SESO
UNIVERSITAIRE FACULTEITEN ST.-IGNATIUS TE ANTWERPEN
STUDIECENTRUM VOOR ECONOMISCH EN SOCIAAL ONDERZOEK

RAILWAY COSTING PROCEDURES

The capital cost of motive power

Evrard CLAESSENS
werknota 7538

februari 1975

Universitaire Faculteiten St.-Ignatius
Prinsstraat 13 - 2000 Antwerpen

D/1975/1169/11

CONTENTS

This working-paper forms part of a research-program about "Theory and Empiry of Railways Planification".

I wish to express my special acknowledgement
to Prof. E. VAN BROEKHOVEN (D.Phil.),
for supervising and commenting this analysis,
and for guiding it within the running research-program.

I am also indebted to ir. A. SOETE (operations manager of Belgian railways) for advisary help with respect to the actual running practice of the Belgian railways, and to ir. W. HEYSELBERGHS for valuable remarks.

Introduction

The railway running practice seems to be more complicated than the proceedings of shipping and road haulage. Most of this statement is true, though operations itselves do not differ strongly. The very difference between railroads and other modes lies in the wide variation of operating procedures, which are simultaneously combined, and which are to be accounted for explicitly.

An outstanding example refers to the "unit of haulage" (the train) and the "load-unit" (the car), which coincide quite perfectly in other modes. This leads us to analyse the operations of locomotives separately, as far as their use and costs are concerned.

Thereby the railway business practice is known to be so ramified that now an "output-unit" concerns some tip-up seat in a commuter-coach, now an iron-ore car within a regular traffic of several millions of tons. Hence, accountants'practice might be complex and thus resort to traditional decision rules. The economic discussion about incremental production however asks for accurate cost calculations, in order to know the right degree of escapability. This requires simulating railway operations, as far as they cause a special cost behaviour. An operation which is of particular interest in the economic calculus is the tractive action of a train (a).

Locomotives are perhaps the "most primary" capital inputs of railway technology. They are neither restricted to fixed routes, nor specialized to the haulage of particular goods (in the sense we commonly think of it with respect to vessels and trucks). As a part of the total capital cost, substitutions with labour can be analysed. Within the capital stock, questions can ask

(a) THE BRITISH TRANSPORT COMMISSION, o.c. (19), pp. 17,18.

for an answer, whether to invest in better infrastructure facilities, in more powerful engines, or in both.

This paper aims to synthesize the technical features, being involved in economic decision making about "motive capital". The analysis will express the findings in terms of direct costs, which can be compared easily by other cost-components. The presented analysis is thus a first step in the tedious systems-analysis of railway practice which will lead to a accurate but simple expression of relevant opportunity costs of the main inputs. The latter topic is essential to be known exactly in calculations with respect to the theory of the firm or in network optimization analysis.

A preliminary choice was to be made about the methodological approach (a). Statistical cost analysis of railway operations, can make itself unusual for getting a sufficiently accurate knowledge of operations cost. It doesn't conduce to a disaggregate level of, say, network links, by present conditions of statistical data information. Moreover, the time-series analysis, mostly used in this respect, often shrouds the short run effects we must know exactly, in order to simulate alternative strategies (b).

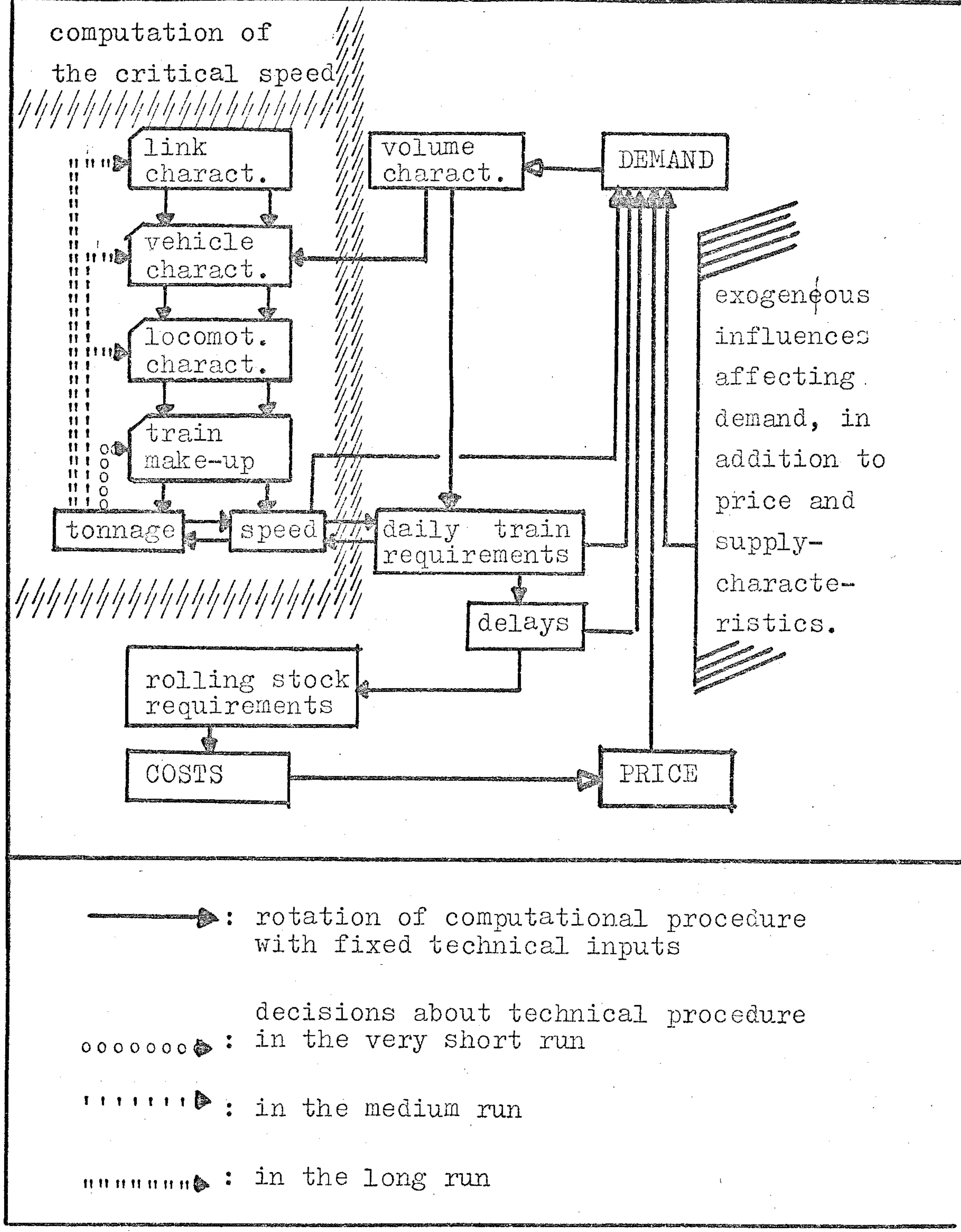
The analysis will start by engineering costing procedures, which are performed numerically. Fixed inputs are essentially the railway constants, to be changed in the long run. The output of the here presented calculations will be expressed in variables which concern the operations decisions of the short run. The results will afterwards be expressed analytically, for easy use in general cost- calculus.

First, the general framework of this particular analysis wil be sketched.

(a) MEYER, J.R., o.c. (1), pp. 39, 41.

(b) CLAESSENS, E. & VAN BROEKHOVEN E., o.c. (20).

Figure 1.1. Computational sheme of demand and supply



1. The general simulation context

The general lay-out, most railway decision-making is based on, is commonly represented as a twofold analysis. Demand is mostly taken as fixed or generated exogenously, and forms as such an input of the supply analysis (a). Optimal supply is then calculated by models which express the main features of the technical performance of existing technologies. The inputs of the most efficient technical solutions are connected with factor prices, which leads to a final cost, with all its important components. About this cost information, the economist has to decide on the pricing policy and practice.

With respect to latter applications, the general sketch will be modified and extended a little to distinguish better between demand and supply (see Figure 1.1.).

Within the supply side the program inputs are ordered hierarchicly; the hierarchy corresponds to the period which is necessary to make decisions about changing engineering features, the program represents as inputs.

Simultaneous effects between demand and supply are included by seeing price and speed as influencing demand; the reciprocal relation between the daily train requirements or delay occurence and demand, is only concerned in the case of regular line-traffic.

Nevertheless these simultaneous effects are in the short run less important than for off-peak passenger traffic. In most problems of present discussion about transportation policy, the very art is confined to finding the minimum cost solution of hauling a given quantity of passengers or goods, defined by a forgoing (or provisional) exogenous demand generation.

(a) MEYER: o.c. (2), appendix B.

The top left area of Figure 1.1. synthesizes the physical part of the scheme, and can be thought also of interest in the purchase decision between railway engineers and the constructors of track, locomotives and rolling stock. The economist is only interested in this calculation as far as he needs to know the inputs of a given technology and the comparative costs of the technical alternatives to be thought out.

This paper is confined to the task of the economist, described above. Physical aspects are expressed by the relevant empirical findings of railway engineering. Corrections will be made to apply existing research to European railways. The result will consist of a simplified relation between basic inputs and their capital costs, which can be used afterwards in the calculations.

2. The technical rail performance model

This section is rather technical; it has the aim to show the technical relationships, tractive behaviour of locomotives are subject to, are basicly simple. Hence, a little amount of variables can be selected to be used in further cost-calculations. The procedure follows existing methodology, with a certain amount of reservations (a).

The rolling speed has a central position in the laws of motion, and determines as such the relation between technical inputs (about track, locomotives and rolling stock) and the train tonnage. Speed influences both the tractive effort of railway engines and the rolling resistance of the train, but in an opposite way. So, speed can be balanced at a maximum, where no acceleration is still possible, and where the constraints of adhesion are fulfilled.

2.1. Tractive effort

In Physics (b) work (W) is both defined as an amount of power (P), exerted during some time (t),

$$\boxed{W = P.t} \quad (1)$$

and also as some forward thrust (F) exerted over some distance (d) :

$$\boxed{W = F.d} \quad (2)$$

By defining speed as : $v = d/t$

we get $\boxed{P = (F.d) / t = F.v}$ (3)

or $\boxed{F = P / v}$ (4)

If abstraction is made of acceleration, the engine is then moving at a uniform speed.

(a) MEYER, o.c. (2), appendix E
(b) RESNICK, o.c. (3), p. 145.

The empirical shape of formula (4) is given by :

$$TE = \frac{248}{V} \sum_{L=1}^{NL} HP(L) \quad x \quad \left[(1.609)^{D2} . \left(\frac{1}{2}\right)^{D3} . \left(\frac{1}{0.7467}\right)^{D1} \right]$$

The right factor allows for measure conversion of variables (a):

if power (HP) is measured in	horsepower (HP)	; D1 = 0
	kiloWatt (kWP)	; D1 = 1
if speed (V) is measured in	miles/hour (m/h)	; D2 = 0
	kilometres (km/h)	; D2 = 1
if tractive effort is expressed in	pounds (lbs)	; D3 = 0
	in kilogram (kg)	; D3 = 1

By way of simplification, all inputs in the "mks-dimesion will by used according to dummy-values, equal to one.

The empirical constant "248" is but valid for European technology (b). The constant "308" proposed by MEYER (c) can be used for countries, where gauge and rolling stock profiles are of greater dimensions than the European system (d).

The "iso-power" curve is expressed by an hyperbole, where effort and speed act as substitutes. For lower speed-levels, the possible tractive effort would cause wheels are slipping; this feature is accounted for by an additional constraint.

(a) RESNICK, o.c. (3) appendix G.

(b) DEBERDT, o.c. (4) p. 3.

(c) MEYER, J.R., o.c. (2), p. 186.

(d) The "European system" is hereafter defined to be a 1.435 m-gauge, with the usual track profile of western continental Europe. Exceptions are countries with wider gauge as Russia, Finland or Spain; higher profiles are usual throughout America. The gauge and profile conditions determine energy losses by mechanical action, which can differ strongly.

2.2. Adhesive effort

Adhesion is a mechanical constraint which expresses the maximum tractive effort to be actually exerted by a locomotive, without wheels slipping. It can be defined to be the relation between the maximum tangent effort of the wheels, and the axle-load:

$$\text{adh. coeff.} = \frac{\text{tangential effort in kg}}{\text{axle-load} \quad \text{in ton}} \quad \text{kg/t} \quad = \mu$$

This unit is not a constant but depends particularly of the load, the speed and the weather circumstances (a). Empirical tests are numerous and different about its measure; values can fluctuate between 0.1 and 0.25 according to the state of the track (b).

We will use in this analysis the curve of "CURTIUS & KNIFFLER", which is a widely used standard in testing new engines (c):

$$\mu = ((7{,}500 \;/\; (V + 44)\;) + 161\;) \text{ kg/t}$$

The here presented formula is a mean which can increase or decrease by about 20 %, whether track is dry or wet (b). Therefore we withhold the equation :

$$EAD(L) = 161 + \left(\frac{7{,}500}{(V+44).1.609^{(1-D2)}}\right)\left(\frac{1}{0.906}\right)^{(1-D4)} . \; 2^{(1-D3)} \; . \; WL(L) \; . \; AXCF \; . \; AWCF$$

(a) VERBEECK H., o.c. (5), pp. 625-663.

(b) DEBERDT M., o.c. (4), p. 8.

(c) KÖRBER J., o.c. (6), pp. 63-72.

where :

- EAD(L): adhesive effort of locomotive "L" in pounds or kg (cfr.D3).
- WL (L): weight of locomotive "L" in US-tons, if D4=0
  mks-tons, if D4=1
- AXCF : adhesive axle-coefficient (see also section 2.6);
  = 1 if full-adhesion locomotives are used
  $<$ 1 cfr. the percentage of driven axles
- AWCF : adhesive weather coefficient; which equals "1", for
  normal track-conditions: $0.8 \leq \text{AWCF} \leq 1.2$

The total adhesive effort of a set of NL locomotives is :

$$\boxed{\text{TAD} = \sum_{L=1}^{NL} \text{EAD(L)}}$$

For each set of locomotives, the values of TAD and TE will be evaluated for different speeds. The lower value expresses the tractive effort which is actually effective, and is to be used in the next calculations. An example of the difference between mechanical and adhesive effort is given in Figure 1.2.

2.3. Net tractive effort

(o.c. (2))

When quoted forces are decreased by the rolling resistance of the locomotive, the "net tractive effort on the couple" is obtained. For one locomotive (L), the level tangent rolling resistance equals :

$$\boxed{\begin{aligned} \text{SSR(L)} = \Bigg(& \frac{29\ \text{AXL(L)}.(0.906)^{D4}}{\text{WL(L)}} + \frac{0.03\ V}{(1.609)^{D2}} + \ldots \\ & + \frac{0.012\ A(L)\ V^2\ (0.906)^{D4}}{(24.D6+5.D7)(0.0925)^{D5}(1.609)^{2D2}.\text{WL(L)}} \Bigg) \frac{1.3(0.906)^{D4}}{(2)^{D3}} \end{aligned}}$$

with :

- A(L) : cross-section area of the locomotive in sq.feet; D5=0
  " metr.; D5=1

  For most European locomotives A(L)= 10 m^2.
- AXL(L): number of driving axles of each locomotive.
- SSR(L): level tangent rolling resistance in lbs/US.ton;D3=D4=0
  kg/mks-ton;D3=D4=1

  This value differs from the first (D6=1) and the second (D7=1) locomotive; in iterations we use:
  D7= absolute value (1-D6).

The total rolling resistance for all locomotives in the train is:

$$\boxed{TSR = \sum_{L=1}^{NL} SSR(L)}$$

The total net tractive effort on the couple is then computed by:
- selecting for each speed and train make-up, the lower value between tractive effort (TE) and adhesion effort (TAD),
- subtracting from the retained value the rolling resistance (TSR), combined with track-condition and locomotive-weight:

$$\boxed{SNET = TE - \sum_{L=1}^{NL} WL(L) \left(TSR + (20.G.NL)(0.906)^{D4}(0.5)^{D3} \right) \frac{1}{(0.906)^{D4}}}$$

with :

SNET : total net tractive effort on the couple of all locomotives,
G : grade of the track in percent.

By accounting separately for each locomotive, we allow a train to be pulled by a set of different locomotives. This is important when on a particular route, only some links need additional tractive help (e.g. because of an extreme gradient).

2.4. The rolling resistance of the train (o.c.(2))

The typology of cars and the classification of goods or demand-segmentation will be set coincident. This is realistic in present technology and will map clearly the cost calculations.

The total weight of a particular car of class K is defined as :

$$\boxed{WGH(K) = \left\{WG(K) + (\ P(K)\ .\ FCL(K)\ )\right\}\ (1/0.906)^{D4}}$$

with :

- WG(K) : weight of an empty car (in tons cfr. D4)
- P(K) : carrying capacity (in tons cfr. D4)
- FCL(K): load factor of the car ($0 \leq FCL \leq 1$)

This weight causes a rolling resistance of :

$$\boxed{TR(K) = \left[116 + \frac{0.045\ V^2}{(1.609)^{2D2}} + WGH(K)\left(1.3 + \frac{0.045\ V}{(1.609)^{D2}} + 20G\right)\right]\left(\frac{1}{2}\right)^{D3}}$$

Summing up for all cars over the different classes, we get the total resistance of the rolling train :

$$\boxed{TTR = \sum_{K=1}^{NK} TR(K)\ .\ C(K)}$$

with :

- C(K) : number of cars in the train, of the same class.
- NK : number of different classes.

2.5. Critical Speed

When comparing the curves of net tractive effort (SNET), and the total resistance of the train (TTR) at increasing speeds, the first resistance will rise and effort will fall for given gradient. The speed which causes TTR to equal SNET, is called the critical speed; constants in this process are the gradient, the train make-up and the power of engines.

If the maximum gradient on a link is taken (G) the critical speed value is also an absolute minimum. By taking the average gradient (GA) the critical speed expresses a mean, which is technically possible; it has however to be corrected for other "organizational" speed-constraints on the link, to obtain the "commercial speed".

2.6. Reduction of the number of input-combinations

The great number of inputs requires long and tedious iterations, when the choice of a railway engineer must be simulated, at least approximately. Additionally, with respect to further cost calculations, some programmatic abstract definition must be settled to enclose any common type of locomotive.

For characteristics of links and vehicles, there is no reason to reduce the number of possible input combinations. Locomotives characteristics will be the object of refined iterations. Therefore the various types need to be reduced to a minimum of technical features.

A class of engine can be characterized a-priori by :

- the traction mode expressed by : DD = 1 for diesels and
  DD = 0 for electric engines.
- the current system involving : DA = 1 for alternating, and
  DA = 0 for direct current.

- the use of motortrains inducing : DM = 1
- diesel shunters and railcars will be indicated (in addition to the diesel-dummy and DM) by : DT = 1

A type of engine has yet been defined more continuously by :

- the power of the locomotive (hereafter in kiloWatt)
- the weight (hereafter in mks-tons), and less continuous
- the number of axles, and how many of them are driven.

The dummies generate no iteration problem, because of their 0/1-value. The continuous variables can easily be assumed, to be technically related. The power level will influence the weight, and weight will require a given number of axles, to avoid excessive axle-load (U.I.C.-standardization defines some 25 mks-tons).

This assumption has been tested for fifty locomotives throughout Europe. Attention has been taken to collect observations of the same gauge and vehicle profile (a).

The most realistic regression fit is :

$$\boxed{\begin{array}{l} WL(L) = 41.16 + \underset{(8.12)}{0.014\ PkW} + \underset{(6.91)}{0.018\ PkW\ DD} \end{array}}$$

with the values of the t-statistic, mentioned between brackets. The number of axles can likely be related to locomotive weight :

$$\boxed{AXL(L) = 0.44 + 0.05\ WL(L) + 0{,}64}$$

(a) e.g.: engines of Russia, Finland, Spain and America were excluded from the sample, because of excessive weight per power-level. As remarked earlier, this fact is caused by other track profiles.
See also THOMA, <u>o.c.</u> (7), p. 271.

For example, a diesel of 2,500 kW, has an expected weight of ca. 121 tons, which require 6.068 axles. This result is realistic. However, because of the discontinuous values of AXL, we will use in iterations a more simple rule :

locomotives		motortrains
WL(L)	AXL(L)	if DM = 1, AXL(L) = 4
-50	3	
50-100	4	
100 +	6	

Note on adhesion weight and driving axles

Up to now we didn't define clearly when to use driving axles and adhesion weight; the relations proposed above, concern only locomotive weight, and the total number of axles.

Adhesion weight is the part of locomotive weight, supported by the driving axles. If all axles are driven, the locomotive is "full-adhesive" and both concepts coincide. In most cases we are dealing with full-adhesion locomotives. An exception to be noticed is the case of motortrains. In Belgian railroads only one axle of the bogie is driven. This fact is accounted for by setting AXL(L) = 2, and by deviding the locomotive weight in the adhesion relation by (see section 2.2.) AXCF (adhesive axle coefficient), where :

	DM = 1	DM = 0
AXCF	0.5	1

In the calculations, the critical speed will not be sensitive to small changes of weight, if speed lies beyond the adhesion constraint. Some railways account for this feature, by ballasting their engines when carrying heavy train-loads. So far we know this is not the case with Western European companies. The estimated equations may so be used without interfering the results.

2.7. Administrative speed limitations

The previous calculations indicate which class and type of locomotive is to be used to attain a "possible" level of maximum and mean speed; the question can likewise be made in terms of link characteristics or train make-up.

In addition to these computations, the final applications must thereby take into account a serie of "administrative speed limitations". The term "administrative" doesn't deny the technical origin of them, but makes only clear that in the short run, these limitations are accepted as such, and known for all concerned links. In the long run the technical aspects of the problem can be analysed and then related to building cost of an incremental change in infrastructure facilities.

An outstanding example of the mentioned limitations are speed constraints, due to excessive curvatives.

Curves in the track alignment cause speed limitations because of increased rolling resistance, given by the formula of Röckl(a):

$$\boxed{w = (k \; / \; (R-k'))} \quad \text{with: } 650 \leq k \leq 500, \quad 55 \leq k' \leq 30$$

depending on the radius (R) and the gauge.
Another speed limitation is necessary to hold the gravity centre between the rails, given the radius and the inclination of the track. A common norm of the railways is in this case (b) :

$$\boxed{V = \sqrt{\frac{R}{11.8}\,(c+130)}}$$

with: V : maximum speed
c : inclination in mm.
R : radius in m.

(a) DEBERDT M., (4), p. 5.
(b) THOMA A., <u>o.c.</u> (7), pp. 170-175.

In the short run these formulas don't enter the discussion, unless by constraints imposed by the railway administration. In the long run their values are to be related to infrastructure costs, which can vary strongly according to a lot of topographical features.

Other speed constraints, which are included in the here discussed subject, concern the technology of points, provisions made around stations and level crossings. They are also given in the short run and changes are to be decided on on the long run.

2.8. An example of application

The previously developed formulas can be combined in various applications. The dependent variables, which are to be iterated, will be choosen accordingly the technical decisions, which enter the discussion.

The train make-up can be assumed to be rather constant; at most iterations will be made for changes in make-up by blocks of say, 5 cars at once. This could be the case in regular merchandise traffic, where some stochastic fluctuations may occur. Passenger traffic on long distance-relations is likely affected by seasonal peaks; the accurate train composition is especially important for luxury international trains (like the TEE-connections) because international clearing is based upon the "driven seat distance"(a). For technical computations, the train composition is mostly a given input whether as a mean or a maximum.
Topics, which seem the subject of refined iterations, are the types of locomotives to be used (speed and power). To obtain a-insight into the tractive behaviour of locomotives, an exemplative program has been tested, which is here presented (see the listing on 2.9).

(a) CAIRE D., o.c., (8), p. 222.

Say, you are a railway engineer (or a researcher imitating his technical decisions); you know there is a given demand on a given relation where track is already available. This demand occurs at some precise moment, or it is to be hauled in a given time period; only in the latter case, some train make-ups can be tested out.

If the demand is expected to be regular in a next future, you will think seriously about the purchase of additional locomotives, whether in connection with a reallocation of existing engines throughout over the network, or not. For given track and vehicle conditions, you wish to get informed about the values of critical speed, for various levels of power. In the case of "common types of locomotives" the weight/power-relationship (section 2.6.) is used. If a specific locomotive is analysed, his characteristics are used as input, and the iteration by changing power values will be dropped.

The program is listed in section 2.9.; the output is visualized in Figure1.2. Four series of inputs are required :

1) program constants; D1 ... D5, NIT.
   The five dummies allow for m asure conversion; unit-values are used if technical variables are expressed in mks-units. If other than mks-units are used for weight and power, the formulas of section 2.6. don't hold; in that case locomotive types cannot be iterated (see NIT).

   NIT : expressed the number of type-iterations by steps of 1000 kW (or horsepower) :

   NIT = 1, a single type is tested out, whose characteristics will be read in statement 1 (also WL and AXL); afterwards the loop around statement 40 will be dropped and also the instruction (DO 501).

2 ≤ NIT ≤ 8 : a class of locomotive will now be tested for different power-values; the starting power level is /(k.1000) + 100 /, which is read as the sole necessary type-input in statement 1. The loop around 501 will test the behaviour by steps of 100 kW or HP(a). This iteration will be repeated cfr. NIT.

2) problem constants; NL, NK, G, GA, DIS, VIN, AWCF, AXCF; constants particular to a problem analysis are the number of locomotives (NL), the different types of cars (NK), the maximum (G) and average (GA) gradient; the distance (DIS) will here not be used. The initial speed (VIN) will mostly be 0 km/h (or M/h) if full information is required about the adhesion constraints.
The tractive performances will be analysed by successive speed increases of 1 km/h up to VIN + 141 (a).
AWCF / 1 if bad weather drcumstances are to be tested.

3) locomotive characteristics: Statement "1" reads :

- the CLASS of locomotive by the constants DD, DA, DM, DT, which have been defined previously. If DD=1 then also DA=1, which will become clear in the cost-analysis.
- the TYPE of locomotive is indicated by the Power(HP), Weight and the number of Axles. If locomotive-types are iterated, weight and axles-input will not be used but computed by the loop around statement 40.

4) car-characteristics: for each class of car the basic technical features are put in by statement "2", and also the number of cars of the same class.

(a) The program iterates always 141 successive speed values and power by steps of 100 kW. By changing the concerned values in the DO-statements and in the output subroutines, the program can be speeded up. Because of too complex input procedures, it is better not to read in these constants separately. See further in section 2.10.

The program output supplies two series of numerical information about the next functional relationships :

1) The relation between tractive effort, train resistance and speed :
   - for one locomotive type if NIT = 1
   - for successive values of 1000 kW according to the value of NIT. The intermediate results of incremental levels of 100 kW are not indicated in CHART 1 neither in MATRIX 1.

   The graph (CHART 1) is represented afterwards by matrix 1 expressing :

   - speeds in column 1
   - train resistance in column 2 and
   - tractive effort-values in the next columns by tresholds of 1.000 kW, according to the initial power-constant :

     (k.1000 + 100) kW,

     and to the number of desired iterations NIT.

2) The relation between critical speed and power.

The previous output is necessary to get understood the whole problem. As concerned to further calculations, only the values of the critical speeds are necessary to be known.in full detail. These are given in CHART 2 and MATRIX 2, by steps of 100 kW; that is, 20 steps if NIT = 2, 30 steps if NIT = 3 etc...
Matrix 2 gives the power-values in the first column and the critical speeds in the second. If NIT = 1, only one row will be represented.

The iteration by power-steps of 100 kW could be thought exagerated; it is nevertheless necessary because deduced cost calculations will be based upon the critical speed values. To get the latter sufficiently present, we had to start with highly detailed power-steps.

The output of CHART 2 is represented as a part of Figure 1.3. (the right-hand-side curves of critical speeds). It 's clear that the critical speeds of electric and diesel-engines differ but by maximum one km/h between 60 and 128 km/h.

Two technical features are to be mentioned at last. Electric locomotives possess the possibility to generate about 20 % more power during an hour; this happens because they pick up their energy from the overhead line, while diesels have to generate their own energy. Electric motors (both for electric engines and for diesels with an electric transmission) cannot run for hours together at speeds below 20 km/h because of overheating the motors. Therefore this area cannot be accepted as a mean running speed for locomotives, except however for shunters and such-like.

2.9. Program listing

```
PAGE   1

// JOB    0001 0F04

LOG DRIVE   CART SPEC   CART AVAIL  PHY DRIVE
  0000        0001        0001        0000
  0001        0F04        0F04        0004
                          0F01        0001
                          0F02        0002
                          0F03        0003
                          0F05        0005

V2 M11   ACTUAL 16K  CONFIG 16K

// * --------------------------------------------------

// * EVRARD CLAESSENS     //  TEL 294

// * --------------------------------------------------

// * RAILROAD      PERFORMANCE MODEL // FULL  ITERATION

// * --------------------------------------------------

// * STUDIECENTRUM VOOR ECONOMISCH EN SOCIAAL ONDERZOEK

// * --------------------------------------------------

// FOR
*ONE WORD INTEGERS
*IOCS(CARD)
*IOCS(1403 PRINTER)
*LIST ALL
      DIMENSION HP(5),AXL(5),WL(5),A(5),DD(5),DA(5),DT(5),IHED(5,8),EAD(
     15),C(10),JHED(10,8),WG(10),P(10),FCL(10),SSR(5),DM(5),WGH(10),TR(1
     20),R(141,12),S(1692),CRIT(100,2),CR(200)
      COMMON MX,MY
      MX=5
      MY=2
C
C READ  LINK, LOCOMOTIVE AND VEHICLE CHARACTERISTICS
C ---------------------------------------------------
      READ(2,100)D1,D2,D3,D4,D5,NIT
  100 FORMAT(5F1.0,3X,I2)
      READ(2,101)NL,NK,G,GA,DIS,VIN,AWCF,AXCF
  101 FORMAT(2I5,6F5.0)
C
      DO 1 L=1,NL
      A(L)=10.
    1 READ(2,102)(IHED(L,I),I=1,8),DD(L),DA(L),DM(L),DT(L),HP(L),WL(L),A
     1XL(L)
  102 FORMAT(8A1,4F1.0,F6.0,F10.0,4F5.0)
      DO 2 K=1,NK
    2 READ(2,103)(JHED(K,I),I=1,8),WG(K),P(K),FCL(K),C(K)
  103 FORMAT(8A1,4F5.0)
C
```

PAGE 2

```
      WRITE(5,904)
  904 FORMAT(1H1,' USED CHARACTERISTICS',/,' --------------------',///)
      WRITE(5,905)NL,NK,G,GA,DIS,VIN,AWCF,AXCF
  905 FORMAT(1H0,' LOCS = ',I2,' , CARS = ',I3,' , MAX.GRADE = ',F5.2,'
     1, MEAN GR = ',F5.2,' , DIST = ',F4.0,' KM ',/,'   VIN = ',F4.0,',
     2AWCF = ',F5.2,', AXCF = ',F5.2,///)
      WRITE(5,907)
  907 FORMAT(1H0,'CAR-TYPE  ','     TARRA','      LOAD','      FULL','
     1  NUMBER')
      DO 90 K=1,NK
   90 WRITE(5,906)(JHED(K,I),I=1,8),WG(K),P(K),FCL(K),C(K)
  906 FORMAT(1H0,8A1,2X,2F10.0,F10.2,F10.0)
      WRITE(5,908)
  908 FORMAT(1H1,'LOC-TYPE    DD DA DM DT    POWER     WEIGHT    AXLES')
      DO 91 L=1,NL
   91 WRITE(5,909)(IHED(L,I),I=1,8),DD(L),DA(L),DM(L),DT(L),HP(L),WL(L),
     1AXL(L)
  909 FORMAT(1H0,8A1,2X,4F3.0,F9.0,F11.3,F9.0)
C
C
C ITERATION  BY CHANGING  LOCOMOTIVE CHARACTERISTICS
C -------------------------------------------------
      IAD=0
      DO 500 IK=1,NIT
      KK=IK+2
      ITER=(NIT-1)
      IF(ITER)39,39,15
   39 IH=1
      NOT=1
      GO TO 40
   15 CONTINUE
      NOT=NIT*10
      DO 501 IL=1,10
      IH=(IK*10)+IL-10
      IB=0
      IC=0
C
```

```
C       COMPUTED WEIGHT + AXLE NUMBER
C       ------------------------------
        DO 40 L=1,NL
        WL(L)=41.16+(0.014*HP(L)+0.018*HP(L)*DD(L))*((1./0.7467)**(1.-D1))
        WLC=WL(L)
        WFR=WLC-50.
        WFF=WLC-100.
        AXCF=1.
        LM=IFIX(DM(L))
        IF(LM-1)35,31,31
 31     AXL(L)=4.
        AXCF=0.5
        GO TO 40
 35     CONTINUE
        IF(WFR)32,32,33
 32     AXL(L)=3.
        GO TO 40
 33     AXL(L)=4.
        IF(WFF)40,40,34
 34     AXL(L)=6.
 40     CONTINUE
        V=VIN
        DO 401 IV=1,141
        IW=IV-1
C
C NET TRACTIVE EFFORT COMPUTATION
C -------------------------------
        THP=0.0
        DO 10 L=1,NL
 10     THP=THP+HP(L)
        TE=((248.*THP)/V)*(1.609**D2)/((2**D3)*(0.7467**D1))
C
C       CHECK ADHESION SUFFICIENCY
C       --------------------------
        TAD=0.
        DO 50 L=1,NL
        EAD(L)=(161.+(7500./((V+44.)*(1.609**(1.-D2)))))*(WL(L)*((1./0.906
       1)**(1.-D4))*(2.**(1.-D3)))*AXCF*AWCF
 50     TAD=TAD+EAD(L)
        DIFF=TAD-TE
        IF(DIFF)51,51,52
 51     TE=TAD
        GO TO 54
 52     CONTINUE
        IF(IAD)53,53,54
 53     AMESH=V-1.
 54     CONTINUE
        D6=0.0
        TSR=0.0
        DO 11 L=1,NL
        SSR(L)=((29.*AXL(L)*(0.906**D4)/WL(L))+(0.03*V/(1.609**D2))+(0.012
       1*A(L)*V*V*(0.906**D4)/(((24.*D6)+(5.*(ABS(D6-1.))))*(0.0929**D5)*(
       21.609**(2.*D2))*WL(L))))*(0.906**D4)/(2.**D3)
        TSR=TSR+SSR(L)
        D6=1.0
 11     CONTINUE
        WLG=0.0
        DO 12 L=1,NL
 12     WLG=WLG+WL(L)
        WLG=(WLG/NL)
        SNET=TE-((WLG/(0.906**D4))*(TSR+(20*G*NL*((0.906**D4)/(2**D3)))))
        IF(IC) 402,402,397
```

```
PAGE   4

C
C TOTAL TRAIN ROLLING RESISTANCE
C ------------------------------
C
 402  CONTINUE
      DO 13 K=1,NK
      WGH(K)=(WG(K)+(P(K)*FCL(K)))*(1./0.906**D4)
      TR(K)=(116.+(0.045*V*V/(1.609**(2.*D2)))+(WGH(K)*(1.3+(0.045*V/(1.
     1609**D2))+(20.*G))))*(1./(2**D3))
      TR(K)=TR(K)*C(K)
 13   CONTINUE
      TTR=0.0
      DO 14 K=1,NK
 14   TTR=TTR+TR(K)
      R(IV,1)=V
      R(IV,2)=TTR
 397  R(IV,KK)=SNET
C
C CRITICAL SPEED COMPUTATION
C -------------------------
      IF(IB-1)398,400,400
 398  IF(SNET-TTR)399,399,400
 399  CRIT(IH,1)=THP
      IAD=1
      IF(V)390,390,391
 390  CRIT(IH,2)=0.0
      GO TO 392
 391  CRIT(IH,2)=R(IW,1)
 392  IB=1
 400  V=V+1
 401  CONTINUE
C
C     CHANGE LOCOMOTIVE CHARACTERISTICS
C     ---------------------------------
      DO 92 L=1,NL
 92   WRITE(5,910)IK,IL,DD(L),DA(L),DM(L),DT(L),HP(L),WL(L),AXL(L)
 910  FORMAT(1H ,'IT =',I2,'/',I2,1X,4F3.0,F9.0,F11.3,F9.0)
      DO 20 L=1,NL
 20   HP(L)=HP(L)+100.
 501  CONTINUE
      IC=1
 500  CONTINUE
```

```
      C
      C
      C  OUTPUT PROCEDURE
      C -----------------
         NAT=NIT+2
         CALL ARRAY(2,141,NAT,141,12,S,R)
         CALL ARRAY(2,NOT,2,100,2,CR,CRIT)
         WRITE(MX,901)
         CALL PLOT(001,S,141,NAT,141,0)
   901   FORMAT(1H1,'  TRACTIVE EFFORT-RESISTANCE PLOT ON SPEED',//)
         WRITE(MX,900)
   900   FORMAT(1H1,' TRACTIVE EFFORT-RESISTANCE AS A FUNCTION OF SPEED',/)
         CALL MXOUT(001,S,141,NAT,0,40,120,1)
         WRITE(MX,902)
   902   FORMAT(1H1,'  CRITICAL SPEED PLOT ON HORSEPOWER ')
         CALL PLOT(002,CR,NOT,2,100,0)
         WRITE(MX,903)
   903   FORMAT(1H1,' CRITICAL SPEED AS A FUNCTION OF HORSEPOWER',///)
         CALL MXOUT(002,CR,NOT,2,0,41,120,1)
   912   FORMAT(1H0,'ADHESION MESH = ',F3.0,///)
         WRITE(MX,912) AMESH
         CALL EXIT
   C ------------------------------------------------
         END
```

```
CORE REQUIREMENTS FOR
 COMMON      2  VARIABLES   8006  PROGRAM   1826

END OF COMPILATION

// XEQ
```

2.10. Comments and restrictions

The program calculates only the tractive behaviour by changing one input, the locomotive power for one class. If different classes are to be compared in the analysis (for example diesel and electric engines), each class is to be simulated separately. It is possible to allow for this class-changes to run them all once in a while, but that would drive up computing time too much.

The necessary computing time is a basic problem which makes it reluctant to use the program too many times. A full run with NIT = 8 asks (a) about 47 minutes; every loop around statement 401, which calculates effort and resistance for different speed-values, requires about 27 seconds. Therefore a short run has been worked out, which gives as the sole output CHART and MATRIX 2.

The short iteration begins identically until the first critical speed has been founded. Taking this value as new initial speed the power level is increased by one step of 100 kW and a second critical speed will be searched. This procedure is followed until all required values are found. The necessary computing time for the short run (VIN=0, NIT=8) amounts to 4 minutes. This is an acceptable value to use the program in the cost calculations. The 141 speed-iterations around stat. nr. 401, are dropped in this run.

The here presented computing scheme can easily be adopted to related computations, because it is written by blocks. We will let however these posibilities neglected here; they are already sketched in Figure 1.1.

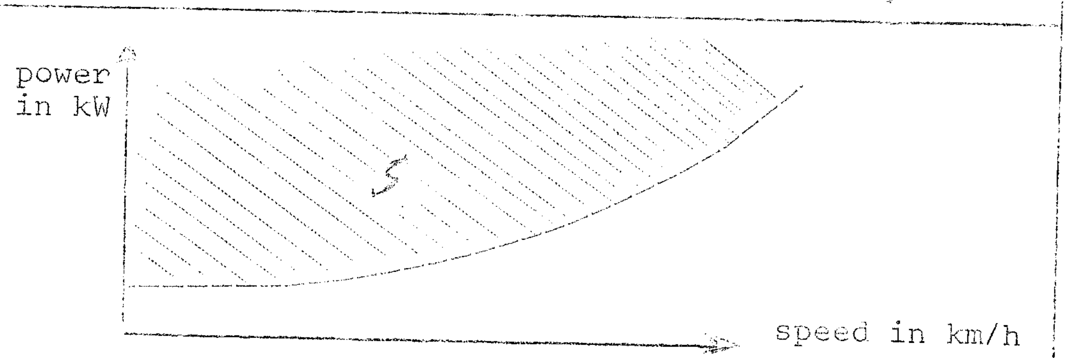
2.11. Convexity of the set of feasible speeds (c)

Up to now, we assigned to ourselves the part of a railway-engineer, when analysing the computational procedure. A physician-re cher had dealt with a more analytical treatment, which was however too tedious to be generalized in the present applications. For simulating general-management decisions, the technical relations ought to be synthesized into a side-condition of the final decision; i.e. whether to run the train or not, and at which speed. The technical performances, we dealt with previously, are now to be seen as a capacity-constraint anyhow (a) in the various programming-approaches (b). By suggesting thus a maximization-problem, a convexity-test is required for the region, which is feasible in the technical sense, previously defined.

The region "S" can be defined to contain the set of all possible speed-power-combinations, which are technically feasible by given train- and track-conditions (see Figure 1.5.).

Figure 1.5. Region "S" of all feasible combinations between speed and the power-level

$$S = \{ (kW, V) \mid V \leq VCRIT,\ kW \geq 0,\ VCRIT \geq 0 \}$$



(a) BAUMOL W.A., o.c. (15), pp. 72-73.
(b) See for example the approaches represented in Figure 1.1.
(c) HADLEY G., o.c. (16), pp. 188-219.

The region "S" is bounded and closed from below by the critical speeds (VCRIT), which are themselves contained by the region (a).

In order to conclude the region "S" be convex in the here presented constraint, it is sufficient to prove the second derivative of the boundary be positive (b). By inspection of the technical relations in sections 2.1. to 2.4., the critical speed-values can be found for a given power-level by solving the effort/resistance-equations for V. This leads us to define (c) :

$$\boxed{S = \{ \{kW, V\} \mid aV^3 + bV^2 + cV \leq d,\ kW \geq 0,\ V \geq 0 \}}$$

where all coefficients a,b,c, and d are positive. Therefore, if

$$\boxed{kW = g(v) \qquad \text{with} \quad v = VCRIT}$$

is differentiable in all orders, the derivatives of the third order are positive; the first and second are likewise positive in the solution space. In this region holds therefore :

$$\boxed{g\,(\lambda\, v_1 + (1-\lambda)\, v_2\,) \leq \lambda g(v_1) + (1-\lambda)\, g(v_2)}$$

$$\text{for } v_1,\ v_2,\ \ldots \geq 0$$

(a) This is obvious because of statement nr. 391 in the program, Speeds are increased by one km/h up to SNET / TTR, for a given V=V°. Afterwards the critical speed-value is set equal to V°-1 (by IW=IV-1) in statement nr. 391.

(b) ZANGWILL W.I., o.c. (18), pp. 27-32.
HADLEY G., o.c. (17), pp. 83-85.

(c) The relations can be summarized in the non-fixed variables as follows :

$$\left.\begin{aligned} SSR &= a''V^2 + b''V + c'' \\ TTR &= a'V^2 + b'V + c' \end{aligned}\right\} \quad \text{and} \quad \left\{\begin{aligned} TE &= d'/V \\ TAD &= d''/V \end{aligned}\right.$$

with a",a',b",b',c",c',d",d' are all positive. Hence for SNET $\geq$ TTR or (TE-SSR-TE) $\geq$ 0 holds :

$$V^3(a'+a'') + V^2(b'+b'') + V(c'+c'') \leq d$$

As indicated in footnote (c) on the previous page, the required properties of the critical speed-boundaries hold but separately for the constraints of tractive effort (d' and g'(v)) or adhesive effort (d" and g"(v)). These boundaries are both convex and possess continuous first derivatives in the solution space, where the second derivative is always positive (a).

The critical speeds which are actually effective in moving a train are computed according to the lower values of TE and TAD (b). Therefore, the region S is the intersection :

$$S = S_1 \cap S_2$$

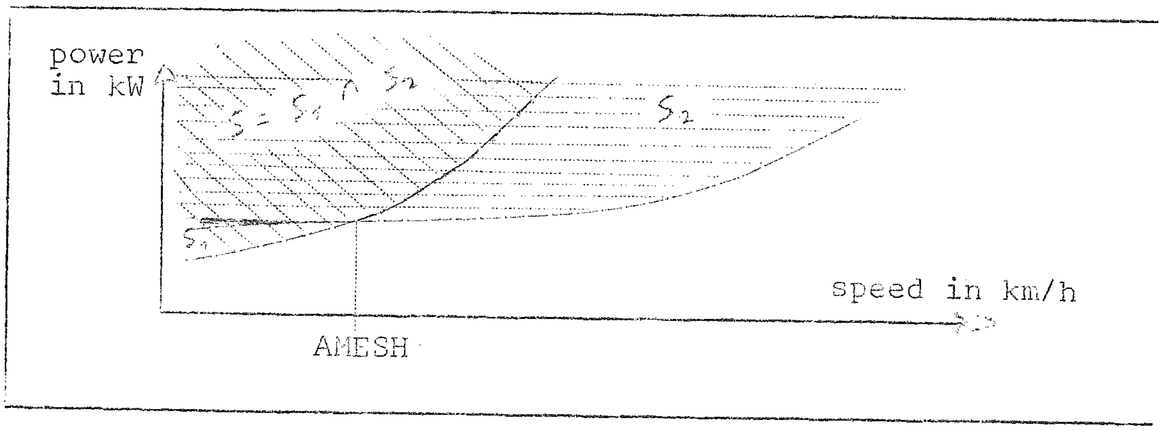
with

$$S_1 = \{(kW, v) \mid kW \geqslant g'(v),\ kW \geqslant 0 \quad v \geqslant 0\}$$

$$S_2 = \{(kW, v) \mid kW \geqslant g''(v),\ kW \geqslant 0 \quad v \geqslant 0\}$$

and is therefore also convex, as indicated in Figure 1.6.

Figure 1.6. Convex hull of critical speeds



(a) This follows from the analytical substitution of the concerned formulas in sections 2.1. to 2.4., for kW and v being non-negative.

The boundary of "S" possesses clearly continuous first derivatives, except at the break-point "AMESH", where for lower speeds the adhesive constraint becomes predominant (a).

Admitted, this break-point will mostly appear at lower speedlevels, it has however full sense by indicating the strength of the adhesive constraint, especially in rainy weather circumstances.

If AMESH = 0, S_1 is a subset of S_2 and $S=S_1$. This situation will be the relevant problem if you like to analyse the maximum speeds provided, you already know the train can start ?

2.12. Conclusion

The here presented calculus has synthesized the relevant technical constraints, which are necessary for defining the technical performance of locomotives, i.e. tractive and adhesive effort for given train composition and track conditions.

The analytical properties of the relationships has proved the feasible region be convex, and the boundary has continuous first derivatives, except at one point, which is however known. The second derivatives are thereby positive in the solution space.

The computed points on the boundary can thus be used for a refined linearization of the constraint-equation in eventual programming approaches. Mostly it will be sufficient to select some of them as mesh-points of the linearized boundary (b). A speed/power combination, which is to be taken as mesh-point, is already known to be the combination power/AMESH. The constraint equation can now be expressed in terms of costs.

(a) TE $>$ TAD (statement nr. 51) for 0 $\leq$ VCRIT $\leq$ AMESH
The critical speedlevel where
TE $\leq$ TAD is computed by statement nr. 53 and 399 (+1).

(b) HADLEY G., (17), pp. 104-116.

3. The capital cost of tractive effort

The previous section stated locomotive power being the predominant determinant of the type of locomotive, which determines the main other characteristics as weight and hence the number of axles; in this context the engine-class is taken as given. As power pretends itself to be a "synthetic" variable of locomotive characteristics, we are interested in testing the hypothesis that the purchase cost of a locomotive-type is also a function of its power.

This means the analysis aims to express solely the "cost of tractive effort". This item is only to be depreciated by obsolescence, and shows no value-decrease due to wear and tear (a). Hence, tractive effort gives rise to an opportunity cost, which is common in the sense of availability over different time periods, and which remains constant during the length of life.

Another property of this viewpoint is tractive effort be so a strictly primary input as a part of the capital cost of running a train. These arguments causes for a direct and continuous cost relation.

The use of a continuous and direct relation might seem unduly sofisticated in common railroad practice. The purchase decision of a new type of engine is likely more discontinuous as power is concerned. Moreover, the length of life (from 20 to 40 years) didn't allow for marginal changes during this period, if changing needs rise.

Arguments as the latter are however too pragmatic. Locomotives are assigned to a wide variety of jobs during their life (use of obsolete engines on secondary links, mixed use for passenger and

(a) LEWIS W.A., o.c. (9), pp. 11-43.

freight haulage, etc.). Rather than concluding all this be decided on discontinuously, it is more important to measure exactly the optimal use of tractive-capital as concerned to these switching purposes. If a high-powered locomotive is inserted for a less important task where its power-capacity is partly unused (for example because the thrunk line needs new engines), the problem arises how to allocate the cost of "unused" tractive capacity on that connection. A first step is here to measure exactly the opportunity cost of capital-waste.

3.1. Specification of the capital/power-relationship

Different assumptions can be stated about :
- the nature of the relation between power and purchase cost,
- the other technical features involving a shift in the regression parameters, which are congruent with the class of locomotives.

The leading continuous regression variables are :
- KNK : purchase cost of locomotives in 1,000 Belgian francs (1971)
- PkW : rated power of the locomotive (in kiloWatt).

The additional technical features, defining the class are :

- DD : <u>the traction system</u> : DD = 1, diesel
  DD = 0, electric
- DA : <u>the current system</u> for electric locomotives
  DA = 1, alternating current (25,000 Volt; 50 Hz.)
  DA = 0, direct current (1,500 Volt).
- DM : <u>the train composition</u>: DM = 1, motortrains and railcars
  DM = 0, pulled trains

  The "pure power investment" in motortrains can be specified in two ways :

  a) The purchase cost of the whole powered unit is taken for observation. In the estimation a dummy "DM" accounts for the estimated "mean coach value", and its coefficient is expected to be positive (about the mean price of a coach).

This approach is obvious if no separate figures are available for the same unit, with and without power. Disadvantages exist however if various types of motortrains are selected in the sample. The estimated "mean coach value" might be a raw proxy, if actual coach prices vary considerably (for example luxurous long-distance motortrains and railcars). Also the estimate may vary according to other regression estimates, which will often cause some imperfections in the estimates.

b) Another method calculates preliminary the "engine cost" of the motortrain, by subtracting the "coach cost" from the cost of the whole powered unit. When the engine cost is taken for observation, "DM" is dropped in the cost-estimation.

- DT : the transmission system : DT = 0 electric transmission
  DT = 1 mechanical transmission.

  Diesel-electric engines need a triple power investment (a diesel-motor generating energy, a dynamo converting it into electric energy and an electric traction-motor). Other transmissions make the relatively expensive dynamo and electric motor redundant.
  Mechanical transmission is at the present moment applied on diesel shunters and railcars; therefore DT will be used to allow for these classes of engines. The coefficient is expected to be negative.

The variables and dummies, specified above, are not only of interest in the capital cost relationship. As power has also an impact upon the weight of locomotives (in addition to DD and DA), the dummies determine thereby the length of life for the related engine.

Some previous trials have proved a linear relationship between purchase cost and power be most efficient for electric locomotives (a). Hence, a fixed construction cost (overheads of constructing firm) exists, and for additional power economies of scale, are present at a constant rate.

Diesels possess a more complex construction. For additional power, besides the electric traction-motor, a more powerful dynamo and diesel motor (generating electric current) are needed. To allow for this feature, two hypothesis-tests are here presented.

A first approach assumes the constant marginal scale effects for electric locomotives, with a different constant term for alternating current. Diesel-technology is accounted for by :

- a different constant term, which is expected to be negative,
- growing marginal scale effects, which are less than those of electric engines.

This hypothesis has been found to be tested most efficiently by an irrational polynomial of the second order (a). The tests account for the two suggested approaches about motortrains. The results are summarized in Table 2.1.

(a) The exponent of $(-\frac{1}{2})$ has been found by iterating various exponents of which $(-\frac{1}{2})$ has given rise to the highest R-value.

Table 2.1. Estimated cost coefficients in the relation between the purchase cost and the power level, given marginal scale effects are increasing for diesels and constant for electric locomotives.

	constants				power variables		F(4,9) F(5,8)
	regress.	DA	DD	DM	PkW	PkW.DD	(R)
1.	3229,80	3080,32 (2.69)	-5620,18 (-2.68)		4.00 (11.41)	364.51 (10.37)	167.31 (0.993)
2.	4104,66	3123.62 (2.68)	-6277,14 (-2.85)	3442.89 (2.93)	3.80 (9.31)	369.37 (10.38)	166.98 (0.933)

Figures between brackets refer to the t-statistics (d.f.=8/9); critical values are 2.26 for $2\frac{1}{2}$ % (one tail), 2.82 for 1 % and 3.25 for $\frac{1}{2}$ %. The critical F(4,9)-value for 1 % is 6.42.
We already argued that the "engine-cost" approach is better for motortrains; therefore the first regression has been withheld (see Figure 2.1.).

The coefficients involving the power-variables are clearly superior to the dummy-coefficients. The power-variables are really to be tested; therefore a higher t-value is necessary, which is present. The lower significance of the dummy-coefficients are not so important. Stronger a-priori statements can be stated, for they concern known technical differences (a). These lower t-values are thereby rather due to the little number of observations, related to the concerned dummy.

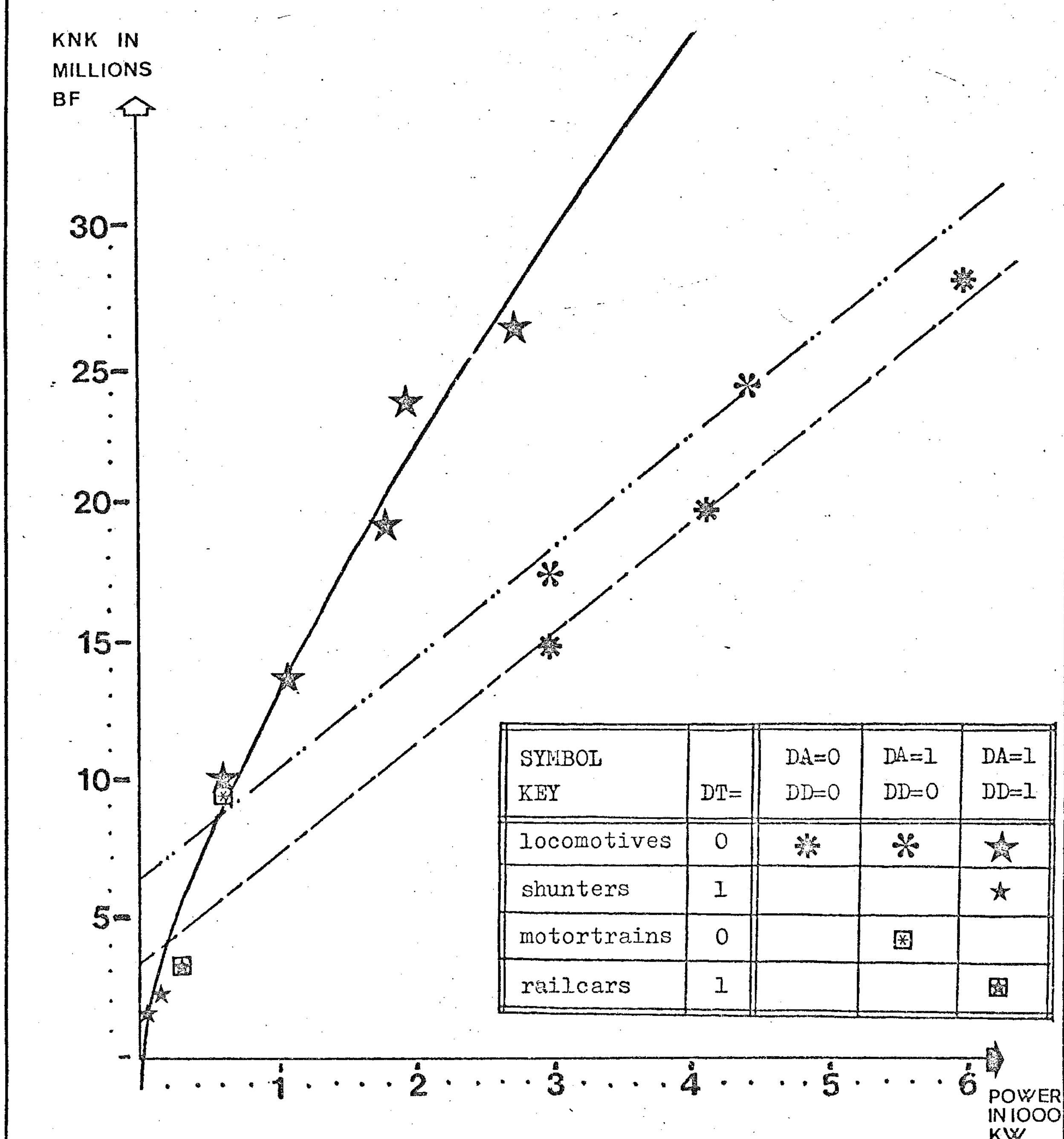
(a) WONNACOTT R.J. & T.H., o.c. (10), p. 67.

A more annoying feature is given rise by the negative constant term which occurs in the diesel-equation. Though this happens only in a short power-range near to the origin, the concerned coefficients cannot be accepted as such. Diesels are known to be produced in a constrained power-range (max. 4,000 HP), because of excessive weight above this level. This is a rather short range to test an hypothesis about scale effects, which is predominant for values, where diesel-observations are not existing (a).

(a) All estimates are obtained by applying O.L.S. One can think seriously about other estimating techniques of equations systems, which can lead in certain circumstances to more efficient results. (See for example THEIL H., o.c. (11), pp. 294-302). Special requirements to the data sample, and the sufficient accuracy of the O.L.S.-results, make the O.L.S.-method obvious for the present calculations.

Figure 2.1. Relation between the power level and the purchase cost of locomotives, given marginal scale effects are increasing for diesels and constant for electric locomotives.

$$KNK = 3229.80 + 3080.32\ DA - 5620.18\ DD + 4.00\ PkW + 364.51\sqrt{PkW.DD}$$



A second approach assumes constant marginal scale effects, both for diesel and electric locomotives, but with different rates. Constant purchase costs for diesels are equal to these of electric engines, in turn of alternating and direct current.

Table 2.2. Estimated cost coefficients in the relation between the purchase cost and the power level, given marginal scale effects are constant both for diesel and electric locomotives, but with a different rate.

		constants			power variables		F(4,9) F(5,8) (R)
	regress.	DA = DD	DT	DM	PkW	PkW.DD	
1.	2702.35	3085.65 (3.16)	-4778.99 (-5.32)		4.12 (16.37)	3.84 (10.17)	239.41 0.995
2.	2436.62	3037.28 (3.06)	-4751.65 (-5.21)	5369.56 (5.96)	4.18 (15.79)	3.94 (9.89)	163,69 0.995
	regress.	DA	DT	DM	PkW	PkW.DD	
3.	4391.72	2510.08 (3.06)	-3506.89 (-3.46)		3.78 (17.86)	4.91 (11.83)	231.54 0.995
4.	4375.59	2502.02 (2.72)	-3510.22 (-3.25)	4615.86 (4.56)	3.78 (15.99)	4.91 (11.12)	144.59 0.995

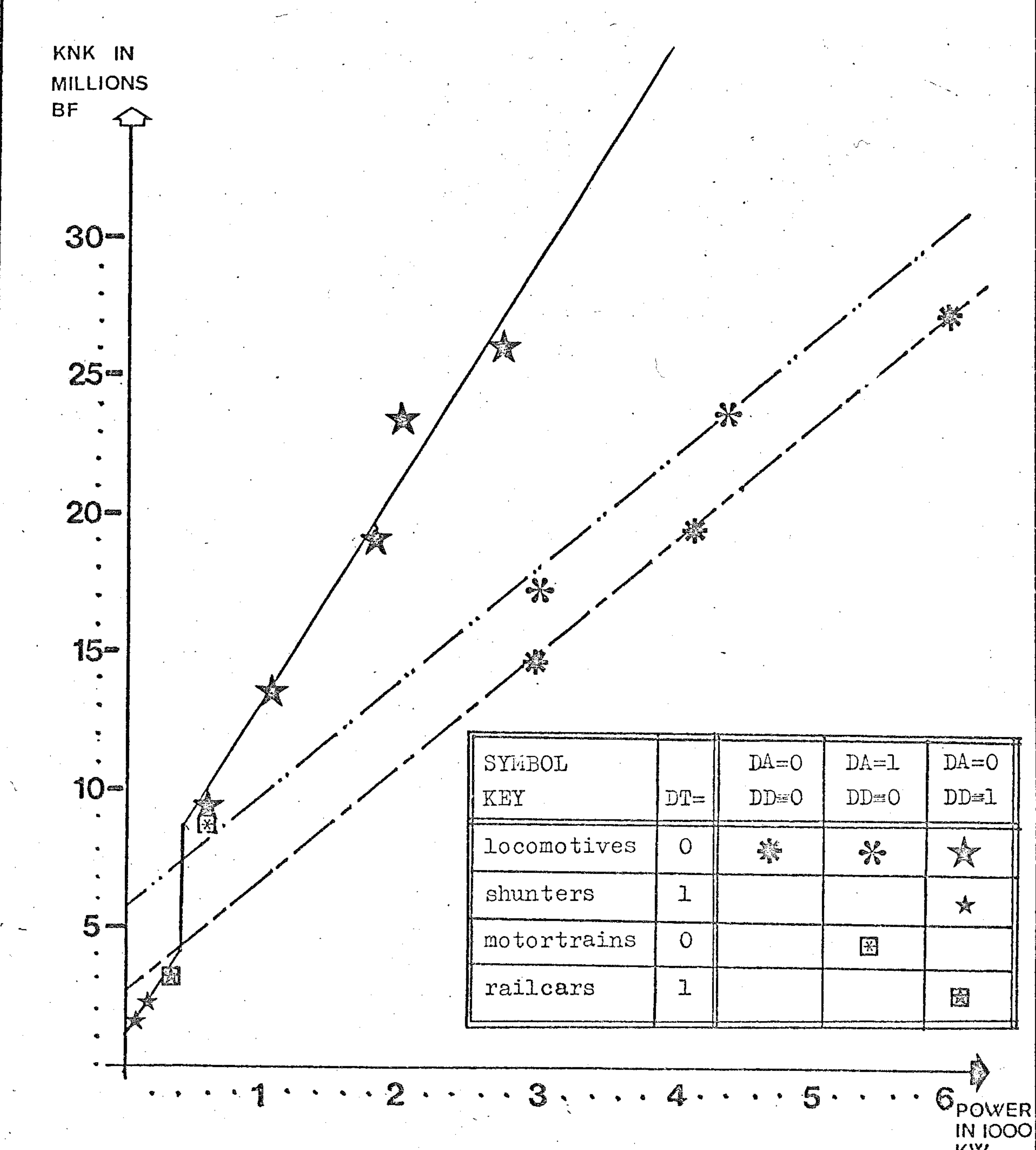
remarks :

a) regressions 1 & 2 allocate to diesels the same constant term as to electric locomotives of alternating current; in 3 & 4 diesels possess the constant of direct current locs; i.e. the regression constant.

b) regressions 2 & 4 take for the purchase cost of motortrains the cost of the powered unit; in these regressions the coefficient of DM accounts for the coach value.
In regressions 1 & 3 the actual value of the engine makes the presence of DM unnecessary.

Figure 2.2. Relation between the power level and the purchase cost of locomotives, given marginal scale effects are constant for diesel and electric locomotives, at different rates.

KNK = 2702.35 + 3085.65 (DA=DD) - 4778.99 DT + 4.12 PkW + 3.84 PkW.DD



This second approach focusses attention upon diesels manufactured by two separate techniques; i.e. for long distance haulage and for operations on shunting yards. The latter are assumed to be cheeper by a constant, measured by the coefficient of DT. The lower resulting fixed cost will affect locomotives in a power-range below 400 kW.

The different approach for shunters must not be seen as a statistical witchcraft, but refers to common practice; shunters are also different accounted for as for length of life and drivers' wages. The dummy "DT" does likewise affect the case of motor-trains, in addition to DM (if selected). When a simplified construction is obvious for motortrains, we are speaking about "diesel-railcars".

The picture of these estimates shows more consistent results. The regression, withheld for latter applications, is the first (see Figure 2.2.). Others indicate some underestimation of the PkW-coefficient, which generates a little degree of auto-correlation in the "electric-equation"; hence, electric locomotives of a high power-level (which really exist) were underestimated in their purchase cost.

Also a realistic difference is shown as for marginal power-coefficients between diesel and electric engines. The additional power investment for diesels rises incrementally to about twice as much as for electric engines. This increases the capital cost for one additional kW from 4,120.- to 7,960.-. Examples are known (3) of shunters, usable by diesel and electric traction; they dispose of doubled power by electric traction, for the dynamo is then used for motor.

The general accuracy of the proposed regression-fit (for both tractions) leads us to think about the power/purchase cost-

(a) HERRMANN O., o.c., (12), pp. 34-35.

relationship, as a rather technological equation. This enables us to apply these results widely, within the constraints we mentioned earlier.

3.2. Adjustments for different international manufacturing condition and for inflation

As suggested before, the data sample was drawn from French engineering figures. This country is an efficient choice because of :

- the great topographical differences throughout the country, which results in a great variation in the power of engines.
- the big size of the company (S.N.C.F.) which induces large series of running locomotives; hence, prices reflect a degree of mass-production.
- the presence of two current systems (direct and alternating) which both take some half of electric train haulage.
- the international market of French manufacturers of rolling stock and locomotives; figures can thus be assumed to be unaffected by some monopsony-power anyhow.
- the marginalistic tradition of French railway engineering; this leads us to accept the assumption of the most efficient technical factor combination, also in the delivering industries.

The results are in agreement to observations in other countries, though the latter did not possess such a wide variation. So we may accept the relative value of the parameters as generally true.

As for the Belgian case, the KNK-values are to be upgraded by :

- PRF : a factor accounti for price-differences, because of other railway technology (for example, another current system, higher lifetime-values). PRF was figured out to be about 1.63.

- FLF : an inflation factor with respect to 1971. In the period 1971-1975 Belgian prices of locomotives have been increased by about 1.46.

The cost of locomotive is therefore :

$$CLC(L) = PRF \times FLF \times KNK = 2{,}38\ KNK.$$

3.3. The annuity-cost

As proposed before, we have defined investments in tractive effort, a capital which depreciates only by obsolescence. Therefore, depreciation due to wear and tear is accounted for by maintenance costs, and is expressed as a function of distance etc.

As locomotives-life is a single function of obsolescence, it can easily be fixed for given technological conditions. For Belgian railways, next figures are used : (a)

locomotive class		LIFE(L) =
electric locomotives and " motortrains	DD = 0 DM = 0,1	40
diesel locomotives and " motortrains	DD = 1 DM = 0,1	25
shunters & railcars	DD = 1, DT = 1 DM = 0,1	20

The user's cost of capital can then be defined to be a capital recovery factor, which is fixed per year. This viewpoint is correct, if locomotives are used in full capacity over their lifetime (b).

(a) DEBERDT M., o.c. (4), pp. 14; the lifetime can differ
In France the figures are resp. 35, 30, 20, o.c. (14).
(b) LEWIS W.A., o.c. (9), pp. 66-67.

In this meaning, the annuity-cost expresses the opportunity cost of capital which is common in the sense of availability in the medium run, and is also fixed for this period. One year of off-capacity use would therefore reduce the lifetime "LIFE(L)" by one year, and the latter annuity expresses then the cost of capital.

The cost calculations assume an equally capacity-use for all years of life; the figures of lifetime can thus be accepted being fixed. The latter rule is realistic in common railway practice, for most important peak-demands are fluctuating within a year (for example the daily commuters'peak in passenger traffic, the seasonal peaks in hollidays-trips and freight services).

A commonly used formula (14) also accounts for a scrap-value :

$$\mathrm{ANN(L)} = \left((\mathrm{CLC(L)} - \mathrm{SCR(L)}) \frac{\mathrm{RENT}}{1 - \dfrac{1}{(1+\mathrm{RENT})^{\mathrm{LIFE(L)}}}} \right) + (\mathrm{SCR(L)}.\mathrm{RENT})$$

where :

CL(L) : purchase-cost of the locomotive (adjusted for inflation).
SCR(L): scrap-value
RENT : discount-rate to be used in the calculations
LIFE(L): lifetime of locomotive.

The formula indicates clearly that the annuity recovers only the value of the locomotive, which is affected by obsolescence. The scrap-value is accounted for as if LIFE(SCRAP) was infinite, and dropped when scrap is sold.

The use of annuity-cost accounting indicates the costs and benefits are discussed, based on the annual/cash-flow, which is a relevant view for medium-run railways running practice.

A last discussion deals with the discount-rate to be used in the calculations. We will not weight the alternative viewpoints here, for most existing contrarieties refer to lumpy investments which are not marginal to a given capital-stock. The analysis will be confined to the topics which are relevant to compute the cost of motive power (a).

The capital recovery factor first expresses the real discount-rate. Adjustments for inflation are already accounted for by CL(L). A second fact is important for Western Europe for most railways are run by nationalized companies. Therefore the stock of motive power possesses a far greater freedom in special availability, than if they were owned by private companies, each running some well defined connections. This may involve lower rates of discount, not because the decision-maker is a public company, but for the future profit-flow from locomotives is quasi riskless, given the whole network (b).

If the electrification policy is performed systematically, the argument also holds for electric locomotives, which need special track conditions.

To define the basic rate, we can base ourselves upon growing government's action in favour of public transportation. This doesn't stand for lowering the capital cost for railways because of expected positive externalities in comparison with motorways or aircraft (this in fact would be arbitrary and methodologically wrong). The arguments focuss rather the degree of certainty of future wealth, when improving today's public transport facilities.

(a) BAUMOL W.A., o.c. (24).

(b) The argument assumes different technologies for different companies, which involves substitutions of motive stock, are impossible. In addition the gradual replacement of obsolete material makes, to some degree, switches over the network possible.

The irreversibilities in land use for trunk motorways, the bargaining-power of the country on the international energy-market, they are all factors which cannot be explicitly expressed as a cost or a benefit, but which determine rather the certainty of future wealth. So they are an element of time preference, revealed by public policy-makers, and not as such an item with a specific cost or benefit.

However, the latter arguments are not solving the whole problem of defining the right rate. We can state the rate be the same as used for infrastructure and rolling stock on a particular relation, with some correction for the lower risk on future returns which are not so evidant for all network-links nor for rolling-stock.

The right discount rate is to be argued when dealing with a project, that is, when discussing a given traffic on a given relation for both can cause discrepancies. The here presented calculations will therefore be performed taking the discount rate as an input, which is variable to some extent.

3.4. Program description

The program, which was presented in section 2.8. computed the technical performances of locomotives operations. It will now be extended for the cost calculations; therefore we use the short run, which was already proposed in section 2.10.

The inputs are :

1) financial constants (read in statement 104-1)

PRF : price-factor accounting for cost differences in railway technology (discussed in section 3.2.).

PRF : inflation factor, accounting for price evolution with respect to 1971 (see also section 3.2.).

RANT : lower level of the rate of discount, and

RTMX : upper level of the rate of discount, to be used in the annuity-calculations.

The program calculates the annuity-costs for RENT=RANT, for RENT=RTMX and for all intermediate rates by steps of 2 %. For example, if RANT= 0,03 and RTMX= 0,07, the output will print the costs at 3,5 and 7 %.

2) program constants (read in statement 100-1)

In addition to the constants already discussed in section 2.8.:

NIT : the number of iterations of locomotive-types around 1000 kW, according to the initial value of HP(L). The separate procedure of calculating the performances of one type of locomotive, if NIT = 1 is here skipped.

NVAR : the number of columns in matrix 2. This number includes one column for the power-values. Thereby for each problem two additional columns are required; one for the critical speeds and one for the purchase costs. NVAR must $\leq$ 10.

JOINT : binary code if the second problem-output is separately printed (JOINT=0) or together with the first (JOINT=1)

3) problem constants: (statement nr. 101-1)

They are all like presented previously in section 2.8. Only AXCF is dropped as input, because the program doesn't test separate locomotive-types (when NIT=1). The number of driven axles is computed by the program itself, according to section 2.6.

A problem is further defined by a "class of locomotive", which is specified in the read-statement nr.1. for the locomotives and in statement nr. 2 for the vehicle characteristics. At the end of the first run of the program, we can allow for changed problem-inputs by the instructions from statement nr. 405 up to 507.

In the here presented example we will test the performances of electric and diesel locomotives, for the cost implications. As expressed in Figure 1.2., we previously had to run the program two times separately; now it will be performed in a double run. The problem constants are first put in for electric engines; they are changed afterwards for diesel by numbers 406-507.

The program output expresses solely the performances of locomotives in terms of critical speed; the relations of tractive and adhesive effort are dropped in the short iteration (see also section 2.10). The program gives two outputs throughout its running time :

1) <u>the relations between power, purchase cost and critical speed</u>

This information is given by matrix 2 and plotted by chart 2. The first column (the base-variable of chart 2) expresses the powerlevels. The second column contains the critical speeds and the third column the purchase cost of the locomotive, which is defined by the power-value of column one.

In our examplative program, the fourth and the fifth column express critical speed and purchase cost of the second tested class of locomotives (diesels). Therefore NVAR = 5. This output is given at the end of the program run.

Figure 1.3. represents the output of CHART 2. As we are using different units on the same graphs, attention must be taken to express the results in realistic comparable scaling.

2) <u>the relation between the critical speeds and the annuity cost</u>

The annuity cost related to the critical speeds are given by matrix 3 and chart 3. This information is printed after each problem-run of the program (before matrix and chart 2). In our example of the difference between electric and diesel-power, a first time the annuities of electric locomotives are printed, the second time the case of diesels is handled. Afterwards matrix 2 is printed.

If JOINT = 1, then the annuities of electric engines are also printed in the matrix 3 of the second problem-run dealing with diesels. The possibility with the JOINT-code is interesting for easy comparison, as shown in Figure 1.4. The total number of computed annuities is restricted to 9. Therefore, if we wish to compute the annuities for more than four different rates, we cannot use JOINT = 1. This is also the case if more than two problems are simultaneously dealt with.

A second restriction to the use of the joint-code deals with the relation between power and critical speeds. Matrix 3 calculates the annuities of critical speeds by relating them both to power, and dropping afterwards the power in the output. For our second run, the annuities of electric engines are so related to the critical speeds of diesels, which differ by maximum one km/h from electric engines. If however the problem-runs were dealing with different track conditions or different train compositions, the joint-code cannot be used (JOINT = 0).

Finally, after matrix 3 the value of AMESH (see section 2.11.) is printed according to the problem-constants.
The parameter-cards to the program contain the required input-characteristics and the symbols to be used in the PLOT-subroutines. In the example which was presented here, three such symbolcards are required :
- first column : blank (b)
- second column up to the ninth: symbols for charts 3 and 2.

For example :

chart 3 (first time) b/3/5/7

(second time)b/3/5/7/T/F/S where T,F and S symbolize the annuities for diesels, if JOINT = 1. (see Figure 1.4.)

chart 2: b/1/A/2/B, where A and B are the symbols for the critical speeds for electric and diesel-types respectively; 1 and 2 stand for the purchase-costs of them.

3.5. Program listing

```
PAGE   1

// JOB    0001 0F04

LOG DRIVE   CART SPEC   CART AVAIL  PHY DRIVE
  0000        0001        0001        0000
  0001        0F04        0F04        0004
                          0F01        0001
                          0F02        0002
                          0F03        0003
                          0F05        0005

V2 M11   ACTUAL 16K  CONFIG 16K

// * ---------------------------------------------------

// * EVRARD CLAESSENS      //  TEL 294

// * ---------------------------------------------------

// * RAILROAD COST PERFORMANCE MODEL  / SHORT ITERATION

// * ---------------------------------------------------

// * STUDIECENTRUM VOOR ECONOMISCH EN SOCIAAL ONDERZOEK

// * ---------------------------------------------------

// FOR
*ONE WORD INTEGERS
*IOCS(CARD,1403 PRINTER)
*LIST ALL
      DIMENSION HP(5),AXL(5),WL(5),A(5),DD(5),DA(5),DT(5),IHED(5,8),EAD(
     15),C(10),JHED(10,8),WG(10),P(10),FCL(10),SSR(5),DM(5),WGH(10),TR(1
     20),CRIT(100,10),CR(1000),CLC(5),ANN(5),ANCT(100,10),AT(1000),LIFE(
     35)
      COMMON MX,MY
      MX=5
      MY=2
C
C READ  LINK, LOCOMOTIVE AND VEHICLE CHARACTERISTICS
C ------------------------------------------------
      READ(2,104)PRF,FLF,RANT,RTMX
 104  FORMAT(4F10.0)
      READ(2,100)D1,D2,D3,D4,D5,NIT,NVAR,JOINT
 100  FORMAT(5F1.0,3X,I2,I3,I2)
      READ(2,101)NL,NK,G,GA,DIS,AWCF
 101  FORMAT(2I5,4F5.0)
C
      DO 1 L=1,NL
      A(L)=10.
 1    READ(2,102)(IHED(L,I),I=1,8),DD(L),DA(L),DM(L),DT(L),HP(L),WL(L)
 102  FORMAT(8A1,4F1.0,F6.0,F10.0,4F5.0)
      DO 2 K=1,NK
 2    READ(2,103)(JHED(K,I),I=1,8),WG(K),P(K),FCL(K),C(K)
 103  FORMAT(8A1,4F5.0)
C
```

PAGE 2

```
      WRITE(5,904)
  904 FORMAT(1H1,' USED CHARACTERISTICS',/,'  --------------------',///)
      WRITE(5,905)NL,NK,G,GA,DIS,AWCF
  905 FORMAT(1H0,' LOCS = ',I2,' , CARS = ',I3,' , MAX.GRADE = ',F5.2,'
     1, MEAN GR = ',F5.2,' , DIST = ',F4.0,' KM  / AWCF = ',F4.2,///)
      WRITE(5,907)
  907 FORMAT(1H0,'CAR-TYPE  ','     TARRA','      LOAD','      FULL','
     1  NUMBER')
      DO 90 K=1,NK
   90 WRITE(5,906)(JHED(K,I),I=1,8),WG(K),P(K),FCL(K),C(K)
  906 FORMAT(1H0,8A1,2X,2F10.0,F10.2,F10.0)
      WRITE(5,908)
  908 FORMAT(1H1,'LOC-TYPE   DD DA DM DT    POWER     WEIGHT    AXLES')
      DO 91 L=1,NL
   91 WRITE(5,909)(IHED(L,I),I=1,8),DD(L),DA(L),DM(L),DT(L),HP(L),WL(L),
     1AXL(L)
  909 FORMAT(1H0,8A1,2X,4F3.0,F9.0,F11.3,F9.0)
C
C
C ITERATION  BY CHANGING  LOCOMOTIVE CHARACTERISTICS
C ------------------------------------------------
      AXCF=1.
      ICR=2
      NRENT=1
  404 CONTINUE
      ICT=ICR+1
      V=0.
      IAD=0
      DO 500 IK=1,NIT
      DO 501 IL=1,10
      IH=(IK*10)+IL-10
  402 CONTINUE
C
C     COMPUTED WEIGHT + AXLE NUMBER
C     -----------------------------
      DO 40 L=1,NL
      WL(L)=41.16+(0.014*HP(L)+0.018*HP(L)*DD(L))*((1./0.7467)**(1.-D1))
      WLC=WL(L)
      WFR=WLC-60.
      WFF=WLC-100.
      AXCF=1.
      LM=IFIX(DM(L))
      IF(LM-1)35,31,31
   31 AXL(L)=4.
      AXCF=0.5
      GO TO 36
   35 CONTINUE
      IF(WFR)32,32,33
   32 AXL(L)=3.
      GO TO 36
   33 AXL(L)=4.
      IF(WFF)36,36,34
   34 AXL(L)=6.
   36 CONTINUE
      SHUNT=HP(L)-400.
      IF(SHUNT)41,42,42
   41 DT(L)=DD(L)+0.7
      DT(L)=IFIX(DT(L))
      GO TO 40
   42 DT(L)=0.
   40 CONTINUE
```

```
PAGE    3
C
C NET TRACTIVE EFFORT COMPUTATION
C --------------------------------
      THP=0.0
      DO 10 L=1,NL
 10   THP=THP+HP(L)
      TE=((248.*THP)/V)*(1.609**D2)/((2.**D3)*(0.7467**D1))
C
C     CHECK ADHESION SUFFICIENCY
C     --------------------------
      TAD=0.
      DO 50 L=1,NL
      EAD(L)=(161.+(7500./((V+44.)*(1.609**(1.-D2)))))*(WL(L)*((1./0.906
     1)**(1.-D4))*(2.**(1.-D3)))*AXCF*AWCF
 50   TAD=TAD+EAD(L)
      DIFF=TAD-TE
      IF(DIFF)51,51,52
 51   TE=TAD
      GO TO 54
 52   CONTINUE
      IF(IAD)53,53,54
 53   AMESH=V-1.
 54   CONTINUE
      D6=0.0
      TSR=0.0
      DO 11 L=1,NL
      SSR(L)=((29.*AXL(L)*(0.906**D4)/WL(L))+(0.03*V/(1.609**D2))+(0.012
     1*A(L)*V*V*(0.906**D4)/(((24.*D6)+(5.*(ABS(D6-1.))))*(0.0929**D5)*(
     21.609**(2.*D2))*WL(L))))*(0.906**D4)/(2.**D3)
      TSR=TSR+SSR(L)
      D6=1.0
 11   CONTINUE
      WLG=0.0
      DO 12 L=1,NL
 12   WLG=WLG+WL(L)
      WLG=(WLG/NL)
      SNET=TE-((WLG/(0.906**D4))*(TSR+(20*G*NL*((0.906**D4)/(2**D3)))))
C
C TOTAL TRAIN ROLLING RESISTANCE
C ------------------------------
C
      DO 13 K=1,NK
      WGH(K)=(WG(K)+(P(K)*FCL(K)))*(1./0.906**D4)
      TR(K)=(116.+(0.045*V*V/(1.609**(2.*D2)))+(WGH(K)*(1.3+(0.045*V/(1.
     1609**D2))+(20.*G))))*(1./(2**D3))
      TR(K)=TR(K)*C(K)
 13   CONTINUE
      TTR=0.0
      DO 14 K=1,NK
 14   TTR=TTR+TR(K)
```

PAGE 4

```
C
C CRITICAL SPEED COMPUTATION
C -------------------------
      IF(SNET-TTR)399,399,400
 399  CRIT(IH,1)=THP
      IAD=1
      IF(V)390,390,391
 390  CRIT(IH,ICR)=0.0
      ANCT(IH,1)=0.0
      GO TO 392
 391  CRIT(IH,ICR)=V-1.
      ANCT(IH,1)=V-1.
 392  CONTINUE
      GO TO 403
 400  V=V+1
      GO TO 402
 403  CONTINUE
C
C COST CALCULATION
C ----------------
C
      IAN=NRENT
      TCST=0.0
      DO 81 L=1,NL
      CLC(L)=2702.35+(3085.65*DA(L))-(4778.99*DT(L))+(4.12*HP(L))+(3.84*
     1HP(L)*DD(L))
      CLC(L)=CLC(L)*PRF*FLF/1000.
 81   TCST=TCST+CLC(L)
      CRIT(IH,ICT)=TCST
      RENT=RANT
 83   CONTINUE
      IAN=IAN+1
      TANN=0.0
      DO 82 L=1,NL
      LIFE(L)=((1.-DD(L))*40.)+(DD(L)*25.)-(DT(L)*DM(L)*5.)
      ANN(L)=(RENT/(1.-(1./((1.+RENT)**LIFE(L)))))*CLC(L)
 82   TANN=TANN+ANN(L)
      ANCT(IH,IAN)=TANN
      RENT=RENT+0.02
      HALT=(RENT-RTMX)*100.+0.5
      KALT=IFIX(HALT)
      IF(KALT)83,83,84
 84   CONTINUE
```

```
C
C     CHANGE LOCOMOTIVE CHARACTERISTICS
C     ---------------------------------
C
      DO 92 L=1,NL
 92   WRITE(5,910)IK,IL,DD(L),DA(L),DM(L),DT(L),HP(L),WL(L),AXL(L)
 910  FORMAT(1H ,'IT =',I2,'/',I2,1X,4F3.0,F9.0,F11.3,F9.0)
      DO 20 L=1,NL
 20   HP(L)=HP(L)+100.
 501  CONTINUE
      IC=1
 500  CONTINUE
C
C
C  OUTPUT PROCEDURE
C ----------------
C
      NOT=NIT*10
      CALL ARRAY(2,NOT,IAN,100,10,AT,ANCT)
      WRITE(MX,911)
 911  FORMAT(1H1,'  CAPITAL COST OF CRITICAL SPEEDS ')
      CALL PLOT(003,AT,NOT,IAN,100,0)
      WRITE(MX,911)
      CALL MXOUT(003,AT,NOT,IAN,0,40,120,1)
      WRITE(MX,912) AMESH
 912  FORMAT(1H0,'ADHESION MESH = ',F3.0,///)
C
C
C     CHANGE PROBLEM INPUTS
C     ---------------------
C
C
      ISTOP=NVAR-ICT
      IF(ISTOP)406,406,405
 405  ICR=ICR+2
      DO 93 L=1,NL
      HP(L)=100.
      DT(L)=1.
      DA(L)=1.
 93   DD(L)=1.
      NL=1
      IF(JOINT)505,505,506
 506  STAP=((RTMX-RANT)*50.)+1.5
      LAP=IFIX(STAP)
      NRENT=NRENT+LAP
      GO TO 507
 505  CONTINUE
      NRENT=1
 507  CONTINUE
      WRITE(5,908)
      GO TO 404
```

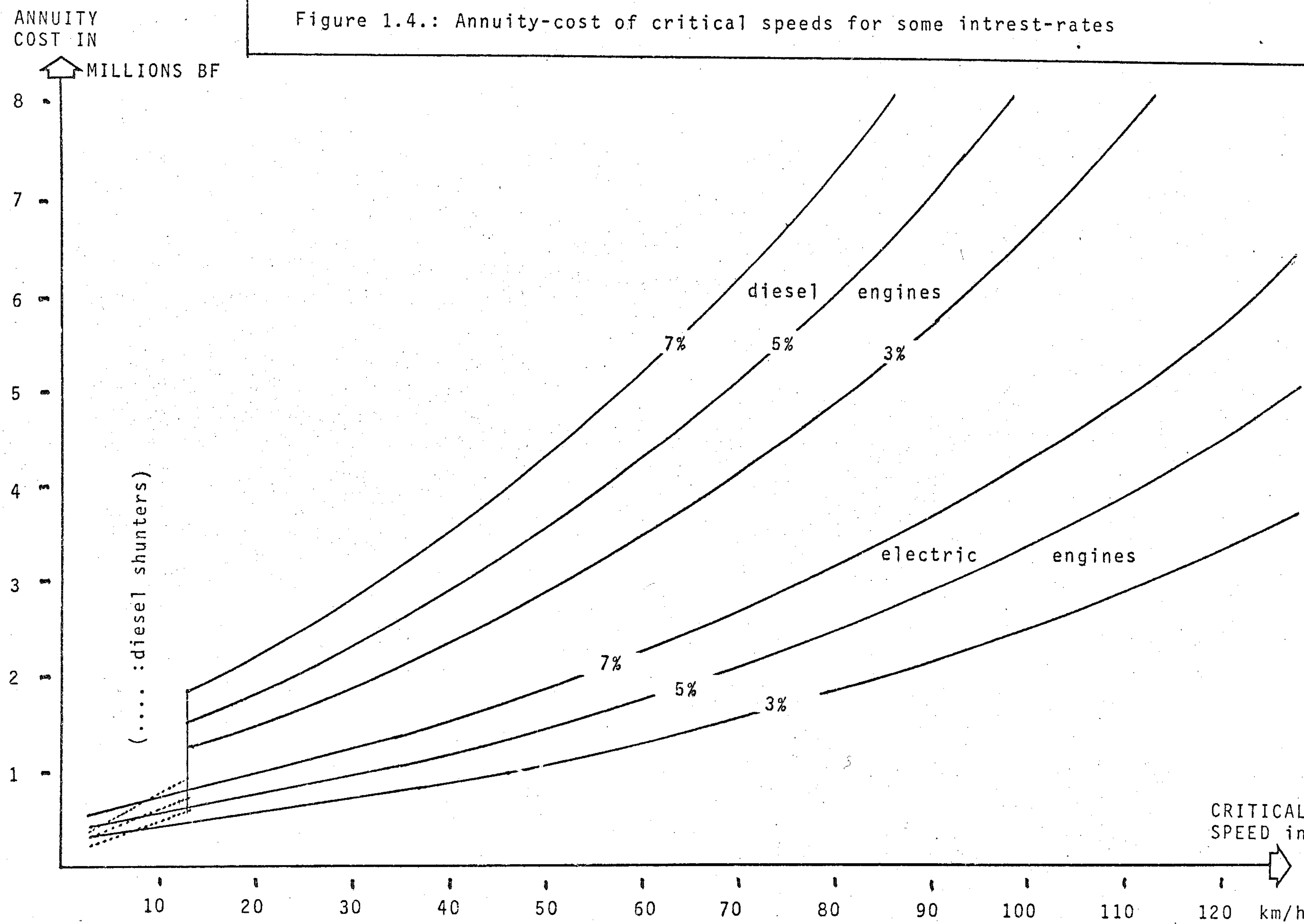
PAGE 6

```
C
C
 406  CONTINUE
      CALL ARRAY(2,NOT,NVAR,100,10,CR,CRIT)
      WRITE(MX,902)
 902  FORMAT(1H1,'  CRITICAL SPEED PLOT ON HORSEPOWER ')
      CALL PLOT(002,CR,NOT,NVAR,100,0)
      WRITE(MX,903)
 903  FORMAT(1H1,' CRITICAL SPEED AS A FUNCTION OF HORSEPOWER',///)
      CALL MXOUT(002,CR,NOT,NVAR,0,41,120,1)
      CALL EXIT
C -----------------------------------------------------------
      END
```

PAGE 9

```
CORE REQUIREMENTS FOR
 COMMON       2  VARIABLES    8484   PROGRAM    2122
```

Figure 1.4.: Annuity-cost of critical speeds for some intrest-rates



3.6. Speed as a power-capital constraint

The basic aim of the proposed calculations was to compute a realistic value of the required capital for motive power, by selecting the relevant technical laws, which enter the discussion. The matrices 2 and 3 of the program, presented in sections 3.5. and 3.6. are providing for the required information.

The simple analytical form of the relationships, of which the output gives the numerical values, can easily be applied in various programming approaches. Some examples have already been sketched in Figure 1.1.

This section will be confined to some applications, which might look trivial by itself, but which get full sense in the context of an integrated problem, whether is be in optimization-problems of the firm, or in network-algorithms.

In section 2.10. the set of feasible speeds was proved to be convex in the solution space (speed and power being non-negative). This property also holds when power is transposed by the cost-relationship to the expression of annuity-costs (a).

The cost of capital is therefore expressed in units which enclose the period of one year and which concern the operations-decision of running a train during that period. In railways practice, the direct allocation of tractive capital to individual passengers'trips is an arbitrary arithmetic. Consumer's choice is assumed mostly to be performed individually, and is not likely to be always expressed by season-tickets of a year.

(a) This follows from the linear cost-transformations, presented in sections 3.1. through 3.3.

In this way, it is necessary to decentralize the operations-decision between the "operations-department" which sells the availability of motive capital to the "commercial-department", which in its turn will use it most efficiently on its network.

By proceeding in this way, the objectives of the operations management can be expressed by a profit maximization problem, subject to a set of technical constraints, which enclose the capacities of infrastructure, rolling-stock and motive capital. The last topic will now be dealt with more specifically.

The boundary of critical speeds possesses the required properties to be used as a constraint in any maximization problem. Obviously it can also be a part of the objective function for any cost-minimization problem, as a particular part of the total costs involved in making a trip.

The analytical expression of the critical speed boundary is already known; its monetary equivalent is :

$$\boxed{ANN = g(v) \qquad \text{with} \quad v = VCRIT}$$

the feasible set being :

$$\boxed{S = \left\{ \{ANN, V\} \mid v \leqslant VCRIT \;\; ; \; V \geqslant 0 \; ; \; ANN \geqslant 0 \right\}}$$

If the boundary is approximated by a linear function, we get :

$$\boxed{ANN = a.VCRIT + b}$$

so that the feasible speed-area is expressed by :

$$\boxed{a.V - ANN \leqslant b}$$

We will now linearize the boundary, more systematically.

First, it could be imagined easily the proposed relationship is a part of a general cost-boundary with similar functions for track- and maintenance-costs. Then, we are actually dealing with two particular terms of a complete boundary as :

$$g_{ij}(x_j) \leqq b_i$$

where all functions "j" are separately expressed. Hence the constraint is a separable function, for it is formed by summing up the required technical relationships.
The presented critical speed-boundary can be linearized approximately by a polygonal function, $\hat{g}_{ij}$, according to some freely choosen mesh-points, "x_{kj}", and a maximum value of x_j, say, "x_{kr}", which is choosen on technological grounds (a).
Any functional value of x_j will then be approximated by a convex combination of the two adjacent mesh-points, of which the real functional values are computed (in our case by the output of the matrices 2 or 3 of the program).

By expressing any x_j as :

$$\boxed{x_j = \sum_{k=0}^{r_j} \lambda_{kj} \cdot x_{kj}}$$

under the restrictions of : $\sum_{k=0}^{r_j} {}_{kj} = 1$

$\lambda_{kj} \geqslant 0$

$\lambda_{sj} \geqslant 0$; $s=k+1$

$\lambda_{lj} \leqslant 0$; $l \neq s$; $l \neq k$

(a) HADLEY G., o.c. (17), pp. 104-116.

The approximate functional value equals :

$$\boxed{\hat{g}_{ij}\ (x_j) = \sum_{k=0}^{r_j} \lambda_{kj} \cdot g_{ij}\ (x_k)}$$

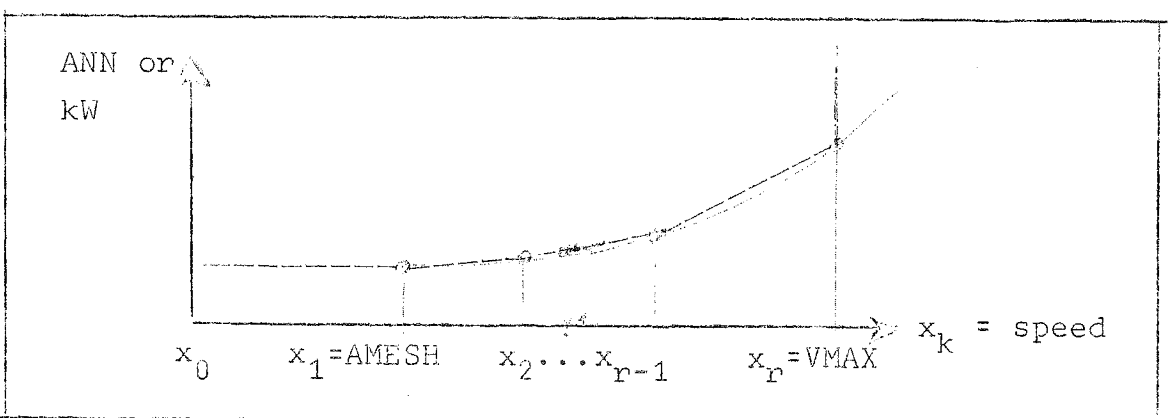
and the linearized constraint is expressed as :

$$\boxed{\sum_{j=1}^{n} \sum_{k=0}^{r_j} g_{ij}\ (x_k).\ \lambda_{kj} \ \lesseqgtr\ b_i}$$ (i, being the constraint sequence)

Together with the other terms in "j", which form the constraint, (or the minimizing objective function), the above expression is to be used in linear programming techniques, with restrictions on the inputs.

In fact, we have not analysed the functional relationships involved to know the value of "$g_{ij}(x_k)$". However, the output of the program provides for sufficiently small intervals, we can freely chooce between (see figure 2.3.).

Figure 2.3. : <u>approximation by a dashed curve</u>



Some mesh-points are necessary to be choosen :

x_1 = AMESH, if the break-point is due to adhesive conditions, $\neq 0$

x_r = VMAX, the maximum administrative speed limitation on the particular connection.

Others are choosen by some evidence, generated by the general conditions of a particular problem. So we can take for

$x_1 \leq x_k \leq x_r$:

- 100 km/h, the maximum speed for general cargo
- 120 km/h, for accelerated cargo services and passenger trains,
- 140 km/h, for express cargo (e.g. containers) and main-line passenger trains,
- 160 km/h, for high-speed passenger trains.
- values beneath the 100 km/h can be taken, according to the track-conditions.

A specific partition of relevant interval of speed is not to be proposed a-priori. First, it is more convenient to apply the same partitioning procedure for infrastructure, rolling stock etc. This reduces the complexity of the problem expression.

Second, the partitioning can change according to the problem. As all parameters of the constraint can change in the long run (by investment in infrastructure facilities), we could have to allow for existing projects in handling the short-run analysis.

As we already mentioned before, the here proposed calculations about the programming approaches are trivial, for we have confined ourselves to the sole use of motive capital. An optimum in this matter can more easily be found by simple substitutions with demand or revenue-equations; also the sense of duality in this single problem will lead us to cost figures we already know. The questions dealt with in this last section, are obviously but useful if investment in motive capital is compared with other allocations which are substitutional in any way.

4. Conclusion

The analysis showed, the requirements of motive capital are easily defined by a series of inputs, which are common knowledge for analysists, who know what is about particular problems and situations they are concerned with. The item which is of particular importance with respect to the marginal effects of it, is maximum train speed, for given track conditions.

A second important feature the analysis dealt with, concerns the class of locomotives. In further cost calculus we are able to distinguish easily the different operations costs with respect to electric or diesel haulage, and also to simulate operations with pulled (or pushed) trains, motortrains, railcars and shunters. All these items are important to be known accurately for defining escapability of operations cost on the calculus level of a separate link, and for policy-objectives concerning :

- the tendency for electrification together with the aim of providing for faster services, due to changing energy-policies (a), railway-relieved projects (b) and a general transport policy (c).
- the growing interest of providing sparsely populated areas for modern transport amenities (a)(b).

In building a successful policy, this particular analysis is a first part of an accurate cost-information.

(a) THE BRITISH TRANSPORT COMMISSION, o.c. (19), pp. 17-18.
(b) CLAESSENS E. & VAN BROEKHOVEN E., o.c. (23), pp. 2,5.
(c) E.E.C. - COMMISSION, o.c. (22), pp. 7,15.

Bibliography and other sources

(1) MEYER J.R., Techniques of Transport Planning, Vol. 1 by, MEYER J.R. & STRASZHEIM R., Pricing and Project Evaluation, The Brockings Institution, Washington D.C., 1971, 343 p.

(2) MEYER J.R., Techniques of Transport Planning, Vol. 2 by, KRESGE D.T. & ROBERTS P.O., Systems Analysis and Simulation Models, The Brookings Institution, Washington D.C., 1971, 228 p.

(3) RESNICK R. & HALLIDAY D., Physics, part 1, Wiley, New-York, 1966, 646 + 63 p.

(4) DEBERDT M., Spoorwegexploitatie, Leuven, 1972, 41 p.

(5) VERBEECK H., Connaissances actuelles de l'adhérence et de son utilisation, Rail International, Brussels, UIC/AICCF, 1973/5.

(6) KOERBER J., Considérations fondamentales concernant l'équipement des engins de traction électriques en moteurs de traction asynchrones, Rail International, o.c., 1975/1.

(7) THOMA A., Kommentar zur Eisenbahn-Bau- und Betriebsordnung, Hestra, Darmstadt, 1969, 311 p.

(8) CAIRE D., Rames automotrices et rames tractées sur les grandes lignes, Chemins de Fer, Paris, AFAC, 1974/5, p. 213-236.

(9) LEWIS W.A., Overhead Costs, Allen & Unwin, 1949, reprinted in Transport, Penguin Ltd., 1968, 325 p.

(10) WONNACOTT R.J. & T.H., Econometrics, Wiley, New-York, 1971, 736 p.

(11) THEIL H., Principles of Econometrics, Wiley, New-York, 1971, 736 p.

(12) HERRMANN O., Dieseltriebfahrzeuge der Scheizerischen Bundesbahnen, Birkhäuser, Basel, 1964, 48 p.

(13) BELGIAN NATIONAL RAILWAYS, operations department, NMBS, Brussels

(14) FRENCH NATIONAL RAILWAYS, operations department, SNCF, Paris.

(15) BAUMOL W.A., Economic Theory and Operations Analysis, Prentice Hall, 3rd ed., London, 1972, 626 p.

(16) HADLEY G., Linear Algebra, Adison-Westley, Reading, 1961, 290 p.

(17) HADLEY G., Non-linear and Dynamic Programming, Adison-Westley, 2°pr., Reading, 1972, 484 p.

(18) ZANGWILL W.I., Non-linear Programming, Prentice Hall, N.J., 1969, 356 p.

(19) THE BRITISH TRANSPORT COMMISSION, Proposals for the Railways, London, H.M.S., 1956, 43 p.

(20) CLAESSENS E. & VAN BROEKHOVEN E., An econometric model of Railroads and Inland Waterways, SESO, Antwerp, 1973, 40 p.

(21) MINISTRY OF TRANSPORT, Railway Policy, London, H.M.S., 1973, 68 p.

(22) COMMISSION OF THE EUROPEAN COMMUNITIES, The Common Transport Policy, Brussels, E.E.C., 1973, 26 p.

(23) CLAESSENS E. & VAN BROEKHOVEN E., Spoorplan '80 - België UFSIA-SESO, Antwerp, 1974, 64 p.

(24) BAUMOL W.J., On the Social Rate of Discount, The American Economic Review, Sept. 1968, pp. 788-802.