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SESO
STUDIECENTRUM VOOR ECONOMISCH EN SOCIAAL ONDERZOEK

RAILROADS and INLAND WATERWAYS
SYNTHESIS

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In the previous articles by J. BUSSCHAERT and E. VAN BROEKHOVEN, demand and supply equations were estimated for both railways and waterways in Belgium (1).

These equations appear to present common variables, indicating the existence of simultaneous interactions between rail and waterways. The object of this essay is to establish a synthesis of the previous research by computing the reduced and final form of this set of simultaneous equations.

Admittedly, the presence of simultaneous equations calls for special estimation techniques, other than the ordinary least squares estimation, applied by BUSSCHAERT and VAN BROEKHOVEN. Nevertheless these estimates will be accepted as such, and our synthesis will be confined to the further algebraic computations on the basis of the structural equations as they were estimated (2).

In §1. the structural equations are repeated, in §2. the reduced form is treated, while §3. contains the final form.

§1. STRUCTURAL EQUATIONS

For the sake of clarity, the symbols, introduced in the previous articles will be redifined:

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(1) See J. BUSSCHAERT and E. VAN BROEKHOVEN, Cost and supply functions in railroad transportation, SESO-werknota nr.7316/851; and J. BUSSCHAERT and E. VAN BROEKHOVEN, An econometric model of Belgian inland waterway transportation, SESO-werknota nr. 7313/851.

(2) The author is indebted to Mr.E. BORGHERS for programming help; see E. BORGHERS, Programmabeschrijvingen - Deel I, SESO-werknota 7321.

Table 1. List of variables: structural model; railroads and waterways

y_j		current endogeneous
y_1	PX	log price railroads' passenger transportation
y_2	QX	log output railroads' passenger transportation
y_3	PF	log price railroads' freight transportation
y_4	QF	log output railroads' freight transportation
y_5	PW	log price inland waterways
y_6	QW	log output inland waterways
y_i^*		lagged endogeneous
y_3^*	PF^*	log price railroads' freight transportation
y_5^*	PW^*	log price inland waterways
x_k		current exogeneous
x_1	DP	log industrial production
x_2	PR	log price Rhine navigation (Rotterdam-Mannheim)
x_4	PC	log price car, private cost of intercity-transportation
x_5	PT	log price car, time-opportunity travelling cost
x_6	DY	log disposable income
x_7	PL	log price, labour input
x_8	PE	log price, energy and maintenance costs
x_9	PZ	log price, capital cost
x_{10}	FR	log number of days of frost
x_k^*		lagged exogeneous
x^*	DP^*	log industrial production
x_2^*	PR^*	log price Rhine navigation (Rotterdam-Mannheim)
x_3^*	WC^*	log waiting capacity in inland waterways

Constant terms and time trends will disappear in this study, as we are not interested in the effect of these variables.

Let us represent the vector of current endogeneous variables by:

$$y_t = (PX, QX, PF, QF, PW, QW)'$$

and the vector of predetermined variables by:

$$x_p \quad (y_{t-1} \quad ; \quad x_t \quad ; \quad x_{t-1})'$$

of which the operating variables, connected with non-zero coefficients in the estimated structural equations are:

$$x = (PF^{\#}, PW^{\#};\ DP, PR, PC, PT, DY, PL, PE, PZ, FR;\ DP^{\#}, PR^{\#}, WC^{\#})'.$$

The structural equations can then be written:

$$\Gamma . y_t = B . x$$

The coefficient matrices have been estimated in the previous SESO-articles to be:

Γ =

j \ i	PX	QX	PF	QF	PW	QW
PX	1.00	-0.27	.	-0.12	.	.
QX	0.27	1.00	.	.	.	.
PF	.	-1.27	1.00	0.88	.	.
QF	.	.	0.72	1.00	-0.49	.
PW	.	.	.	.	1.00	-0.43
QW	.	.	-0.14	.	0.07	1.00

while B equals:

j \ k	PF‡	PW‡	DP	PR	PC	PT	DY	PL	PE	PZ	FR	DP‡	PR‡	WC‡
PX	.	.	.	.	.	.	.	0.89	0.06	0.05	.	.	.	.
QX	.	.	.	.	-0.33	-0.28	-0.72	.	.	.	.	.	.	.
PF	.	.	.	.	.	.	.	0.89	0.06	0.05	.	.	.	.
QF	.	.	0.07	.	.	.	.	.	.	.	.	.	.	.
PW	.	0.70	.	0.04	.	.	.	.	.	.	-0.08	.	-0.07	-0.11
QW	0.45	-0.30	0.19	.	.	.	.	.	.	.	.	0.50	.	.
col.	y_3	y_5	x_1	x_2	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	$x_1^{\ddagger}$	$x_2^{\ddagger}$	$x_3^{\ddagger}$
mat.	B_1		B_2									B_3		

The vertical dotted lines in the matrix B, separate the lagged endogeneous, the current exogeneous and the lagged exogeneous variables from each other. This distinction of course is not important for the computation of the reduced form but is needed when computing the final form.

The separated sub-matrices B_1, B_2 and B_3 are respectively of order 6 x 2, 6 x 9 and 6 x 3.

§2. THE REDUCED FORM

Pre-multiplying the structural matrix by the inverse Γ^{-1}, the reduced form can be computed:

$$y_t = \Gamma^{-1}.B.x = \pi.x$$

To compute the matrix $\pi = \Gamma^{-1}.B$, we first need Γ^{-1}, which is:

Γ^{-1} =

i \ j	PX	QX	PF	QF	PW	QW
PX	1.000	-0.001	-0.212	0.306	0.146	0.063
QX	-0.270	1.000	0.051	-0.083	-0.039	-0.017
PF	-0.875	3.249	2.739	-2.515	-1.197	-0.515
QF	0.605	-2.242	-1.894	2.739	1.303	0.560
PW	-0.051	0.189	0.160	-0.147	0.901	0.387
QW	-0.119	0.441	0.372	-0.342	-0.231	0.901

Postmultiplying this with B, we obtain π, as given in Table 2. This matrix will separately be commented, for the coefficients of lagged endogeneous variables (matrix π_1) and for those of the exogeneous variables (current in matrix π_2 and lagged in matrix π_3).

2.1. Lagged endogeneous influences

The treatment of this sub-matrix π_1 doesn't confront us with a true dynamic model, but only intends to fix some basic ideas about lagged effects of PW and PF.

As is seen from the first two rows in π_1, passenger traffic (QX and PX) does not react very much upon price changes of inland waterways and freight services of rail. It is known already from the structural equations (cfr. the matrix Γ) that passenger output clearly affects the supply price of rail freight, while the reciprocal influences is less important. This feature remains present in the reduced form.

The freight coefficients are indicating an interesting difference between railroads and waterways with respect to their mutual interaction. This is shown in table 3, giving in:

3a) the "single" price elasticities of demand and supply by waterways and railraods;

3b) the "simultaneous" price elasticities of these variables.

REDUCED FORM COEFFICIENTS

Table 3. matrix $\Gamma^{-1}B = \pi = [\pi_1 \vdots \pi_2 \vdots \pi_3]$

k / j	PF‡	PW‡	DP	PR	PC	PT	DY	PI	PE	PZ	FR	DP‡	PR‡	WC‡	k / j
PX	0.028	0.083	0.309	0.006	0.001	-0.001	-0.001	0.701	0.047	0.039	-0.012	0.031	-0.010	-0.016	PX
QX	-0.008	-0.022	-0.083	-0.002	-0.330	-0.280	-0.720	-0.189	-0.013	-0.011	0.003	-0.008	0.003	0.004	QX
PF	-0.232	-0.683	-2.538	-0.048	-1.107	-0.908	-2.334	1.659	0.112	0.093	0.096	-0.257	0.084	0.132	PF
QF	0.252	0.744	2.763	0.052	0.740	0.628	1.614	-1.147	-0.077	-0.064	-0.104	0.280	-0.091	-0.143	QF
PW	0.174	0.514	-0.069	0.036	-0.063	-0.053	-0.136	0.097	0.007	-0.005	-0.072	0.194	-0.063	-0.099	PW
QW	0.405	-0.432	-0.160	-0.009	-0.145	-0.123	-0.317	0.225	0.015	0.013	0.018	0.454	0.016	0.025	QW
col.	$y_3^{\ddagger}$	$y_5^{\ddagger}$	x_1	x_2	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	$x_1^{\ddagger}$	$x_2^{\ddagger}$	$x_3^{\ddagger}$	col.
mat.	π_1		π_2 - impact multipliers - $\pi_{\cdot 2}$									π_3			mat.

The elasticities in table 3a are the coefficients in the structural equations and should be interpreted under the marshallian ceteris paribus conditions about the other endogeneous variables, which affect transport supply and demand and which also appear in the structural equations.

The elasticities in Table 3b however, incorporate the effect of price changes through other endogeneous variables such as the price, set by the competitors. They are obtained from reduced form computation, which explicitly accounts for the reactions of the sole competitor and also for supply conditions. Hence, ceteris paribus conditions only concern influences, exogeneous to the system as a whole.

The difference between rail and waterways is clear:

$$\eta_{wm} = -0.37 \qquad \eta_{ws} = -0.43$$

$$\eta_{fm} = -0.72 \qquad \eta_{fs} = +0.25 \qquad (1)$$

In the case of waterways the simultaneous slope is more elastic. This reflects the fact that deviating demand from rail to waterways results in an increase of marginal cost (and price) of railways, which causes a new increase of demand for waterway transport.

Railways on the other hand act oppositely. Simultaneous price elasticity exceeds single price elasticity. It is even slightly positive. This is partly due to the sharp reaction of the competitor,

$$\frac{\partial PW}{\partial PF^{*}} = 0.74.$$

(1) w: waterways
f: freight services of rail
m: single (marshallian)
s: simultaneous.

Table 3.

a) SINGLE $(I-\Gamma)$ & B_1

	WATERWAYS	RAIL-(FREIGHT)
quantity demanded	$\frac{\partial QW}{\partial PW} + \frac{\partial QW}{\partial PW^{\neq}} = -0.37$ $\frac{\partial QW}{\partial PF} + \frac{\partial QW}{\partial PF^{\neq}} = 0.59$	$\frac{\partial QF}{\partial PF} = -0.72$ $\frac{\partial QF}{\partial PW} = 0.49$
quantity supplied	$\frac{\partial QW}{\partial PW} + \frac{\partial QW}{\partial PW^{\neq}} = 0.68$	$\frac{\partial QF}{\partial PF} = -0.88$

b) SIMULTANEOUS (π_1)

quantity	$\frac{\partial QW}{\partial PW^{\neq}} = -0.43$ $\frac{\partial QW}{\partial PF^{\neq}} = 0.41$	$\frac{\partial QF}{\partial PF^{\neq}} = 0.25$ $\frac{\partial QF}{\partial PW^{\neq}} = 0.74$
int.var.	$\frac{\partial PW}{\partial PF^{\neq}} = 0.74$	$\frac{\partial PF}{\partial PW^{\neq}} = -0.68$
price dyn.	$\frac{\partial PW}{\partial PW^{\neq}} = 0.51$	$\frac{\partial PF}{\partial PF^{\neq}} = -0.23$

Generally, the competition between railroads and waterways is sketched by their mutual price reactions:

$$\frac{\partial PW}{\partial PF^{*}} = 0.74 \qquad \frac{\partial PF}{\partial PW^{*}} = -0.68$$

These values possess opposite signs. This implies directly increasing returns-conditions of the railways. Otherwise, these results were inconsistent with regard to the positive cross-demand slopes, where both sectors were found substitutes.

These simultaneous interactions between rail and waterways can be summarized:

- both industries are closely related, for they are common substitutes;
- this substitution, indicated by cross-demand, is to be connected with the economies of scale of the railways. These conditions bring about an opposite price policy in railway supply, which intersectorial variation clearly indicates;
- at last, railroads are jointly meeting a passenger output, which causes variations in the supply price of their freight sector. This however appear to be a one-way influence.

2.2. Lagged exogeneous variables

The matrix π_3, containing the influence of lagged exogeneous variables shows that among these variables, industrial production is the most decisive one. The higher elasticity of rail output (in comparison with waterways) can again be explained by economies of scale in railroad operations. It is seen indeed that the price of rail transport decreases as a result of increasing demand.

§3. FINAL FORM (1)

Using the conventional expression of the reduced form:

$$y_t = D_1 y_{t-1} + D_2 x_t + D_3 x_{t-1},$$

we have to define the matrix D_1 of order 6 x 6 as our matrix π_1 + four zero-columns, the matrix D_2 of order 6 x 10 as our matrix π_2 + one zero-column (for WC unlagged) and the matrix D_3 of order 6 x 10 as our matrix π_3 + seven zero-columns.

A complete impact of one lag upon the current endogeneous variables is than given by the 6 x 10 matrix:

$$(D_1 D_2 + D_3).$$

Provided these impacts are decreasing over consecutive periods, they attain after an infinite number of periods the cumulated value:

$$(I - D_1)^{-1} \; (D_1 D_2 + D_3)$$

which expresses the interim multipliers.

D_1 is of rank two. Its non-zero characteristic roots are:

$$\lambda_1 = -0.23$$
$$\lambda_2 = \;\; 0.51$$

which both are real and less than unity. This warrants the required outdampening behaviour, and allows us to compute the interim multipliers which are given on Table 4.

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(1) H. THEIL, J.C.G. BOOT; The final form of econometric equation systems, Journal of the International Statistical Institute, vol.30; 2; 1962; pp.136-152.

FINAL FORM COEFFICIENTS

Table 4. Matrix $(1-D_1)^{-1}(D_1D_2+D_3)$ interim multipliers

j \ k	DP	PR	WC	PC	PT	DY	PL	PE	PZ	FR	k / j
PX	-0.064	-0.012	-0.022	-0.049	-0.042	-0.108	0.076	0.005	0.004	-0.005	PX
QX	0.017	0.003	0.006	0.013	0.011	0.029	-0.021	-0.001	-0.001	0.001	QX
PF	0.526	0.098	0.184	0.405	0.344	0.884	-0.628	-0.042	-0.035	0.038	PF
QF	-0.573	-0.107	-0.200	-0.441	-0.374	-0.962	0.684	0.046	0.038	-0.041	QF
PW	-0.396	-0.074	-0.138	-0.305	-0.259	-0.665	0.473	0.032	0.027	-0.028	PW
QW	-0.164	0.053	0.159	-0.111	-0.094	-0.242	0.172	0.012	0.010	0.097	QW

Table 5. Matrix $(1-D_1)^{-1}(D_1D_2+D_3)+D_2$ total multipliers

j \ k	DP	PR	WC	PC	PT	DY	PL	PE	PZ	FR	k / j
PX	0.245	-0.006	-0.022	-0.050	-0.042	-0.108	0.778	0.052	0.044	-0.162	PX
QX	-0.066	0.002	0.006	-0.317	-0.269	-0.691	-0.210	-0.014	-0.012	0.004	QX
PF	-2.011	0.050	0.184	-0.665	-0.564	-0.145	1.031	0.069	0.058	0.133	PF
QF	2.190	-0.055	-0.200	0.299	0.253	0.652	-0.463	-0.031	-0.026	-0.145	QF
PW	-0.465	-0.377	-0.138	-0.367	-0.312	-0.802	0.570	0.038	0.032	-0.100	PW
QW	-0.325	0.043	0.159	-0.256	-0.218	-0.559	0.397	0.027	0.022	0.116	QW

The direct impacts within the same period are already known to be the impact multipliers of D_2. The sum of these joint influences gives us the final form:

$$y = [D_2 + (I-D_1)^{-1} (D_1D_2+D_3)].x$$

These coefficients are given in Table 5. In table 6 they are ranked in descending order of magnitude.

The last required information concerns how interim action behaves in a relevant range of consecutive periods. The matrix of total multipliers:

$$D_2 + (I-D_1)^{-1} (D_1D_2+D_3) = (I-D_1)^{-1}(D_2+D_3)$$

is already known to be the sum of the matrix of impact multipliers D_2 and the successive coefficients matrices $D_1^i(D_1D_2+D_3), i=0...\infty$.

For some crucial variables the coefficients in these matrices will now be plotted, starting from D_2, and cumulating successively with $D_1^i(D_1D_2+D_3)$ for $i=0...4$. In the tables 7a to 7d it clearly appears that after five semestres (i=4) all important fluctuations have already disappeared.

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The most important exogeneous variables are clearly industrial production and disposable income. Both railways and waterways are strongly affected by these variables. The negative influence of disposable income upon passenger transport by rail is easily explained by the relative inferiority of this mode in comparison with car usage. The positive influence upon freight transport by rail is also obvious. It is increased by the economies of scale in this industry, which result in decreasing supply prices. These economies of scale also account for the surprising negative effect of income and production upon inland waterways.

Table 6. Slection of the main multipliers

FROM	TO	TOTAL		IMPACT		INTERIM
DP	QF	1	2.11	1	2.76	-0.57
DP	PF	2	-2.01	2	-2.54	0.53
DY	PF	3	-1.45	3	-2.33	0.88
PL	PF	4	1.03	4	1.66	-0.63
DY	PW	5	-0.80	33	-0.14	-0.67
PL	PX	6	0.78	12	0.70	0.08
DY	QX	7	-0.69	11	-0.72	0.03
PC	PF	8	-0.67	7	-1.11	0.41
DY	QF	9	0.65	5	1.61	-0.96
PL	PW	10	0.57	39	0.10	0.47
PT	PF	11	-0.56	8	-0.91	0.34
DY	QW	12	-0.56	20	-0.32	-0.24
DP	PW	13	-0.47	47	-0.07	-0.40
PL	QF	14	-0.46	6	-1.15	0.68
PL	QW	15	0.40	27	0.23	0.17
PC	PW	16	-0.37	51	-0.06	0.31
DP	QW	17	-0.33	31	-0.15	-0.16
PC	QX	18	-0.32	19	-0.33	-0.01
PT	PW	19	-0.31	52	-0.05	-0.26
PC	QF	20	0.30	10	0.74	-0.44
PT	QX	21	-0.27	22	-0.28	0.01
PT	QF	22	0.25	14	0.63	-0.37
DP	PX	23	0.25	21	0.31	0.06
PT	QW	24	-0.22	35	-0.12	-0.09
PL	QX	25	-0.21	29	-0.19	-0.02
WC	QF	26	-0.20		0.00	-0.20
WC	PF	27	0.18		0.00	0.18
WC	QW	28	0.16		0.00	0.16
PF	QF	29	-0.15	37	-0.10	-0.05
WC	PW	30	-0.14		0.00	-0.14
FR	PF	31	0.13	40	0.10	0.04
FR	QW	32	0.12		0.02	0.10
DY	PX	33	-0.11		-0.001	-0.11
FR	PW	34	-0.10	47	-0.07	-0.03
PE	PF	35	0.07	36	0.11	-0.04
DP	QX	36	-0.07	45	-0.08	0.02
PZ	PF	37	0.06	41	0.09	-0.04
PR	QF	38	-0.06	53	0.05	-0.11
PE	PX	39	0.05	55	0.05	0.01
PC	PX	40	-0.05		0.001	-0.05
PR	PF	40	0.05	54	-0.05	0.10

As an example for cost inputs, the price of labour (PL) indicates an opposite behaviour (Table 7c). As a direct onsequence, both supply prices of railroads and waterways are pulled upwards, but railroads respond more elastically. Indirectly, waterways suddenly meet a sharply increasing demand after the first lag. At the same time their general price level increases also, which induces again a decrease in the quantity demanded.

This last example clearly indicates that time parths are necessary to distill the consistency in the interim action. Of this movement intermediary oscillations mostly are more important than their cumulated value, which fades away the most relevant information. The importance of this interim action for each separate lag proves also the predominant influence from lagged endogeneous action, expressed by π_1.

Table 7a : Effects of DP (industrial production)

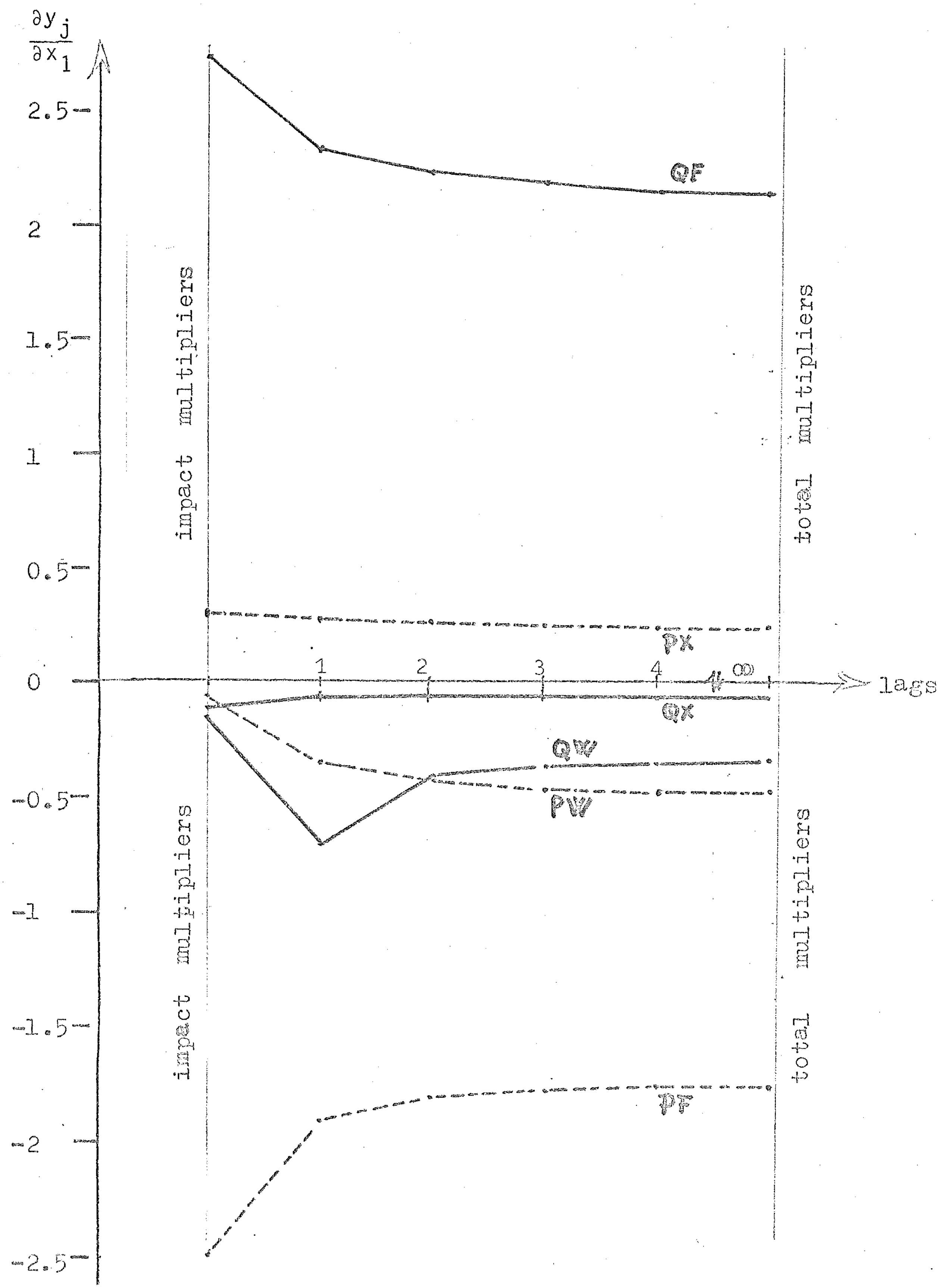


Table 7b : Effects of DY (disposable income)

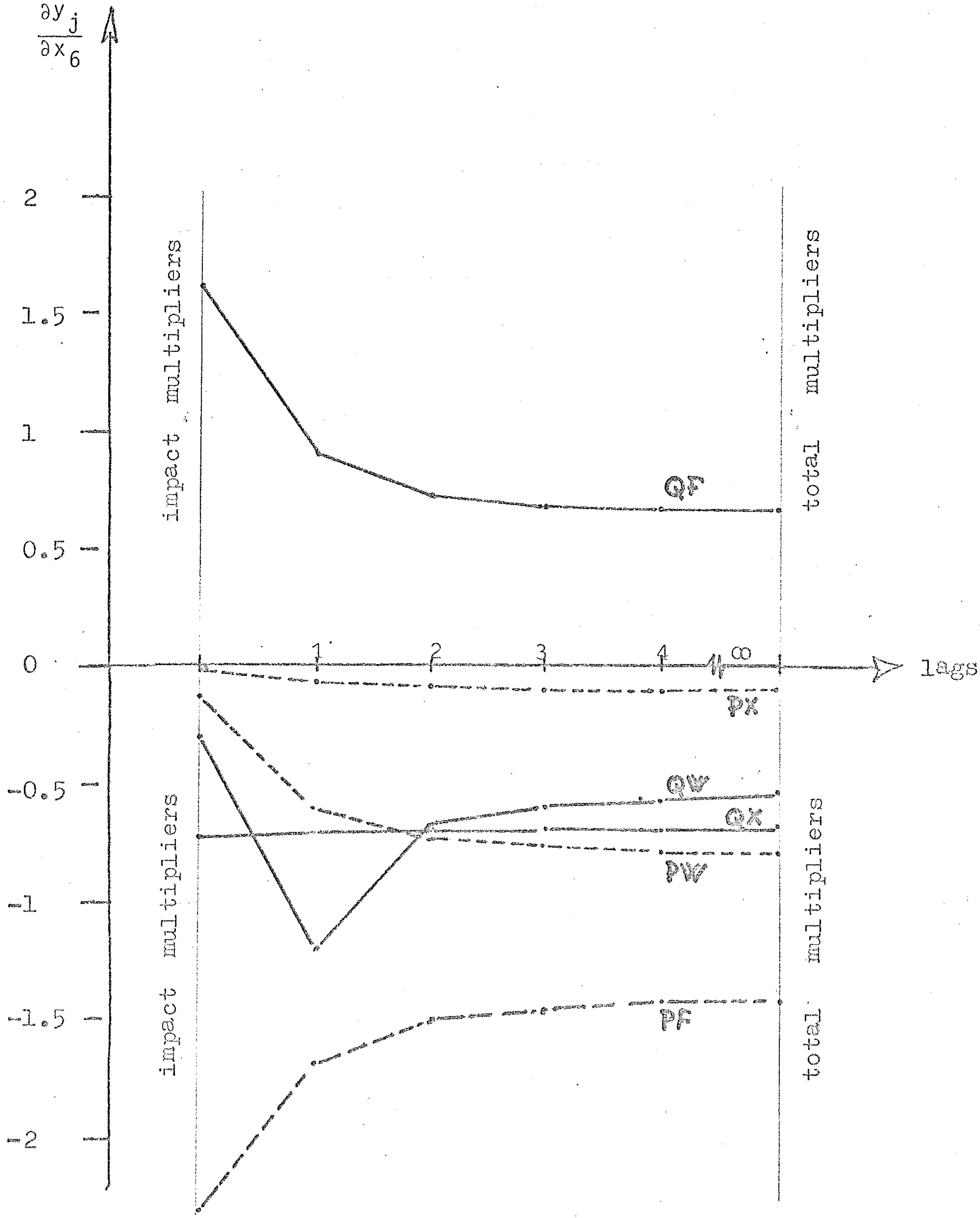


Table 7c : Effects of PL (input price of labour)

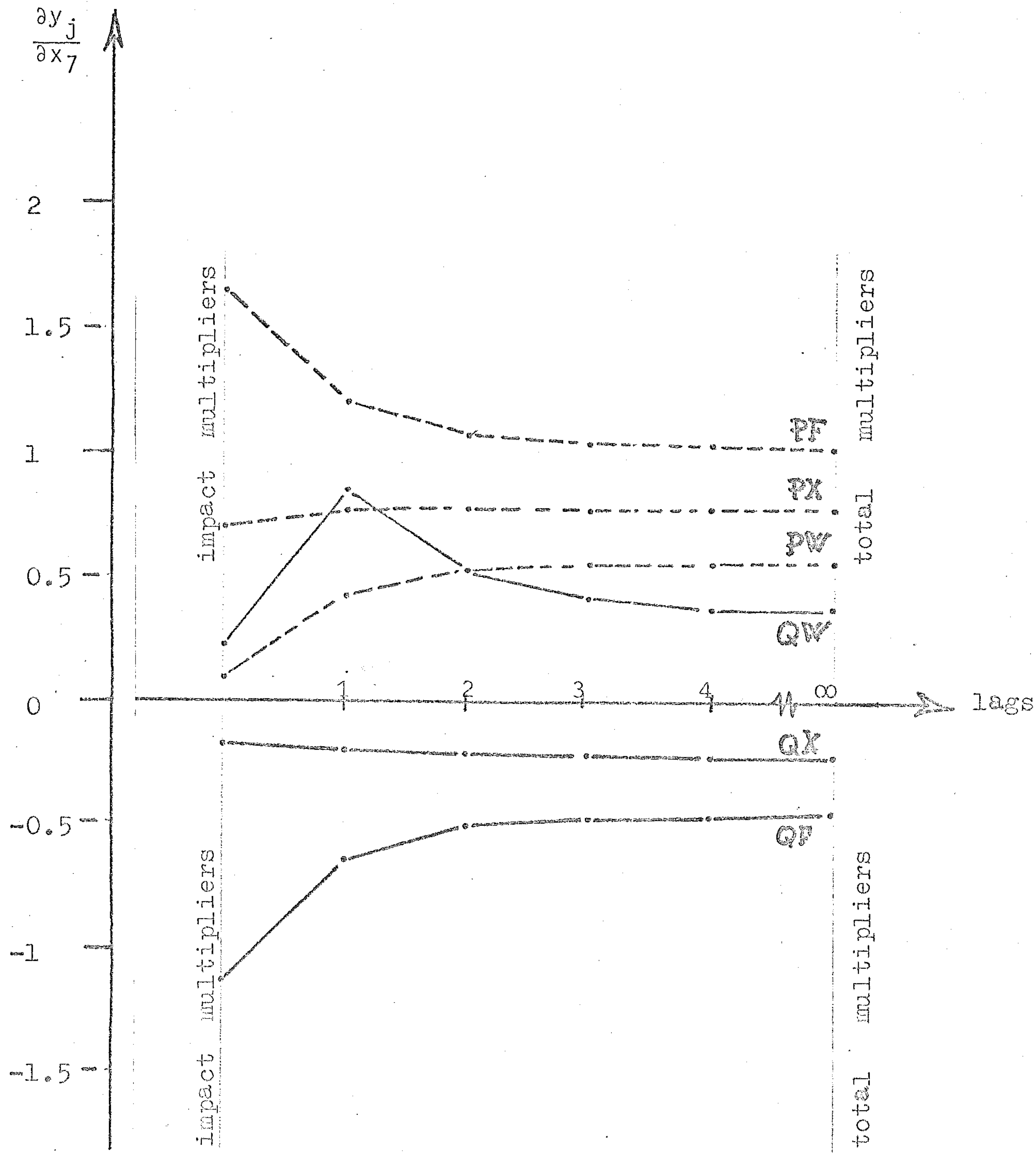
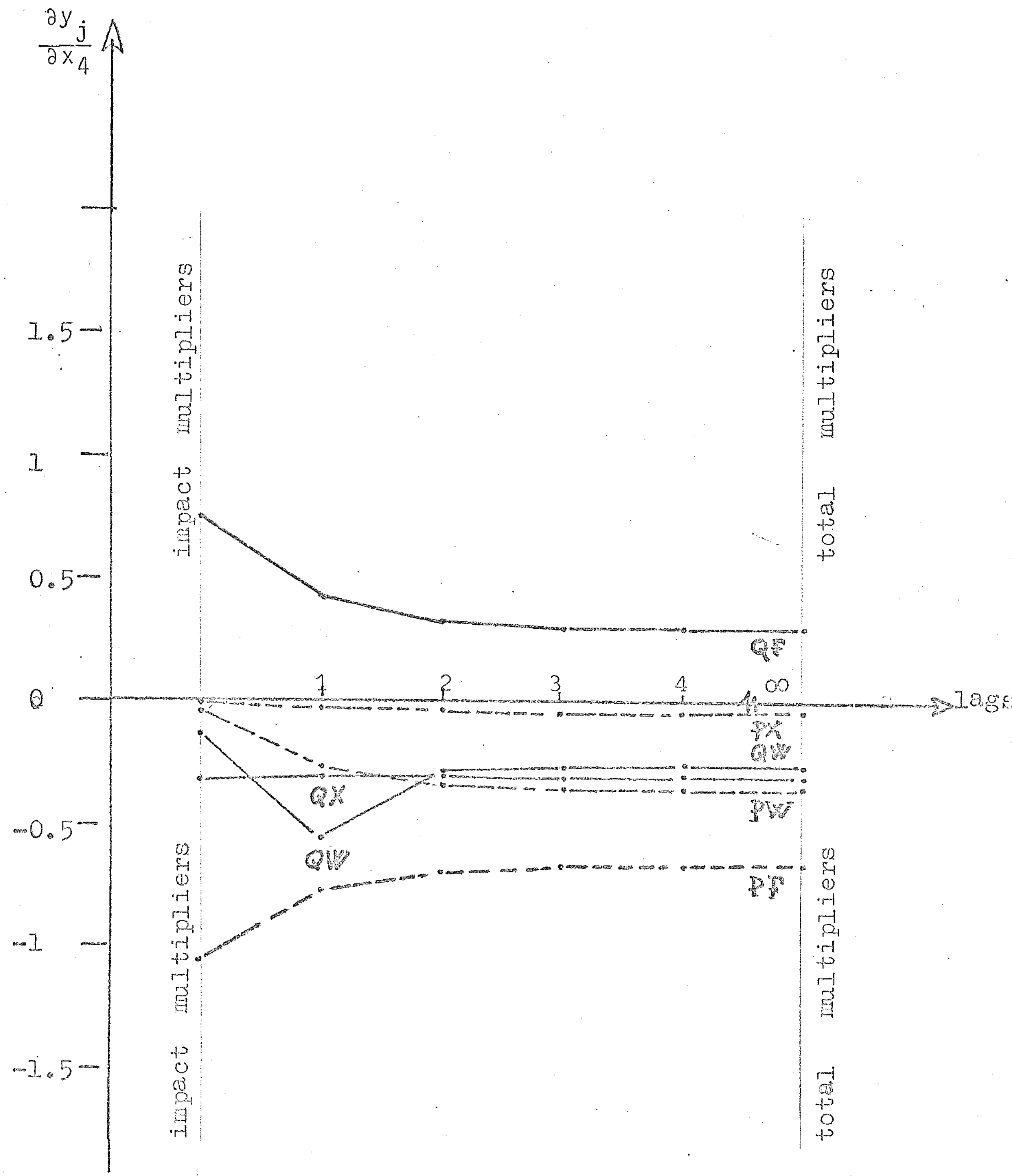


Table 7d : Effects of PC (private cost of car)



§4. CONCLUSIONS

This conclusion is to be seen as an additional one to previous papers. Only variables and actions have been discussed which here are important to understand the behaviour of the interaction between the here concerned industries, rather than the separate industries themselves. This splits the concusion into two parts, the respective policies and the main variables.

The most important variables in this model are industrial production, disposable income together with the cost of car transportation, and finally labour cost as the leading value of cost inputs. They are typically factors which generally affect all equations. Other variables, especially those which concern waterways, are mostly restricted to a particular sector. Their importance thus remains to be structurally important, without great implications in simultaneous action.

Industrial production is a variable which purely express an induction of traffic. It most sensitively affects railroads freight demand, with little effect upon passenger traffic and waterways. An opposite corrollary concerns the labour price, which shows interesting indirect impacts upon the other sectors, especially waterways. The interaction between rail and waterways on the freight market is here shown to be based upon their simultaneous demand conditions. Disposable income at last expresses the negative income effects upon passenger traffic by rail and the rather complex consequences, expecially upon waterways.

Starting from an increase in disposable income, the interactions on the transport market can very typically be described as follows:

1. Passenger traffic decreases according to the negative income elasticity.
2. The resulting excess of railway capacity is transferred to the freight market by a reduction in freight tariffs.
3. This results in decreasing demand for waterway transport.

This of course does not mean that the future waterways will seccessively continue to lose their market share. Other exogeneous variables (such as the price of inputs) clearly have a negative impact upon railways and a positive impact upon waterways.

It should finally be noticed, this research remains preliminary. Further research should imply the reformulation of the structural equations. Some variables which now are treated as exogeneous, could be noted endogeneously, for instance waiting capacity. Other exogeneous variables should be reformulated; for instance, the parallel behaviour of social cost of car transport (private and time opportunity cost) in connection with income could be expressed by a single variable, and also a variable ought to be found which marks the weather conditions better off semestre lags than the present specification of frost days.

These modifications can substantially change the whole picture of the system.

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