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EXISTENCE OF POSITIVE PRICES FOR SRAFFA'S BASICS AND NON-BASICS (1)

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Our purpose is to derive a condition that is both necessary and sufficient for the existence of positive prices for all goods, basics as well as non-basics, for every feasible distribution of the surplus, in Sraffa's model of single-product industries [5, Part I].

As was repeatedly stressed by different authors, e.g. by Burmeister [1 and 2], the application of theorems on non-negative matrices seems the most effective method for a formal approach to Sraffa's propositions. Many mathematical treatments on neo-Ricardian economics use such theorems, but concentrate on systems of basic goods, i.e. products which enter directly or indirectly into the production of all commodities. The existence of all-positive prices in such a system is often mentioned, the proof of which is relatively easy [1, p. 84] as the underlying inputmatrix is indecomposable.
In this paper we shall consider a system where basics and non-basics coexist. Sraffa himself [5, pp. 90-91] realized that in this more general framework the prices of some commodities may become non-positive. Nevertheless he did not offer a rigorous analysis of this situation.

We consider a society which produces a set of n commodities: k basics and n - k non-basics ($1 \leq k < n$); x_j units of commodity j are produced by means of $a_{ij}x_j$ units of commodity i and $l_j x_j$ units of homogeneous labour. The coefficients a_{ij} form the n x n -inputmatrix A; l denotes the 1 x n -vector of direct labour requirements per unit; x and $y = x - Ax$ are n x 1 -vectors of total gross and total net product of the system.
We say that a non-negative (2) n x n -matrix A is decomposable iff (if and only if) there is a non-empty proper subset J of $\{1, 2, .., n\}$ such that $a_{ij} = 0$, $i \notin J$, $j \in J$. Otherwise we shall call A indecomposable (e.g. whenever it is 1 x 1). It is obvious that in our case the inputmatrix is decomposable [4, p.109]. By a suitable numbering of the commodities, this decomposable inputmatrix can be reduced to its so-called normal form [3, p. 75]: (3)

$$\begin{bmatrix} B & D_{01} & D_{02} & \dots & D_{0m} & \dots & D_{0q} \\ 0 & D_1 & D_{12} & \dots & D_{1m} & \dots & D_{1q} \\ 0 & 0 & D_2 & \dots & D_{2m} & \dots & D_{2q} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & D_m & \dots & D_{mq} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & \dots & D_q \end{bmatrix} \qquad /1/$$

(2) Vectors and matrices are called non-negative (denoted by ≥ 0) iff all their elements are non-negative, semi-positive (≥ 0) iff all elements are non-negative and some are positive, positive (> 0) iff all elements are positive.

(3) I repeat that we consider an economy with basics and non-basics. In Gantmacher's notation, this implies $g = 1$ and $s - g > 0$.

In our normal form, the principal submatrix B is an indecomposable $k \times k$ -matrix associated with the k basic goods; the principal submatrices $D_1, D_2, \ldots, D_m, \ldots, D_q$ are indecomposable and are associated with groups of non-basics; in each column at least one submatrix $D_{0g}, D_{1g}, \ldots D_{g-1,g}$ $(g = 1, 2 \ldots, q)$ is semi-positive. The dominant root (Frobenius-Perron eigenvalue) of the matrices A, B, D_m is denoted by f(A), f(B), $f(D_m)$. Let I be the identity matrix and h a numerical parameter. It is well known [4, pp. 95, 102, 107] that the matrix $hI - A$ has a non-negative inverse $(hI - A)^{-1}$ iff $h > f(A)$, that this inverse is positive if A is indecomposable and that the system with inputmatrix A is workable iff $1 > f(A)$.

One of Sraffa's achievements was the construction of an elegant numéraire (his standard composite commodity), but for our purposes nothing essential will be lost if we simply normalize prices so that the value of the actual net product equals one. This net product is distributed post factum among workers (wages) and capitalists (profits).

<u>Definition 1</u>: A $1 \times n$ -vector p is said to be a <u>Sraffa price vector</u> iff there exist non-negative scalars w (wage rate) and r (uniform rate of profits) such that the following equalities hold:

$$pA\,(1 + r) + lw = p \qquad /2/$$

$$py = 1 \qquad /3/$$

A price vector p will also be denoted by $(p_B\ p_{D1}\ p_{D2} \cdots p_{Dm} \ldots p_{Dq})$, the vector of direct labour inputs also by $(l_B\ l_{D1}\ l_{D2} \ldots l_{Dm} \ldots l_{Dq})$.

The notation is obvious: e.g. the $1 \times k$ -vector p_B contains the prices of the basics. Using this notation, the price relations /2/ can be written as :

$$p_B\, B(1 + r) + l_B w = p_B \qquad /4/$$

$$(p_B D_{01} + p_{D1} D_1)\,(1 + r) + l_{D1} w = p_{D1} \qquad /5/$$

more generally, for the mth group of non-basics :

$$(p_B D_{0m} + p_{D1}\, D_{1m} + p_{D2} D_{2m} + \dots + p_{Dm} D_m)(1 + r) + l_{Dm} w = p_{Dm} \qquad /6/$$

We use the following weak (4) <u>assumptions</u>: labour is indispensable to produce any good ($l > 0$), all n commodities have positive gross outputs ($x > 0$) and there is a surplus to be distributed ($y \geq 0$), the system is workable i.e. $f(A) < 1$ and thus a fortiori [3, p. 69] $f(B) < 1$ and $f(D_i) < 1$ all i.

<u>Lemma</u>: The existence of all-positive prices implies that Sraffa's maximum rate of profits (i.e. the value of r if $w = 0$) equals $1/f(B) - 1$.

<u>Proof</u>: If the lemma is false, putting $w = 0$ in equation /4/ implies $p_B B(1 + r) = p_B$ for a vector $p_B > 0$ and $1 + r \neq 1/f(B)$.
This means that the indecomposable matrix B has a positive eigenvector associated with an eigenvalue other than $f(B)$. This is impossible [3, p. 63].

We denote $1/f(B) - 1$ by R. Note that R is positive as $0 < f(B) < 1$. As relations /2/ and /3/ consist of n + 1 equations with n + 2 unknowns, some combinations of w and r make the equations inconsistent.

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(4) I mean: not restrictive from an economic point of view.

Therefore we introduce the following terminology:

Definition 2: A pair $(\bar{w}, \bar{r})$, $\bar{w} \geqq 0$, $0 \leqq \bar{r} \leqq R$, is said to be a feasible distribution iff there exist a Sraffa price vector that solves equations /2/ and /3/ for $w = \bar{w}$ and $r = \bar{r}$.

Note that in definitions 1 and 2, nothing is specified about the sign of the price vector.

We are able now to formulate our theorem that holds in the general neo-Ricardian model outlined above

THEOREM: With any feasible distribution there is associated a positive Sraffa price vector if and only if the normal form of the inputmatrix satisfies:

$$f(B) > f(D_i) \qquad i = 1, 2, \ldots, q \qquad /7/$$

Proof:

a) necessity

We shall prove by contradiction that without condition /7/ no positive Sraffa price vector exists for $w = 0$.
Let D_m be the principal submatrix with the largest dominant root of all D_i. Suppose that a positive vector

$\bar{p} = (\bar{p}_B\ \bar{p}_{D1} \ldots \bar{p}_{Dm} \ldots \bar{p}_{Dq})$ solves equations /2/ and /3/ for $w = 0$ and that nevertheless $f(D_m) \geqq f(B)$.

The lemma implies that $\bar{p}$ is associated with the maximum rate of profits $R = 1/f(B) - 1$. The price equation /6/ for the mth group of non-basics then becomes:

$$\bar{p}_B D_{0m} + \bar{p}_{D1} D_{1m} + \ldots + \bar{p}_{D,m-1} D_{m-1,m} + \bar{p}_{Dm} D_m = f(B) \bar{p}_{Dm}$$

As the sum of the first m terms forms a semi-positive vector, the system $p_{Dm}(f(B)I - D_m) \geq 0$ has a positive solution, viz. $\bar{p}_{Dm}$. As D_m is indecomposable, this implies that $f(B)I - D_m$ is non-negatively invertible [4, p. 107], hence $f(B) > f(D_m)$: contradiction.

b) sufficiency

As $f(D_i) < f(B)$ for all i and as A is decomposable, it is obvious that $f(B) = f(A)$ [3, p. 69]. To the dominant root f(A) of the matrix A there belongs a positive characteristic row vector [3, p. 77]. This row vector, normalized by equation /3/, provides us with a positive Sraffa price vector for the distribution $w = 0$, $r = R$.
We claim that the distributions (w_1, r_1), $w_1 = 0$, $0 \leq r_1 < R$ and (w_2, r_2), $w_2 > 0$, $r_2 = R$ are not feasible.
If $w = w_1$, $r = r_1$, equations /2/ and /3/ become :

$$pA = \frac{1}{1 + r_1} p$$

$$py = 1$$

As $y \geq 0$ and $\frac{1}{1 + r_1} > \frac{1}{1 + R} = f(B) = f(A)$, this system of equations is inconsistent.
If $w = w_2$, $r = r_2$, equation /4/ becomes:

$$p_B (f(B) I - B) = w_2 f(B) 1_B$$

With the dominant root f(B) of the indecomposable matrix B is associated a positive column eigenvector q_B.
Postmultiplying both sides of the last equality by q_B, we obtain: $p_B(f(B)I - B) q_B = w_2 f(B) 1_B q_B$

As the left-hand side is zero and the right-hand side is positive, (w_2, r_2) is not a feasible distribution.

The proof is therefore complete if we can show that condition /7/ also ensures the existence of a positive Sraffa price vector for any given feasible distribution $(\bar{w}, \bar{r})$, $\bar{w} > 0$, $0 \leq \bar{r} < R$. We shall construct such a vector in an inductive way.

The matrix B is indecomposable and $h = 1/(1 + \bar{r}) > 1/(1 + R) = f(B)$, thus $hI - B$ has a positive inverse. Hence $\bar{p}_B = hl_B\bar{w}(hI - B)^{-1}$ is a positive solution of price equation /4/.

The matrix D_1 is indecomposable and $h = 1/(1 + \bar{r}) > f(B) > f(D_1)$, thus $hI - D_1$ has a positive inverse.

Hence $\bar{p}_{D1} = (\bar{p}_B D_{01} + hl_{D1}\bar{w})\ (hI - D_1)^{-1}$, derived from equation /5/, provides us with positive prices for the first group of non-basics.

In the same way we obtain for the mth group of non-basics via relation /6/:

$$\bar{p}_{Dm} = (\bar{p}_B D_{0m} + \bar{p}_{D1}\ D_{1m} + \dots + \bar{p}_{D,m-1} D_{m-1,m} + hl_{Dm}\bar{w})(hI - D_m)^{-1}$$

Here $\bar{p}_{Dm}$ is positive for the same reasons as before.

In this way a positive Sraffa price vector $(\bar{p}_B\ \bar{p}_{D1} \cdots \bar{p}_{Dm} \cdots \bar{p}_{Dq})$, associated with the distribution $(\bar{w}, \bar{r})$, can be constructed.

Q.E.D.

Remark 1: The preceding reasoning reveals that condition /7/ not only guarantees the existence of a positive Sraffa price vector for any given feasible distribution $(\bar{w}, \bar{r})$ with $\bar{w} > 0$ and $0 \leq r < R$, but also its uniqueness.
This uniqueness also holds if $w = 0$ and $r = R$. Assume the contrary: let there be two positive Sraffa price vectors $\bar{p}$ and $\bar{\bar{p}}$. B is indecomposable, hence $\bar{p}_B = \bar{\bar{p}}_B$ must hold [3, p. 53]
Consider equation /5/:

$$\bar{p}_B D_{01} + \bar{p}_{D1} D_1 = f(B) \bar{p}_{D1}$$

$$\bar{\bar{p}}_B D_{01} + \bar{\bar{p}}_{D1} D_1 = f(B) \bar{\bar{p}}_{D1}$$

This implies that $(\bar{p}_{D1} - \bar{\bar{p}}_{D1})(f(B)I - D_1) = 0$

If $\bar{p}_{D1} \neq \bar{\bar{p}}_{D1}$, then $f(B)$ is an eigenvalue of D_1. This contradicts condition /7/, hence $\bar{p}_{D1} = \bar{\bar{p}}_{D1}$.
In the same way, the uniqueness of the other p_{Di} $(i = 2, 3, .., q)$ may be proven.

Remark 2: Define the unit of labour so that total labour lx equals one. Hence, the wage rate equals total wages.
In case of zero profits, their common value equals one. In this case, the Sraffa price vector equals the vector of labour values.
The positivity of these labour values is of course necessary for the prices to be positive for any feasible distribution, but not sufficient as is sometimes erroneously suggested in the literature [1, p. 84]. A simple example may illustrate this. Consider an economy with two basics and one non-basic; gross outputs are $x_1 = 1$, $x_2 = 1$, $x_3 = 1$; net outputs are $y_1 = 1/2$, $y_2 = 0$, $y_3 = 1/2$; direct labour inputs are $l_1 = 1/4$, $l_2 = 1/4$, $l_3 = 1/2$.

The inputmatrix A equals $\begin{bmatrix} 0 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 0 & 1/2 \end{bmatrix}$

In its normal form /1/: $q = 1$, $B = \begin{bmatrix} 0 & 1/2 \\ 1/2 & 0 \end{bmatrix}$, $D_{01} = \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$, $D_1 = [1/2]$

Labour values are all positive: 1/2, 1/2 and 3/2.
Nevertheless, if the whole net product goes to capitalists ($w = 0$), then we find that $1 + R = 2$ and that the price equations have no positive solution.
In the light of our theorem, this is not surprising as $f(B) = f(D_1) = 1/2$.

<u>Remark 3</u>: The proof of the sufficiency part of our theorem shows implicitly that positivity of all prices is ensured even if only one element of l_B is positive and the $n - 1$ other direct labour inputs are zero. We do not insist on this stronger mathematical result, because it does not seem very important from the economic point of view.

APPENDIX: On the reduction of a decomposable inputmatrix to its normal form.

The notion of the normal form of a decomposable matrix turned out to be indispensable to establish our conditions for all-positive prices in a neo-Ricardian economy with basics and non-basics. Our only purpose in this appendix is to sketch a method for the reduction of a matrix to its normal form. Readers who are not interested in this technical problem, may skip this appendix. As this paper deals primarily with economics, we concentrate on the inputmatrix of the neo-Ricardian model with basics and non-basics. This does not diminish the generality of our results, but specialists in numerical analysis or graph theory may find our terminology and some other aspects of our exposition rather peculiar.

We consider an inputmatrix A of the nth order. We always assume the existence of k basics and n - k non-basics ($1 \leq k < n$). We represent the structural relations of the inputmatrix A in a directed graph: every sector is associated with a <u>vertex</u>; there is an <u>arrow</u> (directed edge) <u>from vertex i to vertex j</u> iff commodity i is directly required for the production of commodity j (i.e. iff $a_{ij} > 0$).
There is <u>a path from i to j</u> iff commodity i is directly or indirectly required for the production of commodity j; we also say that <u>sector i is connected with sector j</u> or that <u>j is reachable from i</u>.
With the graph of the inputmatrix A is associated an <u>adjacency matrix M(A)</u>. Its entries are denoted $m_{ij}(A)$ and defined as follows:

$$m_{ij}(A) = 1 \quad \text{iff there is an arrow from i to j}$$
$$\phantom{m_{ij}(A)} = 0 \quad \text{otherwise}$$

We also define the <u>reachability matrix R(A)</u> of the graph. Its entries are denoted $r_{ij}(A)$ and defined as follows:

$$r_{ij}(A) = 1 \quad \text{iff there is a path from } i \text{ to } j$$
$$= 0 \quad \text{otherwise}$$

Let $M^g(A)$ denote the gth power of the matrix M(A) and let its entries be written $m_{ij}^{(g)}(A)$. It is easy to show that $m_{ij}^{(g)}(A) > 0$ iff there is a path of length g (i.e. consisting of g arrows) from i to j. It is obvious that $r_{ij}(A) = 1$ iff $m_{ij}^{(g)}(A) > 0$ for at least one $g \in \{1, 2, \ldots, n\}$.

Commodity i is basic iff i is connected with all $j \in \{1, 2, .., n\}$. Hence the rows of R(A) whose entries are all 1's, indicate the basic sectors. The indices of the k basic sectors form a set, denoted by K.
We then permute (5) and partition A into the following form:

$$\begin{bmatrix} B & C \\ A_0 & D \end{bmatrix} \qquad /8/$$

Here B is a k x k -matrix associated with the k basics.

<u>Lemma 1</u>: The form /8/ has the following properties:

a) B is indecomposable
b) the matrix A_0 is zero
c) the matrix C is semi-positive

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(5) By a permutation of a square matrix, we always mean a permutation of the rows combined with the <u>same</u> permutation of the columns.

Proof:

a) If B is decomposable, there exist two sectors g and h in K (not necessarily distinct), such that there is no path within K from g to h [4, p. 109]. Nevertheless g is a basic sector. Hence, there is a path from g to h which runs via a non-basic sector $m \notin K$. Via basic sector h, this sector m is connected with all sectors. Hence m is basic : contradiction.

b) Suppose $a_{mj} > 0$ for $m \notin K$, $j \in K$. Then m is connected, via basic sector j, with all sectors. Hence m is basic: contradiction.

c) If C is zero, there are no basics: contradiction.

If the matrix D is indecomposable, /8/ is the normal form of A. If not, further manipulations of D (and simultaneously of C, of course) are necessary.
We propose the following procedure for the reduction of the matrix D. Assume that D is a p x p -matrix and that the indices of its sectors form the set P.

1. if p = 1, D is indecomposable- STOP
2. if p > 1, we construct the reachability matrix of D, denoted R(D). Its elements are $r_{ij}(D)$:

   $r_{ij}(D) = 1$ iff there is a path from i to j within P (We shall also say that i is connected with j within P)

   $r_{ij}(D) = 0$ otherwise

2.a) If all elements of R(D) differ from zero, D is indecomposable - STOP
If 2.a) is not relevant, we go to 2.b).

2.b) If R(D), and thus a fortiori D itself, contains a row with all 0's, we permute this row to the last row of D.(6) We then obtain the form:

$$\begin{bmatrix} F & E \\ 0 & D_1 \end{bmatrix}$$

D_1 is a 1 x 1 -matrix (with element 0) and thus indecomposable by our definition.

We recommence the whole procedure for the reduction of the matrix F. If 2.b) is not relevant, we go to 2.c).

2.c) If R(D) contains s rows with all 1's, where $0 < s < p$, D is permuted to the form:

$$\begin{bmatrix} D_1 & E \\ D_0 & F \end{bmatrix} = \begin{bmatrix} D_1 & E \\ 0 & F \end{bmatrix}$$

D_1 is the s x s -matrix associated with the s rows with all 1's. That D_1 is indecomposable and that the matrix D_0 is zero, can be proved in precisely the same way as in the preceding lemma.

We recommence the whole procedure for the reduction of the matrix F.

If 2.c) is not relevant, we go to 2.d).

2.d) Every row of R(D) contains at least one 0 and one 1. /9/

The only two possibilities that remain are 2.d)1. and 2.d)2.

2.d)1. There exists a sector j_0 in P such that $r_{ij_0}(D) = 0$ for all i in P, $i \neq j_0$. /10/

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(6) This row corresponds with a non-basic good that is never used as a means of production.

We then permute D so that the j_0th row becomes the first and we obtain the form:

$$\begin{bmatrix} D_1 & E \\ 0 & F \end{bmatrix}$$

D_1 is a 1 x 1 -matrix and thus indecomposable.
We recommence the whole procedure for the reduction of the matrix F. If 2.d)1. is not relevant, 2.d)2. must hold.

2.d)2. <u>Every sector j_0 in P is such that $r_{ij_0}(D) = 1$ for at least one i in P, $i \neq j_0$</u> /11/

We consider the z rows ($1 \leq z \leq p$) of R(D) with the maximal (7) number of 1's. This maximal number is denoted by d ($0 < d < p$). The indices of these z rows form the set Z. We choose an arbitrary element of Z: the sector h_0.
We form the following subsets of P.

$$\left.\begin{aligned} V_0 &= \{h \in P \mid r_{hh_0}(D) = 0\} \\ V_1 &= \{h \in P \mid r_{hh_0}(D) = 1\} \end{aligned}\right\} \qquad /12/$$

In lemma 2 at the end of this appendix, it is proved that both sets are non-empty. Hence they form a partition of P. This partition is the base for the permutation of the matrix D to the form:

$$\begin{bmatrix} D_{V_1} & E \\ Q & D_{V_0} \end{bmatrix} = \begin{bmatrix} D_1 & E \\ 0 & F \end{bmatrix} \qquad /13/$$

(7) I decided to pick out these rows after a suggestion of C. Van Nuffelen. Of course, he is not responsible for eventual misuse of his suggestion in this context.

The square matrix D_{V_1} is associated with the elements of V_1; D_{V_1} is also denoted by D_1, D_{V_0} by F.
In lemma 3 we shall prove that the matrix $D_{V_1} = D_1$ is indecomposable and that the matrix Q really equals zero.
We recommence the whole procedure for the reduction of the matrix F.

As 1, 2.a), 2.b), 2.c), 2.d)1. and 2.d)2. together exhaust all possible cases and as in every case an effective partition was performed, our procedure must lead up to the normal form in a finite number of steps. To complete our story, we still have to prove two lemma's.

Lemma 2: The sets V_0 and V_1, as defined for case 2.d)2. in relation /12/, are non-empty. (8)

Proof: That V_1 is non-empty, is trivial from /11/ and /12/. Suppose that V_0 is empty and thus $V_1 = P$. Hence, every sector in P is connected with h_0, so that h_0 is connected with itself. As h_0 belongs to Z, it is connected with d sectors $(0 < d < p)$ and thus not connected with at least one sector $h_1 \varepsilon P$. From /11/, h_1 is reachable from at least one element $h_2 \varepsilon P$. As $V_1 = P$, h_2 is connected with h_0 and thus also with the d sectors reachable from h_0. Furthermore h_2 is connected with h_1 and h_0 is not. Hence h_2 is connected with at least $d + 1$ sectors: contradiction.

(8) In the proof of lemma's 2 and 3, "connected" and "reachable" always stand for "connected within P" and "reachable within P", unless the contrary is stated explicitly.

Lemma 3: The form /13/ has the following properties:

a) D_{V_1} is indecomposable

b) the matrix Q is zero

Proof: We first prove the following proposition:

Every sector $t \in Z$ is connected with itself /14/

From /11/, there is a sector $g \neq t$ in P which is connected with t and via t with at least as many sectors as t. If t is not connected with itself, g is connected with more sectors than t : contradiction.

a) Take an arbitrary sector h_1 in V_1. This sector is connected with h_0 and via h_0 with at least as many sectors as h_0. Hence h_1 must also belong to Z. Thus, we may conclude that $V_1 \subset Z$ and also, from /14/, that h_1 is connected with itself.

We claim that this sector h_1 is connected with h_0 within V_1 /15/

Suppose /15/ is false. Then the path from h_1 to h_0 runs via a sector $h_2 \notin V_1$. Hence h_2 is connected with h_0, thus $h_2 \in V_1$: contradiction.

We claim that the sector h_1 is connected with itself within V_1 /16/

If /16/ is false, the path from h_1 to h_1 runs via a sector $h_3 \notin V_1$. Hence h_3 is connected with h_1 and via h_1 with h_0: contradiction.

/15/, /16/ and a lemma of Nikaido [4, p. 109] imply that the indecomposability of D_{V_1} is established as soon as we can prove the following proposition:

The sector h_0 is connected within V_1 with all sectors of V_1 /17/

We prove /17/ by contradiction. If /17/ is false, h_0 is not connected within V_1 with a sector $h_4 \in V_1$.

Then there are two possibilities:

1. A path from h_0 to h_4 runs via a sector $h_5 \notin V_1$. This implies that h_5 is connected with $h_4 \in V_1$ and via h_4 with h_0. Hence $h_5 \in V_1$: contradiction.

2. The sector h_0 is not connected with h_4. $V_1 \subset Z$ and /14/ imply that h_4 is connected with itself and thus $h_4 \neq h_0$.
   As $h_4 \in V_1$, it is connected with h_0 and thus also with the d sectors reachable from h_0. Hence h_4 is connected with at least d + 1 sectors: contradiction.

b) The last part of lemma 3 is proved as follows: take an arbitrary element a_{mj} of the matrix Q ($m \in V_0$, $j \in V_1$); $a_{mj} > 0$ implies that m is connected with j, and via j with h_0. Hence $m \in V_1$: contradiction.

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