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MONOPOLY WELFARE LOSSES: A Methodological Note

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In price theory and welfare economics the adverse effects of monopoly (and monopolistic) pricing on the allocation of resources and aggregate welfare are emphasized. Emirical research on monopolistic pricing behavior and monopolistic business practices in general confirms the hypothesis that industry concentration and monopoly is seriously detering the performance of market economies.

This general belief that monopoly welfare losses are important was shaken by Harold Harberger (1954) who argued that

> "... our economy emphatically does not seem to be monopoly capitalism in big red letters ..."

and that

> "... when we are interested in the big picture of our manufacturing economy, we need not apologize for treating it as competitive, for in fact it is awfully close to being so ...".

Harberger estimated that - in the late 20's which he selected for methodological reasons - the total welfare losses in manufacturing were less than a tenth of a percent of national income.

David Schwarzman (1960) confirmed Harberger's results in using 1954 data. His estimate of monopoly welfare losses was also less than a tenth of a percent of national income. Harvey Leibenstein (1966) surveyed some findings on allocative inefficiency and concludes that

> "... the empirical evidence, ..., certainly suggests that the welfare gains that can be achieved by increasing only allocative efficiency are usually exceedingly small, at least in capitalist economies ..."

and that

> "... they hardly seem worth worrying about ...".

Although Harberger's results were based on kind of "instant economic calculus", his sweeping conclusion apparently got much credit. Recently, A. Bergson (1973) probed some of the methodological problems in measuring monopoly welfare losses.

The purpose of this paper is to review some of these methodological issues and to compare the correct methodological approach with Harberger's approach, provided his empirical assumptions hold.

Section I reviews Harbergers's procedure. Section II deals with a comparison of Marshallian consumer surplus and Hicksian income variation as the measurement of welfare changes. In Section III these alternatives approaches to the measurement of welfare changes are applied to the estimation of monopoly welfare losses.

I. HARBERGER'S PROCEDURE

We learn from classical price theory and welfare economics that the malallocative effect of monopoly results from a divergence between price and marginal costs.

In figure 1.a the industry supply and demand curve is given. The equilibrium production plan in case of perfect competition is obtained at an output q° and price p°. For this price production plan the difference between aggregate willingness to pay (i.e. the area below the demand curve) for q_o and the resource cost (i.e. the area below the marginal cost curve) of q_o is maximum.

Figure 1.b illustrates the case of a monopoly industry. The profit-maximizing price-production plan is p^m, q^m for which marginal revenue equals marginal costs. As one can see, the difference between aggregate willingness to pay and resource cost is not maximum for this price-production plan as it could be increased by the area A'EA if production was carried on to q° and price decreased to p°.

The area enclosed by the demand and marginal cost curve is the traditional measurement of the monopoly welfare loss (1).

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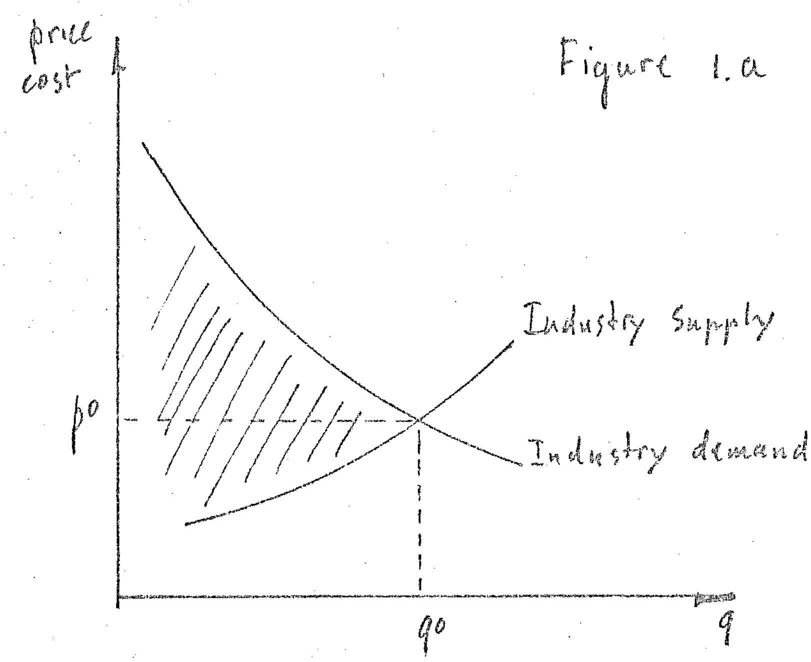
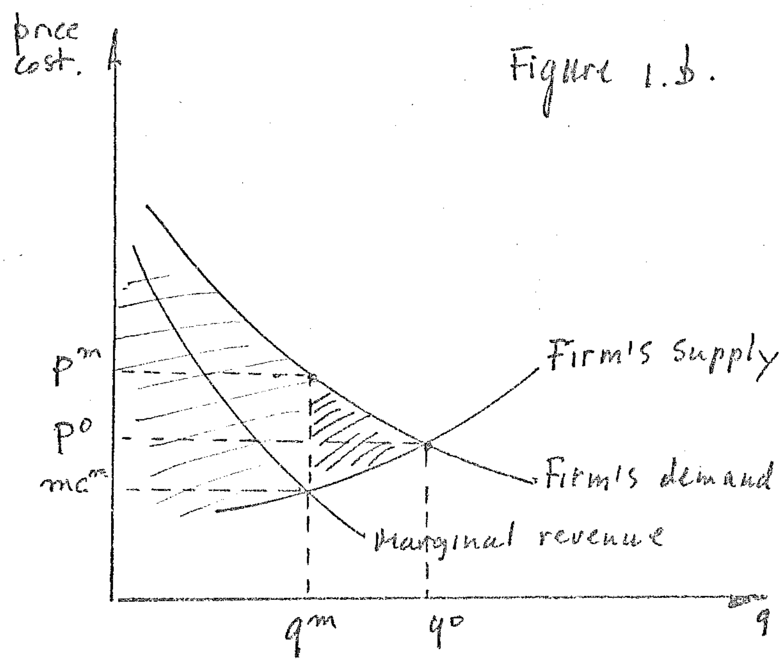
(1) This method was proposed by Hotelling (1938) for the measurements of allocative losses of taxation. Hotelling traces this concept back to Jules Dupuit who developed it for evaluating benefits of public works. The aggregate willingness to pay is equivalent to Marshall's consumers' surplus.


price
cost
p
Figure 1.a
Industry Supply
Industry demand
p^o
q^o
q


price
cost.
p
Figure 1.b.
Firm's Supply
Firm's demand
Marginal revenue
p^m
p^o
mc^m
q^m
q^o
q

This difference between the aggregate willingness to pay (a monetary equivalent of utility) and resource costs is taken as the monetary equivalent of the 'net utility' derived from a certain production.

Harberger takes it as an operating hypothesis that, in the long run, resources are allocated in US manufacturing industries at constant returns to scale. This implies that long-run average costs are constant (for the firm and the industry) and consequently equal to marginal costs).

The reason for this assumption is quite obvious. First, the monopoly welfare loss reduces to

$$\frac{1}{2}\left|(p^m - p^o)(q^m - q^o)\right|$$

if the demand is assumed to be linear. Second, marginal costs are assumed to be equal to average costs, and the latter are less hard to come by than the former.

If there are more products subject to monopolistic pricing, the deadweight losses can be added so that the aggregate loss is

$$\frac{1}{2}\sum_i \left|(p_i^m - p_i^o)\ (q_i^m - q_i^o)\right|$$

This can be expressed in terms of the demand elasticity (ε_i), monopoly excess price-cost ratios (δ_i) and sales values (S_i) so that aggregate welfare losses (W) are approximated by

$$W = \frac{1}{2}\sum_i \left|\delta_i^2\right|\varepsilon_i\ S_i$$

Harberger identifies malallocation by considering returns on invested capital. Under perfect competition returns on invested capital for each firm will be equal. If there is a divergence of perfect competition in some industries, returns on invested capital will differ from the average rate. Industries with high returns will use too few resources, those with low rates will use too much resources. Hence by taking the difference between actual profits and what profits would be at the average rate on return an estimate of excess profits is obtained. The values of excess price-cost ratios /defined as (price - unit cost) ÷ unit cost7 are found by computing the ratios

$$\frac{\text{excess profits}}{\text{sales - excess profits}} \quad \text{for each industry.}$$

With regards to elasticities, Harberger makes the following (heroic) assumption:

> "How high are these elasticities? It seems to me that one need only look at the list of industries ... in order to get the feeling that the elasticities in question are probably quite low. The presumption of low elasticity is further strengthened by the fact that what we envisage is not the substitution of one industry's product against all other products, but rather the substitution of one great aggregate of products (those yielding high rates of return) for another aggregate (those yielding low rates of return). In the light of these considerations, I think an elasticity of unity is about as high as one can reasonably allow for, though a somewhat higher elasticity would not seriously affect the general tenor of my results".

Based on data of accounting capital value, profits and sales for 73 US manufacturing industries (representing about 45 percent of total sales and capital in US manufacturing) for the period 1924-1928, Harberger derives that monopoly welfare losses amount to 59 million dollars - less than one tenth of 1 percent of the national income.

Harberger comments his results in several ways. In general, he says that it should be clear from the outset that "this is not the kind of job one can do with great precision and that the best we can hope for is to get a feeling for the general orders of magnitude that are involved."

In particular, he recognises a number of empirical problems such as the estimates on capital values and monopoly profits. The reason for selecting the period 24-28 is that during and before that period prices were fairly stable so that accounting capital values were close to actual capital values. He also recognises the possibility that monopoly profits are capitalised so that the reported profit rate on capital is an underestimate of the real rate on return.

Also the fact that advertising expenditures are deducted from gross profit bias the reported profit rate downward. However, he argues that this type of bias is not sufficiently important to change the magnitude of his results. Also, in the sample he used there was an overweighting of high-profit industries. However, the bias introduced by using this sample can be considered residual according to Harberger.

Harberger comments on the use of the Hotelling formula, the aggregation problem and on the constant returns hypothesis.

He argues that if one would adopt another operating hypothesis, monopoly welfare losses, it would be decreasing returns to scale. With the latter, the estimates would be smaller.

With regards to the Hotelling formula - only strictly applicable if all industries produce final commodities - he recognises two possibilities with regards to industries producing intermediate products. If they are neglected altogether the resulting monopoly welfare loss would be biased downward. If they are treated as if these intermediate industries where producing final commodities the result would be an overestimate. He prefers the latter assumption as its is safer to produce an overestimate than an underestimate in this case.

Harberger recognised that aggregation biases the estimate of the welfare losses downward, but dismissed the problem by stating that "experiments with hypothetical examples reveal that the probable extent of the bias is small".

II. THE HOTTELLING FORMULA vs. HICKSIAN INCOME VARIATION

The Hotelling formula is used throughout in the analysis of losses from monopoly, taxation, trade tariffs, disequilibria, etc. In fact, this formula is an application of consumer's surplus analysis. Recently, quite some attention has been paid in the literature on the concept of consumer's surplus (1).

(1) See e.g. the survey of J.M. Currie, J.A. Murphy and A. Schmitz, "The Concept of Economic Surplus and its Use in Economic Analysis", Economic Journal, December 1971, pp.741-799 and M.E. Burns, "A Note on the Concept and Measure of Consumer's Surplus", American Economic Review, June 1973, pp.335-344.

Figure 2.a.

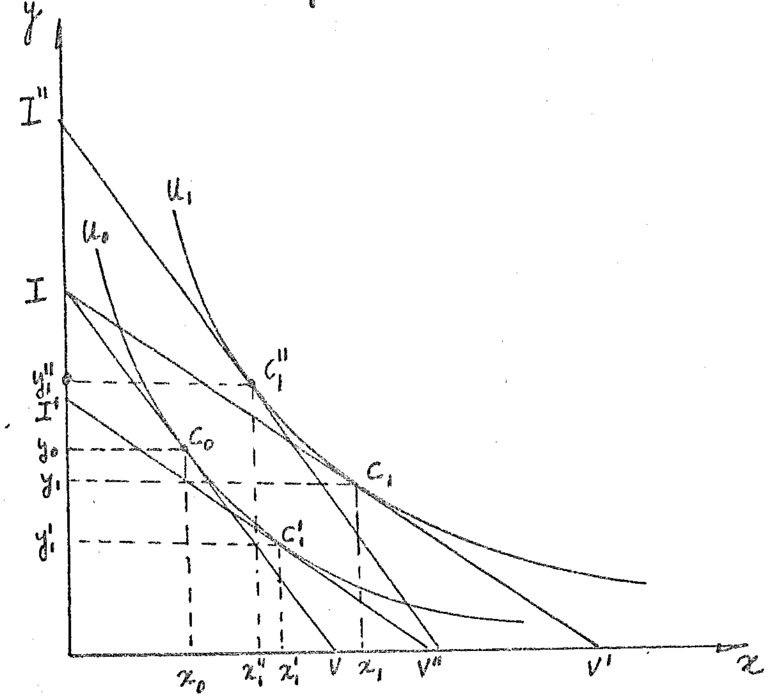
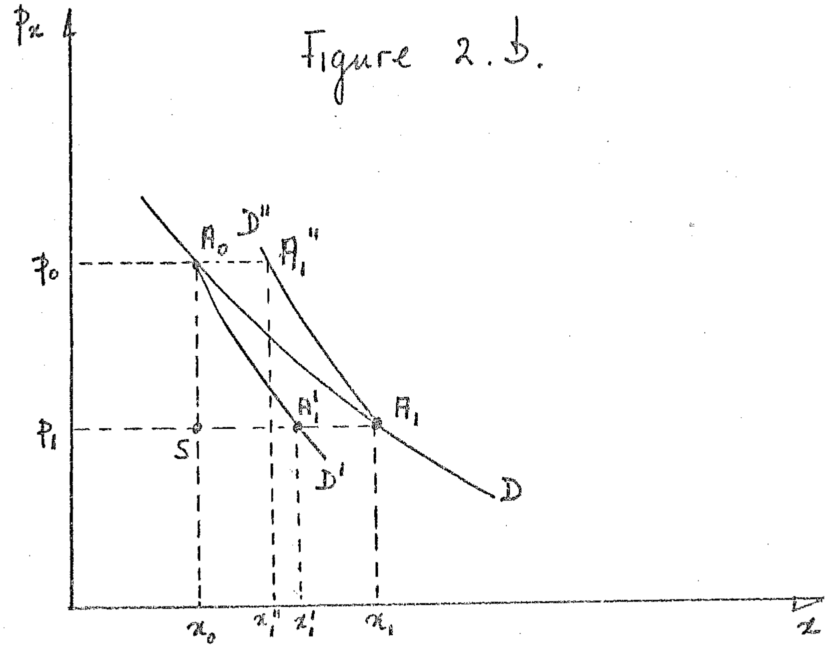


Figure 2.b.



However, few numerical experiments have been done to explore the empirical consequences of adopting one or another concept. In an earlier worknote I performed numerical experiments based on Cobb-Douglas type utility function which indicated that the empirical differences among consumer's surplus concepts were rather unimportant. Bergson (1973) based his analysis on a C.E.S.-specification of utility functions and reported more pronounced differences between concepts.

The consumer's surplus concept depends upon the particular demand concept one adopts. Consider figure 2.a and 2.b. In figure 2.a the consumer's choice of his consumption plan assuming a two-good choice is graphically illustrated. Consider good Y as the numeraire (i.e. its price equals one monetary unit), so that income can be read on the y-axis. Initially the consumer has an income OI. With a particular price for X, say p_o, the budget constraint he faces is given by the line IV. The consumer will maximize his utility - the maximum level being U_o - for the consumption plan C_o (x_o, y_o). A change in price of the commodity X from p_o to p_1 will result in the budget line IV'. The consumer will now maximize his utility - the maximum level being U_1 - for the consumptionplan C_1 (x_1, y_1) (1). The corresponding Marshallian demand curve, is defined as the relation between the price and quantity consumed of a commodity (ceteris paribus) and is given in figure 2.b.

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(1) A normal good is considered (i.e. a good for which the substitution effect is negative). However, the analysis can be developed without difficulties for inferior goods.

The Marshallian demand curve is an expression of the marginal willingness to pay for a certain quantity of commodity X. Consequently, the aggregate willingness to pay is the area below the Marshallian demand curve. The difference between what the consumer wants to pay and what he actually pays is the 'consumer's surplus' and can be considered as the monetary equivalent of the welfare he derives from his consumption. Conventional surplus formula's (Dupuit, Hotelling, etc.) are based on this concept of consumer's surplus.

John Hicks (1956) draw our attention to the fact that each point on the Marshallian demand curve does not necessarily corresponds to the same utility level. Only in those cases, where there is no income effect, different points on the Marshallian demand curve correspond with the same utility level. Hence, the marginal willingness to pay for a particular quantity of a commodity, provided consumers are on the same utility level, cannot be derived from a Marshallian demand curve (except for a few exceptional cases where there is no income effect). Hicks developed the concepts of compensated and equivalent variation (and demand) and both concepts can be used to estimate the utility gain (or loss) from a price change.

Consider again figure 2.a. After the price of commodity X charged from p_o to p_1 and the consumer adjusted his consumption plan from C_o (x_o,y_o) to C_1 (x_1,y_1) he will derive an utility level U_1. In order to bring the consumer back to his utility level prior the price change viz. U_o one would have to reduce his income with II'. This reduction in income is called the equivalent variation. It is the amount of money which the consumer would have to gain (after he purchased x_o at price p_o) in order to find himself on the indifference level he attained purchasing x_1 at price p_1. Consequently, it can be used as the monetary equivalent of an utility change resulting from price shift.

Due to the price change the consumer will have acquired x_1 at price p_1 and will attain the utility level U_1. In order to keep the consumer on his utility level U_1, but with the old price p_o, one would have to increase his income from OI to OI". This increase II" is the 'compensated variation' or the amount of money which the consumer would have to lose, after he purchased x_1 at price p_1 in order to get him at the utility level he had when purchasing x_o at price p_o.

These variation concepts can be used as the monetary expression of a shift in utility as a result from price change, depending upon the point of view one takes. If one considers the shift from the higher utility to the lower as the relevant one, compensating variation should be used. Considering the shift from the lower to the higher level of utility, the equivalent variation is relevant.

The Hicksian compensated demand schedules are associated with the concepts of compensated and equivalent variations.

Consider the case of an equivalent variation.
If the consumer's income is compensated for II' so that he stays on his initial utility level U_o, he will choose a consumption plan $C_1'(x_1', y_1')$. Hence, for various prices for commodity X, and allowing for equivalent income variations

so that the consumer remains at level U_o, the corresponding quantities of X purchased can be calculated. This price-quantity schedule, illustrated in figure 2.b by D'D' is the compensated demand curve. A similar curve for the utility level U_1 and allowing for compensating income variations can be derived, represented in figure 2.b by the curve D"D".

Hence, the consumer's surplus concept viz. the excess of his aggregate willingness to pay over what he actually pays can be defined in terms of compensated demand schedules.

Consider the compensated demand curve D'D'.
After the price change the consumer's purchase will be x_1, derived from the Marshallian demand curve. The gain in surplus will consist of

1) surplus for the first x_o units or the rectangle $p_oA_oSp_1$;
2) a positive surplus for the next $x_1'-x_o$ units or the curvilinear triangle SA_oA_1';
3) a negative surplus for the next (x_1-x_1') units or the curvilinear triangle $A_1'A_1R$.

If one considers the compensated demand schedule D"D" the consumer's surplus will consist of

1) a positive surplus for the first x_o units, viz. the rectangle $p_oA_oSp_1$;
2) the positive surplus for the next (x_1-x_o) units or the curvilinear triangle TSA_1.

From the graphical exposition, it is clear that, depending upon which framework of measurement one uses, the empirical results will differ. In order however to get some idea of the magnitude involved a numerical experiment is performed.

The case of a consumer, purchasing quantities x and y of the commodities X and Y and characterised by a Cobb-Douglas type utility function, is considered in the numerical experiment. A Cobb-Douglas type utility function is used here. Bergson (1973) uses a CES-type utility function. Much can be said on the specification issue.

On the one hand, a Cobb-Douglas specification implies that (Marshallian) demand functions have unitary (own) price elasticities, zero cross-(price) elasticities and unitary income elasticities. Hence, as Harberger assumes unitary (own) price elasticities and zero cross elasticities for his computation of monopoly welfare losses, empirical comparisons based on Cobb-Douglas specifications between his method of measurement and alternative methods make good sense as one uses the same numerical basis.

On the other hand, a CES specification,

> "... has the distinct merit that by varying σ, ... representing the elasticity of substitution between any two consumer's goods, we may allow different degrees of substitutability between products, and to ultimately for varying elasticities of demand for varying elasticities of demand for one or another of our ... goods" (Bergson, p.863).

With regards to the unit income elasticity resulting from a CES specification of the utility function Bergson argues

> "in the absence of empirical data on the comparative income elasticities of ... goods, it may be more of a virtue than a limitation of /CES/ that it implies unitary income elasticities of demand for all products alike" (Bergson p.863).

The following proposition can be made with regard to the specification issue:

1. if one compares Harberger's method (Hotelling formula) with Hicksian methods one has to adopt the same empirical assumptions. Consequently, a Cobb-Douglas specification is appropriate;
2. if one criticizes Harberger's method, inclusive his empirical operating assumptions, a specification such as CES allowing for substitutability is desirable.

I merely attempt here to compare the use of the Hotelling formula with the Hicksian measurements, provided Harberger's empirical operating assumptions hold.

First consider the measurement of the welfare gain for a consumer from a price fall in the two commodity case. Commodity Y is chosen as the numeraire and the price of X changes from p_o to p_1 with 100 λ percent ($\lambda = [p_o - p_1]/p_1$).

In appendix I it is shown that the welfare gain approximations in terms of initial income are:

Hotelling formula: $$h = [(1-\mu)\lambda(2+\lambda)]/[2(1+\lambda)] \quad (1)$$

Marshallian surplus: $$ms = (1-\mu)\log_e(1+\lambda) \quad (2)$$

Compensating variation: $$cv = (1+\lambda)^{(1-\mu)} - 1 \quad (3)$$

Equivalent variation: $$ev = 1 - (1+\lambda)^{-(1-\mu)} \quad (4)$$

where $\mu = y/I$ or the share of the numeraire expenditure in income.

In tables I.a to I.d empirical values of the welfare gain approximations are tabulated for some values of the share in income of expenditures on the numeraire and price fall percentages. In table II.a to II.c the relative difference in percent between Marshallian surplus, compensating variation and equivalent variation versus Hotelling's formula is listed.

Table I.a. Hotelling's approximation of consumer's welfare gain

μ \ λ	.1	.25	.50	1.	2.
0	.1000	.2250	.4167	.7500	1.3333
2	.0764	.1800	.3333	.6000	1.0667
4	.0573	.1350	.2500	.4500	.8000
6	.0382	.0900	.1667	.3000	.5333
8	.0191	.0450	.0833	.1500	.2667

Table I.b. Marshallian consumer's surplus

μ \ λ	.1	.25	.50	1.0	2.00
0	.0953	.2231	.4055	.6931	1.0986
2	.0762	.1785	.3244	.5545	.8789
4	.0572	.1339	.2433	.4159	.6592
6	.0381	.0893	.1622	.2773	.4394
8	.0191	.0446	.0811	.1386	.2197

Table 1.c. Compensating variation in income

μ \ λ	.1	.25	.50	1	2
0	.1000	.2500	.5000	1.0000	2.0000
2	.0792	.1954	.3832	.7411	1.4082
4	.0589	.1433	.2754	.5157	.9332
6	.0389	.0933	.1761	.3195	.6178
8	.0192	.0456	.0845	.1487	.2457

Table 1.d. Equivalent variation in income

μ \ λ	.1	.25	.50	1	2
0	.0909	.2000	.3333	.5000	.6667
2	.0734	.1635	.2770	.4257	.5848
4	.0556	.1253	.2159	.3402	.4827
6	.0374	.0854	.1497	.2421	.3556
8	.0189	.0436	.0779	.1294	.1973

Table II.a. Difference (in percent) between Marshallian surplus and Hotellings' approximation

λ	.10	.25	.50	1	2
	-0.15	-0.83	-2.69	-7.58	-17.60

Table II.b. Differences (in percent) between compensating variation and Hotelling's approximation

μ \ λ	.1	.25	.50	1	2
0	+4.76	+11.11	+20.00	+33.33	+50.00
2	+3.75	+8.58	+14.95	+23.52	+32.02
4	+2.76	+6.12	+10.17	+14.60	+16.65
6	+1.78	+3.74	+5.65	+6.50	+3.47
8	+ .81	+1.42	+1.37	-0.87	-7.85

Table II.c. Difference (in percent) between equivalent variation and Hotelling's approximation

μ \ λ	.1	.25	.50	1	2
0	-4.76	-11.11	-20.00	-33.33	-50.00
2	-3.86	-9.17	-16.89	-29.06	-45.18
4	-2.95	-7.18	-13.62	-24.39	-39.66
6	-2.03	-5.12	-10.17	-19.29	-33.32
8	-1.10	-3.01	-6.53	-13.70	-26.03

Some conclusions are:

1. Marshallian surplus is larger than equivalent variation but smaller than compensating variation (this is always the case for a normal good).

2. For a given price change Hicksian approximations tend to be closer to the Marshallian surplus (and Hotelling's approximation) the smaller the share in expenditures for the commodity

subject to the price change. (This is also a general conclusion. The smaller the income effect, the better Marshallian surplus approximates compensating and equivalent variation.)

3. The smaller price changes - given a share in income - the smaller the difference between different concepts.

4. The empirical effect of using the different concepts is rather small in cases of small variations in price. (One has to bear in mind that this conclusion is particular i. e. in case of unit price and income elasticities, and zero cross elasticities).

III. MONOPOLY WELFARE LOSS MEASUREMENTS

The former experiment merely showed the empirical consequences of adopting a particular concept for a single consumer. However what is the effect on monopoly welfare loss estimates?

Bergson (1973) dealt with this issue and his methods are followed here. However, I use a Cobb-Douglas specification as it is conform with Harberger's empirical assumptions, that one compares the methodological differences.

Consider the following simplified model of our economy. The economy consists of one household, characterised by a Cobb-Douglas type utility function, and behaving as if it were bying its consumption goods in competitive markets (1). Assume that

(1) This assumption is not as unrealistic as it may appear. In a more realistic model there are of course many consumers each having a separate utility function. However, none of them has significant influence on market prices. In such conditions, consumers will maximize their utility subject to a particular (parametric) set of prices and income. All consumers will equalize the ratio of marginal utilities for each pair of commodities to the price ratio. As the latter ratio is the same for all consumers one can safely assume an aggregate ratio or as if one household were buying in competitive markets.

the economy has a linear transformation function and that there are n commodities. Furthermore, production is organised by a number of firms which determine a price for each commodity, not necessarily equal to its resource cost. In this economy, all resources are used and markets cleared. In order to have a particular set of absolute prices, it is assumed that at least one commodity is sold at resource costs.

Finally, the economy is characterised by the following set of equations:

household behaviour	Maximize $U = \prod_{i=1}^{n} x_i^{\alpha_i}$	(1)
	Subject to $I = \sum_i p_i x_i$	(2)
production	$I^{*} = \sum_i c_i x_i$	(3)
industry pricing behaviour	$p_i = \lambda_i c_i$ for $i=1 \ldots n-1$	(4)
	$p_n = c_n = 1$	(5)
equilibrium relation	$\sum c_i x_i^o = I^{*}$	(6)

where U = utility level given $x_1 \ldots x_n$

x_i = quantity of commodity i (x_i^o = actual quantity of i)

I = income (I^o = actual income; I^{*} = income in perfect competitive equilibrium; I' = income after compensation)

$c_i (i=1 \ldots n)$ = resource cost of commodity i

$p_i (i=1 \ldots n)$ = price of commodity i (p_i^o = actual price of i)

$\lambda_i = p_i / c_i$.

If prices are $(p_1^o, p_2^o, \ldots p_{n-1}^o, 1)$ of which some are different from their respective resource costs $(c_1, c_2, \ldots c_{n-1}, 1)$ there is a monopoly welfare loss. With the price vector p^o the consumers will have an utility level U_o, whereas they could attain U_1 with p=c. The compensating variation is the appropriate measurement of this welfare loss i.e. the additional income required to put consumers on the utility level U_1 if the price vector were p^o.

In order to calculate this compensating variation one proceeds as follows (for the complete proof, see Appendix II).

Actual production-consumption $x^o=(x_1^o \ldots x_n^o)$ is found from

$$\text{maximize} \quad U = \prod_i x_i^{\alpha_i} \qquad (7)$$

$$\text{subject to} \quad I^o = \sum_i p_i^o x_i \qquad (8)$$

One x^o is found the value of $I^{\#}$ can be estimated, according to (6). Next, the maximal utility level U_1 associated with a price vector p=c and income $I^{\#}$ is estimated. One solves

$$\text{maximize} \quad U = \prod_i x_i^{\alpha} \qquad (9)$$

$$\text{subject to} \quad I^{\#} = \sum_i c_i x_i \qquad (10)$$

for x and compute U_1.

The minimal income, I_1', that the household requires, provided the actual monopolistic pricing practices, to attain the utility level U_1, is found from

$$\text{minimize} \quad I = \sum_i p_i^o x_i \qquad (11)$$

$$\text{subject to} \quad U_1 = \prod_i x_i^{\alpha_i} \qquad (12)$$

Compensating variation is derived from $I_1 - I_o$.

In Appendix II it is proven that compensating variation in terms of initial income, equals

$$cv = \left(\prod_{i=1}^{n} \lambda_i^{\mu_i}\right)\left(\sum_{i=1}^{n} \mu_i \lambda_i^{-1}\right) - 1 \qquad (13)$$

where μ_i = share of expenditures for commodity i in total income
λ_i = price-cost ratio for commodity i.

According to the Hotelling formula and provided the price elasticity is -1, the monopoly welfare loss equals

$$h = \frac{1}{2} \sum_{i=1}^{n} (\lambda_i - 1)^2 \mu_i \qquad (14)$$

Direct comparison of (13) and (14) does not indicate what the sign the difference has. However, most numerical examples show that (14) leads to larger estimates than (13).

E.g. in table III.a and III.b the estimates of compensating variation and the Harberger approximation are tabulated for a two-good economy and different values of the monopoly degree and its importance in the economy.

From this table one concludes that for the particular cases considered, the Harberger approximation overestimates the welfare gains. This result probably holds in general. A second conclusion is that the monopoly welfare losses (compensating variation) do not necessarily increase with the relative importance of the monopolistic sector. Third, the effect of aggregation is apparent if one considert also table III.c. For each aggregate degree of monopoly (or average price-cost ratio) several welfare loss estimates are possible.

Table III.a. Compensating variation in percent of income for a two-good economy (1)

μ_2 \ λ_2	1.05	1.10	1.15	1.20	1.25
.8	.0192	.0741	.1608	.2760	.4170
.6	.0287	.1097	.2367	.4041	.6071
.4	.0285	.1083	.2323	.3944	.5893
.2	.0189	.0713	.1520	.2566	.3814

Table III.b. Harberger approximation in percent of income for a two-good economy (1)

μ_2 \ λ_2	1.05	1.10	1.15	1.20	1.25
.8	.1000	.4000	.9000	1.6000	2.5000
.6	.0750	.3000	.6750	1.2000	1.8750
.4	.0500	.2000	.4500	.8000	1.2500
.2	.0250	.1000	.2250	.4000	.6250

(1) λ_2 = price-cost ratio for the monopolistic commodity
μ_2 = expenditive share for the monopolistic commodity
$\lambda_1 = 1$
$\mu_1 = 1 - \mu_2$

Table III.c. Average price-cost ratio for a two-good economy (1)

μ_2 \ λ_2	1.05	1.10	1.15	1.20	1.25
.8	1.04	1.08	1.12	1.16	1.20
.6	1.03	1.06	1.09	1.12	1.15
.4	1.02	1.04	1.06	1.08	1.10
.2	1.01	1.02	1.03	1.04	1.05

From the previous dicussion, I conclude that if one adopts Harberger's empirical assumptions but corrects his method of measurement by using compensating variation, one obtains lower monopoly welfare loss estimates. This does not imply that Harberger's general conclusion holds i.e. that monopoly welfare losses are inconsequential. Several of his empirical assumptions are subject to serious criticism. Their effect on the final estimate is an unsettled question.

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(1) See footnote on p.23.

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APPENDIX I. DERIVATION OF WELFARE GAIN FORMULAS FOR COBB-DOUGLAS SPECIFICATION OF UTILITY

Assume the consumer has a Cobb-Douglas type utility function

$$U = y^{\beta} \prod_{i=1}^{n} x_i^{\alpha_i} \qquad (I.1)$$

where U = utility level

y = quantity of numeraire commodity

x_i = quantity of commodity i and i=1...n

β, α_i = constants for which $0<\frac{\alpha}{\beta}i<1$

If the prices of commodity i is p_i and the consumer's income I the budget equation is

$$I = y + \sum_{i=1}^{n} p_i x_i \qquad (I.2)$$

The first-order conditions for utility maximation subject to the budget constraint are

$$\frac{\partial \Lambda}{\partial x_k} = \alpha_k \, x_k^{\alpha_k - 1} \, y^{\beta} \prod_{\substack{i=1 \\ i \neq k}}^{n} x_i^{\alpha_i} - \lambda p_k = 0 \quad \text{for } k=1...n \qquad (I.3)$$

$$\frac{\partial \Lambda}{\partial y} = \beta y^{\beta - 1} \prod_{i=1}^{n} x_i^{\alpha_i} - \lambda = 0 \qquad (I.4)$$

$$\frac{\partial \Lambda}{\partial \lambda} = I - y - \sum_{i=1}^{n} p_i x_i = 0 \qquad (I.5)$$

where Λ is the Lagrange-function and λ the Lagrange-multiplier.

From (I.3), (I.4) and (I.5) the Marshallian demand curves are derived. From (I.3) and (I.4) the multiplier λ is eliminated. The resulting equation and (I.5) lead to a solution for x_k and y.

The Marshallian demand curves are

$$y = \left(\frac{\beta}{\beta + \sum_{i=1}^{n} \alpha_i}\right) I = \mu I \qquad (I.7)$$

and

$$x_i = \left(\frac{\alpha_i}{\beta + \sum_{i=1}^{n} \alpha_i}\right) I \, p_i^{-1} \qquad \text{for } i=1...n \qquad (I.8)$$

It is clear that for all curves (own) price elasticities and income elasticities are unit; cross-price elasticities are zero.

If e.g. for commodity k the price falls from p_k^o to p_k' with 100 λ percent $[\lambda=(p_k^o-p_k')p_k']$, Marshallian consumer surplus is defined as the money gain for the first x_k^o units and the excess of willingness to pay over what is paid for the next $(x_k'-x_k^o)$ units, or

$$MS = (p_k^o-p_k')x_k^o + \left[\int_{x_k^o}^{x_k'} p_k \, dx - p_k'(x_k' - x_k^o)\right] \qquad (I.9)$$

From (I.8) one knows that

$$p_k = \left(\frac{\alpha_k}{\beta + \sum_{i=1}^{n} \alpha_i}\right) I \, x_k^{-1} \qquad (I.10)$$

$$x_k^o = \left(\frac{\alpha_k}{\beta + \sum_{i=1}^{n} \alpha_i}\right) I \, (p_k^o)^{-1} \qquad (I.11)$$

$$x_k' = \left(\frac{\alpha_k}{\beta + \sum_{i=1}^{n} \alpha_i}\right) I \, (p_k')^{-1} \qquad (I.12)$$

Solving (I.9) and using (I.10), substituting quantities by using (I.11) and (I.12) leads to

$$MS = I\left(\frac{\alpha_k}{\beta + \sum_{i=1}^{n} \alpha_i}\right) \log_e\left(\frac{p_k^o}{p_k'}\right) \qquad (I.13)$$

or

$$MS = I\left(\frac{\alpha_k}{\beta + \sum_{i=1}^{n} \alpha_i}\right) \log_e(1+\lambda) \qquad (I.14)$$

Marshallian surplus in terms of income is found as

$$ms = \left(\frac{\alpha_k}{\beta + \sum_{i=1}^{n} \alpha_i}\right) \log_e(1+\lambda) \qquad (I.15)$$

Relation (2) on p. 15 for the two commodity case can be directly derived from (I.15).

Hotelling's approximation is defined as

$$H = \tfrac{1}{2}(p_k^o - p_k')(x_k^o + x_k') \qquad (I.15.a)$$

Substituting (I.11) and (I.12) in (I.15.a) leads to

$$H = \frac{I}{2}\left(\frac{\alpha_k}{\beta + \sum_{i=1}^{n} \alpha_i}\right)(p_k^o - p_k')\left(\frac{1}{p_k'} + \frac{1}{p_k'}\right) \qquad (I.15.b)$$

In terms of the relative price change, this becomes

$$H = \frac{I}{2}\left(\frac{\alpha_k}{\beta + \sum_{i=1}^{n} \alpha_i}\right) \cdot \frac{\lambda(2+\lambda)}{(1+\lambda)} \qquad (I.15.c)$$

or

$$h = \frac{H}{I} = \frac{1}{2}\left(\frac{\alpha_k}{\beta + \sum_{i=1}^{n} \alpha_i}\right) \cdot \frac{\lambda(2+\lambda)}{(1+\lambda)} \qquad (I.15.d)$$

For the two commodity case equation (1) on p. 15 is easily derived from (I.15.d).

Hicks' compensated variation is found by the difference in the minimal income I" that the consumer should have to attain an utility U with a price level for x_k at p_k^o, and his initial income I'. All other prices are held constant, or $p_i = p_i^*$ for all $i \neq k$.

The income I" is found from the following optimization.

$$\text{minimize} \quad I'' = y + \sum_{\substack{i=1 \\ i \neq k}}^{n} p_i^* x_i + p_k^o x_k \qquad (I.16)$$

$$\text{subject to} \quad U_1 = y^\beta \prod_{i=1}^{n} x_i^{\alpha_i} \qquad (I.17)$$

The utility level U_1 corresponds with the utility derived from consumption at

- prices for $i \neq k$ at $p_i = p_i^{\#}$
- the price for k at $p_k = p_k'$
- income at I

Hence the level of U_1 in terms of prices and income is given by

$$U_1 = \left(\frac{I}{\beta+\Sigma\alpha_i}\right)^{(\beta+\Sigma\alpha_i)} \left[\bar{\alpha}_k (p_k')^{-1}\right]^{\alpha_k} \beta^{\beta} \prod_{\substack{i=1 \\ i \neq k}}^{n} \left[\bar{\alpha}_i (p_i^{\#})^{-1}\right]^{\alpha_i} \qquad \text{(I.18)}$$

First-order conditions for (I.16) subject to (I.17) and with Λ'' denoting the Lagrange function and λ'' the multiplier, are

$$\frac{\partial \Lambda''}{\partial x_j} = p_j^{\#} - \lambda'' \alpha_j x_j^{\alpha_j - 1} y^{\beta} \prod_{\substack{i=1 \\ i \neq j}}^{n} x_i^{\alpha_i} = 0 \quad \text{for all } j \neq k \qquad \text{(I.19)}$$

$$\frac{\partial \Lambda''}{\partial x_k} = p_k^{o} - \lambda'' \alpha_k x_k^{\alpha_k - 1} y^{\beta} \prod_{\substack{i=1 \\ i \neq k}}^{n} x_i^{\alpha_i} = 0 \qquad \text{(I.20)}$$

$$\frac{\partial \Lambda''}{\partial y} = 1 - \lambda'' \beta y^{\beta-1} \prod_{i=1}^{n} x_i^{\alpha_i} = 0 \qquad \text{(I.21)}$$

$$\frac{\partial \Lambda''}{\partial \lambda''} = U_1 - y^{\beta} \prod_{i=1}^{n} x_i^{\alpha_i} = 0 \qquad \text{(I.22)}$$

Second-order conditions are fullfilled if the bordered Hessian of Λ'' is negative (1).

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(1) It can be shown that if $0<\alpha_i<1$ and $0<\beta<1$ this conditions hold .

Solving equations (I.19), (I.20 and (I.21) one obtains the following Hicksian compensated demands

$$y = U_1^{1/(\beta+\Sigma\alpha_i)} \beta^{\Sigma\alpha_i/(\beta+\Sigma\alpha_i)} \left(\frac{\alpha_k}{p_k^o}\right)^{-\alpha_k/(\beta+\Sigma\alpha_i)} \prod_{\substack{i=1 \\ i\neq k}}^{n} \left(\frac{\alpha_i}{p_i^{*}}\right)^{-\alpha_i/(\beta+\Sigma\alpha_i)} \qquad (I.23)$$

$$x_i = \left(\frac{\alpha_i}{\beta}\right) \frac{y}{p_i^{*}} \quad \text{for all } i\neq k \qquad (I.24)$$

$$x_k = \left(\frac{\alpha_k}{\beta}\right) \frac{y}{p_k^o} \qquad (I.25)$$

Substituting (I.18) in (I.23), (I.24) and (I.25) one obtains

$$y = \left(\frac{\beta}{\beta+\Sigma\alpha_i}\right) I \cdot \left(\frac{p_k^o}{p_k'}\right)^{\alpha_k/(\beta+\Sigma\alpha_i)} \qquad (I.26)$$

$$x_i = \left(\frac{\alpha_i}{p_i^{*}}\right)\left(\frac{1}{\beta+\Sigma\alpha_i}\right) I \cdot \left(\frac{p_k^o}{p_k'}\right)^{\alpha_k/(\beta+\Sigma\alpha_i)} \qquad (I.27)$$

$$x_k = \left(\frac{\alpha_k}{q_k^o}\right)\left(\frac{1}{\beta+\Sigma\alpha_i}\right) I \cdot \left(\frac{p_k^o}{p_k'}\right)^{\alpha_k/(\beta+\Sigma\alpha_i)} \qquad (I.28)$$

The compensated income can be computed by substituting (I.26), (I.27) and (I.28) in (I.16) which leads to

$$I'' = I\left(\frac{p_k^o}{p_k'}\right)^{\alpha_k/(\beta+\Sigma\alpha_i)} \qquad (I.29)$$

The compensated variation is $I''-I$ which leads to

$$CV = I \left[\left(\frac{p_k^o}{p_k'}\right)^{\alpha_k/(\beta+\Sigma\alpha_i)} - 1 \right] \qquad (I.30)$$

or

$$CV = I \left[(1+\lambda)^{\alpha_k/(\beta+\Sigma\alpha_i)} - 1 \right] \qquad (I.31)$$

In terms of initial income this results in

$$cv = (1+\lambda)^{\alpha_k/(\beta+\Sigma\alpha_i)} - 1 \qquad (I.32)$$

The formula (3) used for the two commodity case is easily derived.

Determining equivalent variation is similar to the procedure for compensating variation. The outcome of this calculus is

$$EV = I \left[1 - \left(\frac{p_k'}{p_k^o}\right)^{\alpha_k/(\beta+\Sigma\alpha_i)} \right] \qquad (I.33)$$

or

$$EV = I \left[1 - \left(\frac{1}{1+\lambda}\right)^{\alpha_k/(\beta+\Sigma\alpha_i)} \right] \qquad (I.34)$$

and in terms of initial income

$$ev = 1 - \left(\frac{1}{1+\lambda}\right)^{\alpha_k/(\beta+\Sigma\alpha_i)} \qquad (I.35)$$

from which (4) is derived.

APPENDIX II. MONOPOLY WELFARE LOSS

First, the actual production-consumption vector x_i^o is computed. One solves the following maximization problem

$$\text{maximize} \quad U = \prod_i x_i^{\alpha_i} \qquad \text{(II.1)}$$

$$\text{subject to} \quad I^o = \sum_i p_i^o x_i \qquad \text{(II.2)}$$

commodity n is the numeraire or $p_n^o = c_n = 1$.

The Lagrangian is

$$\Lambda = \prod_i x_i^{\alpha_i} + \lambda(I^o - \sum_i p_i^o x_i) \qquad \text{(II.3)}$$

First order conditions are

$$\frac{\partial \Lambda}{\partial x_i} = \alpha_i x_i^{\alpha_i - 1} \prod_{\substack{j=1 \\ j \neq i}}^{n} x_j^{\alpha_j} - \lambda p_i^o = 0 \quad \text{for } i=1\ldots n-1 \qquad \text{(II.4)}$$

$$\frac{\partial \Lambda}{\partial x_n} = \alpha_n x_n^{\alpha_n - 1} \prod_{\substack{j=1 \\ j \neq i}}^{n} x_j^{\alpha_j} - \lambda = 0 \qquad \text{(II.5)}$$

$$\frac{\partial \Lambda}{\partial \lambda} = I^o - \sum_{i=1}^{n} p_i^o x_i = 0 \qquad \text{(II.6)}$$

Solving this set of equatios leads to

$$x_i^o = \left(\frac{\alpha_i}{\sum_i \alpha_i}\right) \cdot \left(\frac{I^o}{p_i^o}\right) \quad \text{for } i=1\ldots n \qquad \text{(II.7)}$$

As $$\left(\frac{\alpha_i}{\sum_i \alpha_i}\right) = \frac{p_i^\circ x_i^\circ}{I^\circ} = \mu_i \qquad \text{(II.8)}$$

with μ_i the income share of commodity holds (II.7) can be written as

$$x_i^\circ = \mu_i \frac{I_o}{p_i^\circ} \qquad \text{(II.9)}$$

The value of I^* is estimated from the equilibrium relation (6) or

$$I^* = \sum c_i \; x_i^\circ \qquad \text{(II.10)}$$

which leads to

$$I^* = I^\circ \sum_{i=1}^{n} \left(\frac{\mu_i}{\lambda_i}\right) \qquad \text{(II.11)}$$

The utility level associated with a perfect competitive equilibrium is determined from

$$\text{maximize} \quad U = \prod_{i=1}^{n} x_i^{\alpha_i} \qquad \text{(II.12)}$$

$$\text{subject to} \quad I^* = \sum_{i=1}^{n} c_i x_i \qquad \text{(II.13)}$$

The Lagrangian is

$$\Lambda' = \prod_{i=1}^{n} x_i^{\alpha_i} + \lambda'(I^* - \sum_{i=1}^{n} c_i x_i) \qquad \text{(II.14)}$$

First order conditions are

$$\frac{\partial \Lambda'}{\partial x_i} = \alpha_i x_i^{\alpha_i - 1} \prod_{\substack{j=1 \\ j \neq i}}^{n} x_j^{\alpha_j} - \lambda' c_i = 0 \quad \text{for } i=1...n-1 \tag{II.15}$$

$$\frac{\partial \Lambda'}{\partial x_n} = \alpha_n x_n^{\alpha_n - 1} \prod_{\substack{j=1 \\ j \neq n}}^{n} x_j^{\alpha_i} - \lambda' = 0 \tag{II.16}$$

$$\frac{\partial \Lambda'}{\partial \lambda'} = I^{*} - \sum_{i=1}^{n} c_i x_i = 0 \tag{II.17}$$

This set of equation leads to the following values of x's

$$x_i = \left(\frac{\alpha_i}{\sum_i \alpha_i}\right) \frac{I^{*}}{c_i} \quad \text{for } i=1...n \tag{II.18}$$

$$x_i = \mu_i \frac{I^{*}}{c_i} \tag{II.19}$$

The utility level U_1 associated with those values is formed by substituting (II.19) in (II.12) or

$$U_1 = \prod_{i=1}^{n} \left(\mu_i \frac{I^{*}}{c_i}\right)^{\alpha_i} \tag{II.20}$$

or

$$U_1 = (I^{*})^{\Sigma \alpha_i} \prod_{i=1}^{n} \left(\frac{\mu_i}{c_i}\right)^{\alpha_i} \tag{II.20'}$$

The minimum income consumers require to obtain this utility level, with the actual set of prices is derived from

$$\text{minimize} \quad I = \sum_{i=1}^{n} p_i^o \, x_i \tag{II.21}$$

$$\text{subject to} \quad U_1 = \prod_{i=1}^{n} x_i^{\alpha_i} \tag{II.23}$$

The Lagrangian function is

$$\Lambda'' = \sum_{i=1}^{n} p_i^o x_i + \lambda''(U_1 - \prod_{i=1}^{n} x_i^{\alpha_i}) \tag{II.24}$$

First-order conditions lead to

$$\frac{\partial \Lambda''}{\partial x_i} = p_i^o - \lambda'' \alpha_i x_i^{\alpha_i - 1} \prod_{\substack{j=1 \\ i \neq j}}^{n} x_j^{\alpha_j} = 0 \quad \text{for } i=1 \ldots n-1 \tag{II.25}$$

$$\frac{\partial \Lambda''}{\partial x_n} = 1 - \lambda'' \alpha_n x_n^{\alpha_n - 1} \prod_{\substack{j=1 \\ j \neq n}}^{n} x_j^{\alpha_j} = 0 \tag{II.26}$$

$$\frac{\partial \Lambda''}{\partial \lambda''} = U_1 - \prod_{i=1}^{n} x_i^{\alpha_i} = 0 \tag{II.27}$$

From this set of equations the x's are solved. This results in

$$x_i = \alpha_i p_i^{o\,-1} U_1^{1/\Sigma\alpha_i} \prod_{i=1}^{n} \alpha_i^{-\mu_i} p_i^{\mu_i} \quad \text{for } i=1 \ldots n \tag{II.28}$$

From (II.28) one derives

$$p_i x_i = \alpha_i U_1^{1/\Sigma\alpha_i} \prod_{i=1}^{n} \alpha_i^{-\mu_i} p_i^{\mu_i} \quad \text{for } i=1\ldots n \tag{II.29}$$

Summation of (II.29) over all i's leads to

$$I' = U^{1/\Sigma\alpha_i} (\Sigma\alpha_i) \prod_{i=1}^{n} \alpha_i^{-\mu_i} p_i^{\mu_i} \tag{II.30}$$

Substituting (II.20') in (II.30) results in

$$I' = I^{*} (\Sigma\alpha_i) \prod_{i=1}^{n} (\lambda_i^{\mu_i} \mu_i^{\mu_i} \alpha_i^{-\mu_i}) \tag{II.31}$$

and since

$$\mu_i^{\mu_i} \alpha_i^{-\mu_i} = 1/\Sigma\alpha_i \tag{II.32}$$

(II.31) reduces to

$$I' = I^{*} \prod_{i=1}^{n} (\lambda_i^{\mu_i}) \tag{II.33}$$

Substituting (II.13) in (II.33) results in

$$I' = I^{\circ} \left(\sum_{i=1}^{n} \frac{\mu_i}{\lambda_i}\right)\left(\prod_{i=1}^{n} \lambda_i^{\mu_i}\right) \tag{II.34}$$

Compensated variation in terms of I° equals

$$cv = \frac{I'-I^{\circ}}{I} = \left(\sum_{i=1}^{n} \frac{\mu_i}{\lambda_i}\right)\left(\prod_{i=1}^{n} \lambda_i^{\mu_i}\right) - 1$$