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STUDIECENTRUM VOOR ECONOMISCH EN SOCIAAL ONDERZOEK

A DISEQUILIBRIUM ANALYSIS OF THE COMMODITY MARKET, EMPLOYMENT, AND THE HOURS OF WORK

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The purpose of this paper is to construct a model for markets in disequilibrium when prices are exogeneous. We consider the commodity and the labour market. In order to analyse rationings on employment and on the average hours of work, the labour market is divided in two submarkets. The number of workers and the average hours of work per worker are separately treated so that we have three markets.

We consider a closed economy without government. The decisions are taken by consumers and producers. The producer supplies the commodities and demands a number of workers and also an average number of hours of work per worker. As we do not allow stock formation, the production and the supply of commodities will coincide. The consumer demands commodities, decides whether or not he will enter the labour market and supplies a number of hours of work. For the producer, the Walrasian quantities are determined by profit maximization subject to a production function. For the consumer, they are determined by utility maximization subject to a budget constraint. The Walrasian functions are represented as follows:

y^S commodity supply
y^D commodity demand
x_1^S labour force participation
x_1^D number of workers demanded
x_2^S average hours of work supplied
x_2^D average hours of work per worker demanded

In a first part the spill-over between the three markets are analysed in a rather general way. In a second part the effective demand and supply functions are determined, starting from specific functions. For the producer we use a Cobb-Douglas production function and for the consumer a specific form of the Johanssen utility function. The purpose of this part is to construct functions which can be used for estimation. Where possible the spill-over coefficients are determined and discussed.

1. The different disequilibrium regimes and the spill-over effects

When we assume that prices and wages are not sufficiently flexible to bring the labour and commodity market into equilibrium, we cannot use the Walrasian demand and supply functions any longer. As there may exist excess demand or excess supply in the different markets, we have to consider the effective demand and supply functions. Following Clower (1965) and Benassy (1975, 1976, 1977, 1978) we define the effective demand (supply) in a specific market as the demand (supply) taking into account the quantity constraints in all the other markets (1). If there is no rationing in the other markets the effective demand (supply) will coincide with the Walrasian or notional demand (supply).

1.1. The different disequilibrium regimes

As we have three markets, there will exist eight possible disequilibrium regimes which are given in table I. The realised transactions $\bar{y}$, $\bar{x}_1$ and $\bar{x}_2$ are assumed to be the minimum of effective demand and effective supply (2). If there is any rationing in a market, this will influence the demand and the supply on the other markets. This influence is called the spill-over effect (Patinkin (1949, 1956)).

Table I.

I	II	III	IV
$\bar{y} = y^{D'} < y^{S'}$ $\bar{x}_1 = x_1^{D'} < x_1^{S'}$ $\bar{x}_2 = x_2^{D'} < x_2^{S'}$	$\bar{y} = y^{D'} < y^{S'}$ $\bar{x}_1 = x_1^{D'} < x_1^{S'}$ $x_2^{D'} > x_2^{S'} = \bar{x}_2$	$\bar{y} = y^{D'} < y^{S'}$ $x_1^{D'} > x_1^{S'} = \bar{x}_1$ $\bar{x}_2 = x_2^{D'} < x_2^{S'}$	$\bar{y} = y^{D'} < y^{S'}$ $x_1^{D'} > x_1^{S'} = \bar{x}_1$ $x^{D'} > x_2^{S'} = \bar{x}_2$
V	VI	VII	VIII
$y^{D'} > y^{S'} = \bar{y}$ $\bar{x}_1 = x_1^{D'} < x_1^{S'}$ $\bar{x}_2 = x_2^{D'} < x_2^{S'}$	$y^{D'} > y^{S'} = \bar{y}$ $\bar{x}_1 = x_1^{D'} < x_1^{S'}$ $x_2^{D'} > x_2^{S'} = \bar{x}_2$	$y^{D'} > y^{S'} = \bar{y}$ $x_1^{D'} > x_1^{S'} = \bar{x}_1$ $\bar{x}_2 = x_2^{D'} < x_2^{S'}$	$y^{D'} > y^{S'} = \bar{y}$ $x_1^{D'} > x_1^{S'} = \bar{x}_1$ $x_2^{D'} > x_2^{S'} = \bar{x}_2$

(1) For an introduction in the theory of disequilibrium economics we refer to Malinvaud (1977), Barro and Grossman (1976), Müllbauer and Portes (1978).

(2) It can also be assumed that the realised transactions are the minimum of the Walrasian demand and supply (cfr. Clower (1965)).

1.2. The spill-over effects

In an economy with n markets we can define the effective demand on market i, as the demand taking into account all the rationings in the other markets. Generally, we can write the effective demand (D') as a function of the rationings in the other markets (Q) and of other factors (P)

$$D_i' = f(Q,P)$$

where $\quad Q = (Q_1, Q_2, \ldots, Q_{i-1}, Q_{i+1}, \ldots, Q_n)$

The spill-over effect from a rationing in market $j \neq i$ on the effective demand on market i, is then defined as

$$\frac{\partial D_i'}{\partial Q_j}$$

In our model we can consider several spill-over effects.

1.2.1. The spill-over effects for the producer

1.2.1.1. The spill-over from the commodity market to the labour market, $\frac{\partial x_1^{D'}}{\partial \bar{y}}$ and $\frac{\partial x_2^{D'}}{\partial \bar{y}}$

When the producer is confronted with a quantity constraint in the commodity market, this means that he cannot realise his Walrasian supply. As we assumed no stock formation, the producer will have to bring his production below the Walrasian production. Therefore he will need less labour input. The effective demand for workers will be smaller than the Walrasian demand. The same holds for the effective demand for average hours of work per worker.

1.2.1.2. <u>The spill-over effect from a rationing on the number of workers in the commodity market and on the demand for hours</u> $\frac{\partial y^{S'}}{\partial x_1}$ and $\frac{\partial x_2^{D'}}{\partial \bar{x}_1}$

When the producer is rationed on the number of workers, this can have two consequences. If he continues with the Walrasian average hours per worker, his effective production will fall below his Walrasian production and so will his effective supply. On the other hand, the producer can try to meet his Walrasian supply, by compensating the rationed number of workers by an increase of the average hours of work per worker. This effective demand for hours will be greater than the Walrasian one. In this case the producer will select that procedure which results in his highest profit.

1.2.1.3. <u>The spill-over effect from a rationing on the average hours of work per work on the commodity market and on the demand for workers</u>, $\frac{\partial y^{S'}}{\partial \bar{x}_2}$ and $\frac{\partial x_1^{D'}}{\partial \bar{x}_2}$

For this si'uation we can follow the same reasoning as in 1.2.1.2. but starting from a rationing on the average hours of work.

1.2.2. The spill-over effects for the consumer

1.2.2.1. <u>The spill-over effect from the commodity market to the labour market</u>, $\frac{\partial x_1^{S'}}{\partial \bar{y}}$ and $\frac{\partial x_2^{S'}}{\partial \bar{y}}$

If the consumer is rationed in the commodity market, that is if he cannot buy what he wants, some consumers will not find it necessary to work any longer and they will leave the labour market. The effective labour participation will be smaller than the Walrasian one. Others will supply less hours and so the effective supply of hours will also be smaller than the Walrasian one. We can, however, state that the spill-over effects will be rather small because, when rationed on the commodity market, the consumer can save the part of his income which he cannot spend.

1.2.2.2. The spill-over effect from a rationing on the number of workers on the commodity market and on the supply of hours of work, $\frac{\partial y^{D'}}{\partial x_1}$ and $\frac{\partial x_2^{S'}}{\partial \bar{x}_1}$

If there is a rationing on the number of workers this means that there is unemployment. In a household where someone who wants to work but is unable to find any work or where someone becomes unemployed, the reactions on the commodity demand and the supply of hours will depend upon the unemployment insurance. If high, then the effective commodity demand and the effective supply of hours will not differ very much from the Walrasian ones. If the insurance is low or even zero, the effective demand will be smaller than the Walrasian demand. How much smaller will depend upon the lost income. It is possible that other members of the household want to compensate this loss by supplying more hours than the Walrasian quantity.

1.2.2.3. The spill-over effect from a rationing on the average hours of work per worker on the commodity market and on the labour force participation, $\frac{\partial y^{D'}}{\partial \bar{x}_2}$ and $\frac{\partial x_1^{S'}}{\partial \bar{x}_2}$

We can follow the same reasoning as in 1.2.2.2. but starting from a rationing on the average hours of work per worker.

2. Disequilibrium analysis for a specific model

We will concentrate our disequilibrium analysis and the study of the spill-over effects on a specific model. For the producer we will consider a Cobb-Douglas production function and for the consumer a special form of the Johansson utility function.

2.1. The producer side of the model

We consider a representative producer (or a representative body of producers) who has for each period t ($t=0,\ldots,\infty$) a production function in the number of workers and in the average hours of work per worker. We assume that the periods between sample points follow each other fast enough to hold the capital stock fixed for each period. We use a Cobb-Douglas production function with Hicks-neutrality

$$y_t = A\, e^{\lambda t}\, x_{1t}^{a}\, x_{2t}^{b}$$

We assume that the production is subject to stochastic influences caused by weather changes, deficiencies, breakdowns, etc. If the stochastic component is additive, we have

$$\underline{y}_t = y_t + \underline{\varepsilon}_t$$

where $E(\underline{\varepsilon}_t)$ is assumed to be zero so that $E(\underline{y}_t) = y_t$.

The producer is going to maximize the expected present value of his net profit over an infinite time horizon. We assume that the net profit in period t is the result of:

- the revenue $p_t\underline{y}_t$ from selling the output $\underline{y}_t$ at the price p_t;
- the labour cost.

  The producer has to pay wages $w_t x_{1t} x_{2t}$, where w_t is the gross hourly wage rate. He also has to pay contributions to the social security. These contributions are a fraction of the wages. They are calculated according to legally given coefficients (1).
  We assume that the producer has to pay an average percentage s_t of the total wage bill $w_t x_{1t} x_{2t}$.
  Their are also contributions which are independent of the hours of work, but vary with the number of workers. We assume that the average cost per worker is z_t.
  The total labour cost is

$$(1+s_t)w_t x_{1t} x_{2t} + z_t x_{1t}$$

- other costs as capital costs, net depreciation, etc., are represented by c_t;

(1) The Social Security contributions are calculated on different wage levels. The coefficients differ for the several branches of the Social Security. These are, however, plans for changing this system in 1981. The contributions will then be calculated on the total wages or on total sales according to one coefficient.

- taxes to be paid over the gross profit $p_t y_t - w_t(1+s_t)x_{1t}x_{2t} - z_t x_{1t} - c_t$. We assume that the producer pays taxes according to an average tax rate u_t

The net profit in period t is given by

$$(1-u_t)\left[p_t y_t - w_t(1+s_t)x_{1t}x_{2t} - z_t x_{1t} - c_t\right]$$

let k_t be the discount factor for period t and

$$k_t = 1 \qquad \text{for } t=o$$

$$= \prod_{\theta=o}^{t} \frac{1}{1+r_\theta} \text{ for } t > o \quad (r_\theta = \text{discount rate at the end of period } \theta)$$

The present value of the net profit is

$$\sum_{t=o}^{\infty} k_t(1-u_t)\left[p_t y_t - w_t(1+s_t)\, x_{1t}x_{2t} - z_t x_{1t} - c_t\right]$$

The expected value is

$$E\{\sum_{t=o}^{\infty} k_t(1-u_t)\left[p_t y_t - w_t(1+s_t)x_{1t}x_{2t} - z_t x_{1t} - c_t\right]\}$$

$$= \sum_{t=o}^{\infty} k_t(1-u_t)\left[p_t E(y_t) - w_t(1+s_t)x_{1t}x_{2t} - z_t x_{1t} - c_t\right]$$

$$= \sum_{t=o}^{\infty} k_t(1-u_t)\left[p_t y_t - w_t(1+s_t)x_{1t}x_{2t} - z_t x_{1t} - c_t\right]$$

2.1.1. The Walrasian supply and demand functions

To obtain the Walrasian supply and demand funtions of the producer, we have to solve following programme

$$\max_{y_t, x_{1t}, x_{2t}} \sum_{t=0}^{\infty} k_t(1-u_t)\left[p_t y_t - w_t(1+s_t)x_{1t}x_{2t} - z_t x_{1t} - c_t\right]$$

$$\text{s.t.} \qquad y_t = A{}^{\lambda}e^{\ t}\, x_{1t}^{a}\, x_{2t}^{b} \qquad \forall t=0,\ldots,\infty$$

Assuming an interior solution, we have to impose some additional conditions on the parameters a and b (1), viz.

$$a + b < 1 \qquad 0 < a < 1$$
$$a > b \qquad 0 < b < 1$$

The first-order conditions are

$$p_t a \frac{y_t}{x_{1t}} = w_t(1+s_t)x_{2t} + x_t \qquad /1/$$

$$p_t b \frac{y_t}{x_{2t}} = w_t(1+s_t)x_{1t} \qquad /2/$$

$$y_t = A\, e^{\lambda t}\, x_{1t}^{a}\, x_{2t}^{b} \qquad /3/$$

Solving /2/ for x_{1t} and substituting in /1/ gives

$$\begin{cases} \dfrac{a}{b}\, x_{2t} w_{+}(1+s_t) = w_t(1+s_t)x_{2t} + z_t \\[2ex] x_{1t} = b\, \dfrac{p_t y_t}{w_t(1+s_t)x_{2t}} \\[2ex] y_t = A\, e^{\lambda t}\, x_{1t}^{a}\, x_{2t}^{b} \end{cases}$$

<=>

$$\begin{cases} x_{2t}^{D} = \dfrac{b}{a-b}\, z_t w_t^{-1}(1+s_t)^{-1} & /4/ \\[2ex] x_{1t} = b\, \dfrac{p_t A e^{\lambda t}\, x_{1t}^{a}\, x_{2t}^{b}}{w_t(1+s_t)x_{2t}} & /5/ \\[2ex] y_t = A\, e^{\lambda t}\, x_{1t}^{a}\, x_{2t}^{b} & \end{cases}$$

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(1) See Appendix 1.

Substituting /4/ into /5/ and solving for x_{1t}, gives

$$x_{1t}^{D} = A^{\frac{1}{1-a}} (a-b)^{\frac{1-b}{1-a}} b^{\frac{b}{1-a}} e^{\frac{\lambda t}{1-a}} p_t^{\frac{1}{1-a}} w_t^{-\frac{b}{1-a}} (1+s_t)^{-\frac{b}{1-a}} z_t^{-\frac{1-b}{1-a}} \qquad /6/$$

The Walrasian commodity supply is found by substituting /4/ and /6/ in /3/

$$y_t^{s} = A^{\frac{1}{1-a}} (a-b)^{\frac{a-b}{1-a}} b^{\frac{b}{1-a}} e^{\frac{\lambda t}{1-a}} p_t^{\frac{a}{1-a}} w_t^{-\frac{b}{1-a}} (1+s_t)^{-\frac{b}{1-a}} z_t^{-\frac{a-b}{1-a}} \qquad /7/$$

After taking logarithms of /4/, /6/ and /7/ we find loglinear relationships

$$\ln y_t^{s} = \frac{1}{1-a} \ln A + \frac{a-b}{1-a} \ln(a-b) + \frac{b}{1-a} \ln b + \frac{\lambda t}{1-a} + \frac{a}{1-a} \ln p_t - \frac{b}{1-a} \ln w_t$$

$$- \frac{b}{1-a} \ln(1+s_t) - \frac{a-b}{1-a} \ln z_t \qquad /8/$$

$$\ln x_{1t}^{D} = \frac{1}{1-a} \ln A + \frac{1-b}{1-a} \ln(a-b) + \frac{b}{1-a} \ln b + \frac{\lambda}{1-a} t + \frac{1}{1-a} \ln p_t$$

$$- \frac{b}{1-a} \ln w_t - \frac{b}{1-a} \ln (1+s_t) - \frac{1-b}{1-a} \ln z_t \qquad /9/$$

$$\ln x_{2t}^{D} = \ln b - \ln(a-b) + \ln z_t - \ln w_t - \ln (1+s_t) \qquad /10/$$

The commodity supply is an increasing function of the price and is decreasing in labour costs. The demand for workers is a decreasing function of labour cost. The demand for hours will decrease as wages increase, but will increase as the cost per worker increases.

2.1.2. The effective demand and supply functions

After the Walrasian demand and supply functions, we will determine the effective demand and supply functions and pay some attention to the spill-over effects.

2.1.2.1. <u>The producer is rationed in the commodity market</u>

In this case, the effective commodity supply coincides with the Walrasian supply. The effective demand for workers and for hours can be found by solving the following programme

$$\max \sum_{t=0}^{\infty} k_t(1-u_t)\left[p_t y_t - w_t(1+s_t)x_{1t}x_{2t} - z_t x_{1t} - c_t\right]$$

$$\text{s.t.} \quad y_t = A\, e^{\lambda t} x_{1t}^{a}\, x_{2t}^{b} \qquad t=0,\ldots,\infty$$

$$y_t = \bar{y}_t$$

This results in (1):

$$\ln x_{2t}^{D'} = \ln b - \ln(a-b) - \ln w_t - \ln(1+s_t) + \ln z_t = \ln x_{2t}^{D} \qquad /11/$$

$$\ln x_{1t}^{D'} = -\frac{1}{a}\ln A - \frac{b}{a}\ln b + \frac{b}{a}\ln(a-b) - \frac{\lambda}{a}t + \frac{b}{a}\ln w_t + \frac{b}{a}\ln(1+s_t) - \frac{b}{a}\ln z_t + \frac{1}{a}\ln \bar{y}_t \qquad /12/$$

We can remark that the Walrasian demand for average hours of work per worker remains unaffected by the rationing on the commodity market(2). This is not the case for the demand for workers. The spill-over elasticity is given by

$$\frac{\partial \ln x_{1t}^{D'}}{\partial \ln \bar{y}_t} = \frac{1}{a}$$

As we know $0 < a < 1$ we can conclude

$$1 < \frac{1}{a}$$

(1) The derivation of the effective demand and supply functions is given in Appendix 2.

(2) This is caused by the specific form of the profit function. We have a cost for the men hours and a cost for the number of men. If we should introduce a cost for the hours of work, indepent of the number of men, the effective demand for hours would differ from the Walrasian demand.

In order to have a better insight in the spill-over effect, we will rewrite the effective demand for workers as follows

$$\ln x_{1t}^{D'} = \ln x_{1t}^{D} - \ln x_{1t}^{D} + \ln x_{1t}^{D'}$$

$$= \ln x_{1t}^{D} - (\frac{1}{1-a} + \frac{1}{a}) \ln A - (\frac{b}{1-a} + \frac{b}{a}) \ln b - (\frac{1-b}{1-a} - \frac{b}{a}) \ln(a-b) - \frac{1}{1-a}\ln p_t$$

$$- (\frac{\lambda}{1-a} + \frac{\lambda}{a})t + (\frac{b}{1-a} + \frac{b}{a})\ln w_t + (\frac{b}{1-a} + \frac{b}{a})\ln(1+s_t) + (\frac{1-b}{1-a} - \frac{b}{a})\ln z_t$$

$$+ \frac{1}{a} \ln \bar{y}_t$$

$$= \ln x_{1t}^{D} - \frac{1}{a}(\ln y_t^S - \ln \bar{y}_t)$$

We can clearly see that the effective demand for workers is equal to the Walrasian demand minus 1/a times the rationed quantity (Walrasian quantity - realised transactions).

2.1.2.2. <u>The producer is rationed on the number of workers</u>

The effective demand for workers is equal to the Walrasian demand.

$$\ln x_{1t}^{D'} = \ln x_{1t}^{D}$$

By maximizing the profit function subject to the production function and to $x_{1t}^{D} = \bar{x}_{1t}$, we find the effective demand for average hours of work per worker and the effective commodity supply

$$\ln x_{2t}^{D'} = \frac{1}{1-b} \ln A + \frac{1}{1-b} \ln b + \frac{\lambda}{1-b} t + \frac{1}{1-b} \ln p_t - \frac{1}{1-b} \ln w_t - \frac{1}{1-b} \ln (1+s_t)$$
$$- \frac{1-a}{1-b} \ln \bar{x}_{1t} \qquad /13/$$

$$\ln y_t^{S'} = \frac{1}{1-b} \ln A + \frac{b}{1-b} \ln b + \frac{\lambda}{1-b} t + \frac{b}{1-b} \ln p_t - \frac{b}{1-b} \ln w_t - \frac{b}{1-b} \ln(1+s_t)$$
$$+ \frac{a-b}{1-b} \ln \bar{x}_{1t} \qquad /14/$$

These functions can be rewritten as

$$\ln x_{1t}^{D'} = \ln x_{2t}^{D} + \frac{1-a}{1-b} (\ln x_{1t}^{D} - \ln \bar{x}_{1t})$$

$$\ln y_t^{S'} = \ln y_t^{S} - \frac{a-b}{1-b} (\ln x_{1t}^{D} - \ln \bar{x}_{1t})$$

The spill-over elasticity for the effective demand for hours is given by

$$\frac{\partial \ln x_{2t}^{D'}}{\partial \ln \bar{x}_{1t}} = - \frac{1-a}{1-b}$$

As we know that $0 < a < 1$, $0 < b < 1$ and $a > b$, we have

$$- 1 < - \frac{1-a}{1-b} < 0$$

The effective demand for average hours of work per worker will be greater than the Walrasian demand.

The difference between the effective demand for hours and the Walrasian demand will be less than proportionate to the rationed quantity of the number of workers. The spill-over elasticity for the effective commodity supply is

$$\frac{\partial \ln y_t^{S'}}{\partial \ln \bar{x}_{1t}} = \frac{a-b}{1-b}$$

with $0 < \frac{a-b}{1-b} < 1$

Although the effective demand for hours is greater than the Walrasian demand, the effective commodity supply will be less than the Walrasian supply. The producer tries to compensate the lower number of workers by demanding more hours, but this compensation is only partly.

2.1.2.3. The producer is rationed on the average hours of work per worker

The effective demand for the average hours of work per worker coincides with the Walrasian demand

$$\ln x_{2t}^{D'} = \ln x_{2t}^{D}$$

The effective demand for workers and the effective supply of commodities are found by maximizing profit subject to the production function and to $x_{2t} = \bar{x}_{2t}$.

$$\ln x_{1t}^{D'} = \frac{1}{1-a}\ln A + \frac{1}{1-a}\ln a + \frac{\lambda}{1-a}t + \frac{1}{1-a}p_t + \frac{b}{1-a}\ln \bar{x}_{2t} - \frac{1}{1-a}\ln[w_t(1+s_t)\bar{x}_{2t}+z_t] \qquad /15/$$

$$\ln y_t^{S'} = \frac{1}{1-a}\ln A + \frac{a}{1-a}\ln a + \frac{\lambda}{1-a}t + \frac{a}{1-a}\ln p_t + \frac{b}{1-a}\ln \bar{x}_{2t} - \frac{a}{1-a}\ln[w_t(1+s_t)\bar{x}_{2t}+z_t] \qquad /16/$$

These functions are not log-linear in the realised transactions. The spill-over cannot be easily isolated as in the previous case. We shall consider the original functions (before logarithms).

$$x_{1t}^{D'} = A^{\frac{1}{1-a}}\, a^{\frac{1}{1-a}}\, e^{\frac{\lambda t}{1-a}}\, p_t^{\frac{1}{1-a}}\, \bar{x}_{2t}^{\frac{b}{1-a}}\, [w_t(1+s_t)\bar{x}_{2t}+z_t]^{-\frac{1}{1-a}}$$

$$y_t^{S'} = A^{\frac{1}{1-a}}\, a^{\frac{a}{1-a}}\, e^{\frac{\lambda t}{1-a}}\, p_t^{\frac{a}{1-a}}\, \bar{x}_{2t}^{\frac{b}{1-a}}\, [w_t(1+s_t)\bar{x}_{2t}+z_t]^{-\frac{1}{1-a}}$$

We can now consider the spill-over effects

$$\frac{\partial x_{1t}^{D'}}{\partial \bar{x}_{2t}} = A^{\frac{1}{1-a}}\, a^{\frac{1}{1-a}}\, e^{\frac{\lambda t}{1-a}}\, p_t^{\frac{1}{1-a}} \Big\{ \frac{b}{1-a}\, \bar{x}_t^{\frac{b}{1-a}-1}\, [w_t(1+s_t)\bar{x}_{2t}+z_t]^{-\frac{1}{1-a}}$$

$$+ \left(-\frac{1}{1-a}\right)[w_t(1+s_t)\bar{x}_{2t}+z_t]^{-\frac{1}{1-a}-1}\, w_t(1+s_t)\bar{x}_{2t}^{\frac{b}{1-a}} \Big\}$$

$$\frac{\partial x^{D'}_{1T}}{\partial \bar{x}_{2t}} = A^{\frac{1}{1-a}} a^{\frac{1}{1-a}} e^{\frac{\lambda t}{1-a}} p_t^{\frac{1}{1-a}} \frac{1}{1-a} \bar{x}_{2t}^{\frac{b}{1-a} - 1} \{w_t(1+s_t)\bar{x}_{2t}+z_t\}^{-\frac{1}{1-a} - 1}$$

$$\{b\, w_t(1+s_t)\bar{x}_{2t} + bz_t - w_t(1+s_t)\bar{x}_{2t}\}$$

$$= A^{\frac{1}{1-a}} a^{\frac{1}{1-a}} e^{\frac{\lambda t}{1-a}} p_t^{\frac{1}{1-a}} \frac{1}{1-a} \bar{x}_{2t}^{\frac{b}{1-a} - 1} \{w_t(1+s_t)\bar{x}_{1t}+z_t\}^{-\frac{1}{1-a} -1}$$

$$\{(b-1)w_t(1+s_t)\bar{x}_{2t} + bz_t\}$$

In the same way we have

$$\frac{\partial y^{S'}_t}{\partial \bar{x}_{2t}} = A^{\frac{1}{1-a}} a^{\frac{a}{1-a}} e^{\frac{\lambda t}{1-a}} p_t^{\frac{a}{1-a}} \frac{1}{1-a} \bar{x}_{2t}^{\frac{b}{1-a} - 1} \{w_t(1+s_t)\bar{x}_{2t}+z_t\}^{-\frac{a}{1-a} - 1}$$

$$\{(b-a)w_t(1+s_t)\bar{x}_{2t} + bz_t\}$$

The sign of the spill-over effects will be determined by the last factor of the functions, as all the other factors are positive.

Therefore, we may write

$$\frac{\partial x^{D'}_{1t}}{\partial \bar{x}_{2t}} \gtrless 0$$

$$<=>$$

$$bz_t - (1-b)w_t(1+s_t)\bar{x}_{2t} \gtrless 0$$

$$<=>$$

$$b\ z_t \gtrless (1-b)w_t(1+s_t)\bar{x}_{2t}$$

$$<=>$$

$$\frac{b}{1-b} \frac{z_t}{w_t(1+s_t)} \gtrless \bar{x}_{2t}$$

and

$$\frac{\partial y_t^{S'}}{\partial \bar{x}_{2t}} \gtreqless 0$$

$$<=>$$

$$bz_t - (a-b)w_t(1+s_t)\bar{x}_{2t} \gtreqless 0$$

$$<=>$$

$$bz_t \gtreqless (a-b)w_t(1+s_t)\bar{x}_{2t}$$

$$<=>$$

$$\frac{b}{a-b}\,\frac{z_t}{w_t(1+s_t)} \gtreqless \bar{x}_{2t}$$

$$<=>$$

$$x_{2t}^D \gtreqless \bar{x}_{2t} \qquad (1)$$

We can summarize as follows:

$\bar{x}_{2t}$		$\bar{x}_{2t} = \frac{b}{1-b}\,\frac{z_t}{w_t(1+s_t)}$		$x_{2t}^D = \frac{a}{1-b}\,\frac{z_t}{w_t(1+s_t)}$	
$\frac{d\ y^{S'}}{d\ \bar{x}_{2t}}$	+	+	+	0	-
$\frac{d\ x_{1t}^{D'}}{d\ \bar{x}_{2t}}$	+	0	-	-	-

We can illustrate this graphically on figure 1.

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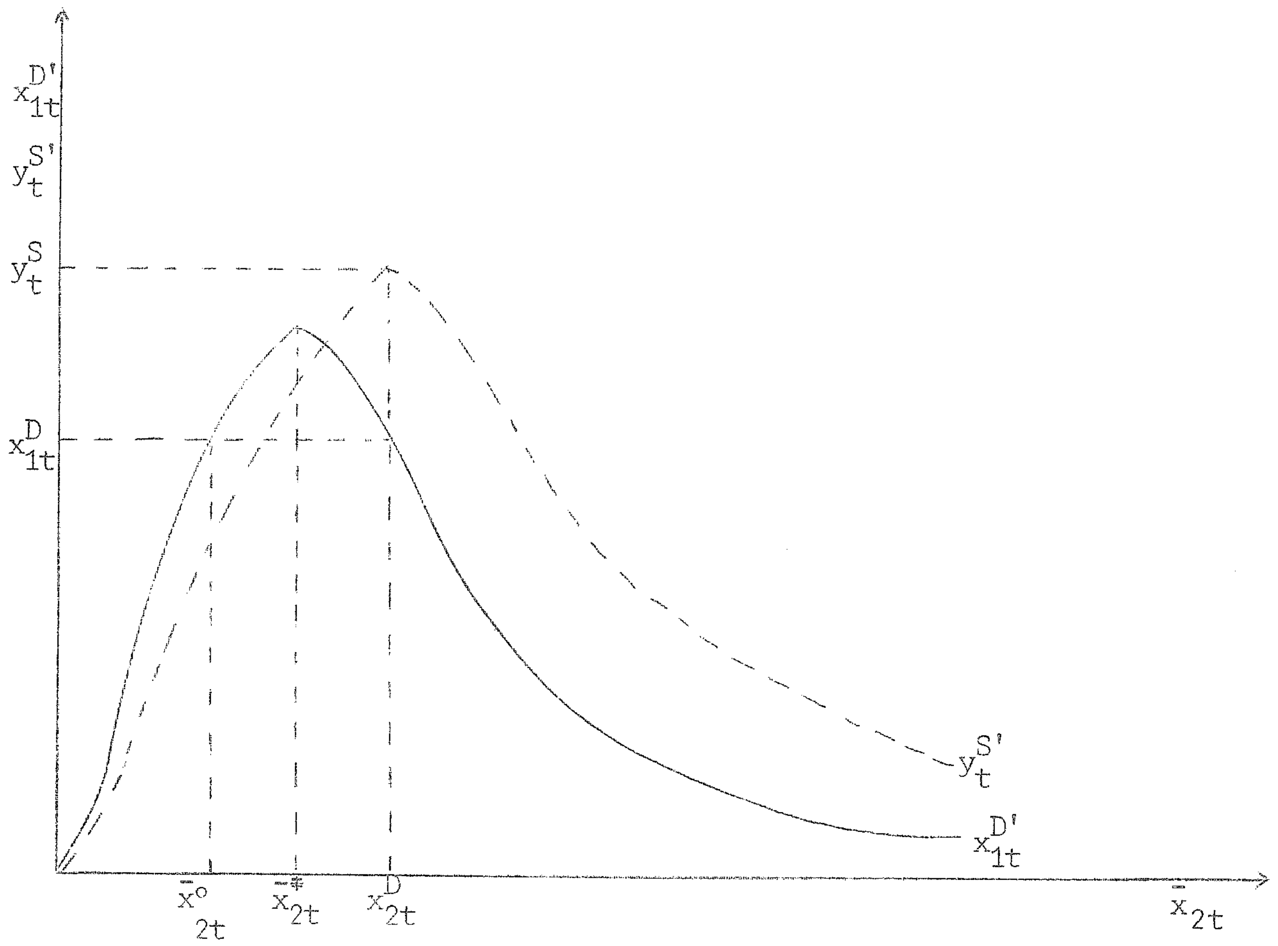
(1) When we use the minimum condition $\bar{x}_{2t}=\min\{x_{2t}^S, x_{2t}^D\}$ we can state that $\bar{x}_{2t} \leq x_{2t}^D$ and therefore $y_t^{S'}/\bar{x}_{2t}$ will allways be nonnegative. Working with the condition $\bar{x}_{2t}=\min\{x_{2t}^{S'}, x_{2t}^{D'}\}$, $\bar{x}_{2t}$ can be greater than x_{2t}^D if $x_{2t}^{D'} > x_{2t}^D$ and $x_{2t}^D < \bar{x}_{2t} \leq x_{2t}^{D'}$.

Figure 1



If the number of hours of work per worker is set below $\bar{x}^o_{2t}$, unemployment will increase.

Another way to get round the non-loglinearity is to use a Taylor expansion.

Let $f(\ln \bar{x}_{2t}) = \ln\{w_t(1+s_t)\bar{x}_{2t}+z_t\}$

Using a first degree Taylor expansion around $\ln x^D_{2t}$ gives

$$f(\ln \bar{x}_{2t}) = \ln\{w_t(1+s_t)x^D_{2t}+z_t\} + \frac{w_t(1+s_t)}{w_t(1+s_t)x^D_{2t}+z_t}(\ln \bar{x}_{2t} - \ln x^D_{2t})$$

As $x^D_{2t} = \frac{b}{a-b} \frac{z_t}{w_t(1+s_t)}$ we can write

$$f(\ln \bar{x}_{2t}) = \ln \frac{a}{a-b} + \ln z_t + \frac{a-b}{a} \frac{w_t(1+s_t)}{z_t} \ln \bar{x}_{2t}$$
$$- (\frac{a-b}{a} \ln \frac{b}{a-b}) \frac{w_t(1+s_t)}{z_t} - \frac{a-b}{a} \frac{w_t(1+s_t)}{z_t} \ln \frac{w_t(1+s_t)}{z_t}$$

The effective functions are then

$$\ln x^{D'}_{1t} = \frac{1}{1-a} \ln A + \frac{1}{1-a} \ln a - \frac{1}{1-a} \ln \frac{a}{a-b} + \frac{\lambda}{1-a} t + \frac{1}{1-a} \ln p_t - \frac{1}{1-a} \ln z_t$$
$$+ \{\frac{a-b}{a(1-a)} \ln \frac{b}{a-b}\} \frac{w_t(1+s_t)}{z_t} + \frac{a-b}{a(1-a)} \frac{w_t(1+s_t)}{z_t} \ln \frac{w_t(1+s_t)}{z_t}$$
$$+ \frac{abz_t - (a-b)w_t(1+s_t)}{(1-a)a} \ln \bar{x}_{2t} \qquad /17/$$

$$\underset{\omega\nu}{\ln} y^{S'}_t = \frac{1}{1-a} \ln A + \frac{a}{1-a} \ln a - \frac{a}{1-a} \ln \frac{a}{a-b} + \frac{\lambda}{1-a} t + \frac{a}{1-a} \ln p_t - \frac{a}{1-a} \ln z_t$$
$$+ (\frac{a-b}{1-a} \ln \frac{b}{a-b}) \frac{w_t(1+s_t)}{z_t} + \frac{a-b}{1-a} \frac{w_t(1+s_t)}{z_t} \ln \frac{w_t(1+s_t)}{z_t}$$
$$+ \frac{bz_t - (a-b)w_t(1+s_t)}{1-a} \ln \bar{x}_{2t} \qquad /18/$$

The sign of the spill-over coefficients is no longer a function of $\bar{x}_{2t}$. We ıcannot directly use this spill-over coefficients. In estimation, they can always be used to determine the real sign of the spill-over as we can solve a and b from them.

2.1.2.4. <u>The producer is rationed in the commodity market and on the number of workers</u>

The effective commodity supply is equal to /14/, the effective demand for workers coincide with /12/. The effective demand for hours is

$$\ln x_{2t}^{D'} = -\frac{1}{b}\ln A - \frac{\lambda}{b}t - \frac{a}{b}\ln \bar{x}_{1t} + \frac{1}{b}\ln \bar{y}_t \qquad /19/$$

The effective demand for average hours of work per worker contains two spill-over effects

$$\frac{\partial \ln x_{2t}^{D'}}{\partial \ln \bar{x}_{1t}} = -\frac{a}{b} \qquad -\frac{a}{b} < -1$$

$$\frac{\partial \ln x_{2t}^{D'}}{\partial \ln \bar{y}_t} = \frac{1}{b} \qquad 1 < \frac{1}{b}$$

As $a < 1$ we have $\frac{1}{b} > \frac{a}{b}$

If both $\ln \bar{x}_{1t}$ and $\ln \bar{y}_t$ change with the same quantity the spill-over effect from the commodity market will dominate.

2.1.2.5. <u>The producer is rationed on the commodity market and on the average hours of work per worker</u>

The effective commodity supply is equal to /16/ and the effective demand of hours of work is equal to /11/.
The effective demand for workers is

$$\ln x_{1t}^{D'} = -\frac{1}{a}\ln A - \frac{\lambda}{a}t - \frac{b}{a}\ln \bar{x}_{2t} + \frac{1}{a}\ln \bar{y}_t \qquad /20/$$

In this case the effective demand for workers contains two spill-over effects

$$\frac{\partial \ln x_{1t}^{D'}}{\partial \ln \bar{x}_{2t}} = -\frac{b}{a} \qquad -1 < -\frac{b}{a} < 0$$

$$\frac{\partial \ln x_{1t}^{D'}}{\partial \ln \bar{y}_t} = \frac{1}{a} \qquad \frac{1}{a} > 1$$

If both $\ln \bar{x}_{2t}$ and $\ln \bar{y}_t$ change with the same quantity the spill-over effect from the commodity market will dominate.

2.1.2.6. The producer is rationed on the number of workers and on the average hours of work per worker

The effective demand for workers is equal to /15/ and the effective demand for hours is equal to /13/.
The effective commodity supply is

$$\ln y_t^{S'} = \ln A + \lambda\, t + a \ln \bar{x}_{1t} + b \ln \bar{x}_{2t} \qquad /21/$$

The spill-over effects on the effective commodity supply are

$$\frac{\partial \ln y_t^{S'}}{\partial \ln \bar{x}_{1t}} = a \qquad 0 < a < 1$$

$$\frac{\partial \ln y_t^{S'}}{\partial \ln \bar{x}_{2t}} = b \qquad 0 < b < 1$$

with $a < b$.

The spill-over elasticities are simply the elasticities of output with respect to the number of workers and the average hours of work per worker.

2.1.2.7. The producer is rationed in all the markets

The effective functions are given by /19/, /20/ and /21/.
They contain two spill-over effects.

2.2. The consumer side of the model

We consider a representative consumer (or a body of consumers) who has for every period $t=0,\dots,\infty$ a utility function

$$U_t = \frac{\beta_1}{\alpha_1}\left(\frac{y_t}{\beta_1}\right)^{\alpha_1} + \frac{\beta_2}{\alpha_2}\left(\frac{x_{2t}}{\beta_2}\right)^{\alpha_2} + \frac{\beta_3}{\alpha_3}\left(\frac{M_t/P_{c,t}}{\beta_3}\right)^{\alpha_3}$$

where $\alpha_1, \alpha_2, \alpha_3 < 1$

$\beta_1, \beta_3 > 0$

$\beta_2 < 0$

The budget restriction is given by

$$y_t P_{c,t} + M_t = \{w_t(1-q_t)x_{2t} + N_t\}(1-v_t) + M_{t-1}$$

with

$P_{c,t}$	the consumer price index
M_t	the nominal stock of money at the end of period t
q_t	the average coefficient for calculating the contribution to the social security
N_t	non-labour income
v_t	average personal income tax rate

We will assume that money has an indirect utility. The utility function for the consumer over the period $0,\dots,\infty$ will be given by

$$U = \sum_{t=0}^{\infty} \ell_t U_t$$

The factor ℓ_t is a discount factor which gives less importance to future utilities.

2.2.1. The Walrasian demand and supply function

In order to determine the Walrasian commodity demand and the supply of average hours of work, we have to solve following programme

$$\text{Max} \quad U = \sum_{t=0}^{\infty} \ell_t \{ \frac{\beta_1}{\alpha_1}(\frac{y_t}{\beta_1})^{\alpha_1} + \frac{\beta_2}{\alpha_2}(\frac{x_{2t}}{\beta_2})^{\alpha_2} + \frac{\beta_3}{\alpha_3}(\frac{M_t/P_{c,t}}{\beta_3})^{\alpha_3} \}$$

$$\text{s.t.} \quad P_{c,t}y_t + M_t = w_t(1-q_t)(1-v_t)x_{2t} + (1-v_t)N_t + M_{t-1}$$

$$t=0,\ldots,\infty$$

The Lagrange function, with μ_t as multiplier, is

$$\mathcal{L} = \sum_{t=0}^{\infty} \ell_t \{ \frac{\beta_1}{\alpha_1}(\frac{y_t}{\beta_1})^{\alpha_1} + \frac{\beta_2}{\alpha_2}(\frac{x_{2t}}{\beta_2})^{\alpha_2} + \frac{\beta_3}{\alpha_3}(\frac{M_t/P_{c,t}}{\beta_3})^{\alpha_3} \}$$

$$+ \sum_{t=0}^{\infty} \mu_t \{ w_t(1-q_t)(1-v_t)x_{2t} + (1-v_t)N_t + M_{t-1} - P_{c,t}y_t - M_t \}$$

The first order conditions are

$$\ell_t (\frac{y_t}{\beta_1})^{\alpha_1 - 1} = P_{c,t}\, \mu_t \qquad /2.1/$$

$$\ell_t (\frac{x_{2t}}{\beta_2})^{\alpha_2 - 1} = -w_t(1-q_t)(1-v_t)\, \mu_t \qquad /2.2/$$

$$\ell_t (\frac{M_t/P_{c,t}}{\beta_3})^{\alpha_3 - 1} \frac{1}{P_{c,t}} = \mu_t \qquad /2.3/$$

$$P_{c,t}y_t + M_t = w_t(1-q_t)(1-v_t) + (1-v_t)N_t + M_{t-1} \qquad /2.4/$$

Substituting 2.3 in 2.1 and 2.2 gives

$$\begin{cases} (\frac{y_t}{\beta_1})^{\alpha_1 - 1} = (\frac{M_t/P_{c,t}}{\beta_3})^{\alpha_3 - 1} \\ (\frac{x_{2t}}{\beta_2})^{\alpha_2 - 1} = -\, w_t(1-q_t)(1-v_t)(\frac{M_t/P_{c,t}}{\beta_3})^{\alpha_3 - 1} \frac{1}{P_{c,t}} \end{cases}$$

$$\Leftrightarrow \begin{cases} y_t = \beta_1\, \beta_3^{-\frac{1-\alpha_3}{1-\alpha_1}} (\frac{M_t}{P_{c,t}})^{\frac{1-\alpha_3}{1-\alpha_1}} \\ x_{2t} = -\beta_2\, \beta_3^{-\frac{1-\alpha_3}{1-\alpha_2}} \{ \frac{w_t}{P_{c,t}} (1-q_t)(1-v_t) \}^{-\frac{1}{1-\alpha_2}} (\frac{M_t}{P_{c,t}})^{\frac{1-\alpha_3}{1-\alpha_2}} \end{cases}$$

<=>

$$\begin{cases} \ln y_t^D = \ln \beta_1 - \dfrac{1-\alpha_3}{1-\alpha_1} \ln \beta_3 + \dfrac{1-\alpha_3}{1-\alpha_1} \ln \dfrac{M_t}{P_{c,t}} & /22/ \\ \ln x_{2t}^S = \ln(-\beta_2) - \dfrac{1-\alpha_3}{1-\alpha_2} \ln \beta_3 - \dfrac{1}{1-\alpha_2} \ln\{\dfrac{w_t}{P_{c,t}} (1-q_t)(1-v_t)\} + \dfrac{1-\alpha_3}{1-\alpha_2} \ln \dfrac{M_t}{P_{c,t}} & /23/ \end{cases}$$

If we assume that the money stock is known, we have found the Walrasian commodity demand and supply of average hours of work per worker.

We still have to find the supply of the number of workers or the labour force participation. Leonell C. aAndersen (1978) constructed a model of labour force participation. He starts from the assumption that the decision to be in the labour market depends on an individual's reservation wage rate relative to his decision wage rate. Andersen gives following definitions

"..., the reservation wage rate is an individual's marginal" "rate of substitution of goods and services for leisure when" "all available time is allocated to leisure. The decision" "wage rate is the perceived amount of goods and services" "the individual can purchase if a unit of time is shifted" "from leisure to work." (1)

When the decision wage rate is greater than the reservation wage rate, the individual will be in the labour market.

Andersen states for the reservation wage rate

$$RW_{ij} = f_{ij}(N_j^*, WE_j^*, v_j^*, q_j^*, B_j^*, N_j - N_j^*, WE_j - WE_j^*, v_j - v_j^*, q_j - q_j^*, B_j - B_j^*)$$

where RW_{ij} is the reservation wage rate of the i^{th} number of the j^{th} household

WE_j is the perceived average real wage rate of employed members of the j^{th} household

B_j is the size of the j^{th} household

The asterisk indicates the permanent component of the variable. The transitory component is the difference between the actual value and the permanent component.

(1) p.10

For the decision wage rate it is assumed that

$$DW_{ij} = s_{ij}\{ (\frac{W}{P_c})^*_j, v^*_j, q^*_j, (\frac{W}{P_c})_j - (\frac{W}{P_c})^*_j, v_j - v^*_j, q_j - q^*_j \}$$

where DW_{ij} is the decision wage rate of the i^{th} member in the j^{th} household.

The individual's participation in the labour market (LF_{ij}) is then

$$\begin{aligned} LF_{ij} &= 1 \quad \Leftrightarrow \quad RW_{ij} < DW_{ij} \\ &= 0 \quad \Leftrightarrow \quad RW_{ij} \geq DW_{ij} \end{aligned}$$

The total number of members of labour force age (14 years and over) in a household participating in the labor market (LF_j) is the sum of the total number of individuals whose decision wage rates are not smaller than their reservation wage rates. Andersen gives following function

$$LF = h_j[N^*, (\frac{W}{P_c})^*, v^*, q^*, B^*, N-N^*, \frac{W}{P_c} - (\frac{W}{P_c})^*, v-v^*, q-q^*, B-B^*, PL]$$

He assumes that the relationship is linear in natural logarithms.

For the formation of perceptions of permanent components we assume the following adjustment process (1).

$$(1-\delta D)N^*_t = \lambda N_{t-1} \qquad \text{with } \delta = 1-\lambda$$

$$(1-\delta D)(\frac{W}{P_c})^*_t = \lambda(\frac{W}{P_c})_{t-1}$$

$$(1-\delta D)v^*_t = \lambda v_{t-1}$$

$$(1-\delta D)q^*_t = \lambda q_{t-1}$$

$$(1-\delta D)B^*_t = \lambda B_{t-1}$$

The function LF is

(1) D is the delay operator such that $D^i X_t = X_{t-i}$.

$$\ln LF_t = \alpha_0 + \alpha_1 \ln N_t^{*} + \alpha_2 \ln\left(\frac{w_t}{P_{c,t}}\right)^{*} + \alpha_3 \ln v_t^{*} + \alpha_4 \ln q_t^{*} + \alpha_5 \ln B_t^{*}$$

$$+ \alpha_6(\ln N_t - \ln N_t^{*}) + \alpha_7\left\{\ln\left(\frac{w_t}{P_{c,t}}\right)_t - \ln\left(\frac{w_t}{P_{c,t}}\right)^{*}\right\} + \alpha_8(\ln v_t - \ln v_t^{*})$$

$$+ \alpha_9(\ln q_t - \ln q_t^{*}) + \alpha_{10}(\ln B_t - \ln B_t^{*}) + \alpha_{11}\ln PL_t$$

$<=>$

$$(1-\delta D)\ln LF_t = (1-\delta)\alpha_0 + \lambda_1 \ln N_{t-1} + \lambda\alpha_2 \ln\frac{w_{t-1}}{P_{c,t-1}} + \lambda\alpha_3 \ln v_{t-1} + \lambda\alpha_4 \ln q_{t-1}$$

$$+\lambda\alpha_5 \ln B_{t-1} + \alpha_6(\ln N_t - \ln N_{t-1}) + \alpha_7\left(\ln\frac{w_t}{P_{c,t}} - \ln\frac{w_{t-1}}{P_{c,t-1}}\right)$$

$$+ \alpha_8(\ln v_t - \ln v_{t-1}) + \alpha_9(\ln q_t - \ln q_{t-1}) + \alpha_{10}(\ln B_t - \ln B_{t-1})$$

$$+ \alpha_{11}\ln PL_t - \delta\alpha_{11}\ln PL_{t-1}$$

$<=>$

$$\ln LF_t = \lambda\alpha_0 + \alpha_1\lambda \ln N_{t-1} + \alpha_2\lambda\ln\frac{w_{t-1}}{P_{c,t-1}} + \alpha_3\lambda\ln v_{t-1} + \alpha_4\lambda\ln q_{t-1} + \alpha_5\lambda\ln B_{t-1}$$

$$+\alpha_6(\ln N_t - \ln N_{t-1}) + \alpha_7\left(\ln\frac{w_t}{P_{c,t}} - \ln\frac{w_{t-1}}{P_{c,t-1}}\right) + \alpha_8(\ln v_t - \ln v_{t-1})$$

$$+\alpha_9(\ln q_t - \ln q_{t-1}) + \alpha_{10}(\ln B_t - \ln B_{t-1}) + \alpha_{11}\ln PL_t$$

$$- (1-\lambda)\alpha_{11} \ln PL_{t-1} + (1-\lambda)\ln LF_{t-1}$$

We shall consider this function as the labour force participation function

$$\ln x_{1t}^{S} = a_0 + a_1\ln N_{t-1} + a_2\ln\frac{w_{t-1}}{P_{c,t-1}} + a_3\ln v_{t-1} + a_4\ln q_{t-1} + a_5\ln B_{t-1}$$

$$+ a_6(\ln N_t - \ln N_{t-1}) + a_7\left(\ln\frac{w_t}{P_{c,t}} - \ln\frac{w_{t-1}}{P_{c,t-1}}\right) + a_8(\ln v_t - \ln v_{t-1})$$

$$+ a_9(\ln q_t - \ln q_{t-1}) + a_{10}(\ln B_t - \ln B_{t-1}) + a_{11}\ln PL_t + a_{12}\ln PL_{t-1}$$

$$+ a_{13}\ln LF_{t-1}$$

2.2.2. The effective demand and supply functions

In order to have functions that are linear in natural logarithms we cannot always use a direct derivation from maximization of the utility function. We shall have to make additional assumptions.

2.2.2.1. The consumer is rationed in the commodity market

The effective commodity demand is equal to the Walrasian demand. The quantity of money is in this case the Walrasian quantity $\left(\frac{M_t}{P_{c,t}}\right)^W$. So we can write

$$\ln y_t^{D'} = \ln y_t^D = \ln \beta_1 - \frac{1-\alpha_3}{1-\alpha_1} \ln \beta_3 + \frac{1-\alpha_3}{1-\alpha_1} \ln \left(\frac{M_t}{P_{c,t}}\right)^W$$

In order to derive the effective supply of hours of work we make the following assumption. The rationing on the commodity market is reflected in the money stock. We assume

$$\ln \left(\frac{M_t}{P_{c,t}}\right) = \ln \left(\frac{M_t}{P_{c,t}}\right)^W + \gamma_1 (\ln y_t^D - \ln \bar{y}_t)$$

As the consumer can buy less commodities, he will save more so that we assume $\gamma_1 > 0$.
We can now replace $\ln y_t^D$ to become:

$$\ln\left(\frac{M_t}{P_{c,t}}\right) = \ln\left(\frac{M_t}{P_{c,t}}\right)^W + \gamma_1 \ln \beta_1 - \gamma_1 \frac{1-\alpha_3}{1-\alpha_1} \ln \beta_3 + \gamma_1 \frac{1-\alpha_3}{1-\alpha_1} \ln\left(\frac{M_t}{P_{c,t}}\right)^W - \gamma_1 \ln \bar{y}_t$$

This expression will now be replaced into the supply of average hours

$$\ln x_{2t}^{S'} = \ln(-\beta_2) - \frac{1-\alpha_3}{1-\alpha_2} \ln \beta_3 - \frac{1}{1-\alpha_2} \ln\{\frac{w_t}{P_{c,t}} (1-q_t)(1-v_t)\} + \frac{1-\alpha_3}{1-\alpha_2} \frac{1-\alpha_1+\gamma_1-\gamma_1\alpha_3}{1-\alpha_1} \ln\left(\frac{M_t}{P_{c,t}}\right)^W$$

$$+ \frac{1-\alpha_3}{1-\alpha_2} \gamma_1 \ln \beta_1 - \frac{1-\alpha_3}{1-\alpha_2} \gamma_1 \frac{(1-\alpha_3)}{1-\alpha_1} \ln \beta_3 - \frac{1-\alpha_3}{1-\alpha_2} \gamma_1 \ln \bar{y}_t$$

<=>

$$\ln x_{2t}^{S'} = \ln(-\beta_2) - \frac{(1-\alpha_3)(1-\alpha_1+\gamma_1-\alpha_3\gamma_1)}{(1-\alpha_1)(1-\alpha_2)} \ln \beta_3 + \frac{1-\alpha_3}{1-\alpha_2} \gamma_1 \ln \beta_1$$

$$+ \frac{(1-\alpha_3)(1-\alpha_1+\gamma_1-\alpha_3\gamma_1)}{(1-\alpha_1)(1-\alpha_2)} \ln\left(\frac{M_t}{P_{c,t}}\right)^{w} - \frac{1}{1-\alpha_2} \ln\left\{\frac{w_t}{P_{c,t}}(1-q_t)(1-v_t)\right\}$$

$$- \frac{1-\alpha_3}{1-\alpha_2} \gamma_1 \ln \bar{y}_t \qquad /25/$$

The spill-over elasticity is $-\frac{1-\alpha_3}{1-\alpha_2} < 0$ (of course under the assumption that $\gamma_1 > 0$)

If there is a stronger rationing, this means lower realised transactions, the effective supply of hours will rise. This suggests that our assumption was wrong and that $\gamma_1 < 0$. For the effective labour force participation we will state in an ad hoc way that

$$\ln x_{1t}^{S'} = \ln x_{1t}^{S} + \gamma_2(\ln y_t^{D} - \ln \bar{y}_t)$$

We do not want to make any assumption about the sign of γ_2 although we think $\gamma_2 < 0$.

Replacing the Walrasian quantities gives

$$\ln x_{1t}^{S'} = a_0 + \gamma_1 \ln \beta_1 - \gamma_1 \frac{1-\alpha_3}{1-\alpha_1} \ln \beta_3 + a_1 \ln N_{t-1} + a_2 \ln \frac{w_{t-1}}{P_{c,t-1}} + a_3 \ln v_{t-1}$$

$$+ a_4 \ln q_{t-1} + a_5 \ln B_{t-1} + a_6(\ln N_t - \ln N_{t-1}) + a_7\left(\ln \frac{w_t}{P_{c,t}} - \ln \frac{w_{t-1}}{P_{c,t-1}}\right)$$

$$+ a_8(\ln v_t - \ln v_{t-1}) + a_9(\ln q_t - \ln q_{t-1}) + a_{10}(\ln B_t - \ln B_{t-1}) + a_{11} \ln PL_t$$

$$+ a_{12} \ln PL_{t-1} + a_{13} \ln x_{1,t-1}^{S} + \frac{1-\alpha_3}{1-\alpha_1} \ln \frac{M_t}{P_{c,t}} - \gamma_2 \ln \bar{y}_t \qquad /26/$$

The spill-over elasticity is $-\gamma_2$.

2.2.2.2. <u>The consumer is rationed on his labour force participation</u>
In this situation the effective labour force participation coincides with the Walrasian one.
We shall assume that the number of unemployed influences the money stock in the following way

$$\ln\left(\frac{M_t}{P_{c,t}}\right) = \ln\left(\frac{M_t}{P_{c,t}}\right)^W + \gamma_3(\ln x_{1t}^S - \ln \bar{x}_{1t})$$

We think it is reasonable to assume that an increase in unemployment leads to a decrease in the money stock and so $\gamma_3 < 0$.

We have for the effective functions

$$\ln y_t^{D'} = \ln \beta_1 - \frac{1-\alpha_3}{1-\alpha_1}\ln \beta_3 + \frac{1-\alpha_3}{1-\alpha_1}\ln\left(\frac{M_t}{P_{c,t}}\right)^W + \gamma_3\frac{1-\alpha_3}{1-\alpha_1}\ln x_{1t}^S - \gamma_3\frac{1-\alpha_3}{1-\alpha_1}\ln \bar{x}_{1t}$$

$$\ln x_{2t}^{S'} = \ln(-\beta_2) - \frac{1-\alpha_3}{1-\alpha_2}\ln \beta_3 + \frac{1}{1-\alpha_2}\ln\frac{W_t}{P_{c,t}}(1-q_t)(1-v_t) + \frac{1-\alpha_3}{1-\alpha_2}\ln\left(\frac{M_t}{P_{c,t}}\right)^W$$
$$+ \frac{1-\alpha_3}{1-\alpha_2}\gamma_1\ln x_{1t}^S - \frac{1-\alpha_3}{1-\alpha_2}\gamma_1\ln \bar{x}_{1t}$$

<=>

$$\ln y_t^{D'} = \ln \beta_1 - \frac{1-\alpha_3}{1-\alpha_1} 1 \quad \beta_3 + a_0\gamma_3\frac{1-\alpha_3}{1-\alpha_1} + \frac{1-\alpha_3}{1-\alpha_1}\ln\left(\frac{M_t}{P_{c,t}}\right)^W + a_1\gamma_3\frac{1-\alpha_3}{1-\alpha_1}\ln N_{t-1}$$
$$+ a_2\gamma_3\frac{1-\alpha_3}{1-\alpha_1}\ln\frac{W_{t-1}}{P_{c,t-1}} + a_3\gamma_3\frac{1-\alpha_3}{1-\alpha_1}\ln v_{t-1} + a_4\gamma_3\frac{1-\alpha_3}{1-\alpha_1}\ln q_{t-1} + a_5\gamma_3\frac{1-\alpha_3}{1-\alpha_1}\ln B_{t-1} +$$
$$+ 6\gamma_3\frac{1-\alpha_3}{1-\alpha_1}(\ln N_t - \ln N_{t-1}) + a_7\gamma_3\frac{1-\alpha_3}{1-\alpha_1}\left(\ln\frac{W_t}{P_{c,t}} - \ln\frac{W_{t-1}}{P_{c,t-1}}\right) + a_8\gamma_3\frac{1-\alpha_3}{1-\alpha_1}(\ln v_t -$$
$$\ln v_{t-1}) + a_9\gamma_3\frac{1-\alpha_3}{1-\alpha_1}(\ln q_t - \ln q_{t-1}) + a_{10}\gamma_3\frac{1-\alpha_3}{1-\alpha_1}(\ln B_t - \ln B_{t-1})$$
$$+ a_{11}\gamma_3\frac{1-\alpha_3}{1-\alpha_1}\ln PL_t + a_{12}\gamma_3\frac{1-\alpha_3}{1-\alpha_1}\ln PL_{t-1} + a_{13}\gamma_3\frac{1-\alpha_3}{1-\alpha_1}\ln x_{1,t-1}^S - \frac{1-\alpha_3}{1-\alpha_1}\gamma_3\ln \bar{x}_{1t}$$

$$\ln x_{2t}^{S'} = \ln(-\beta_2) - \frac{1-\alpha_3}{1-\alpha_2}\ln\beta_3 + a_0\gamma_3\frac{1-\alpha_3}{1-\alpha_2} + \frac{1-\alpha_3}{1-\alpha_2}\ln(\frac{M_t}{P_{c,t}})^w - \frac{1}{1-\alpha_2}\ln\frac{w_t}{P_{c,t}}(1-q_t)(1-v_t)$$

$$+ a_1\gamma_3\frac{1-\alpha_3}{1-\alpha_2}\ln N_{t-1} + a_2\gamma_3\frac{1-\alpha_3}{1-\alpha_2}\ln\frac{w_t}{P_{c,t}} + a_3\gamma_3\frac{1-\alpha_3}{1-\alpha_2}\ln v_{t-1} + a_4\gamma_3\frac{1-\alpha_3}{1-\alpha_2}\ln q_{t-1}$$

$$+ a_5\gamma_3\frac{1-\alpha_3}{1-\alpha_2}\ln B_{t-1} + a_6\gamma_3\frac{1-\alpha_3}{1-\alpha_2}(\ln N_t - \ln N_{t-1}) + a_7\gamma_3\frac{1-\alpha_3}{1-\alpha_1}(\ln\frac{w_t}{P_{c,t}} - \ln\frac{w_{t-1}}{P_{c,t-1}})$$

$$+ a_8\gamma_3\frac{1-\alpha_3}{1-\alpha_2}(\ln v_t - \ln v_{t-1}) + a_9\gamma_3\frac{1-\alpha_3}{1-\alpha_2}(\ln q_t - \ln q_{t-1}) + a_{10}\gamma_3\frac{1-\alpha_3}{1-\alpha_2}(\ln B_t -$$

$$\ln B_{t-1}) + a_{11}\gamma_3\frac{1-\alpha_3}{1-\alpha_2}\ln PL_t + a_{12}\gamma_3\frac{1-\alpha_2}{1-\alpha_2}\ln PL_{t-1} + a_{13}\gamma_3\frac{1-\alpha_3}{1-\alpha_2}\ln x_{1,t-1}^S$$

$$- \frac{1-\alpha_3}{1-\alpha_2}\gamma_3\ln\bar{x}_{1t} \qquad /28/$$

The spill-over elasticities are both greater than zero.

2.2.2.3. <u>The consumer is rationed on the average hours of work</u>

The effective supply of hours is the Walrasian one

$$\ln x_{2t}^{S'} = \ln x_t^S = \ln(-\beta_2)\frac{1-\alpha_3}{1-\alpha_2}\ln\beta_3 - \frac{1}{1-\alpha_2}\ln\frac{w_t}{P_{c,t}}(1-q_t)(1+v_t) + \frac{1-\alpha_3}{1-\alpha_2}\ln(\frac{M_t}{P_{c,t}})^w$$

To find the effective commodity demand we will make an assumption that is analogue to that in 2.2.2.1.

$$\ln(\frac{M_t}{P_{c,t}}) = \ln(\frac{M_t}{P_{c,t}})^w + \gamma_4(\ln x_{2t}^S - \ln\bar{x}_{2t})$$

with $\gamma_4 < 0$.

We can then write

$$\ln y_t^{S'} = \ln\beta_1 - \frac{1-\alpha_3}{1-\alpha_1}\ln\beta_3 + \frac{1-\alpha_3}{1-\alpha_1}\ln(\frac{M_t}{P_{c,t}})^w + \gamma_4\frac{1-\alpha_3}{1-\alpha_1}\ln x_{2t}^S - \gamma_4\frac{1-\alpha_3}{1-\alpha_1}\ln\bar{x}_{2t}$$

<=>

$$\ln y_t^{S'} = \ln \beta_1+\gamma_4\frac{1-\alpha_3}{1-\alpha_1}\ln(-\beta_2)-\frac{(1-\alpha_3)(1-\alpha_2+\gamma_4-\alpha_3\gamma_4)}{(1-\alpha_1)(1-\alpha_2)}\ln \beta_3-\gamma_4\frac{1-\alpha_3}{(1-\alpha_1)(1-\alpha_2)}\ln\frac{w_t}{P_{c,t}}(1-q_t)$$

$$(1-v_t)+\frac{(1-\alpha_3)(1-\alpha_2+\gamma_4-\alpha_3\gamma_4)}{(1-\alpha_2)(1-\alpha_1)}\ln(\frac{M_t}{P_{c,t}})^w-\gamma_4\frac{1-\alpha_3}{1-\alpha_1}\ln \bar{x}_{2t} \qquad /29/$$

The spill-over elasticity is $-\gamma_4 \dfrac{1-\alpha_3}{1-\alpha_1} > 0$.

For the effective labour force participation we will introduce, as in 2.2.2.1., an ad hoc function.

$$\ln x_{1t}^{S'} = \ln x_{1t}^S + \gamma_5(\ln x_{2t}^S - \ln \bar{x}_{2t})$$

<=>

$$\ln x_{1t}^{S'} = a_0 + \gamma_5\ln(-\beta_2)-\gamma_5\frac{1-\alpha_3}{1-\alpha_2}\ln\beta_3+a_1\ln N_{t-1}+a_2\ln\frac{w_{t-1}}{P_{c,t-1}}+a_3\ln v_{t-1}+a_4\ln q_{t-1}+$$

$$a_5\ln B_{t-1}+a_6(\ln N_t-\ln N_{t-1})+a_7(\ln\frac{w_t}{P_{c,t}}-\ln\frac{w_{t-1}}{P_{c,t-1}})+a_8(\ln v_t-\ln v_{t-1})+$$

$$a_9(\ln q-\ln q_{t-1})+a_{10}(\ln B_t-\ln B_{t-1})+a_{11}\ln PL_t+a_{12}\ln PL_{t-1}+a_{13}\ln x_{1t-1}^{S'}$$

$$-\frac{\gamma_5}{1-\alpha_2}\ln\frac{w_t}{P_{c,t}}(1-q_t)(1-v_t)+\gamma_5\frac{1-\alpha_3}{1-\alpha_2}\ln(\frac{M_t}{P_{c,t}})^w-\gamma_5\ln \bar{x}_{2t} \qquad /30/$$

The sign of γ_5 is not quite clear and we will not make any assumption about it.

2.2.2.4. The consumer is rationed in the commodity market and on the labour force participation

The effective commodity demand is the same as /27/ and the effective labour force participation is equal to /26/.

For the effective supply of hours of work, we make following assumption

$$\ln(\frac{M_t}{P_{c,t}}) = \ln(\frac{M_t}{P_{c,t}})^w + \gamma_6(\ln y_t^D-\ln \bar{y}_t) + \gamma_7(\ln x_{1t}^S - \ln \bar{x}_{1t})$$

The effective supply of hours becomes

$$\ln x_{2t}^{S'} = \ln(-\beta_2) - \frac{1-\alpha_3}{1-\alpha_2}\ln\beta_3 - \frac{1}{1-\alpha_2}\ln\frac{w_t}{P_{c,t}}\,(1-q_t)(1-v_t) + \frac{1-\alpha_3}{1-\alpha_2}\ln\Big(\frac{M_t}{P_{c,t}}\Big)^w + \gamma_6\Big(\frac{1-\alpha_3}{1-\alpha_2}\Big)\ln y_t^D + \gamma_7\frac{1-\alpha_3}{1-\alpha_2}\ln x_{1t}^S - \gamma_6\frac{1-\alpha_3}{1-\alpha_2}\ln\bar{y}_t - \gamma_7\frac{1-\alpha_3}{1-\alpha_2}\ln\bar{x}_{1t}$$

After substituting $\ln y_t^D$ and $\ln x_{1t}^S$ this gives

$$\begin{aligned}\ln x_{2t}^{S'} = {} & \ln(-\beta_2) + \gamma_6\frac{1-\alpha_3}{1-\alpha_1}\ln\beta_1 - \frac{(1-\alpha_3)(1-\alpha_1+\gamma_6-\gamma_6\alpha_3)}{(1-\alpha_1)(1-\alpha_2)}\ln\beta_3 + \frac{1-\alpha_3}{1-\alpha_2}\gamma_7 a_0 - \frac{1}{1-\alpha_2}\ln\frac{w_t}{P_{c,t}}\,(1-q_t) \\ & (1-v_t) + \frac{(1-\alpha_3)(1-\alpha_1+\gamma_6-\gamma_6\alpha_3)}{(1-\alpha_1)(1-\alpha_2)}\ln\Big(\frac{M_t}{P_{c,t}}\Big)^w + \gamma_7 a_1\frac{1-\alpha_3}{1-\alpha_2}\ln N_{t-1} + \gamma_7 a_2\frac{1-\alpha_3}{1-\alpha_2}\ln\frac{w_{t-1}}{P_{c,t-1}} \\ & + \gamma_7 a_3\frac{1-\alpha_3}{1-\alpha_2}\ln v_{t-1} + \gamma_7 a_4\frac{1-\alpha_3}{1-\alpha_2}\ln q_{t-1} + \gamma_7 a_5\frac{1-\alpha_3}{1-\alpha_2}\ln B_{t-1} + \gamma_7 a_6\frac{1-\alpha_3}{1-\alpha_2}(\ln N_t - \ln N_{t-1}) \\ & + \gamma_7 a_7\frac{1-\alpha_3}{1-\alpha_2}\Big(\ln\frac{w_t}{P_{c,t}} - \ln\frac{w_{t-1}}{P_{c,t-1}}\Big) + \gamma_7 a_8\frac{1-\alpha_3}{1-\alpha_2}(\ln v_t - \ln v_{t-1}) + \gamma_7 a_9\frac{1-\alpha_3}{1-\alpha_2}(\ln q_t - \ln q_{t-1}) \\ & + \gamma_7 a_{10}(\ln B_t - \ln B_{t-1}) + \gamma_7 a_{11}\frac{1-\alpha_3}{1-\alpha_2}(\ln PL_t) + \gamma_7 a_{12}\frac{1-\alpha_3}{1-\alpha_2}\ln PL_{t-1} \\ & + \gamma_7 a_{13}\frac{1-\alpha_3}{1-\alpha_2}\ln x_{1t-1}^S - {}_6\frac{1-\alpha_3}{1-\alpha_2}\ln\bar{y}_t - \gamma_7\frac{1-\alpha_3}{1-\alpha_2}\ln\bar{x}_{1t} \qquad /31/\end{aligned}$$

We have two spill-over elasticities

$$\frac{\partial\ln x_{2t}^{S'}}{\partial\ln\bar{y}_t} = -\gamma_6\,\frac{1-\alpha_3}{1-\alpha_2} \quad \text{and} \quad \frac{\partial\ln x_{2t}^{S'}}{\partial\ln\bar{x}_{1t}} = -\gamma_7\,\frac{1-\alpha_3}{1-\alpha_2}$$

2.2.2.5. The consumer is rationed in the commodity market and on the hours of work

For the effective commodity demand we have the same function as /29/.
The effective supply of hours of work is equal to /25/.
For the effective labour force participation we assume

$$\ln x_{1t}^{S'} = \ln x_{1t}^{S} + \gamma_8(\ln y_t^D - \ln \bar{y}_t) + \gamma_9(\ln x_{2t}^S - \ln \bar{x}_{2t})$$

Substituting $\ln y_t^D$ and $\ln x_{2t}^S$ gives

$$\ln x_{1t}^{S'} = a_0 + \gamma_8 \ln\beta_1 + \gamma_9 \ln(-\beta_2) - (1-\alpha_3)\frac{\gamma_8+\gamma_9-\alpha_2\gamma_8-\alpha_3\gamma_9}{(1-\alpha_1)(1-\alpha_2)}\ln\beta_3 - \frac{\gamma_9}{1-\alpha_2}\ln\frac{w_t}{P_{c,t}}(1-q_t)(1-v_t)$$

$$+(1-\alpha_3)\frac{\gamma_8+\gamma_9-\alpha_2\gamma_8-\alpha_3\gamma_9}{(1-\alpha_1)(1-\alpha_2)}\ln\left(\frac{M_t}{P_{c,t}}\right)^w + a_1\ln N_{t-1} + a_2\ln\frac{w_{t-1}}{P_{c,t-1}} + a_3\ln v_{t-1}$$

$$+a_4\ln q_{t-1} + a_5\ln B_{t-1} + a_6(\ln N_t - \ln N_{t-1}) + a_7\left(\ln\frac{w_t}{P_{c,t}} - \ln\frac{w_{t-1}}{P_{c,t-1}}\right) + a_8(\ln v_t -$$

$$\ln v_{t-1}) + a_9(\ln q_t - \ln q_{t-1}) + a_{10}(\ln B_t - \ln B_{t-1}) + a_{11}\ln PL_t + a_{12}\ln PL_{t-1}$$

$$+a_{13}\ln x_{1t-1}^{S'} - \gamma_8\ln\bar{y}_t - \gamma_9\ln\bar{x}_{2t} \qquad /32/$$

We have again two spill-over elasticities

$$\frac{\partial \ln x_{1t}^{S'}}{\partial \ln \bar{y}_t} = -\gamma_8 \quad \text{and} \quad \frac{\partial \ln x_{1t}^{S'}}{\partial \ln \bar{x}_{2t}} = -\gamma_9$$

2.2.2.6. <u>The consumer is rationed on both the labour force participation and the average hours of work</u>

The effective labour force participation is the same as /30/ and the effective supply of hours is equal to /28/. For the effective commodity demand we make the following assumption

$$\ln\frac{M_t}{P_{ct}} = \ln\left(\frac{M_t}{P_{c,t}}\right)^w + \gamma_{10}(\ln x_{1t}^S - \ln\bar{x}_{1t}) + \gamma_{11}(\ln x_{2t}^S - \ln\bar{x}_{2t})$$

The effective commodity demand can be written as

$$\ln y_t^D = \ln\beta_1 - \frac{1-\alpha_3}{1-\alpha_1}\ln\beta_3 + \frac{1-\alpha_3}{1-\alpha_1}\gamma_{10}\ln x_{1t}^S + \frac{1-\alpha_3}{1-\alpha_1}\gamma_{11}\ln x_{2t}^S - \gamma_{10}\frac{1-\alpha_3}{1-\alpha_1}\ln\bar{x}_{1t}$$

$$-\gamma_{11}\frac{1-\alpha_3}{1-\alpha_1}\ln\bar{x}_{2t}$$

<= >

$$\ln y_t^{D'} = \ln \beta_1 + \gamma_{10}\frac{1-\alpha_3}{1-\alpha_1}a_0 + \gamma_{11}\frac{1-\alpha_3}{1-\alpha_1}\ln(-\beta_2) - \frac{(1-\alpha_3)(1-\alpha_2+\gamma_{11}-\gamma_{11}\alpha_3)}{(1-\alpha_1)(1-\alpha_2)}\ln \beta_3$$

$$-\frac{\gamma_{10}(1-\alpha_3)}{(1-\alpha_1)(1-\alpha_2)}\ln\frac{w_t}{P_{c,t}}(1-q_t)(1-v_t) + \frac{(1-\alpha_3)(1-\alpha_2+\gamma_{11}-\gamma_{11}\alpha_3)}{(1-\alpha_1)(1-\alpha_2)}\ln(\frac{M_t}{P_{c,t}})^w$$

$$+\gamma_{10}a_1\frac{1-\alpha_3}{1-\alpha_1}\ln N_{t-1} + \gamma_{10}a_2\frac{1-\alpha_3}{1-\alpha_1}\ln\frac{w_{t-1}}{P_{c,t-1}} + \gamma_{10}a_3\frac{1-\alpha_3}{1-\alpha_1}\ln v_{t-1} + \gamma_{10}a_4\frac{1-\alpha_3}{1-\alpha_1}\ln q_{t-1}$$

$$+\gamma_{10}a_5\frac{1-\alpha_3}{1-\alpha_1}\ln B_{t-1} + \gamma_{10}a_6\frac{1-\alpha_3}{1-\alpha_1}(\ln N_t - \ln N_{t-1}) + \gamma_{10}a_7\frac{1-\alpha_3}{1-\alpha_1}(\ln\frac{w_t}{P_{c,t}} - \ln\frac{w_{t-1}}{P_{c,t-1}})$$

$$+\gamma_{10}a_8\frac{1-\alpha_3}{1-\alpha_1}(\ln v_t - \ln v_{t-1}) + \gamma_{10}a_9\frac{1-\alpha_3}{1-\alpha_1}(\ln q_t - \ln q_{t-1}) + \gamma_{10}a_{10}\frac{1-\alpha_3}{1-\alpha_1}(\ln B_t -$$

$$B_{t-1}) + \gamma_{10}a_{11}\frac{1-\alpha_3}{1-\alpha_1}\ln PL_t + \gamma_{10}a_{12}\frac{1-\alpha_3}{1-\alpha_1}\ln PL_{t-1} + \gamma_{10}a_{13}\frac{1-\alpha_3}{1-\alpha_1}\ln x^S_{1,t-1}$$

$$-\gamma_{10}\frac{1-\alpha_3}{1-\alpha_1}\ln \bar{x}_{1t} - \gamma_{11}\frac{1-\alpha_3}{1-\alpha_1}\ln \bar{x}_{2t} \qquad /33/$$

The spill-over elasticities

$$\frac{\partial y_t^{D'}}{\partial \bar{x}_{1T}} = -\gamma_{10}\frac{1-\alpha_3}{1-\alpha_1} \quad \text{and} \quad \frac{\partial y_t^{D'}}{\partial \bar{x}_{2r}} = -\gamma_{11}\frac{1-\alpha_3}{1-\alpha_1}$$

2.2.2.7. The consumer is rationed in all the markets

We have an effective commodity demand which is the same as /33/. The effective labour force participation coincides with /32/ and the effective supply of hours is equal to /31/.

Conclusion

To study the commodity market, employment and the hours of work we used a disequilibrium analysis. In a first part we have defined the different disequilibrium regimes. Some of these regimes occur frequently, others are rather rare. The emphasis is placed on the influences from a disequilibrium in one market on the other markets. Those spill-over effects are first analysed in a rather general way. In a second part this is done for specific functional forms. The sign of the spill-over coefficients cannot always be determined and depends upon the chosen specifications. It appears that the spill-over coefficient can have a changing sign. This is the case when the producer is rationed on the average hours of work. The effective demand for workers may rise or fall with a rise in the rationing on the hours. Therefore a reduction in the number of hours of work will not necessarily lead to an increasing employment. This example illustrates the importance of a disequilibrium analysis. We also derived the effective demand and supply functions starting from a Cobb-Douglas production function and a specific form of the Johanssen utility function. These functions will be used for estimation.

We have eight regimes and rather complicated functions. We suppose that the greatest problem in estimation will be the identification of the model. It will be necessary to make some simplifying assumptions concerning the number of regimes and the functional form of effective demand and supply.

This paper serves as a starting point and a guide to the construction of a disequilibrium model suitable for estimation.

Bibliography

ANDERSEN L.C. (1978) "An Explanation of Movements in the Labor Force Participation Rate, 1957-76. Federal Reserve Bank of St.-Louis Review. August, pp.7-21.

BARRO & GROSSMAN (1971) "A General Disequilibrium Model of Income and Employment", American Economic Review. March, pp.82-93.

——————— (1974) "Suppressed Inflation and the Supply Multiplier". Review of Economic Studies. January, pp.87-104.

——————— (1976) Money, Employment, and Inflation. Cambridge University Press, Cambridge

——————— (1978) "Consumtion, Income, and Liquidity" in Schwödiauer (ed.). Equilibrium and Disequilibrium in Economic Theory. Proceedings of a Conference Organized by the Institute of Advanced Studies, Vienna, Austria, July 3-5, 1974. D. Reidel Publishing Company, Dordrecht, Boston, pp.565-591.

BARTEN (1977) "The System of Consumer Demand Functions Approach : A Review", Econometrica. January, pp.23-51.

BELOUS & LEVITAN (1977) Shorter Hours, Shorter Weeks : Spreading the Work to Reduce Unemployment. John Hopkins University Press, Baltimore, London.

BENASSY J.P. (1975) "Neo-Keynesian Disequilibrium Theory in a Monetary Economy", Review of Economic Studies. October, pp.503-523.

——————— (1976) "Théorie du déséquilibre et fondements micro-économiques de la macroéconomie", Revue Economique, September, pp.755-804.

——————— (1977) "On Quantity Signals and the Foundations of Effective Demand Theory", Scandinavian Journal of Economics. Vol.79, nr.2, pp.147-168.

——————— (1978) "A Neo-Keynesian Model of Price and Quantity Determination in Disequilibrium" in Schwödiauer (ed.), Equilibrium and Disequilibrium in Economic Theory. Proceedings of a Conference organized by the Institute of Advanced Studies, Vienna, Austria, July 3-5, 1974. D. Reidel Publishing Company, Dordrecht, Boston, pp.511-544.

CLOWER R.C. (1965) "The Keynesian Counterrevolution : a Theoretical Appraisal", in Hahn & Brechling (eds.) The Theory of Interest Rates. London.

DE WACHTER M. (1980) Arbeidsparticipatie- en arbeidsaanbodbeslissingen. Een literatuuroverzicht. SESO-rapport 8098, SESO-UFSIA, Antwerpen, March.

FELDSTEIN M.S. (1967) "Specification of the Labour Input in the Aggregate Production Function", Review of Economic Studies. pp.375-386.

ITO T. (1980) "Methods of Estimation for Multimarket Disequilibrium Models", Econometrica. January, pp.97-125.

KUH E. (1966) "Unemployment, Production Functions, and Effective Demand", Journal of Political Economy. June, pp.238-249.

MALINVAUD E. (1976) The Theory of Unemployment Reconsidered. Oxford, Basil Blackwell.

MEERSMAN H. (1980) Goederen- en arbeidsmarkt in onevenwicht : een micro- en macro-economische analyse. SESO-rapport 8097, SESO-UFSIA, Antwerpen, March.

MUELLBAUER & PORTES (1978) "Macroeconomic Models with Quantity Rationing", Economic Journal. December, pp.788-821.

PATINKIN D. (1949) "Involuntary Unemployment and the Keynesian Supply Function", Economic Journal. September, pp.360-383.

——————— (1956) Money, Interest, and Prices. Row, Peterson and Company, Illinois, New York.

PLASMANS J.E.J. (1975) Production Investment Behavior. Tilburg University Press, The Netherlands.

——————— (1975) "Een heterogeen arbeidsvraagmodel", Maandschrift Economie. October-November, pp.24-38.

REYNOLDS (1949,1978) Labor Economics and Labor Relations. Prentice Hall, Inc., Englewood Cliffs, New Yersey.

REZA (1975) "Labour Supply and Demand, Absenteeism, and Union Behavior", Review of Economic Studies. April, pp.237-247.

WABE J.S. (ed.) (1974) Problems in Manpower Forecasting. Saxon House, Westmead, Farnborough, Hants., England & Lexington Books, Lexington, Mass., USA.

Appendix 1

Starting with the profit function as defined in 2.1.

$$\pi_o = \sum_{t=o}^{\infty} k_t(1-u_t)\ [p_t y_t - w_t(1+s_t)x_{1t}x_{2t} - z_t x_{1t} - c_t]$$

and using the Cobb-Douglas production function

$$y_t = A\ e^{\lambda t}\ x_{1t}^{a} x_{2t}^{b} \qquad \forall\ t = o, \ldots, \infty$$

we will prove that, under perfect competition, an interior solution of the profit maximization will imply

$$o < a < 1,\ o < b < 1,\ a + b < 1 \text{ and } a > b.$$

In a market with perfect competition the producer has to solve the following programme

$$\underset{y_t, x_{1t}, x_{2t}}{\text{Max}} \sum_{t=o}^{\infty} k_t(1-u_t)\ [p_t y_t - w_t(1+s_t)x_{1t}x_{2t} - z_t x_{1t} - c_t]$$

$$\text{s.t.} \quad y_t = A\ e^{\lambda t}\ x_{1t}^{a} x_{2t}^{b} \qquad \forall\ t = o, \ldots, \infty$$

$$y_t,\ x_{1t},\ x_{2t} \geq o \qquad \forall\ t = o, \ldots, \infty$$

This is equivalent to

$$\underset{x_{1t},\ x_{2t}}{\text{Max}} \sum_{t=o}^{\infty} k_t(1-u_t)\ [p_t A\ e^{\lambda t}\ x_{1t} x_2 t - w_t\ (1+s_t)\ x_{1t}x_{2t}$$

$$- z_t x_{1t} - c_t]$$

$$\text{s.t.} \quad x_{1t},\ x_{2t} \geq o \qquad \forall\ t = o, \ldots, \infty$$

From the Kuhn-Tucker conditions we have for every $t = o, \ldots, \infty$

$$p_t A\, e^{\lambda t}\, a\, x_{1t}^{a-1}\, x_{2t}^{b} - w_t\, (1+s_t) x_{2t} - z_t \leq o$$

$$x_{1t} \left[p_t A\, e^{\lambda t}\, a\, x_{1t}^{a-1}\, x_{2t}^{b} - w_t\, (1+s_t) x_{2t} - z_t \right] = o$$

$$p_t A\, e^{\lambda t}\, b\, x_{1t}^{a}\, x_{2t}^{b-1} - w_t\, (1+s_t) x_{1t} \leq o$$

$$x_{2t} \left[p_t A\, e^{\lambda t}\, b\, x_{1t}^{a}\, x_{2t}^{b-1} - w_t\, (1+s_t) x_{1t} \right] = o$$

As we assume an interior solution, we must have for every $t = o, \ldots, \infty$

$$\begin{cases} p_t A\, e^{\lambda t}\, a\, x_{1t}^{a-1}\, x_{2t}^{b} - w_t\, (1+s_t)\, x_{2t} - z_t = o \\ p_t A\, e^{\lambda t}\, b\, x_{1t}^{a}\, x_{2t}^{b-1} - w_t\, (1+s_t)\, x_{1t} = o \end{cases}$$

$\Leftrightarrow$

$$\begin{cases} p_t\, a\, \dfrac{y_t}{x_{1t}} = w_t\, (1+s_t)\, x_{2t} + z_t \\ p_t\, b\, \dfrac{y}{x_{2t}} = w_t\, (1+s_t)\, x_{1t} \end{cases}$$

$\Leftrightarrow$

$$\begin{cases} a = \dfrac{w_t\, (1+s_t)\, x_{1t} x_{2t} + z_t x_{1t}}{p_t y_t} \\ b = \dfrac{w_t\, (1+s_t)\, x_{1t} x_{2t}}{p_t y_t} \end{cases}$$

$$a > o, \quad b > o, \quad a > b$$

The second-order conditions for a maximum give additional constraints

Considering the Hessian matrix for period t

$$H_t = \begin{bmatrix} \frac{\partial^2 \pi}{\partial x_{1t}^2} & \frac{\partial^2 \pi}{\partial x_{1t} \partial x_{2t}} \\ \frac{\partial^2 \pi}{\partial x_{2t} \partial x_{1t}} & \frac{\partial^2 \pi}{\partial x_{2t}^2} \end{bmatrix} = p_t y_t \begin{bmatrix} \frac{a(a-1)}{x_{1t}^2} & \frac{ab}{x_{1t}x_{2t}} \\ \frac{ab}{x_{1t}x_{2t}} & \frac{b(b-1)}{x_{2t}^2} \end{bmatrix}$$

H_t is negative definite

$\Leftrightarrow$

1) $o < a < 1$ and $o < b < 1$

2) $ab(a-1)\ (b-1) - a^2b^2 > o$

$\Leftrightarrow$

$a+b < 1$

Feldstein, however, stated in his article "Specification of the Labour Input in the Aggregate Production Function" (1967) that the elasticity of output with respect to the number of men must be smaller than the elasticity of output with respect tot the average hours.

He explains this by two reasons

We quote (p.377)

"First, an increase in hours increases the flow of "capital services as well. Because depreciation plus "interest rises less than in proportion to the number "of hours that the capital stock is used, longer hours "lower the cost per unit of capital services and may "cause net output to rise more than in proportion "to hours. Second, a number of hours during each "working week may be regarded as a fixed "labour cost with no corresponding output ... "These paid but non-productive hours do not "rise proportionately with the number of hours "officially worked. An increase in the length "of the official working week therefore entails "a more than proportionate increase in the number of hours actually worked; this raises η_H "relative to η_N"(1).

(1) $\eta_H = b \quad \eta_N = a$

The first reason has clearly nothing to do with the production function which gives a technological relation. The second reason must be outweighted with the fact that increasing average hours worked decrease the output per man hour, caused by fatigue.

Appendix 2 : Derivation of the effective demand and supply functions

1. The producer side of the model

1.1. The producer is rationed in the commodity market

$$\text{Max} \sum_{t=o}^{\infty} k_t(1-u_t)\ [p_t y_t - w_t(1+s_t)x_{1t}x_{2t} - z_t x_{1t} - c_t]$$

$$\text{s.t.} \quad y_t = A\ e^{\lambda t}\ x_{1t}^a x_{2t}^b \qquad t = o, \ldots, \infty$$

The Lagrange function is

$$L = \sum_{t=o}^{\infty} k_t(1-u_t)\ [p_t y_t - w_t(1+s_t)x_{1t}x_{2t} - z_t x_{1t} - c_t]$$

$$+ \sum_{t=o}^{\infty} \mu_t(A\ e^{\lambda t} x_{1t}^a x_{2t}^b - y_t) + \sum_{t=o}^{\infty} \eta_t(\bar{y}_t - y_t)$$

where μ_t and η_t are the Lagrange Multipliers for $t = o, \ldots, \infty$
The first-order conditions are given by

$$k_t(1-u_t)p_t = \mu_t + \eta_t \qquad t = o, \ldots, \infty \qquad (1.1.1)$$

$$k_t(1-u_t)[w_t(1+s_t)x_{2t} + z_t] = \eta_t\ a\ \frac{y_t}{x_{1t}} \qquad (1.1.2)$$

$$k_t(1-u_t)[w_t(1+s_t)x_{1t}] = \mu_t\ b\ \frac{y_t}{x_{2t}} \qquad (1.1.3)$$

$$y_t = A\ e^{\lambda t}\ x_{1t}^a x_{2t}^b \qquad (1.1.4)$$

$$y_t = \bar{y}_t \qquad (1.1.5)$$

We do not need the first equation to solve the system for x_{1t} and x_{2t}

Solving (1.1.3) for μ_t gives

$$\mu_t = k_t(1-u_t)\,\frac{w_t(1+s_t)x_{1t}x_{2t}}{b\,y_t}$$

and substituting in (1.1.2) gives

$$w_t(1+s_t)x_{2t} + z_t = \frac{a}{b}\,w_t(1+s_t)x_{2t}$$

$$\Leftrightarrow$$

$$x^{D'}_{2t} = z_t\,w_t^{-1}\,(1+s_t)^{-1}\,\frac{b}{a-b} \qquad (1.1.6)$$

$$\Leftrightarrow$$

$$\ln x^{D'}_{2t} = \ln b - \ln(a-b) + \ln z_t - \ln w_t - \ln(1+s_t)$$

Substituting (1.1.5) and (1.1.6) in (1.1.4) and solving for x_{1t} gives the effective demand for men

$$x^{D'}_{1t} = A^{-\frac{1}{a}}\,b^{-\frac{b}{a}}\,(a-b)^{\frac{b}{a}}\,e^{-\frac{\lambda t}{a}}\,w_t^{\frac{b}{a}}\,z_t^{\frac{-b}{a}}\,(1+s_t)^{\frac{b}{a}}\,\bar{y}_t^{\frac{1}{a}}$$

$$\Leftrightarrow$$

$$\ln x^{D'}_{1t} = -\frac{1}{a}\ln A - \frac{b}{a}\ln b + \frac{b}{a}\ln(a-b) - \frac{\lambda}{a}t + \frac{b}{a}\ln w_t - \frac{b}{a}\ln z_t + \frac{b}{a}\ln(1+s_t) + \frac{1}{a}\ln \bar{y}_t$$

1.2. The producer is rationed on the number of workers

We can immediately write the Lagrange function with multipliers μ_t and η_t

$$L = \sum_{t=o}^{\infty} k_t(1-u_t)\left[p_t y_t - w_t(1+s_t)x_{1t}x_{2t} - z_t x_{1t} - c_t\right] + \sum_{t=o}^{\infty} \mu_t\left[A\,e^{\lambda t}x_{1t}^{a}x_{2t}^{b} - \bar{y}_t\right] + \sum_{t=o}^{\infty} \eta_t(\bar{x}_{1t} - x_{1t})$$

The first-order conditions for a maximum are

$$k_t(1-u_t)p_t = \mu_t \qquad (1.2.1)$$

$$k_t(1-u_t)[w_t(1+s_t)x_{2t} + z_t] = \mu_t A\, e^{\lambda t} a\, x_{1t}^{a-1}\, x_{2t}^{b} - \eta_t \qquad (1.2.2)$$

$$k_t(1-u_t)\, w_t(1+s_t)x_{1t} = \mu_t\, A\, e^{\lambda t} b\, x_{1t}^{a}\, x_{2t}^{b-1} \qquad (1.2.3)$$

$$y_t = A\, e^{\lambda t}\, x_{1t}^{a}\, x_{2t}^{b} \qquad (1.2.4)$$

$$x_{1t} = \bar{x}_{1t} \qquad (1.2.5)$$

Substituting (1.2.1) and(1.2.5) in (1.2.3) and solving for x_{2t} gives

$$x_{2t}^{D'} = A^{\frac{1}{1-b}}\, b^{\frac{1}{1-b}}\, e^{\frac{\lambda t}{1-b}}\, p_t^{\frac{1}{1-b}}\, w_t^{-\frac{1}{1-b}}\, (1+s_t)^{-\frac{1}{1-b}}\, \bar{x}_{1t}^{-\frac{1-a}{1-b}} \qquad (1.2.6)$$

$\Leftrightarrow$

$$\ln x_{2t}^{D'} = \frac{1}{1-b}\ln A + \frac{1}{1-b}\ln b + \frac{\lambda t}{1-b} + \frac{1}{1-b}\ln p_t - \frac{1}{1-b}\ln w_t - \frac{1}{1-b}\ln(1+s_t) - \frac{1-a}{1-b}\ln \bar{x}_{1t}$$

Substituting (1.2.6) together with (1.2.5) in (1.2.4) gives the effective commodity supply

$$y_t^{s'} = A^{\frac{1}{1-b}}\, b^{\frac{b}{1-b}}\, p_t^{\frac{b}{1-b}}\, w_t^{-\frac{b}{1-b}}\, (1+s_t)^{-\frac{b}{1-b}}\, \bar{x}_{1t}^{\frac{a-b}{1-b}}$$

$\Leftrightarrow$

$$\ln y_t^{s'} = \frac{1}{1-b}\ln A + \frac{b}{1-b}\ln b + \frac{\lambda t}{1-b} + \frac{b}{1-b}\ln p_t - \frac{b}{1-b}\ln w_t - \frac{b}{1-b}\ln(1+s_t) + \frac{a-b}{1-b}\ln \bar{x}_{1t}$$

1.3. The producer is rationed on the average hours of work per worker

The Lagrange function for this problem is given by

$$L = \sum_{t=o}^{\infty} k_t(1-u_t)\left[p_t y_t - w_t(1+s_t)\, x_{1t}x_{2t} - z_t x_{1t} - c_t\right]$$
$$+ \sum_{t=o}^{\infty} \mu_t(A\, e^{\lambda t} x_{1t}^a x_{2t}^b - y_t) + \sum_{t=o}^{\infty} \eta_t(x_{2t} - \bar{x}_{2t})$$

The first-order conditions for a maximum are

$$k_t(1-u_t)p_t = \mu_t \tag{1.3.1}$$

$$k_t(1-u_t)\left[w_t(1+s_t)x_{2t} + z_t\right] = \mu_t\, A\, e^{\lambda t} a\, x_{1t}^{a-1}\, x_{2t}^{b} \tag{1.3.2}$$

$$k_t(1-u_t)\, \bar{w}_t(1+s_t)\, x_{1t} = \mu_t\, A\, e^{\lambda t} b\, x_{1t}^{a}\, x_{2t}^{b-1} - \eta_t \tag{1.3.3}$$

$$y_t = A\, e^{\lambda t}\, x_{1t}^{a}\, x_{2t}^{b} \tag{1.3.4}$$

$$x_{2t} = \bar{x}_{2t} \tag{1.3.5}$$

Substituting (1.3.1) and (1.3.5) in (1.3.2) and solving for x_{1t} gives the effective demand for workers

$$x_{1t}^{D'} = A^{\frac{1}{1-a}}\, a^{\frac{1}{1-a}}\, e^{\frac{\lambda t}{1-a}}\, p_t^{\frac{1}{1-a}}\, \bar{x}_{2t}^{\frac{b}{1-a}} \left[w_t(1+s_t)\, \bar{x}_{2t} + z_t\right]^{-\frac{1}{1-a}}$$
$$\Leftrightarrow \tag{1.3.6}$$
$$\ln x_{1t}^{D'} = \frac{1}{1-a}\ln A + \frac{1}{1-a}\ln a + \frac{\lambda}{1-a}t + \frac{1}{1-a}\ln p_t + \frac{b}{1-a}\ln \bar{x}_{2t}$$
$$- \frac{1}{1-a}\ln\left[w_t(1+s_t)\, \bar{x}_{2t} + z_t\right]$$

Substituting (1.3.5) and (1.3.6) in (1.3.4) gives

$$y^{s'} = A^{\frac{1}{1-a}}\, a^{\frac{a}{1-a}}\, e^{\frac{\lambda t}{1-a}}\, p_t^{\frac{a}{1-a}}\, \bar{x}_{2t}^{\frac{b}{1-a}} \left[w_t(1+s_t)\, \bar{x}_{2t} + z_t\right]^{-\frac{a}{1-a}}$$
$$\Leftrightarrow$$
$$\ln y^{s'} = \frac{1}{1-a}\ln A + \frac{a}{1-a}\ln a + \frac{\lambda}{1-a}t + \frac{a}{1-a}\ln p_t + \frac{b}{1-a}\ln \bar{x}_{2t}$$
$$- \frac{a}{1-a}\ln\left[w_t(1+s_t)\, \bar{x}_{2t} + z_t\right]$$

1.4. The producer is rationed in the commodity market and on the number of workers

For the effective commodity supply we have to consider only the rationing on the number of workers. Therefore we can refer to 1.2.

For the effective demand for average hours of work per worker is found by solving the following programme

$$\operatorname{Max} \sum_{t=o}^{\infty} k_t(1-u_t) \left[p_t y_t - w_t(1+s_t)\, x_{1t} x_{2t} - z_t x_{1t} - c_t\right]$$

$$\text{s.t. } y_t = A\, e^{\lambda t}\, x_{1t}^{a}\, x_{2t}^{b}$$

$$y_t = \bar{y}_t$$

$$x_{1t} = \bar{x}_{1t}$$

This gives

$$\bar{y}_t = A\, e^{\lambda t}\, \bar{x}_{1t}^{a}\, x_{2t}^{b}$$

$$\Leftrightarrow$$

$$x_{2t}^{b} = \bar{y}_t\, A^{-1}\, e^{-\lambda t}\, \bar{x}_{1t}^{-a}$$

$$\Leftrightarrow$$

$$x_{2t}^{D'} = A^{-\frac{1}{b}}\, e^{-\frac{\lambda t}{b}}\, \bar{x}_{1t}^{-\frac{a}{b}}\, \bar{y}_t^{\frac{1}{b}}$$

$$\Leftrightarrow$$

$$\ln x_{2t}^{D'} = -\frac{1}{b} \ln A - \frac{\lambda}{b} t - \frac{a}{b} \ln \bar{x}_{1t} + \frac{1}{b} \ln \bar{y}_t$$

1.5. The producer is rationed in the commodity market and on the average hours of work per worker

The effective commodity is given in 1.3., the effective demand for average hours of work is found as in 1.1.

For the effective demand for workers we have to solve the following programme

$$\text{Max} \sum_{t=o}^{\infty} k_t(1-u_t) \left[p_t y_t - w_t(1+s_t)x_{1t}x_{2t} - z_t x_{1t} - c_t\right]$$

$$\text{s.t. } y_t = A\, e^{\lambda t}\, x_{1t}^{a}\, x_{2t}^{b}$$

$$y_t = \bar{y}_t$$

$$x_{2t} = \bar{x}_{2t}$$

We have

$$\bar{y}_t = A\, e^{\lambda t}\, x_{1t}^{a}\, \bar{x}_{2t}^{b}$$

$\Leftrightarrow$

$$x_{1t} = A^{-\frac{1}{a}}\, e^{-\frac{\lambda}{a}t}\, \bar{x}_{2t}^{-\frac{b}{a}}\, \bar{y}_t^{\frac{1}{a}}$$

$\Leftrightarrow$

$$\ln x_{1t}^{D'} = -\frac{1}{a}\ln A - \frac{\lambda}{a}t - \frac{b}{a}\ln \bar{x}_{2t} + \frac{1}{a}\ln \bar{y}_t$$

1.6. The producer is rationed on the average hours of work per worker and on the number of workers

The effective demand for workers is given in 1.3., the effective demand for average hours of work is given in 1.2.
The effective commodity supply is nothing else than the production function with given inputs $\bar{x}_{1t}$ and $\bar{x}_{2t}$

$$y_t^{s'} = A\, e^{\lambda t}\, \bar{x}_{1t}^{a}\, \bar{x}_{2t}^{b}$$

$\Leftrightarrow$

$$\ln y_t^{s'} = \ln a + \lambda t + a \ln \bar{x}_{1t} + b \ln \bar{x}_{2t}$$

1.7. The producer is rationed in all the markets

The effective commodity supply is found as in 1.6., the effective demand for workers as in 1.5. and the effective demand for hours as in 1.4.