STUDIECENTRUM VOOR ECONOMISCH EN SOCIAAL ONDERZOEK

TIME ALLOCATION AND THE LINEAR EXPENDITURE SYSTEM

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1. Since the Becker-Lancaster approach (1965-66) to consumer-demand, there has been a large body of literature on consumption technology. Part of it was intended to build in consumption technology into the classical consumption models. More specifically, the allocation of time has been introduced in various models. Mostly this introduction was limited to the time-variable as just one more argument in the utility function and a time-restriction. Nevertheless this approach has proved to be useful.

This way of handling the time input is the most obvious one, when demand functions are directly specified. But if one starts off from a functionally specified utility function, consistency asks for the introduction of the time component by functionally specifying the consumption technology. In this contribution, we have tried to apply this reasoning to the Linear Expenditure System, derived from the Stone-Geasy utility function and an appropriate consumption technology. A first empirical exercise for Belgian data is made to a simplified version.

2. We start from the utility function on consumption activities or commodities :

$$u^{*} = \sum_{k=1}^{m} \varepsilon_k \ln a_k \qquad (1)$$

where a_k are the different commodities. The parameters ε_k in this function can be normalized such that they add up to unity.

$$\sum_{k=1}^{m} \varepsilon_k = 1 \qquad (2)$$

The consumption technology consists of m production functions, where commodities as output are related to market goods and time categories as input. The production functions are of the Cobb-Douglas type, but only in the uncommitted consumption, i.e. after the minimum required or committed consumption goods and time are substracted.

$$a_k = \prod_{i=1}^{n} (q_i - \gamma_i)^{\alpha_{ki}} \prod_{j=1}^{\theta} (t_j - \delta_j)^{\beta_{kj}} \qquad k : 1 \ldots m \quad (3)$$

In this most general form, q_i are the market goods and t_j the time categories : γ_i and δ_j are the committed goods and time. These production functions have to be linear homogeneous in the uncommitted quantities in order to derive the Linear Expenditure System.

$$\sum_i \alpha_{ki} + \sum_j \beta_{kj} = 1 \qquad k : 1 \ldots m \tag{4}$$

We further remember that in this model the elasticity of substitution between uncommitted time and goods is one, which allows a substitution, we indeed believe to be rather strong for many commodities (1).

The committed quantities are interpreted here as minimum required quantities of market goods and time input for producing an activity; e.g. If you want to go to sail, you certainly need at least the smallest type of dinghy and minimum half an hour for rigging and unrigging the boot.
It then is assumed that :

$$q_i - \gamma_i \geq 0 \qquad i : 1 \ldots n$$

$$t_j - \delta_j \geq 0 \qquad j : 1 \ldots \theta \tag{5}$$

We now construct the utility function $u(q,t)$, which is in fact a combination of tastes and technology $u^{*}\left[a(q,t)\right]$:

$$u = \sum_{k=1}^{m} \sum_{i=1}^{n} \varepsilon_k \alpha_{ki} \ln(q_i - \gamma_i) + \sum_{k=1}^{m} \sum_{j=1}^{\theta} \varepsilon_k \beta_{kj} \ln(t_j - \delta_j) \tag{6}$$

It clearly takes the form of an extended Stone-Geary utility function. Maximizing with respect to the income and time restrictions :

$$\sum_{i=1}^{n} p_i q_i = wt_a + y$$

$$t_a + \sum_{i=1}^{\theta} t_j = T \tag{7}$$

where p_i are the market prices of goods, t_a labor time, y the non-labor income and T total available time, yields the first order conditions :

(1) In another contribution, we have estimated the average elasticity of substitution between time and goods, and found a value of 1.73 (Késenne, 1980).

$$\sum_{k=1}^{m} \frac{(\varepsilon_k \alpha_{ki})}{q_i - \gamma_i} - \lambda p_i = 0 \qquad i : 1 \ldots n$$

$$\sum_{k=1}^{m} \frac{(\varepsilon_k \beta_{kj})}{t_j - \delta_j} - \lambda w = 0 \qquad j : 1 \ldots \theta \qquad (8)$$

$$wT + y - \sum_{i=1}^{n} p_i q_i - \sum_{j=1}^{\theta} wt_j = 0$$

After summing over i and j, we find for the inverse of the lagrange multiplier λ the supernumerous full income :

$$\lambda^{-1} = wT + y - \sum_{i=1}^{n} p_i \gamma_i - \sum_{j=1}^{\theta} w\delta_j \qquad (9)$$

such that the Linear Expenditure System can be written as :

$$p_i q_i = (\sum_k \varepsilon_k \alpha_{ki}) \lambda^{-1} + \gamma_i p_i \qquad i : 1 \ldots n$$

$$w\, t_j = (\sum_k \varepsilon_k \beta_{kj}) \lambda^{-1} + \delta_j w \qquad j : 1 \ldots \theta \qquad (10)$$

3. It is clear however that after estimating this demand system, there are serious identification problems involved in finding the parameters, appearing in the utility function and the consumption technology. One solution to this problem is the simplification of the consumption technology to input separability, i.e. each good and time category can only be used in the production of one commodity :

$$a_i = (q_i - \gamma_i)^{\alpha_i} (t_i - \delta_i)^{1-\alpha_i} \qquad i : 1 \ldots n \qquad (11)$$

Since in this case $m = n = \theta$, we get the demand system :

$$p_i q_i = \varepsilon_i \alpha_i \lambda^{-1} + \gamma_i p_i$$

$$i : 1 \ldots n \qquad (12)$$

$$w\, t_i = \varepsilon_i (1-\alpha_i) \lambda^{-1} + \delta_i w$$

In this version, the tastes and technology can be identified because of the linear homogeneity of the production function. In some empirical applications (Abbott and Ashenfelter, 1976), a variant of this model is used, without making the separation of tastes and technology. The approach here could allow for a possible interpretation of their estimation results, but since mostly total consumption time or leisure is taken as just one category, there again is an identification problem. Abbott and Ashenfelter were using as demand equation for leisure :

$$w \sum_{i=1}^{n} t_i = \left(\sum_{i=1}^{n} \varepsilon_i (1-\alpha_i) \right) \lambda^{-1} + \left(\sum_{i=1}^{n} \delta_i \right) w \qquad (13)$$

This identification problem can be solved by assuming all production functions having the same parameters α and $(1-\alpha)$ such that the demand systems become :

$$p_i q_i = \alpha \varepsilon_i \lambda^{-1} + \gamma_i p_i$$
$$w \sum_{i=1}^{n} t_i = (1-\alpha)\lambda^{-1} + \left(\sum_{i} \delta_i \right) w \qquad (14)$$

Abbott and Ashenfelter then found for $(1-\alpha)$ a value of .121 such that the implicit consumption technology in their model is :

$$a_i = (q_i - \gamma_i)^{.879} (t_i - \delta_i)^{.121} \qquad i : 1 \ldots n$$

The calculated parameters in their implicit commodity utility function are then given in table I.

table I

durables	ε_1	.271
food	ε_2	.183
clothing	ε_3	.152
other non-durables	ε_4	.029
housing services	ε_5	.086
transp. services	ε_6	.113
other services	ε_7	.162

4. One possible application of the model, developed above is describing how the consumer allocates leisure activities over leisure time. Therefore we distinguish between weekly leisure (t_1) and holidays (t_2). This can shed light on the demand structure for these time categories and help to explain the consumer preference. We start from two activities and two goods, leisure - (a_1) and non-leisure activities (a_2) and the associated goods (q_1 and q_2). The non-leisure activity is supposed being done during the semi-leisure time (t_2), which is assumed constant over the sample period. The model then is :

$$u^* = \varepsilon_1 \ln a_1 + \varepsilon_2 \ln a_2 \qquad \text{with } \varepsilon_1 + \varepsilon_2 = 1$$

$$a_1 = (q_1-\gamma_1)^{\alpha} (t_1-\delta_1)^{\beta 1} (\gamma_2-\delta_2)^{\beta 2} \qquad \text{with } \alpha + \beta_1 + \beta_2 = 1$$

$$a_2 = (q_2 - \gamma_2) \tag{15}$$

$$p_1q_1 + p_2q_2 = wt_a + y$$

$$t_1 + t_2 + t_a = T$$

The simple production function for a_2 follows from the fact that the constant semi-leisure time can be lifted out of the model. Total available time then is reduced by the semi-leisure time.

From this model we can derive the demand system

$$p_1q_1 = \varepsilon_1\alpha\lambda^{-1} + \gamma_1p_1$$

$$p_2q_2 = \varepsilon_2\lambda^{-1} + \gamma_2p_2$$

$$wt_1 = \varepsilon_1\beta_1\lambda^{-1} + \delta_1w \tag{16}$$

$$wt_2 = \varepsilon_1\beta_2\lambda^{-1} + \delta_2w$$

where λ^{-1} is the supernumerous full-income :

$$\lambda^{-1} = wT + y - p_1\gamma_1 - p_2\gamma_2 - w(\delta_1+\delta_2) \tag{17}$$

It is easily seen that after estimating this system, we can identify the parameters of the utility function and the consumption technology.

The data are time series from 1953 till 1975, taken from the Belgian National Accounts, published by the N.I.S. The data on leisure time, we received from the Planning Bureau and are completed by information from the "N.I.S.-Sociale Statistieken". The total available time, we put at 52 weeks a year or 8736 hours. The semi-leisure time, we estimated 4368 hours a year, based on a social investigation for Belgium (Van Mechelen, 1969). The leisure good consists of expenditures on amusement, hotels, restaurants and café's, books, newspapers and periodicals, radio and television, and, what is called, other leisure activities. The wage rate is a calculated average hourly compensation for the whole Belgian active population (2). The list of data is given in an appendix.

An adapted two step Zellner procedure is used to estimate the Linear Expenditure System (16), because of the non-linearities in parameters. For the matrix of contemporaneous covariances of the error term, we took the most simple form of symmetrically distributed estimation of this sum-constraint model.

$$\Sigma = (I_4 - \frac{1}{4} i_4 i_4') \qquad (18)$$

Because of the expected positive autocorrelation, the system is estimated in first differences. The results are given in table II. The imposed value for γ_1 is the minimum value for q_1 in the sample. The non-negativity constraint (5) was not fulfilled for each year, when estimated unrestricted.

table II

		stand. errors	R^2
$\varepsilon_1\alpha$	.1026	.0644	.9484
ε_2	.4289	.2665	.9901
$\varepsilon_1\beta_1$	.3323	.9951	.9951
$\varepsilon_1\beta_2$	.1362	.0979	.9659
γ_1	10921.	0.	
γ_2	18215.	17182.	.9664
δ_1	2078.	84.	
δ_2	47.	47.	

(2) A more detailed description of the calculation of the wage rate is given in Késenne (1979).

The linear homogeneity restriction or the consumption technology allows to calculate the numerical values for the parameters in the utility function and the leisure production function.

table III

ε_1	.5711
ε_2	.4289
α	.1796
β_1	.5819
β_2	.2385

In table IV the derived elasticities for the year 1974 in the demand equations are given.

table IV Elasticities 1974

	Full income real	price of leisure goods		price of non-leisure goods		wage rate	
		non-comp.	comp.	non-comp.	comp.	non-comp.	comp.
leisure good	2.2549	-.5894	-.4868	-.0804	.8748	-1.0215	.1782
non-leisure good	1.0128	-.0211	.0249	-.9520	-.5231	- .4577	.0781
weekly leisure	.7086	-.0148	.0175	-.0252	.2749	- .3779	-.0016
holidays	2.1934	-.0457	.0541	-.0780	.8509	-1.8289	-.6642

5. The parameters of the utility function on commodities reveal that on the average the leisure activities considered are more satisfying than the non leisure activities. The marginal rate of substitution is :

$$\frac{da_1}{da_2} = -.7510 \frac{a_1}{a_2} \qquad (19)$$

This preference is enforced by the lower acitivity level of leisure compared with non-leisure.

The marginal productivity of uncommitted weekly leisure is for each year much higher than this of uncommitted holidays.

$$\frac{\partial a_1}{\partial(t_1 - \delta_1)} = \frac{.5819}{t_1 - 2078}\, a_1 > \frac{\partial a_1}{\partial(t_2 - \delta_2)} = \frac{.2385}{(t_2 - 47)}\, a_1 \qquad (20)$$

A possible explanation is that still a major part of leisure activities are done each day after work or during the weekends.
Also the time input is rather important compared with the input of market goods. The marginal productivity of uncommitted market goods is rather low, such that more market goods cannot increase utility the way more consumption time can do.
We further remember that the elasticity of substitution between the uncommitted quantities of time and goods equals one. This explains why the uncompensated wage elasticity in the demand function for the leisure good is negative and the compensated is positive. The price of time indeed is a considerable (shadow) cost in doing a lot of consumption activities. The (direct) income effect of a wage change increases nominal full income, but the real full income change is considerably smaller, due to the negative (indirect) income effect of a change in the price of time. The compensated wage effect is positive because substitution between time and market goods in the production of activities is possible. Since the price of time has increased relatively to the price of goods, during the last 20 years, the consumer has substituted time for market goods.
The income elasticity is much higher for the leisure good than for the non-leisure good, which could be expected. More holidays are preferred to shortening the workweek. This does not contradict the results for the consumption technology. Leisure activities are all taken together as one composite in this model, while we expect that a separation of leisure activities, associated with holidays, like tourism, would reveal that these activities are preferred when economic welfare further increases.
Because of the highly aggregate level of this empirical application, these results have to be taken with a lot of reserve.

6. We have tried to show that the Linear Expenditure System opens a few possibilities for what is called the new approach to consumer demand. By functionally specifying a Cobb-Douglas type of consumption technology, more insight is offered in the undescribed process between consumer preferences and market demand. Particularly with respect to the allocation of time, this method can be made useful. Unfortunately, there is at least one weak point in this model. The Stone-Geary utility function assumes strong

separability of independence of preferences. This implies, among other things, that the marginal utility of a market good is not influenced by the time, spent on the consumption of that good, which is of course unsatisfactory. Strong separability, as Abbott and Ashenfelter (1976) have shown, is a sufficient condition for the separate treatment of the income-leisure choice and the allocation of the chosen income. Barnett (1976) has tested that this separate treatment is not appropriate. This, however, is the price, paid for the elegant simplicity of the Linear Expenditure System.

Appendix

in hours a year

year	average labor time	average weekly leisure	average holiday leisure
1953	2311	1949	108
1954	2310	1938	120
1955	2305	1925	138
1956	2309	1903	156
1957	2294	1914	160
1958	2294	1912	162
1959	2284	1920	164
1960	2254	1946	168
1961	2245	1951	172
1962	2244	1948	176
1963	2242	1946	180
1964	2214	1962	192
1965	2168	1996	204
1966	2133	2007	228
1967	2088	2050	230
1968	2072	2062	234
1969	2028	2094	246
1970	1998	2121	249
1971	1959	2155	254
1972	1932	2172	264
1973	1887	2205	276
1974	1871	2205	292
1975	1840	2216	312

Source : Planning Bureau, Brussels
Cah. Econ. de Bruxelles, 1976
Sociale Statistieken NIS

in B FR.

year	average nominal wage rate	average non-labor income	(average) non-leisure good at const. prices	price-index non-leisure good	(average) leisure good at const. prices	price-index leisure good
1953	30.3543	11 082.7	107 104	0.695387	10 921.5	.618330
1954	31.3535	12 204.7	110 481	0.704191	10 998.1	.621169
1955	33.0575	14 196.9	118 015	0.696397	11 536.2	.624917
1956	34.1011	13 542.8	119 329	0.709864	11 635.1	.651052
1957	38.0109	14 670.0	126 623	0.736762	12 742.9	.673014
1958	36.7817	14 085.3	121 650	0.733842	12 918.3	.711470
1959	38.9729	14 961.3	125 075	0.756836	12 947.2	.719413
1960	41.2019	15 770.8	131 176	0.754198	13 476.9	.720318
1961	43.0895	16 564.7	133 427	0.772293	14 010.1	.732016
1962	44.9743	16 978.5	137 633	0.780834	14 084.7	.740694
1963	49.1240	17 109.1	144 123	0.807285	14 370.1	.758310
1964	53.1343	16 978.2	146 829	0.833746	15 164.6	.804452
1965	58.9797	18 208.5	152 925	0.869907	15 626.7	.834872
1966	63.6686	19 754.3	157 477	0.901835	15 589.6	.868631
1967	68.2724	21 002.8	161 268	0.922293	16 262.4	.911253
1968	74.0114	23 255.2	169 887	0.947246	16 616.6	.943743
1969	80.5914	25 813.3	176 399	0.976538	17 609.0	.964951
1970	85.7417	28 667.4	181 234	1.00000	18 745.5	1.00000
1971	95.5768	30 651.7	187 937	1.04633	19 682.6	1.07928
1972	108.274	31 879.6	198 499	1.09317	20 987.2	1.14703
1973	124.736	37 437.3	211 538	1.16068	22 699.1	1.20205
1974	143.482	47 977.8	216 910	1.31729	23 871.8	1.28603
1975	162.840	51 392.6	214 599	1.48082	23 898.0	1.39070

Bron : National Accounts, NIS

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