) - T^{CA}$$

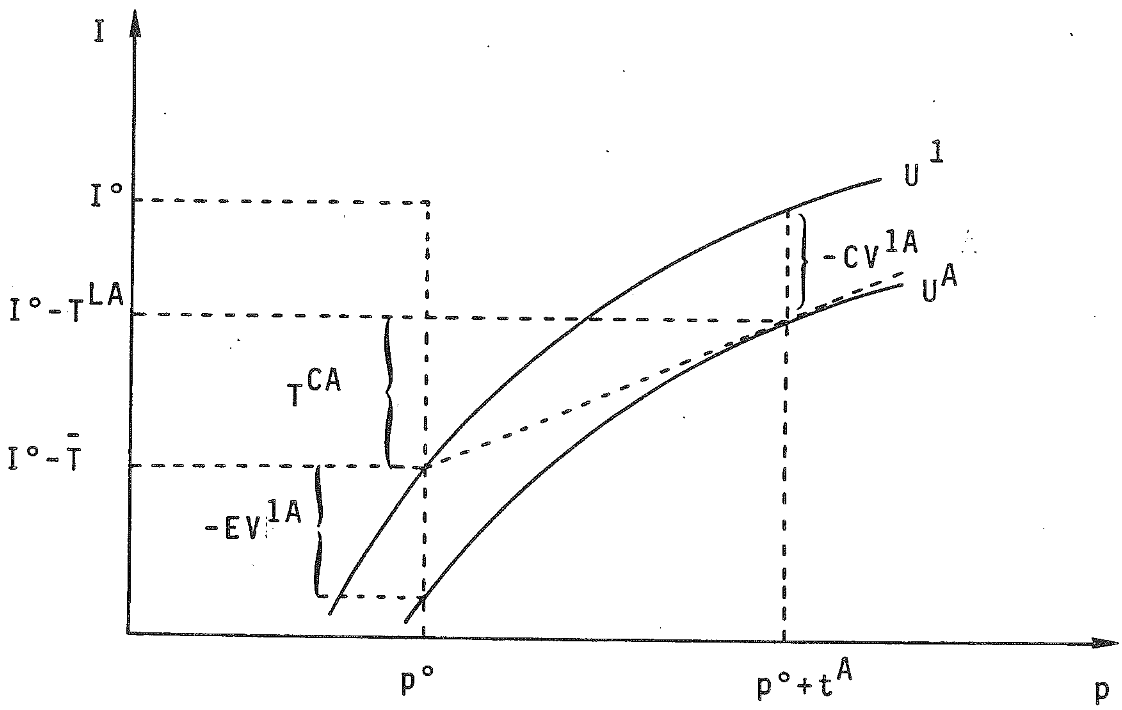
$$(11) \qquad -CV^{1A} = E(p^\circ + t^A, U^1) - E(p^\circ + t^A, U^A)$$
$$= E(p^\circ + t^A, U^1) - E(p^\circ, U^1) - T^{CA}$$

where, in both cases,

$$(12) \qquad T^{CA} = t^A x(p^\circ + t^A, I^\circ - T^{LA})$$

Both measures are illustrated on figure 1.

Figure 1



The curves U^1 and U^A represent the expenditure functions $E(p,U^1)$ and $E(p,U^A)$, or equivalently, they represent the two contours of $V(p,I)$ corresponding to the values U^1 and U^A. $E(p,U^1)$ passes through the point $(p^\circ, I^\circ - \bar{T})$, while $E(p,U^A)$ passes through the point $(p^\circ + t^A, I^\circ - T^{LA})$. The tax revenue from commodity taxation, T^{CA}, can be obtained geometrically as follows. By Shephard's lemma,

$$\frac{\partial E(p^\circ + t^A, U^A)}{\partial p} = h(p^\circ + t^A, U^A) = h(p^\circ + t^A, V(p^\circ + t^A, I^\circ - T^{LA}))$$

$$= x(p^\circ + t^A, I^\circ - T^{LA})$$

where h is th Hicksian compensated demand function. It then follows that

$$T^{CA} = (p^\circ + t^A - p^\circ)\,\frac{\partial E(p^\circ + t^A, U^A)}{\partial p} = t^A x(p^\circ + t^A, I^\circ - T^{LA}).$$

In other words, the tax revenue T^{CA} can be obtained by multiplying the slope of $E(p,U^A)$ in the point $(p^\circ + t^A, I^\circ - T^{LA})$ by $t^A (= p^\circ + t^A - p^\circ)$.

(10) is the loss measure as defined by Kay {2}, Pazner and Sadka {4}, and Zabalza {5}. (11) is the new loss measure proposed by Zabalza.

Let now (t^B, T^{LB}) be another tax structure satisfying (6) and (7), resulting in state B

$$(p^\circ + t^B, I^\circ - T^{LB}), \text{ with utility } U^B = V(p^\circ + t^B, I^\circ - T^{LB})$$

Using (10) and (11) we can calculate $-EV^{1B}$ and $-CV^{1B}$. It is then clear from equivalence (5) from section I that

(13) $$-EV^{1A} \gtrless -EV^{1B} \iff U^A \lessgtr U^B$$

This means that the deadweight loss as defined by the EV will be greatest for that tax structure which yields the lowest utility. In this sense it is a reliable loss measure. On the other hand, referring to our rejection of (4), it is clear that the equivalence

(14) $$- CV^{1A} \gtrless - CV^{1B} \iff U^A \lessgtr U^B$$

does not hold. Hence, the loss measure defined by the CV is not reliable. An example of a case where the dead weight loss as measured by the CV, is greatest for that tax structure which yields the highest utility is given in section III.

Let us now consider the second approach. In this approach, we assume that the tax revenue resulting from a tax structure (t,T^L) is fully returned to the consumer in a lump sum way. This result in a state

(15) $$(p°+t,\ I°+T^C),\ \text{with utility level } V(P°+t,\ I°+T^C)$$

where T^C is now given by

(16) $$T^C = tx(p°+t, I°+T^C)$$

Note that this value of T^C will, in general, differ from the value defined in (7). States of type (15) can then be compared with the initial state 0. The dead weight loss of a tax structure (t,T^L) is then defined as the loss that remains if all tax revenue is returned to the consumer. In other words, it is the loss that would remain after the transition from state 0 to a state of type (15).

Let $(t^a,\ T^{La})$ be a tax structure satisfying (16), resulting in a state a

$$(p°+t^a,\ I°+T^{Ca}),\ \text{with utility } U^a = V(p°+t^a,\ I°+T^{Ca})$$

where

$$T^{Ca} = t^a x(p°+t^a,\ I°+T^{Ca})$$

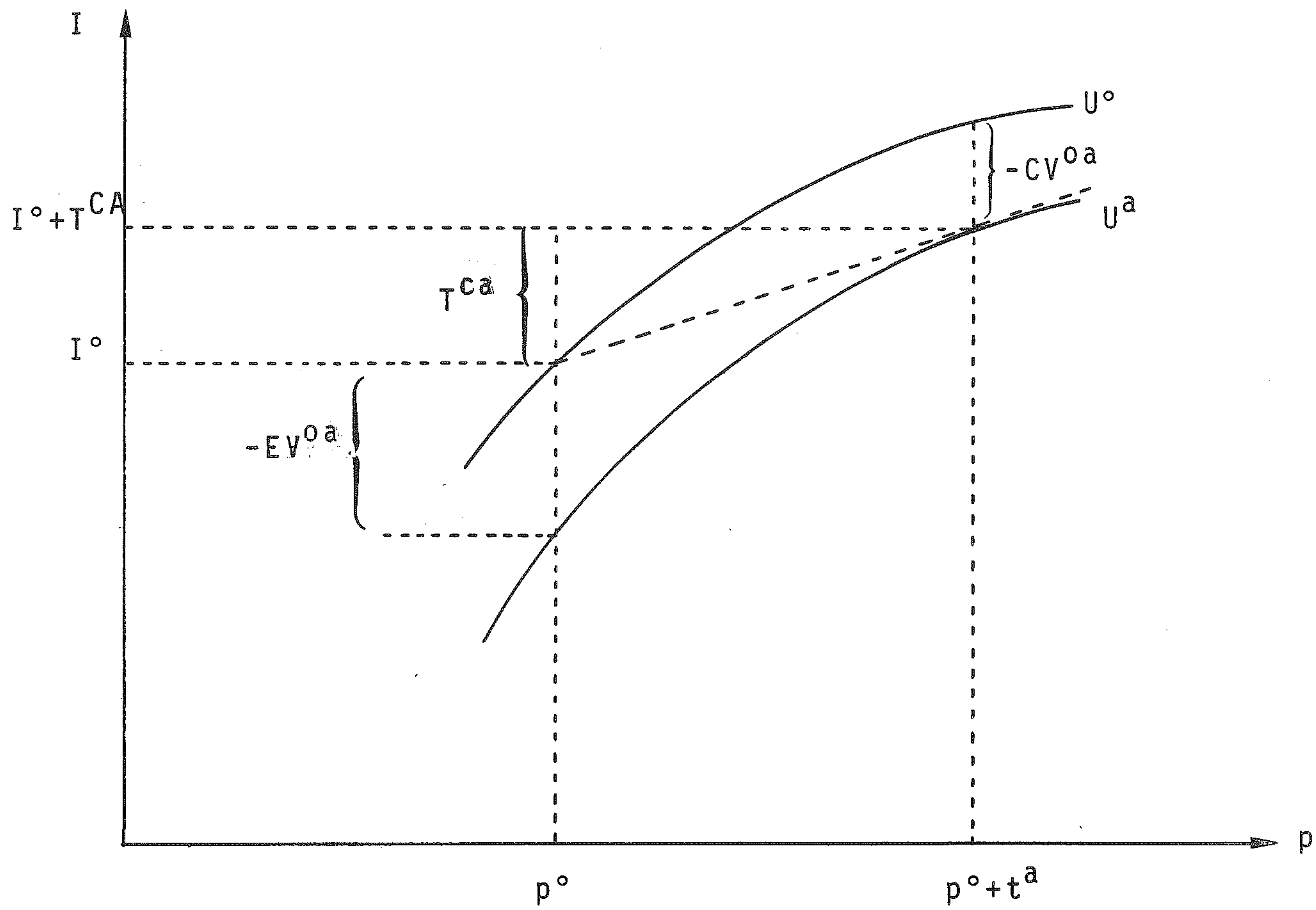
The dead weight loss of taxation can then be measured either as (the negative of) the EV, or (the negative of) the CV, resulting from the transition from state 0 to state a. Applying (2) and (1), this leads to the following definitions

$$(17) \qquad -EV^{oa} = E(p°,U°) - E(p°,U^a)$$
$$= E(p°+t^a,U^a) - E(p°,U^a) - T^{ca}$$

$$(18) \qquad -CV^{oa} = E(p°+t^a,U°) - E(p°+t^a,U^a)$$
$$= E(p°+t^a,U°) - E(p°,U°) - T^{ca}$$

These measures are illustrated on figure 2.

Figure 2



Referring again to our discussion in section I, it is clear that (17) is a reliable loss measure, while (18) is not. An example of this latter possibility is given in section III.

Comparing the second lines of (10), (11), (17) and (18), it is clear that the four given measures are all of the form

$$\int_{p^\circ}^{p^\circ+t} h(p,U)dp - T^c$$

They only differ in the value given to U, and in the calculation of T^c. It is clear that, for a given taxstructure, all four measures will in general differ.

When reading the relevant literature, it is also striking that sometimes arguments belonging to one approach are used to rationalize loss measures belonging to the other approach.

III An example

Consider the following direct utility function

$$U(z,x_1,x_2) = \lg(zx_1)+x_2, \text{ where } (z,x_1,x_2)\in R^3_+$$

The corresponding indirect utility function is given by

$$V(p_1,p_2,I) = \lg\{\frac{p_2^{\,2}}{p_1}\} + \frac{I}{p_2} - 2$$

The expenditure function is of the form

$$E(p_1,p_2,U) = 2\,p_2 - p_2 \lg\{\frac{p_2^{\,2}}{p_1}\} + p_2U$$

Define state o as

$$(p_1^\circ,p_2^\circ,I^\circ) = (1,1,4), \text{ with utility } U^\circ=V(1,1,4)=2$$

Following the first approach, assume that a tax revenue $\bar{T} = 0$

is required. It is easy to check that the following tax structures satisfy this requirement

$$t_1^A = 1,\ t_2^A = 1,\ T^{LA} = -2$$

$$t_1^B = -\frac{22}{31},\ t_2^B = -\frac{1}{4},\ T^{LB} = 2$$

After some calculations one then obtains

$$T^{CA} = 2 \qquad T^{CB} = -2$$

$$U^A = 1.693 \qquad U^B = 1.328$$

$$-EV^{1A} = .307 \qquad -EV^{1B} = .672$$

$$-CV^{1A} = .614 \qquad -CV^{1B} = .504$$

U^1 (see(9)) is given by $U^1 = V(1,1,4-0)=2$. We see that $-EV^{1A}$ and $-EV^{1B}$ behave decently: $U^A > U^B$ and $-EV^A < -EV^{1B}$. However, the CV measures behave perversely: $U^A > U^B$ while $-CV^{1A} > -CV^{1B}$.

Let us now follow the second approach. Consider again two tax structures

$$t_1^a = 1 \qquad t_2^a = 1$$

$$t_1^b = -\frac{22}{31},\ t_2^b = -\frac{1}{4}$$

The values given to T^L are irrelevant in this approach. One then obtains the following results

$$T^{ca} = 2 \qquad T^{cb} = -2$$

$$U^a = 1.693 \qquad U^b = 1.328$$

$$-EV^{\circ a} = .307 \qquad -EV^{\circ b} = .672$$

$$-CV^{\circ a} = .614 \qquad -CV^{\circ b} = .504$$

Again, the EV measures behave decently, while the CV measures behave perversely.

REFERENCES

{1} DIAMOND, P.A. and McFADDEN, D. (1974). Some uses of the expenditure function in public finance. Journal of Public Economics, 3, 3-21.

{2} KAY, J.A. (1980). The dead weight loss from a tax system. Journal of Public Economics, 13, 111-120

{3} PAUWELS, W. (1978). The possible perverse behavior of the compensating variation as a welfare ranking. Zeitschrift für Nationalökonomie , 38, 369-378

{4} PAZNER, E.A. and SADKA, E. (1980). Excess Burden and economic surplus as consistent welfare indicators, Public Finance, 35, 439-449.

{5} ZABALZA, A. (1982) Compensating and equivalent variations, and the dead weight loss of Taxation, Economica, 49, 355-359.