BENEFITS OF DREDGING THROUGH
REDUCED TIDAL WAITING

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Economic benefits of dredging tidal harbour approaches can be grouped under two headings: 1. economies of size in sea transportation and 2. reductions in waiting time. Methods of estimating economies of size in sea transportation are well known. Engineering cost functions for sea transportation were developed by T.D. HEAVER (1970) and R.O. GOSS & C.D. JONES (1977). Statistical cost functions were recently estimated by B. DE BORGER & W. NONNEMAN (1980). In this paper I focus on the savings in waiting time. I present some formulas to approximate reductions in waiting time due to increased depth of a tidal approach. In section 1 the method is explained. In section 2 the use of the formulas is illustrated.

1. Method

For practical purposes, I assume that vessels arrive at random over tidal periods. Whether or not a vessel can be 'served' will depend upon tidal rise versus the required depth for safe entry of the vessel. Over time, available depth in the approach is approximated by the following cosinusoidal function (°)

\[ h_t = h^0 + r \cos \left( \frac{t \pi}{6} \right) / 2 \]  (1)

where \( h_t \) = available depth at time \( t \), \( h^0 \) = average depth, \( r \) = tidal range, \( t \) = time in hours.

This available depth has to be compared with the required depth of a vessel arriving at time \( t \). Required full-load draught is approximated by the following formula (See UNCTAD (1977), p 70)

\[ \text{\textsuperscript{\textcircled{0}}} \text{One should verify this assumption in each particular case. In cases where the observed pattern differs substantially from this cosine-function, a suitable mathematical proxy should be used.} \]
\[ d_t = \sqrt{S_t} + 6.5 \]  \hspace{1cm} (2)

where \( d_t \) = required depth in metres for safe entry and \( S_t \) = vessel size in thousand ton deadweight. This formula is a fairly good approximation of required draught to within one metre over the range 10,000 to 500,000 dwt for dry and liquid bulk carriers, taking into account the effects of squat, pitch and bed clearance.

By comparing (1) and (2) maximum vessel size is easily derived as

\[ S = (h_0 + \frac{r}{2} - 6.5)^2 \]  \hspace{1cm} (3)

All vessels with a size lower than \( S \) are not subject to tidal waiting and can use the approach at any time. The value of \( S \) is also derived by solving (1) and (2) at minimum available depth or

\[ S = (h_0 - \frac{r}{2} - 6.5)^2 \]  \hspace{1cm} (4)

Consequently, vessels within the range \( S \) and \( \bar{S} \) are potentially subject to waiting until rise in tide is sufficient.

Consider diagram 1. In panel (a) of this diagram the relation between vessel size and required depth is drawn. Panel (b) represents the assumed tidal cycle. Suppose a vessel of size \( S \) arrives. It requires a depth of \( d \) as can be derived from panel (a).

This depth is available from time 0 to \( \phi \) and from time \( 12 - \phi \) till time 12 during a tidal cycle. As it was assumed that vessels arrive at random over the cycle, four possibilities must be considered.

1. A vessel arrives during interval \([0, \phi]\) of a tidal cycle. The probability of such an arrival is \( \phi/12 \). If a vessel arrived during this interval it can be served without any waiting time.

2. A vessel arrived during interval \([\phi, 6]\) of a tidal cycle. The probability of such an arrival is \((6 - \phi)/12\). If a vessel arrived during this interval it will have to wait till time \( 12 - \phi \) for service. As expected arrival time is \((\phi + 6)/2\) average waiting time in this situation will be \( 9 - 3\phi/2 \).
3. An arrival may occur during interval \([5, 12 - \phi]\) with a probability of \((6 - \phi)/12\). In this event it will have to wait till time \(12 - \phi\) for service. Average waiting time will be \(3 - \phi/2\).

4. If an arrival occurs during interval \([12 - \phi, 12]\) of the tidal cycle, no waiting will be incurred.

Diagram 1. Required vs available depth.

(a) Required depth vs vessel size  
(b) Available depth vs time

Expected waiting for a vessel of size \(S\) over a tidal cycle is calculated by the sum of products of the probability of an arrival during a particular interval and average waiting time in the event an arrival occurs during that interval.

The following results on average waiting is obtained

\[
W = \begin{cases} 
0 & \text{if } S < \underline{S} \\
6 - 2 + 2/6 & \text{if } \underline{S} \leq S < \overline{S} \\
\infty & \text{if } S \geq \overline{S}
\end{cases}
\]  

(5)
where \( W \) = expected waiting time and \( \phi = \) the 'tidal window' for a vessel of size \( S \). The tidal window \( \phi \) is a function of vessel size \( S \) as the following equality holds

\[
d = \sqrt{S} + 6.5 = h_\phi = h^o + r \cos (\phi \pi / 6)/2
\]

from which \( \phi \) is solved as

\[
= 6 \arccos \left( \frac{\sqrt{S} + 6.5 - h^o}{r/2} \right) / \pi
\]  

(6)
2. Example

Suppose one wants to evaluate the consequences of dredging one additional metre on waiting times for various sizes of vessels in an approach with an average depth of 12 metres and a tidal range of 5 metres.

Using formulas (3) and (4) with $h^o = 12$ (13 with the project) and $r = 5$ the boundaries for tidal waiting are established. Without the project maximum vessel size is 64,000 dwt; with the project this is increased to 81,000 dwt. Without the project vessels below 9,000 dwt are not subject to waiting; with the project this lower boundary is 16,000 dwt.

Using formula (6) tidal windows are computed over the relevant range of vessel sizes. The result is plotted in diagram 2, panel (a) where $\phi$ shows the result without the project and $\phi_1$ with the project.

Once tidal windows are found for various vessel sizes, expected waiting time is estimated by means of formula (5). In panel (b) of diagram 2 expected waiting time vs vessel size is shown.

Panel (c) represents the time savings in hours for various vessel sizes. Time savings vary more than proportional with vessel size. As costs per unit of time also vary more than proportional with vessel size total benefits of a dredging program due to reduced waiting time might be much larger than benefits through economies of size. It may well be the case that no benefits at all from size economies will be realised in view of the large expected waiting time for the largest vessel sizes. In such cases, time savings alone might justify a program.
Diagram 2. Effects of dredging on waiting time

(a) Tidal window

(b) Expected waiting time

(c) Time saving
Abstract

Tidal waiting (for random arrivals and a sinusoidal tide pattern) can be approximated by simple formulas. Such formulas may be useful to estimate benefits of dredging programs due to reduce tidal waiting. Even if no economies of size should be expected, a dredging program may still be justified by time savings as they are especially important for the larger and most time-expensive vessels.
References

