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The Optimal Supply and Income of Physicians - A Dynamic Analysis

W. Pauwels*

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Universitaire Faculteiten St.-Ignatius
Prinsstraat 13 - 2000 Antwerpen
D/1982/1169/14

ABSTRACT

In this paper a welfare economic analysis is made of the market for physicians. The model is formulated as a control problem where the number of physicians per head of the population is the state variable, and where the income of a physician is the control variable. The government's objective is to maximize the present value of the future net surpluses. It is shown that, in a steady-state equilibrium, the consumer's marginal willingness to pay for the services of physicians should exceed the marginal cost of these services by a factor which is inversely proportional to the elasticity of the supply of new physicians with respect to their income. If one introduces time lags into the supply of new physicians, the size of this elasticity is decreased, so that the discrepancy between marginal willingness to pay and marginal cost is increased.

In this paper a welfare economic analysis is made of the market for physicians [1]. The optimal supply and income of physicians are determined within a dynamic context. The model is formulated as a control problem where the number of physicians per head of the population is the state variable, and where the income of a physician is the control variable.

As the supply of new physicians is sensitive to the income of physicians, manipulating physicians' incomes will change the number of physicians. This number of physicians directly affects the consumers' aggregate willingness to pay for their services. On the other hand, manipulating physicians' incomes also involves costs: the cost of schooling new physicians, and the total wage bill paid to existing physicians. How then should the government control income so as to maximize the aggregate net social benefit ? This is the question we are asking.

The paper is organized as follows. Section one presents the basic model. This model is solved and analysed in section two. A model with a more complicated supply structure is investigated in section three. The final section contains some concluding remarks.

1. The basic model

Let $N(t)$ denote the total stock of physicians existing at time t, and let $B(t)$ represent the total population at time t. The state variable of the model, $z(t)$, is then definad as the number of physicians per head of the population [2], i.e., $z(t) = N(t)/B(t)$. Assume that population grows at a constant exponential rate r. If we then differentiate $z(t)$ with respect to time, we obtain

$$\dot{z}(t) = r\, z(t) + \frac{\dot{N}(t)}{B(t)} \qquad (1)$$

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1 It will be clear from what follows that the same analysis can be applied to several other professions as well.

2 We will assume in this paper that there is a constant relationship between the stock variable $z(t)$ and the flow of services generated by this stock.

Assuming that the stock of physicians, N(t), depreciates at a constant exponential rate δ, we can write

$$\dot{N}(t) = -\delta\, N(t) + G(t) \tag{2}$$

where G(t) represents the flow of newly graduated physicians at time t. Combining (1) and (2) gives

$$\dot{z}(t) = -(r+\delta)\; z(t) + \frac{G(t)}{B(t)} \tag{3}$$

We now have to explain the behaviour of G(t)/B(t). It is clear that more new students will enter medical schools if the expected incomes of physicians are relatively high. We will therefore assume that G(t)/B(t) depends on the expected income of physicians. For the moment we will neglect the time lag between the time of entering a medical school and the time of graduation. We will also assume that expected income is equal to actual income. Both assumptions will be relaxed in section three. We can then write

$$\frac{G(t)}{B(t)} = f(y(t)) \tag{4}$$

where y(t) is the income of a physician at time t. The function f will have the following properties: for all $y \geqslant 0$

$$\begin{aligned} &0 \leqslant f(y) \leqslant 1 \\ &f'(y) > 0 \\ &f''(y) < 0 \end{aligned} \tag{5}$$

As G(t)/B(t) is a fraction, the values of f should also be fractions. Also, increasing y will increase the supply of new physicians, but at a decreasing rate.

Combining (3) and (4) gives us the equation of motion

$$\dot{z}(t) = -(r+\delta)\; z(t) + f(y(t)) \tag{6}$$

One could now take the point of view that y(t) is an endogenous variable, whose behavior over time is determined by market forces. In order to complete the description of the dynamics of the market for physicians, one would then have to specify a differential equation incorporating the determinants of $\dot{y}(t)$. In this respect, it is reasonable to assume that $\dot{y}(t)$ depends negatively on $z(t)$: a decrease in $z(t)$ will make it easier for physicians to obtain an increase in $y(t)$, and vice versa.

A typical finding in this context is the emergence of a cyclical behaviour: high values of z will lower y; low values for y will attract fewer students to medical schools, and z will decrease; the decrease in z allows y to rise; this again raises z, etc. An empirical investigation of such a cycle in Belgium was made by W. Nonneman in [4] [3]. A simple model consistent with this cyclical behaviour is as follows.

$$\frac{\dot{z}(t)}{z(t)} = -a + b\, y(t) \tag{7}$$

$$\frac{\dot{y}(t)}{y(t)} = c - d\, z(t) \tag{8}$$

Where a,b,c and d are positive parameters. Equation (7) describes the supply of new physicians, and is similar to (6). Equation (8) describes the income formation : the rate of change of y depends negatively on z. System (7) and (8) is a Volterra-Lotka system of differential equations. See e.g. G.Gandolfo [3, pp. 409-416]. Its solution gives a cyclical behaviour of z and y which is consistent with empirical observations.

In this paper we will take a different point of view. Instead of assuming that $y(t)$ is determined endogenously, we will assume that $y(t)$ is completely controlled by the government. Also, instead of using a descriptive approach, we will take a normative approach. Indeed, the government is supposed to control $y(t)$ so as to maximize a social welfare function.

How can such a social welfare function be specified ? We will consider both social benefits and social costs. On the benefit side, let $\phi(z)$ be a function giving, for each value of z, consumers' aggregate willingness to pay for z. This is given by the relevant area under the aggregate demand curve for z. This demand curve is given by $\phi'(z)$ (see figure 1). As the demand curve is negatively sloped, we also have, for all $z \geqslant 0$ that

$$\phi''(z) < 0 \tag{9}$$

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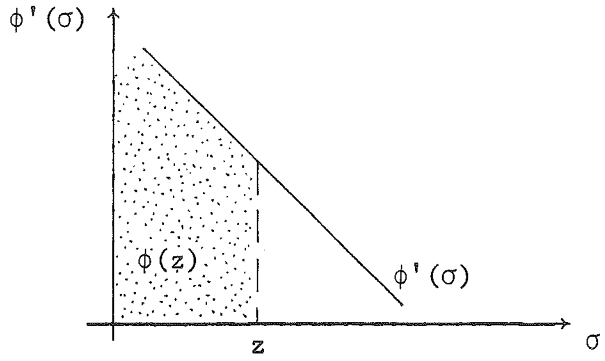
3 For similar analyses, see R.B. Freeman [1] and [2].

so that ϕ is strictly concave.

Figure 1 :



On the cost side, we have, first, the social cost of schooling new physicians. Let this "investment cost" be represented by a function C(f(y)) where

$$C'(f(y)) > 0 \quad \text{and} \quad C''(f(y)) > 0 \tag{10}$$

for all $f(y) \geqslant 0$. It follows that

$$\frac{dC}{dy} = C'(f(y))\, f'(y) > 0$$

In the following section we will also assume that

$$\frac{d^2C}{dy^2} = C''(f(y))\, f'(y)^2 + C'(f(y))\, f''(y) > 0 \tag{11}$$

so that C is strictly convex not only in f(y) (see (10)), but also in y.

The second component on the cost side is given by the total wage bill to be paid to physicians. This "operating cost" is given by z(t)y(t).

The objective functional of the government can now be specified as

$$\int_0^\infty \{\phi(z(t)) - C[f(y(t))] - z(t)\, y(t)\}\, e^{-\rho t}\, dt \tag{12}$$

where ρ is the government's rate of discount. (12) represents the present value of all future net social benefits.

2. Solution of the basic model

Our problem is now to find a trajectory y(.) over the interval $[0,\infty)$ such that (12) is maximized, subject to (6), given an initial condition $z(0) = z_0$. In what follows, we will neglect the nonnegativity constraint $y(t) \geqslant 0$. The current value Hamiltanian for this problem is given by

$$H[z(t),y(t),\lambda(t),t] = e^{-\rho t}\{\phi(z(t)) - C[f(y(t))] - z(t)\, y(t) - (r+\delta)\, z(t)\, \lambda(t) + \lambda(t)\, f(y(t))\} \qquad (13)$$

We then obtain the following necessary optimality conditions. If $y^*(.)$ is an optimal control trajectory with corresponding state trajectory $z^*(.)$, then

(a) there exists a non-zero trajectory $\lambda^*(.)$ satisfying the adjoint equation

$$\dot{\lambda}^*(t) = (\rho + r + \delta)\, \lambda^*(t) - [\phi'(z^*(t)) - y^*(t)], \ t \in [0,\infty) \qquad (14)$$

(b) for all $t \in [0,\infty)$, $y^*(t)$ maximizes

$$H[z^*(t),\, y(t), \lambda^*(t), t] \qquad (15)$$

In view of (5) and (11), we know that $y^*(t)$ is the unique global (interior) maximum of (15) if and only if

$$\lambda^*(t)\, f'(y^*(t)) = C'[f(y^*(t))]f'(y^*(t)) + z^*(t) \qquad (16)$$

The adjoint variable $\lambda^*(t)$ can be interpreted as the net social value of a unit of z. For $\lambda^*(t) \neq 0$, (14) can be written as

$$\frac{\dot{\lambda}^*(t)}{\lambda^*(t)} = (\rho + r + \delta) - \frac{\phi'(z^*(t)) - y^*(t)}{\lambda^*(t)} \qquad (17)$$

which is a well known equation in capital theory. The expression

$$\frac{\phi'(z^*(t)) - y^*(t)}{\lambda^*(t)}$$

can be interpreted as the gross social rate of return on the stock of "capital" $z^*(t)$. It is the demand price for $z^*(t)$, $\phi'(z^*(t))$, minus the operating cost, $y^*(t)$, divided by the price of $z^*(t)$. Equation

(17) then says that the rate of change of the price of $z^*(t)$ is equal to the difference between the opportunity rate of return, $\rho + r + \delta$, and the actual rate of return.

Condition (16) can be interpreted as saying that the marginal social revenue of an increase in y should be equal to the marginal social cost of an increase in y. This marginal social cost consists of an investment cost and an operating cost.

We can now add to (14) the transversality condition

$$\lambda^*(t) \to \hat{\lambda} = \frac{\phi'(\hat{z}) - \hat{y}}{\rho + r + \delta} \quad \text{for} \quad t \to \infty \tag{18}$$

where $\hat{z}$ and $\hat{y}$ are the steady-state values of z and y. As, by (5), (9) and (11), the Hamiltonian H is concave in z and y, we get the following sufficiency result. If the trajectories $y^*(.)$, $z^*(.)$ and $\lambda^*(.)$ satisfy (6), (14), (18) and (16), then $y^*(.)$ is a solution of the control problem.

From equations (6), (14) and (16) it is clear that the steady-state solution of the model is given by the values $\hat{z}$, $\hat{y}$ and $\hat{\lambda}$ which satisfy

$$-(r + \delta)\hat{z} + f(\hat{y}) = 0$$

$$(\rho + r + \delta)\hat{\lambda} - [\phi'(\hat{z}) - \hat{y}] = 0$$

$$\hat{\lambda} f'(\hat{y}) - C'(f(\hat{y})) f'(\hat{y}) - \hat{z} = 0$$

In this system of equations, $\hat{\lambda}$ can be eliminated. We can also define

$$\eta(y) = \frac{y f'(y)}{f(y)}$$

as the elasticity of f with respect to y. The steady-state values of z and y can then be obtained from

$$(r + \delta)\hat{z} = f(\hat{y}) \tag{19}$$

$$\phi'(\hat{z}) = (\rho + r + \delta) C'(f(\hat{y})) + \hat{y} + \frac{\rho + r + \delta}{r + \delta} \cdot \frac{\hat{y}}{\eta(\hat{y})} \tag{20}$$

The meaning of (19) is obvious. In a steady-state equilibrium the gross supply of new physicians must equal the number of new physicians required to compensate for population growth and depreciation. Equation (20) is more interesting. It says that the demand price for physicians, $\phi'(\hat{z})$, must equal the sum of the opportunity cost of the marginal increase of investment outlays required to school new physicians, $(\rho + r + \delta)\, C'(f(\hat{y}))$, plus the operating cost (income) of a physician, $\hat{y}$, plus a term which is proportional to $\hat{y}$ and inversely proportional to the elasticity $\eta(\hat{y})$. This last term is, perhaps, somewhat unexpected. The first two terms on the RHS of (20) can be interpreted as the total marginal cost of $\hat{z}$. It then follows from (20) that the excess of the demand price of $\hat{z}$ over its total marginal cost should be greater (smaller) the smaller (greater) the value of $\eta(\hat{y})$. In case this elasticity tends to infinity, there should be no excess of the demand price over total marginal cost. It is easy to see that, if the last term in (20) would be neglected, the resulting solution of (19) and (20) would involve values of z and y which would be too large. We have here a case where demand price should not equal marginal cost.

The complete behaviour of the optimal solution is summarized on the phase diagram of figure 2. From (6) we can derive all combinations of $z^*(t)$ and $y^*(t)$ for which $\dot{z}^*(t) = 0$. The slope the resulting curve has the following properties

$$\left.\frac{dy^*}{dz^*}\right|_{\dot{z}^*=0} = \frac{r+\delta}{f'(y^*)} > 0$$

$$\left.\frac{d^2y^*}{dz^{*2}}\right|_{\dot{z}^*=0} = -\frac{(r+\delta)^2\, f''(y^*)}{f'(y^*)^3} > 0$$

To obtain the differential equation of $\dot{y}^*(t)$, we differentiate (16) with respect to time. Using then (6), (14) and (16) in this expression we obtain

$$\dot{y}^*(t) = \frac{1}{C''[f(y^*(t))]f'(y^*(t))^2 - \frac{z^*(t)f''(y^*(t))}{f'(y^*(t))}} \{(\rho+r+\delta)C'[f(y^*(t))]f'(y^*(t))$$
$$+ (\rho+r+\delta)z^*(t) - [\phi'(z^*(t)) - y^*(t)]f'(y^*(t)) - f(y^*(t)) + (r+\delta)z^*(t)\}$$

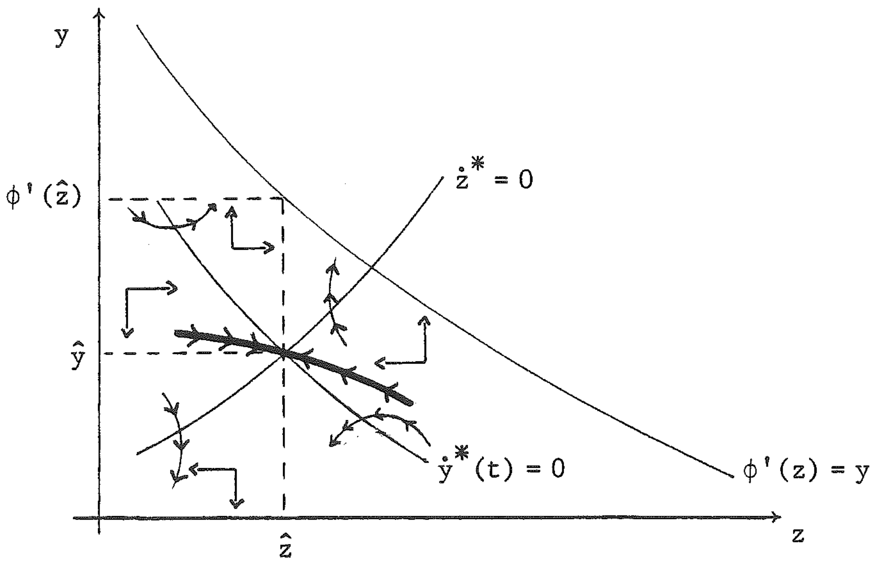
From this equation we can derive all combinations of $z^*(t)$ and $y^*(t)$ for which $\dot{y}^*(t) = 0$. The slope of the resulting curve is given by

$$\left.\frac{dy^*}{dz^*}\right|_{\dot{y}^*} = \frac{\phi''(z^*(t))f'(y^*(t)) - 2r - 2\delta - \rho}{(\rho+r+\delta)\{C''[f(y^*(t))]f'(y^*(t))^2 + C'[f(y^*(t))]f''(y^*(t)) - \dots \frac{\phi'(z^*(t)) - y^*}{\rho+r+\delta} f''(y^*(t))\}}$$

Under assumptions (9) and (11), this expression is negative.

The heavy line in figure 2 indicates the optimal trajectories of z^* and y^* for all possible initial values z_0 of z^*.

Figure 2 :



3. The introduction of time lags in the supply structure

When deriving equation (6), we assumed that the response of the supply of physicians to expected income is immediate. We also assumed that expected income always equals actual income. These two assumptions will now be relaxed.

Let L denote the duration of medical studies, and let ε be the fraction of the number of entering medical students who will, in fact, graduate. We can then write

$$G(t) = \varepsilon\, E(t - L) \qquad (21)$$

where E(t-L) is the number of students entering medical schools at time t-L. Using (21), we can write

$$\frac{G(t)}{B(t)} = \varepsilon \frac{E(t-L)}{B(t)} = \varepsilon \frac{B(t-L)}{B(t)} \frac{E(t-L)}{B(t-L)} = \varepsilon e^{-rL} \frac{E(t-L)}{B(t-L)} \qquad (22)$$

The ratio E(t-L)/B(t-L) will now be explained by the behaviour of y in the past. Indeed, when making a decision on whether or not to enter a medical school, students will take into account the past behaviour of y. This is formulized by putting

$$\frac{E(t-L)}{B(t-L)} = \int_{-\infty}^{t-L} w(t-L-\sigma)\, f(y(\sigma))\, d\sigma \qquad (23)$$

where w is a weighting function with

$$\int_{-\infty}^{t-L} w(t-L-\sigma)\, d\sigma = 1$$

Define now

$$\bar{w}(t-\sigma) \begin{cases} = 0 & \text{for } t-L \leqslant \sigma \leqslant t \\ = w(t-L-\sigma) & \text{for } -\infty < \sigma \leqslant t-L \end{cases} \qquad (24)$$

Using (22), (23) and (24) in (3), we obtain the equation of motion

$$\dot{z}(t) = -(r+\delta)z(t) + \varepsilon e^{-rL} \int_{-\infty}^{t} \bar{w}(t-\sigma)\, f(y(\sigma))\, d\sigma \qquad (25)$$

which is now an integro-differential equation.

It will be useful to define a new state variable v(t) as

$$v(t) = \varepsilon e^{-rL} \int_{-\infty}^{t} \bar{w}(t-\sigma)\, f(y(\sigma))\, d\sigma \qquad (26)$$

(25) can then be written as

$$\dot{z}(t) = -(r+\delta)\, z(t) + v(t) \qquad (27)$$

Differentiating (26) with respect to time, and recalling from (24) that $\bar{w}(0) = 0$, we obtain

$$\dot{v}(t) = \varepsilon e^{-rL} \int_{-\infty}^{t} \bar{w}'(t-\sigma)\, f(y(\sigma))\, d\sigma \qquad (28)$$

As v(t) represents the realized supply of new physicians at time t, (12) should now be written as

$$\int_0^\infty \{\phi(z(t)) - C(v(t)) - z(t)\, y(t)\}\, e^{-\rho t}\, dt \qquad (29)$$

where, as in (10), we will assume that for all $v \geqslant 0$

$$C'(v) > 0 \quad \text{and} \quad C''(v) > 0 \qquad (30)$$

Our new control problem then consists of finding a trajectory y(.) defined over the interval $[0,\infty)$, such that (29) is maximized, subject to (27) and (28), given the behaviour of y in the interval $(-\infty,0)$. To solve this problem, we will make use of the results of S.P. Sethi [6][4]. The current value Hamiltonian is given by

$$\begin{aligned} H[z(t),v(t),y(t),\lambda_1(t),\lambda_2(\sigma\geqslant t),t] = {} & e^{-\rho t}\{\phi(z(t)) - C(v(t)) - z(t)y(t) \\ & - (r+\delta)\lambda_1(t)z(t) + \lambda_1(t)v(t) \\ & + \varepsilon e^{-rL} f(y(t)) \int_t^\infty e^{-\rho(\sigma-t)} \lambda_2(\sigma)\bar{w}'(\sigma-t)d\sigma\} \end{aligned} \qquad (31)$$

The following result gives the necessary optimality conditions. If $y^*(.)$ is an optimal control trajectory with corresponding state trajectories $z^*(.)$ and $v^*(.)$, then

(a) there exist non zero trajectories $\lambda_1^*(.)$ and $\lambda_2^*(.)$ satisfying the adjoint equations

$$\dot{\lambda}_1^*(t) = (\rho+r+\delta)\lambda_1^*(t) - [\phi'(z^*(t)) - y^*(t)] \qquad (32)$$

$$\dot{\lambda}_2^*(t) = \rho\,\lambda_2^*(t) + C'(v^*(t)) - \lambda_1^*(t) \qquad (33)$$

for $t \in [0,\infty)$

(b) for all $t \in [0,\infty)$, $y^*(t)$ maximizes

$$H[z^*(t),v^*(t),y(t),\lambda_1^*(t),\lambda_2^*(\sigma\geqslant t),t] \qquad (34)$$

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4 A model with a very similar mathematical structure is analysed in [5]

As (34) is strictly concave in y (by (5)), it follows that $y^*(t)$ will be the unique global maximum of (34) if and only if

$$\varepsilon\, e^{-rL}\, f'(y^*(t)) \int_t^\infty e^{-\rho(\sigma-t)} \lambda_2^*(\sigma)\, \bar{w}'(\sigma-t)\, d\sigma = z^*(t) \qquad (35)$$

The interpretation of $\lambda_1^*(t)$ is the same as the interpretation of $\lambda^*(t)$ in the previous model. Also, (32) is the same as (14). The adjoint variable $\lambda_2^*(t)$ can be interpreted as the net social value of a new physician. For $\lambda_2^*(t) \neq 0$, (33) can be written as

$$\frac{\dot{\lambda}_2^*(t)}{\lambda_2(t)} = \rho - \frac{\lambda_1^*(t) - C'(v^*(t))}{\lambda_2^*(t)}$$

The expression $$\frac{\lambda_1^*(t) - C'(v^*(t))}{\lambda_2^*(t)}$$

can be interpreted as the rate of return on a new physician. It is the difference between the value of an existing physician minus the marginal cost of "producing" a new physician, divided by the value of a new physician.

The LHS of equation (35) can be interpreted as the marginal social revenue generated by an increase in y. This marginal social revenue is given by the present value of the future values of new physicians resulting from the increase in y. How the supply of these new physicians is spread over time depends, of course, on the form of the weighting function $\bar{w}$. Equation (35) can then be interpreted as requiring that the marginal social revenue generated by an increase in y should equal the increase in the operating cost due to the increase in y.

We can now add to equations (32) and (33) the transversality conditions

$$\lambda_1^*(t) \to \hat{\lambda}_1 = \frac{\phi'(\hat{z}) - \hat{y}}{\rho + r + \delta} \quad \text{for} \quad t \to \infty \qquad (36)$$

$$\lambda_2^*(t) \to \hat{\lambda}_2 = \frac{\hat{\lambda}_1 - C'(\hat{v})}{\rho} \quad \text{for} \quad t \to \infty \qquad (37)$$

where $\hat{z}$, $\hat{y}$ and $\hat{v}$ are the steady-state values of z, y and v. By (5),

(9) and (30), the Hamiltonian H is concave in z, v and y. We then obtain the following sufficiency result. If the trajectories $y^*(.)$, $z^*(.)$, $v^*(.)$, $\lambda_1^*(.)$ and $\lambda_2^*(.)$ satisfy (27), (28), (32), (36), (33), (37) and (35), then $y^*(.)$ is a solution of the control problem.

From equations (25), (26), (32), (33) and (35), it is clear that the steady-state solution of the model is given by the values $\hat{z}$, $\hat{v}$, $\hat{y}$, $\hat{\lambda}_1$ and $\hat{\lambda}_2$ which satisfy

$$-(r+\delta)\hat{z} + \varepsilon\, e^{-rL} f(\hat{y}) = 0 \qquad (38)$$

$$\hat{v} - \varepsilon\, e^{-rL} f(\hat{y}) = 0 \qquad (39)$$

$$(\rho + r + \delta)\hat{\lambda}_1 - [\phi'(\hat{z}) - \hat{y}] = 0 \qquad (40)$$

$$\rho\, \hat{\lambda}_2 + C'(\hat{v}) - \hat{\lambda}_1 = 0 \qquad (41)$$

$$\varepsilon\, e^{-rL} f'(\hat{y})\, \hat{\lambda}_2 \int_0^\infty e^{-\rho x}\, \bar{w}'(x)\, dx - \hat{z} = 0 \qquad (42)$$

The integral in (42) can be simplified as follows. First, it is clear that

$$\int_0^\infty e^{-\rho x}\, \bar{w}'(x)\, dx = \rho \int_0^\infty e^{-\rho x}\, \bar{w}(x)\, dx$$

Secondly, recalling the definition of $\bar{w}$ from (24), we know that

$$\int_0^\infty e^{-\rho x}\, \bar{w}(x)\, dx = \int_L^\infty e^{-\rho x}\, \bar{w}(x)\, dx$$

Performing a change of variable

$$\sigma = x - L$$

and recalling that $\bar{w}(\sigma+L) = w(\sigma)$ we finally obtain

$$\int_0^\infty e^{-\rho x}\, \bar{w}'(x)\, dx = \rho\, e^{-\rho L} \int_0^\infty e^{-\rho\sigma}\, w(\sigma)\, d\sigma$$

(42) can then be written as

$$\varepsilon\, e^{-rL} f'(\hat{y})\, \hat{\lambda}_2\, \rho\, e^{-\rho L} \int_0^\infty e^{-\rho\sigma}\, w(\sigma)\, d\sigma - \hat{z} = 0 \qquad (43)$$

From system (38) - (41) and (43), $\hat{\lambda}_1$, $\hat{\lambda}_2$ and $\hat{v}$ can be eliminated so that we are left with the following two equations

$$(r+\delta)\hat{z} = \varepsilon\, e^{-rL}\, f(\hat{y}) \tag{44}$$

$$\phi'(\hat{z}) = (\rho+r+\delta)C' + \hat{y} + \frac{\rho+r+\delta}{r+\delta} \cdot \frac{\hat{y}}{\eta(\hat{y})} \cdot \frac{e^{\rho L}}{\int_0^\infty e^{-\rho x}\, w(x)\, dx} \tag{45}$$

in two unknowns $\hat{z}$ and $\hat{y}$. This system has to be compared with system (19), (20). The interpretation of (44) is obvious. Comparing (45) with (20), we see that the two equations differ by the expression

$$\frac{e^{\rho L}}{\int_0^\infty e^{-\rho x}\, w(x)\, dx} \tag{46}$$

by which the last term on the RHS, representing the elasticity of the supply of new physicians, is multiplied. As (46) is greater than 1, the introduction of time lags increases the importance of the last term in (45). Put differently, the elasticity $\eta(\hat{y})$ is decreased from $\eta(\hat{y})$ in (20) to

$$\eta(\hat{y})\, e^{-\rho L} \int_0^\infty e^{-\rho x}\, w(x)\, dx$$

in (45). This implies that the excess of the demand price over marginal cost should increase because of the time lags.

An especially appealing form for the weighting function w is a gamma distribution

$$w(x) = \Gamma(x|\alpha,\beta) = \frac{1}{\alpha!\ \beta^{\alpha+1}}\, x^{\alpha}\, e^{-\frac{x}{\beta}} \tag{47}$$

This distribution is unimodal, and provides great flexibility through its two parameters α and β. Increases in α postpone the peak of the density, while increases in β extend the range over which the density is non negligible. If $\alpha = 0$, (47) reduces to the exponential distribution.

If we then use (47) in (46), we obtain

$$\frac{e^{\rho L}}{\int_0^\infty e^{-\rho x}\, w(x)\, dx} = e^{\rho L}(1 + \rho\beta)^{\alpha+1} \tag{48}$$

If we substitute (48) into (45), then the system of equations (44),(45) contains seven parameters, viz., L, ε, α, β, δ, r and ρ. We can then calculate how the steady-state solution $(\hat{z},\hat{y})$ changes as one of these parameters changes. The results of these calculations are given in table 1.

Table 1 : effects of an increase in the variables running vertically upon those running horizontally

	$\hat{z}$	$\hat{y}$
L	<0	?
ε	>0	<0
α	<0	<0
β	<0	<0
δ	<0	?
r	<0	?
ρ	<0	<0

In order to understand these results, let us call equation (44) the supply equation, and equation (45) the pricing equation. In $(\hat{z},\hat{y})$ space, the supply equation has a positive slope, while the pricing equation has a negative slope.

An increase in L shifts the supply equation upwards, and the pricing equation downwards. This results in a decrease of $\hat{z}$. The effect on $\hat{y}$ is ambiguous. An increase in ε shifts the supply equation downwards, with no effect on the pricing equation. Hence, there must be an increase in $\hat{z}$ and a decrease in $\hat{y}$. Increases in α or β shift the pricing equation downwards, while leaving the supply equation unaffected. The result is a decrease in both $\hat{z}$ and $\hat{y}$. Increases in δ or r shift the supply equation upwards, while the effect on the pricing equation is not clear. Calculations show that $\hat{z}$ must decrease, but that the effect on $\hat{y}$ is ambiguous. Finally, an increase in ρ shifts the pricing equation downwards, with no effect on the supply equation. Hence, both $\hat{z}$ and $\hat{y}$ must decrease.

4. Concluding remarks

In this paper, an optimal pricing policy for the services of physicians was derived, using a dynamic model where the supply of new physicians depends on their expected income. It was shown that, if the government wants to maximize the present value of future net surpluses, the consumers' marginal willingness to pay for the services of physicians should exceed the marginal cost of these services by a factor which is inversely proportional to the elasticity of the supply of new physicians with respect to their income. If one introduces time lags into the supply of new physicians, the size of this elasticity is decreased, so that the discrepancy between marginal willingness to pay and marginal cost is increased.

Finally, as already stated in footnote 1, it is clear that the same model can be applied to other professions as well. One only has to change the interpretation of the functions ϕ, C and f.

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