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SUBSTITUTION IN CONSUMPTION
AN APPLICATION TO THE ALLOCATION OF TIME

Stefan KESENNE

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Universitaire Faculteiten Sint-Ignatius
Prinsstraat 13 - 2000 Antwerpen
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In this article, we have constructed a general model, where the commodity-price-effect and the good-price-effect in a Becker-Lancaster-Model are related, and the relevance of factor-substitution in consumption technology can be verified. A first application is made to the allocation of time for Belgian data.

Substitution in Consumption, an application to the allocation of time*

STEFAN KESENNE
UNIVERSITY OF ANTWERP

1. Introduction

In his book "The Harried Leisure Class" (1970) Staffan Linder rejects the general idea of an excess supply of consumption time in the more developped countries today. He asserts, to the contrary, that, by increasing labor productivity and economic welfare, and in spite of decreasing labour time, people struggle with a shortage of consumption time. A rational consumer acts as if he is equalizing the marginal utility of all time units, in consumption and in labour, in order to reach the optimal allocation of time.

Since higher labour productivity has increased the marginal utility of labour time, measured by the hourly wage rate, consumption time is forced to increase its marginal utility in order to equal the wage rate, which is to be considered as the market price of leisure. This means that the consumer has to be very economical with the available consumption time. He will try to consume as many as possible market goods per unit of time. With Belgian data we found that people are using nearly five times as many non-essentials in real value per unit of leisure time in 1975 compared with 1953. These non-essentials include the total private consumption minus the social minimum income, calculated for Belgium.

In the Becker approach (1965) commodities (or consumption activities) are produced by means of market goods and time, considered as production factors or inputs. If the market price of time equals the

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hourly wage rate, it is apparent that in the last 25 years the price of time has increased much more then the price of private consumption goods. One can ask whether this substitution between time and goods in the consumption of commodities is due to the relative more expensive time, i.e. a factor substitution effect. It is clear however that for many other commodities, factor substitution will be hardly possible, because time and goods are complementary. One cannot read a whole book in 10 minutes. So the consumer will also drop commodities that are very time-intensive, since they have tremendously raised in price.

We clearly have to distinguish two types of substitution, commodity substitution and factor substitution. In a somewhat broader sense, this can be considered as a separation of tastes and technology, which is confounded in traditional demand theory, where changes in demand, due to changes in technology, are also described as changes in tastes. The reason simply is that the utility function in traditional demand theory represents a combination of consumer preferences and consumption technology, since the utility function in the commodity-space is translated into the good-space.

In the following paragraph, we try to set up a general model, where these substitution effects can be separated and introduce some assumptions in order to simplify the exposition and to verify empirical validity.

2. A general model

There is a need to relate the effect of a commodity price change on the demand for that commodity to the effect of a price change of goods on the demand for goods. This relation can expose to what extend the latter effect is due to a commodity substitution or a factor substitution.

Let us suppose a well-behaved utility function on m commodities :

$$u(a) \qquad (1)$$

These commodities are produced by means of market goods [(1)]. This is represented by m concave and differentiable productionfunctions on n market goods :

$$a = a(x) \qquad (2)$$

The consumption of market goods is restricted by the budget equation

$$p_x' x = m \qquad (3)$$

where p_x is the appropriate price-vector and m the budget.

The optimum conditions for the consumer, with given budget and prices, are found by maximizing the Lagrangean :

$$u(a) + \lambda'(a(x) - a) + \mu(m - p_x' x). \qquad (4)$$

The first order conditions are :

$$\frac{\partial u}{\partial a} - \lambda = 0 \qquad (5.1)$$

$$\left[\frac{\partial a(x)}{\partial x}\right]' \lambda - \mu p_x = 0 \qquad (5.2)$$

$$a(x) - a = 0 \qquad (5.3)$$

$$m - p_x' x = 0 \qquad (5.4)$$

The shadow prices of commodities can be defined by the ratio of the marginal utilities of commodities to the marginal utility of income, i.e.

(1) For simplicity, time is considered as one of the market goods.

$$p_a = \frac{\lambda}{\mu} \qquad (6)$$

So we can rewrite the first order conditions by

$$\frac{\partial u}{\partial a} = \mu p_a \qquad (5.1')$$

$$\left[\frac{\partial a(x)}{\partial x}\right]' p_a = p_x \qquad (5.2')$$

Substitution of (5.2') in (5.4) yields :

$$m - p_a' \frac{\partial a(x)}{\partial x} x = 0 \qquad (5.4')$$

If we suppose the production function (2) to be linear homogenous, the Euler conditions says that :

$$\frac{\partial a(x)}{\partial x} x = a \qquad (7)$$

So (5.4') becomes :

$$m - p_a' a = 0 \qquad (5.4'')$$

which is the budget constraint in terms of commodities.

The solution of equations (5.1') and (5.4''), under the condition that the prices of commodities are independent of the consumer preferences, yields the demand equations for commodities :

$$a = a^{*}(p_a, m) \qquad (8)$$

Pollak and Wachter (1975) proved that absence of joint production and linear homogeneity of the production functions are the necessary and sufficient conditions for the independence.

Under these conditions, (5.1'), (5.4'') and (5,3) can be rewritten as :

$$a(x) - a^{*}(p_a, m) = 0 \qquad (9)$$

which, together with (5.2')

$$\left[\frac{\partial a(x)}{\partial x}\right]' p_a - p_x = 0$$

form the first order conctions for an optimum.

To analyse the influence of a price change of goods, i.e. a factorprice change, on the demand for goods, we differentiate both equations partially with respect to p'_x :

$$\frac{\partial a^*}{\partial p'_a} \cdot \frac{\partial p_a}{\partial p'_x} - \frac{\partial a}{\partial x'} \cdot \frac{\partial x}{\partial p'_x} = 0 \qquad (10.1)$$

$$\frac{\partial a'}{\partial x} \cdot \frac{\partial p_a}{\partial p'_x} + Z \frac{\partial x}{\partial p'_x} - I_n = 0$$

where the matrix Z of order $(n \times n)$ is

$$Z = A(p_a \otimes I_n)$$

Here the matrix A is written as a row of m matrices :

$$\left[\frac{\partial^2 a_k}{\partial x \partial x'} \right] \quad k : 1 \ldots m$$

Alternatively we can write the matrix Z as :

$$\left[\sum_{k=1}^{m} \left(\frac{\partial^2 a_k}{\partial x_i \partial x_j} \; p_{ak} \right) \right] \qquad i,j : 1 \ldots n$$

The solution of the model, after substituting $\frac{\partial p_a}{\partial p'_x}$ from the first equation into the second one, and solving for $\frac{\partial x}{\partial p'_x}$ is :

$$\frac{\partial x}{\partial p'_x} = \left[\frac{\partial a'}{\partial x} \left(\frac{\partial a^*}{\partial p'_a} \right)^{-1} \frac{\partial a}{\partial x'} + Z \right]^{-1} \qquad (11)$$

The matrix that has to be inverted on the right side of the equation is clearly composed of two terms. The first one is a function of the commodity substitution $\left(\frac{\partial a^*}{\partial p'_a} \right)$, the second one is a function of the consumption technology, where the factor substitution is of crucial interest [2].

3. A 2 commodity - 4 good model

The price effect derived in (11) is not very convenient for further analysis. In order to get more insight in the relation between commodity substitution and factor substitution, we will proceed on a more specific basis.

One specific assumption is that each production-factor can only be used for the production of one commodity. They cannot be used alternatively (input separability).

Less restrictive is the assumption of only two commodities and four goods.

We then have the utility function on two commodities

$$u(a_1,a_2) \qquad (12)$$

and the linear homogeneous production functions,

$$\begin{aligned} a_1 &= a_1(x_1,x_2) \\ a_2 &= a_2(x_3,x_4) \end{aligned} \qquad (13)$$

with optimum conditions :

$$\begin{aligned} a_1(x_1,x_2) &= a_1^*(p_{a1},p_{a2},m) \\ a_2(x_3,x_4) &= a_2^*(p_{a1},p_{a2},m) \end{aligned} \qquad (14)$$

$$p_{a1}\frac{\partial a_1}{\partial x_1} = p_1 \qquad p_{a1}\frac{\partial a_1}{\partial x_2} = p_2$$

$$p_{a2}\frac{\partial a_2}{\partial x_3} = p_3 \qquad p_{a2}\frac{\partial a_2}{\partial x_4} = p_4$$

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(2) In appendix I the relation between $\frac{\partial x}{\partial m}$ and $\frac{\partial a}{\partial m}$ has been derived for time-allocation problems, where m is the so called "full income" i.e.

$$m = wT + y$$

The sum of non-wage income y and the product of total available time T and the wage rate w, being also the price of time.

Differentiating these conditions partially with respect to p_1 results in a six equation model that can be solved, after a few substitutions, based on the property of linear homogeneity of the production functions (cfr. appendix II). The solution can be made more elegant by transforming it to elasticities and using the following substitutions :

$$k_1 = \frac{p_1 x_1}{p_1 x_1 + p_2 x_2} \quad \text{and} \quad k_2 = \frac{p_3 x_3}{p_3 x_3 + p_4 x_4} \qquad (15)$$

i.e. the share of the cost of x_i in the cost of producing commodity j.

The solution then is :

$$e_{p_1}^{x_1} = k_1 \eta_{11} - (1 - k_1)\sigma_1$$

$$e_{p_1}^{x_2} = k_1(\eta_{11} + \sigma_1)$$

$$e_{p_1}^{x_3} = k_1\, \eta_{21} \qquad (16)$$

$$e_{p_1}^{x_4} = k_1\, \eta_{21}$$

$$e_{p_1}^{p_{a1}} = k_1$$

$$e_{p_1}^{p_{a2}} = 0$$ (3)

where σ_1 is the elasticity of factor substitution in commodity 1 and

$\eta_{ij} = \frac{p_{aj}}{a_i} \cdot \frac{\partial a_i}{\partial p_{aj}}$: the price elasticity of the demand for commodities.

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(3) $\frac{\partial p_{a2}}{\partial p_1} = 0$ because good 1 cannot be used for the production of commodity 2. p_{a2} is only dependent of the way x_3 and x_4 are combined in the production of commodity 2 and this combination is only dependent on the price ratio p_3 / p_4 .

The complete matrix of partial price elasticities is now easily derived :

$$E_p^x = \begin{bmatrix} k_1\eta_{11}-(1-k_1)\sigma_1 & (1-k_1)(\eta_{11}+\sigma_1) & k_2\eta_{12} & (1-k_2)\eta_{12} \\ k_1(\eta_{11}+\sigma_1) & (1-k_1)\eta_{11}-k_1\sigma_1 & k_2\eta_{12} & (1-k_2)\eta_{12} \\ k_1\eta_{21} & (1-k_1)\eta_{21} & k_2\eta_{22}-(1-k_2)\sigma_2 & (1-k_2)(\eta_{22}+\sigma_2) \\ k_1\eta_{21} & (1-k_1)\eta_{21} & k_2(\eta_{22}+\sigma_2) & (1-k_2)\eta_{22}-k_2\sigma_2 \end{bmatrix} \quad (17)$$

The price elasticity of the demand for market goods can be written as a simple function of the elasticity of factorsubstitution and the price elasticity of the demand for commodities (4).

By Euler's Rule for homogeneous demand functions, this matrix also tells us that the income elasticities (ε_i) are :

$$\varepsilon_1 = \varepsilon_2 = -\eta_{11} - \eta_{12}$$

$$\varepsilon_3 = \varepsilon_4 = -\eta_{21} - \eta_{22}$$

since these income elasticities simply are minus the sum of all price elasticities.

This result can be extended in a straightforward way to more commodities, where each commodity has only two inputs. In case of more than two productionfactors, partial elasticities of substitution have to be introduced (cfr. R.G.D. Allen, 1969). For the simple application to the allocation of time in the next paragraph, we only need two productionfactors, consumption time and consumption goods.

(4) Part of this solution we also found in Gilbert Ghez and Gary Becker (1973), and in R.G.D. Allen (1969).

4. Allocation of time

We introduce time categories in the model as production-factors for commodities. The simplified model can be reformulated as :

$$u(a_1,a_2)$$

$$a_1 = a_1(q_1,t_1) \qquad (18)$$

$$a_2 = a_2(q_2,t_2)$$

with the appropriate time and income restrictions:

$$p_1q_1 + p_2q_2 = wt_a + y \qquad (19)$$

$$t_1 + t_2 = T$$

After splitting up the total wage effect in a sum of a direct income effect and a price effect, the resulting price effect is still of a somewhat different nature. Since the wage rate is the price of both t_1 and t_2, the price and the demand of a_1 and a_2 are both directly affected. With the above matrix (17) in hand, this price effect of a wage change can easily be calculated by adding the columns 2 and 4 i.e.

$$e_w^{q_1} = (1 - k_1)\,\sigma_1 + (1 - k_1)\eta_{11} + (1 - k_2)\eta_{12}$$

$$e_w^{t_1} = -k_1\sigma_1 + (1 - k_1)\eta_{11} + (1 - k_2)\eta_{12} = e_w^{q_1} - \sigma_1$$

$$e_w^{q_2} = (1 - k_2)\sigma_2 + (1 - k_1)\eta_{21} + (1 - k_2)\eta_{22} \qquad (20)$$

$$e_w^{t_2} = -k_2\sigma_2 + (1 - k_1)\eta_{21} + (1 - k_2)\eta_{22} = e_w^{q_2} - \sigma_2$$

The wage effect in a time-allocation model seems now to be far from simple. There is first of all the direct income effect, since the wage rate is a part of the income. The uncompensated price effect of a wage change is composed of at least three terms. If only two production factors are involved for each commodity, the first term with σ is a function of the factorsubstitution between time and market goods. The two other terms, if only two commodities are considered, are functions of the commodity-price effects. These commodity price effects can then be separated in a classical (indirect) income effect and a compensated price effect or substitution effect, following Slutsky. Then we are still left with Houthakkers analysis of the consumer preferences to find the general and specific substitution effect. In this simplified model we can end up with a separation of the total wage effect in eight sub-effects.

5. An empirical exercise

As already mentioned in the introduction, the Belgian people are used nearly five time as many non essentials per unit of leisure time in 1975 compared with 1953. We are interested whether this shift is due to a factor substitution in consumption activities, i.e. doing the same activities as before, but using less time and more goods or to what extend this is due to a decrease in consumption of time-intensive activities, where substitution is hardly possible. In this very simple exercise however, we will only try to estimate the overall-elasticity of substitution in leisure-activities.

Starting from the utility function

$$u(a_1,a_2) \qquad (21)$$

where a_1 is the leisure-activity and a_2 the semi-leisure-activity, like sleaping, eating, transport and social obligations, we consider linear homogeneous CES-production functions over market goods and time categories :

$$\begin{aligned} &a_1(q_1,t_1) \\ &a_2(q_2,t_2) \end{aligned} \qquad (22)$$

The time-income constraint is

$$p_1q_1 + p_2q_2 + w(t_1 + t_2) = wT + y = m$$

where the difference $T - (t_1 + t_2)$ equals the labour time.

We now make the assumption that semi-leisure good q_2 and semi-leisure time t_2 are constant over the sample period, since they only represent the most essential consumption goods and the time strictly needed to consume them.

So we are left with a two equation demand model after maximizing the utility function subject to the technology and the budget restriction. Since the model is also sum-constraint, we are allowed to drop one more equation, such that there is only one left for estimation.

Taking it in log-linear form :

$$\ln t_1 = \alpha + \beta \ln m + \gamma \ln p_1 + \delta \ln p_2 + \lambda \ln w + u \qquad (24)$$

where : $\gamma = k_1(\eta_{11} + \sigma_1)$

$$\delta = k_2\, \eta_{12}$$

$$\lambda = (1 - k_1)\eta_{11} - k_1\sigma_1 + (1 - k_2)\eta_{12} \qquad (24')$$

This results in :

$$\ln t_1 = \alpha + \beta \ln m + \eta_{11} \left[k_1 \ln p_1 + (1-k_1) \ln w \right]$$
$$+ \eta_{12}\left[k_2 \ln p_2 + (1-k_2) \ln w \right] + \sigma_1\left[k_1(\ln p_1 - \ln w)\right] + u$$

where $k_1 = \dfrac{p_1q_1}{p_1q_1 + wt_1}$ and $k_2 = \dfrac{p_2q_2}{p_2q_2 + wt_2}$.

We further notice that by our assumptions η_{21}, η_{22} and σ_2 have to equal zero in the model.

The data we used are time series (1953-1975) from the Belgian National Accounts; the data on leisure time we received from the Planning Bureau and are completed by information from the "Sociale Statistieken" (N.I.S.). The total available time (T) we put on 8 736 hours a year. The semi-leisure time we estimated 4 368 hours, based on a social investigation for Belgium (Van Mechelen, 1969). The average amount spend on essentials was estimated on 110.000 BF a year in 1970. The price index of essentials is based on a basket of essential goods and services. (Food, clothing, houserent, Energy and Transport) cfr. appendix III. Most of other data are taken from calculations, made in an earlier study on time-allocation (Késenne, 1979).

Under the usual assumptions about the error term, the above relation was estimated by O.L.S. Because of the serious auto-correlation in the errors, we did a reestimation in first differences. The results are given in table 1.

TABLE 1

(standard errors between brackets)

β	η_{11}	η_{12}	σ_1	R^2	D.W.
.85	-1.62	.74	1.73	.74	1.72
(.31)	(.42)	(.14)	(.44)		

TABLE 2

(standard errors between brackets)

β	γ	δ	λ
.85	.02	.20	-1.10
(.31)	(.004)	(.04)	(.33)

Considering the good fit and the significant estimates, the results are very satisfying. The factorsubstitutioneffect (σ_1) between leisure-time and leisure goods is very high. This means that consumers are very sensitive for the tremendous rise in price of leisure-time in the last decennia and are adapting their consumption behavior, in the sense that more consumption goods are consumed per unit of time. However, this highly restricted and simplified model cannot tell us very much about the commodity substitution between leisure activities, since only one leisure activity is taken into account.

In table 2 we have calculated the parameters in relation (24) using the estimates in table 1; k_1 and k_2 were put equal to its average value over the sample i.e. $\bar{k}_1 = .2021$; $\bar{k}_2 = .2711$.

The wage elasticity λ is an uncompensated price elasticity, which is clearly negative for the demand for leisure time. The direct income effect of a wage change has to be added to yield the total wage effect. Since the direct income effect of a wage change is only somewhat smaller then the income effect, we see that the total wage elasticity is still negative, but not significantly different from zero.

The importance of the price effect of time is verified in this exercise. The income effect of a wage change has never been denied, but also the price effect clearly has to be taken into account. Its strong negativity follows, as seen in (24'), both from substitution between time and goods in one or more consumptionactivities and from substitution between time-intensive and good-intensive consumptionactivities, where this factor-substitution is technically impossible.

6. Conclusion

The separation of the commodity substitution and the factorsubstitution seems interesting enough to investigate. We have shown that under fairly widespread assumptions on preferences and technology, these substitution effects can be estimated without specifying exactly the activities that are considered. The difficulties of specification and measurement of commodities has always been one of the strongest objections against the new approach to consumer theory.

In a more detailed empirical application, it must be possible to estimate both the factorsubstitution effects and the commodity substitution effects or -price effects between different consumption activities.

Appendix I

In time allocation problems, where the nominal wage rate is considered as the market price of time, the time-income budget is the "full income"

$$m = wT + y$$

which is exogenous in the model.

Equation (10.1) then becomes :

$$\frac{\partial a^*}{\partial p_a'} \cdot \frac{\partial p_a}{\partial p_x'} + \frac{\partial a}{\partial m} \left(\frac{\partial m}{\partial p_x}\right)' - \frac{\partial a}{\partial x'} \cdot \frac{\partial x}{\partial p_x'} = 0 \qquad (10.3)$$

where $\frac{\partial m}{\partial p_x} = e_1 \frac{\partial m}{\partial w} = e_1 T$ if consumption time is the first good in the vector x. A change in the nominal wage rate always causes a direct income effect on the demand for goods, equaling

$$\left.\frac{\partial x}{\partial w}\right|_{dp_a = 0} = \frac{\partial x}{\partial m} T$$

From the solution of (10.2) and (10.3) we can easily derive that :

$$\frac{\partial x}{\partial m} = \left[\frac{\partial a'}{\partial x} \left(\frac{\partial a^*}{\partial p_a'}\right)^{-1} \frac{\partial a}{\partial x'} + Z\right]^{-1} \frac{\partial a'}{\partial x} \left(\frac{\partial a^*}{\partial p_a'}\right)^{-1} \frac{\partial a}{\partial m}$$

The solution of (10.2) and (10.3) also reveals that solution (11) is perfectly general and applicates to the time allocation model if we clearly distinguish the price effect of a wage change.

Appendix II

The six equation model, after differentiating the optimum condition (14) partially with respect to p_1, is :

$$* \quad \frac{\partial a_1}{\partial x} \cdot \frac{\partial x}{\partial p_1} + \frac{\partial a}{\partial x_2} \cdot \frac{\partial x_2}{\partial p_1} - \frac{\partial a_1}{\partial p_{a1}} \cdot \frac{\partial p_{a1}}{\partial p_1} - \frac{\partial a_1}{\partial p_{a2}} \cdot \frac{\partial p_{a2}}{\partial p_1} = 0$$

$$* \quad \frac{\partial a_2}{\partial x_3} \cdot \frac{\partial x_3}{\partial p_1} + \frac{\partial a_2}{\partial x_4} \cdot \frac{\partial x_4}{\partial p_1} - \frac{\partial a_2}{\partial p_{a1}} \cdot \frac{\partial p_{a1}}{\partial p_1} - \frac{\partial a_2}{\partial p_{a2}} \cdot \frac{\partial p_{a2}}{\partial p_1} = 0$$

$$* \quad p_{a1} \frac{\partial a_1}{\partial x_1^2} \cdot \frac{\partial x_1}{\partial p_1} + p_{a1} \frac{\partial^2 a_1}{\partial x_1 \partial x_2} \cdot \frac{\partial x_2}{\partial p_1} + \frac{\partial a_1}{\partial x_1} \cdot \frac{\partial p_{a1}}{\partial p_1} = 1$$

$$* \quad p_{a1} \frac{\partial^2 a_1}{\partial x_2 \partial x_1} \cdot \frac{\partial x_1}{\partial p_1} + p_{a1} \frac{\partial^2 a_1}{\partial x_2^2} \cdot \frac{\partial x_2}{\partial p_1} + \frac{\partial a_1}{\partial x_2} \cdot \frac{\partial p_{a1}}{\partial p_1} = 0$$

$$* \quad p_{a2} \frac{\partial^2 a_2}{\partial x_3^2} \cdot \frac{\partial x_3}{\partial p_1} + p_{a2} \frac{\partial^2 a_2}{\partial x_3 \partial x_4} \cdot \frac{\partial x_4}{\partial p_1} + \frac{\partial a_2}{\partial x_3} \cdot \frac{\partial p_{a2}}{\partial p_1} = 0$$

$$* \quad p_{a2} \frac{\partial^2 a_2}{\partial x_4 \partial x_3} \cdot \frac{\partial x_3}{\partial p_1} + p_{a2} \frac{\partial^2 a_2}{\partial x_4^2} \cdot \frac{\partial x_4}{\partial p_1} + \frac{\partial a_2}{\partial x_4} \cdot \frac{\partial p_{a2}}{\partial p_1} = 0$$

For linear homogeneous production functions the elasticity of substitution can be written as : (Allen R.G.D., 1969)

$$\sigma_1 = \frac{\frac{\partial a_1}{\partial x_1} \cdot \frac{\partial a_1}{\partial x_2}}{a_1 \frac{\partial^2 a_1}{\partial x_1 \partial x_2}}$$

and :

$$\frac{\partial^2 a_1}{\partial x_1^2} = \frac{-x_2}{x_1} \cdot \frac{\partial^2 a_1}{\partial x_1 \partial x_2}$$

such that all second order partial derivatives can be written in function of the elasticity of substitution.

If we further write :

$$\eta_{ij} = \frac{p_{aj}}{a_i} \cdot \frac{\partial a_i}{\partial p_{aj}}$$

the six equation system becomes :

$$* \quad -\eta_{11} \frac{a_1}{p_{a1}} \cdot \frac{\partial p_{a1}}{\partial p_1} - \eta_{12} \frac{a_1}{p_{a2}} \cdot \frac{\partial p_{a2}}{\partial p_1} + \frac{p_1}{p_{a1}} \cdot \frac{\partial x_1}{\partial p_1} + \frac{p_1}{p_{a1}} \cdot \frac{\partial x_2}{\partial p_1} = 0$$

$$* \quad -\eta_{21} \frac{a_2}{p_{a1}} \cdot \frac{\partial p_{a1}}{\partial p_1} - \eta_{22} \frac{a_2}{p_{a2}} \cdot \frac{\partial p_{a2}}{\partial p_1} + \frac{p_3}{p_{a2}} \cdot \frac{\partial x_3}{\partial p_1} + \frac{p_4}{p_{a2}} \cdot \frac{\partial x_4}{\partial p_1} = 0$$

$$* \quad a_1 \sigma_1 \frac{\partial p_{a1}}{\partial p_1} - \frac{x_2}{x_1} p_2 \frac{\partial x_1}{\partial p_1} + p_2 \frac{\partial x_2}{\partial p_1} = \sigma_1 \frac{a_1 p_a}{p_1}$$

$$* \quad a_1 \sigma_1 \frac{\partial p_{a1}}{\partial p_1} + p_1 \frac{\partial x_1}{\partial p_1} - \frac{x_1}{x_2} p_1 \frac{\partial x_2}{\partial p_1} = 0$$

$$* \quad a_2 \sigma_2 \frac{\partial p_{a2}}{\partial p_1} \quad \frac{-x_4}{x_3} p_4 \frac{\partial x_3}{\partial p_1} + p_4 \frac{\partial x_4}{\partial p_1} = 0$$

$$* \quad a_2 \sigma_2 \frac{\partial p_{a2}}{\partial p_1} + p_3 \frac{\partial x_3}{\partial p_1} - \frac{x_3}{x_4} p_3 \frac{\partial x_4}{\partial p_1} = 0$$

Solving this system by elementary operations yields :

$$\frac{\partial x_1}{\partial p_1} = \frac{x_1\, p_{a1}\, a_1\, (x_1\, p_1\, \eta_{11} - x_2\, p_2\, \sigma_1)}{p_1\, (x_1\, p_1 + x_2\, p_2)^2}$$

$$\frac{\partial x_2}{\partial p_1} = \frac{p_{a1}\, a_1\, (\eta_{11} + \sigma_1)\, x_1\, x_2}{(p_1\, x_1 + p_2\, x_2)^2}$$

$$\frac{\partial x_3}{\partial p_1} = \frac{x_3\, x_1\, \eta_{21}\, a_2\, p_{a2}}{(p_1\, x_1 + p_2\, x_2)\, (p_3\, x_3 + p_4\, x_4)}$$

$$\frac{\partial x_4}{\partial p_1} = \frac{x_1\, x_4\, \eta_{21}\, a_2\, p_{a2}}{(p_1\, x_1 + p_2\, x_2)\, (p_3\, x_3 + p_4\, x_4)}$$

$$\frac{\partial p_{a1}}{\partial p_1} = \frac{x_1\, p_{a1}}{(x_1\, p_1 + x_2\, p_2)}$$

$$\frac{\partial p_{a2}}{\partial p_1} = 0$$

Appendix III Data

	Leisure time (1)(t_1)	Average (2) Full Income (m)	Price-index non-essentials(p_1)	Price-index(3) essentials(p_2)	Average(2) nominal wage-rate(w)	k_1	k_2
53	2,057	143,670	.6883	.6738	30.3543	.1023	.3586
54	2,058	149,157	.6968	.6948	31.3535	.1128	.3582
55	2,603	157,592	.6900	.6915	33.0575	.1635	.3450
56	2,059	162,496	.7046	.7145	34.1011	.1631	.3454
57	2,074	180,701	.7309	.7364	38.0109	.2092	.3279
58	2,074	174,748	.7317	.7342	36.7817	.1883	.3345
59	2,084	185,195	.7533	.7504	38.9729	.2088	.3265
60	2,114	195,741	.7510	.7453	41.2019	.2343	.3130
61	2,123	204,780	.7685	.7546	43.0895	.2488	.3060
62	2,124	213,426	.7771	.7743	44.9743	.2552	.3024
63	2,126	231,683	.8028	.7938	49.1240	.2765	.2892
64	2,154	249,069	.8310	.8204	53.1343	.2794	.2800
65	2,200	275,832	.8667	.8613	58.9797	.2835	.2689
66	2,235	297,859	.8988	.8941	63.6686	.2867	.2613
67	2,280	319,217	.9213	.9061	68.2724	.2910	.2505
68	2,296	346,537	.9469	.9275	74.0114	.3050	.2399
69	2,340	377,836	.9755	.9663	80.5914	.3055	.2319
70	2,370	403,187	1.0000	1.0000	85.7417	.3069	.2270
71	2,409	448,131	1.0495	1.0327	95.5768	.3117	.2139
72	2,436	504,822	1.0983	1.0938	108.2740	.3140	.2028
73	2,481	582,284	1.1647	1.1692	124.7360	.3179	.1910
74	2,497	674,709	1.3142	1.3033	143.4820	.3257	.1862
75	2,528	762,677	1.4718	1.4544	162.8400	.3019	.1836

(1) Hours a year

(2) in Belgian francs (for calculations see: Késenne (1979))

(3) based on prices of food (1), clothing (4), Houserent (5), Energy (6a) and transport (10c) (Belgian National Accounts - NIS)

References

R.G.D. ALLEN (1969). "Mathematics for Economists". Mc Millan, London.

G.S. BECKER (1965). "A theory of the allocation of time", The Economic Journal, sept.

G. CHEZ and G. BECKER (1973). "The allocation of time and goods over the life cycle". Columbia Univ. Press, New York.

S. KESENNE (1979). "Tijdsallocatie en Vrijetijdsbesteding". Tijdschrift voor Economie en Management, nr. 2

S. LINDER (1970), "The Harried Leisure Class". Columbia Univ. Press, New York.

R. POLLAK and M. WACHTER (1975). "The Relevance of the household production functions and its implications for the allocation of time". Journal of Political Economy. April.

F. VAN MECHELEN (1969). "Vrijetijdsbesteding in Vlaanderen". Antwerpen, Uitg. S.m. Ontwikkeling.

N.I.S. (1970 + 1976) "Statistische Studiën", nr. 21 + 45, Brussels.