STUDIECENTRUM VOOR ECONOMISCH EN SOCIAAL ONDERZOEK

COST AND SUPPLY FUNCTIONS IN RAILROAD TRANSPORTATION

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1. AIM OF THIS STUDY (1)

The main goal of this study is, to establish the cost function of the railroad industry and find out whether or not the industry considered is operating under conditions of increasing returns to scale, and to make a statement about the nature of the change in costs provided that output were increased by a small amount.

There are some distinctions between this and other railroad cost studies:

1° We wanted to estimate the separate effects on total costs of passenger transportation on the one hand and freight transportation on the other, under the restriction that the allocation of total costs to either kinds of service was not made available by the Belgian Railroad Administration. Some authors did not have to encounter this difficulty; see e.g. G.H. Borts in /1/. Other authors handled the problem by establishing a fixed proportion between the two outputs; see e.g. L.R. Klein /2/ and Von Nördling /3/. The solution which was used in this study will be presented in section 2.

2° In the case of Belgium, the railroad transportation industry consists only of a single supplier, according to a law of July 23d. 1926. This law also prohibits the entry of other firms to the industry, and stipulates the articles of association, from which it is clear that this monopoly is managed and controlled by the State and pursuists an exploitation, ruled by principles of economic efficiency, yet under the restriction that the general economic intrests of the country are safeguarded. Under these circumstances it will be clear that a) the policy which is followed by the railroad company will not precisely fall in line with the monopolistic model of the market. Yet

(1) Other econometric railroad cost studies have been included in the bibliography.

it is not clear which commercial policy will be followed the market.
b) The policy actually followed may be shifting constantly, and these shifts might correlate with the consecutive administrations. These matters will be touched upon in the final section of this study.

3° The cost figures that were released for this study only refer to aggregates of the national track. It was impossible to obtain cross-section results of the explcitation costs of single lines. Obviously one could only study <u>time series</u> of aggregates. In fact, this is a disadvantage, since a kind of "regression fallacy" may occur. We are not referring here to the difficulty which was pointed out by G.H. Borts in <u>/1/</u>. He remarked, that if capacity decisions are made incorrectly, joint density between costs and output might show biased parameters. In our case this regression fallacy is unlikely to occur because we only work with operating costs, capital outlays could not be considered (see section 3).

Instead, in our study important biases may occur on account of the use of time series. Changes in factor prices, or the occurance of technical change in the course of the time series might bias both the long run (see figure 1.a) and the short run cost elasticities (see figure 1.b).

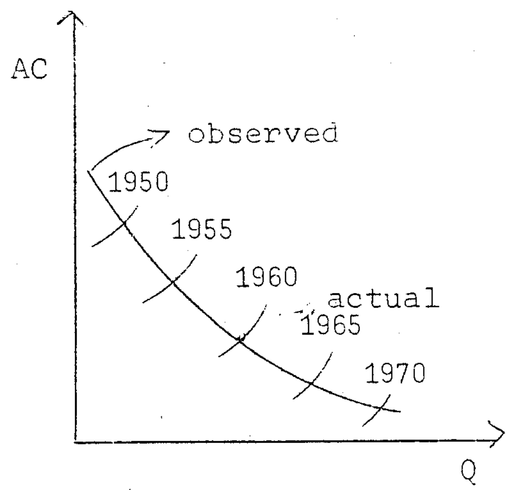

Figure 1.a

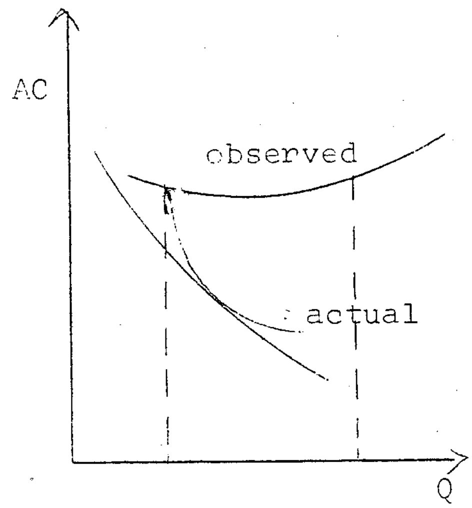

Figure 1.b

In this study, allowance will be made for this difficulty in that factor prices as well as a variable accounting for shifts in the cost curve due to technical progress appear in the cost function as a result of the specification of the production function.

2. THE MODEL

2.1. Assumptions

(a) The Railroad Company in Belgium exploits a network of regular connections, which have not changed basically for a long number of years. In the course of the company's history, one will perceive periods of expansion as well as periods of contraction of the network as such, as well as of its capability of supplying services. However, these expansions and contractions have come off quite gradually; whereas the main characteristic of the network, notably the c-bweb-like pattern with Brussels as its center, has always been preserved. More particularly, this means that the creation of new connections between industry poles (e.g. between Ruhr and Antwerp, or between Liège and Antwerp or between Antwerp-Ghent-Kortrijk-Lille) have never been envisaged. Obviously, this way of doing business will most certainly have caused the drain of a good deal of profits. This last remark is important for the present study, in that it implies that our model cannot be based on the assumption that the industry concerned is maximizing its output at ruling transport prices. In other words the Belgian Railway Company doesn't quite follow the monopolistic nor the competitive model. The picture of management leaves rather the impression, that the Belgian Railroad Company exploits a (given) network that the authorities have imposed on it.

Under these circumstances, one can proceed on the assumption that the company intends to minimize its costs, for given levels of output X^o and Y^o.

(b)
It is known, that the Belgian Railroad Company produces two outputs: passenger and freight transportation. For the production of these outputs, it combines three input factors: labour, energy (gasoil and electricity) and capital. If the outputs are represented by X and by X and by Y respectively, and the input factors by x, y and z, the

essentials of the existence of joint production, may be expressed mathematically in this way:

$$\frac{\delta X}{\delta x} > 0 \qquad \frac{\delta X}{\delta y} > 0 \qquad \frac{\delta X}{\delta z} > 0$$

$$\frac{\delta Y}{\delta x} > 0 \qquad \frac{\delta Y}{\delta y} > 0 \qquad \frac{\delta Y}{\delta z} > 0$$

This is very simple. But, a complicated problem is the statement about complementarity or substitution in production. It is difficult, as an outsider, to say what will be the effect on $\frac{\delta X}{\delta x}$ for instance, when the levels of Y, y or z would be increased with a trifle etc. Will the effect be positive, zero, or negative? We do think that the effect of the level of the input factorsenergy or capital on the marginal productivity of labour (etc.) will most likely be possitive. So, we assume complementarity in production. We further believe the law of diminishing returns to hold; i.e. we think the marginal product of a factor decreases with rising output levels.

2.2. The model

Our model has to allow for the assumptions stipulated in subsection 2.1., as well as for the fact that we want to know the separate cost effects of changes in passenger or changes in freight transportation levels. We therefore suggest:

to minimize costs $C = p_x.x + p_y.y + p_z.z$

when the production function $X^{\alpha}Y^{\beta} = x^a.y^b.z^c.e^{\varepsilon t}$ holds.

X, Y = the outputs

x, y, z = the inputs

p_x, p_y, p_z = the prices of productive inputs.

One will notice, that the production function is of the Cobb-Douglas type, generalized for the case of two outputs. Technical progress was assumed to be neutral in the Hicks-sense.

This function has some more, and some less attractive properties,

(1) The analysis of this function gives occasion to an investigation of a cost function which potentially provides a plausible fit to the data.

(2) The connection between the coefficients of the cost function and the coefficients of the production relation is quite simple. It is even so that the estimation of the cost function suffices for the deduction of the coefficients of the production function, and of the factor demand equations.

(3) For the estimation, OLS can be used. Vinod (1), who proceded from the same model in his study of the estimation problems of joint production, applied canonical analysis. The results before us show that the problem can be approached in a simpler way from an econometric viewpoint and in a more interesting way, from an economic viewpoint.

(4) The elasticity of substitution is fixed at a not so plausible level.

In order to determine the minimum of $C=p_x.x+p_y.y+p_z.z$ under the subsidiairy condition that $F=X^{-\alpha}Y^{-\beta}x^a y^b z^c e^{\varepsilon t}-1=0$, the method of the Lagrangean multiplier is used. So, we can write the expression:

$$W = p_x.x + p_z.z + p_y.y - \lambda(F).$$

The first order conditions of the minimum of this function with respect to the variables x, y and z are:

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(1) H.O. Vinod, Economics of joint production, Econometrica, vol.36, April 1968.

$$\begin{cases} \dfrac{\delta W}{\delta x} = p_x - \lambda F'_x \\ \dfrac{\delta W}{\delta y} = p_y - \lambda F'_y \\ \dfrac{\delta W}{\delta z} = p_z - \lambda F'_z \\ \dfrac{\delta W}{\delta(-\lambda)} = F = X^{-\alpha}.Y^{-\beta}.x^a.y^b.z^c.e^{\varepsilon t} - 1 = 0 \end{cases} \qquad (2.2.1.)$$

where F'_x = the partial derivative of the function F to x, i.e., the marginal product of the factor x. (F'_y and F'_z, analogously to F'_x).

One can see, that the optimum is characterized by the ratio's $\frac{F'_x}{p_x} = \frac{F'_y}{p_y} = \frac{F'_z}{p_z}$ and by the fact that the production function is satisfied.

Now

Now F'_x, F'_y and F'_z, or the partial derivates of the production function can be written as

$$F'_x = \frac{a}{x}F \qquad F'_y = \frac{b}{y}F \qquad F'_z = \frac{c}{z}F$$

Substitution of these partial derivatives in the optimum conditions gives:

$$\begin{cases} c.p_x.x \qquad\qquad -a.p_z.z=0 \\ \qquad\quad c.p_y.y \quad -b.p_z.z=0 \\ x^a.y^b.z^c.e^{\varepsilon t} \qquad\quad =X^{\alpha}Y^{\beta} \end{cases} \qquad (2.2.2)$$

This system can be solved for instance by substitution of

$x=\frac{ap_z}{cp_x}z$ and $y=\frac{bp_z}{cp_y}.z$ in the third equation which gives an expression in z

$$X^{\alpha}Y^{\beta} = \left(\frac{ap_z}{cp_x}\right)^a . \left(\frac{bp_z}{cp_y}\right)^b . z^{(a+b+c)} . e^{\varepsilon t}$$

$$z = \left(\frac{X^{\alpha} . Y^{\beta}}{A}\right)^{\frac{1}{a+b+c}} \qquad \text{where } A = \left(\frac{a.p_z}{c.p_x}\right)^a . \left(\frac{b.p_z}{c.p_y}\right)^b . e^{\varepsilon t}$$

and when z is re-substituted in the system of optimum equations, one will find x and y as:

$$\left\{\begin{array}{l} x = \dfrac{ap_z}{cp_x} . \left(\dfrac{X^{\alpha}Y^{\beta}}{A}\right)^{\frac{1}{a+b+c}} \\[2ex] y = \dfrac{bp_z}{cp_y} . \left(\dfrac{X^{\alpha}Y^{\beta}}{A}\right)^{\frac{1}{a+b+c}} \end{array}\right. \qquad (2.2.3)$$

which are in fact the factor demand equations.

Inserting the new expressions for x, y and z in the cost equation

$$C = p_x.x + p_y.y + p_z.z \qquad (2.2.4)$$

gives: $$C = \left(p_x.\frac{ap_z}{cp_x} + p_y.\frac{bp_z}{cp_y} + p_2\right)\left(\frac{X^{\alpha}Y^{\beta}}{A}\right)^{\frac{1}{a+b+c}} \qquad (2.2.5)$$

Now, the expression

$$\frac{\left(p_x.\dfrac{ap_z}{cp_x} + p_y.\dfrac{bp_z}{cp_y} + p_z\right)}{A^{\frac{1}{a+b+c}}}$$

can be simplified to

$$(a+b+c)\left(a^{\frac{-a}{a+b+c}} . b^{\frac{-b}{a+b+c}} . c^{\frac{-c}{a+b+c}}\right)(p_x)^{\frac{a}{a+b+c}} . (p_y)^{\frac{b}{a+b+c}} . (p_z)^{\frac{c}{a+b+c}} . e^{\frac{\varepsilon t}{a+b+c}}$$

So that the cost function can be written as:

$$C = B.p_x^{a'}.p_y^{b'}.p_z^{c'}.X^{\alpha'}.Y^{\beta'}.e^{\varepsilon' t} \qquad (2.2.6)$$

where it is very important to notice that $a'+b'+c'=1$

and where otherwise $B = (a+b+c)(a^{\frac{-a}{a+b+c}}.b^{\frac{-b}{a+b+c}}.c^{\frac{-c}{a+b+c}})$

$$a' = \frac{a}{a+b+c} \qquad b' = \frac{b}{a+b+c} \qquad c' = \frac{c}{a+b+c}$$

$$\alpha' = \frac{\alpha}{a+b+c} \qquad \beta' = \frac{\beta}{a+b+c} \qquad \varepsilon' = \frac{\varepsilon}{a+b+c}$$

The estimation of this cost function can be done by means of a dubbel logarithmic form, at least, if one is willing to add the disturbance term U to the cost function as e^u with u normally distributed.

One obtains: $\log C = \log B + a'\log p_x + b'\log p_y + c'\log p_z + \alpha'\log X +$

$$\beta'\log Y + \varepsilon' t + u$$

A typical feature of this estimation is the restriction of the coefficients. One will therefore have to apply a method of least squares with this restriction on the coefficients as a subsidiairy condition. From the estimates of $\hat{a}'$, $\hat{b}'$, $\hat{\alpha}'$ and $\hat{\beta}'$, the coefficients a, b, c, α, and β can be deduced.

This model is capable of generalization. On the one hand one may increase the number of outputs, on the other hand one can introduce (at least in principle) a larger number of production factors, without affecting the essence of the problem.

3. THE DATA

With respect to the estimation of the cost function, we could dispose of time series, with an overall length of 19 years (1952-1970) (*). The data that were used for the calculations, are presented in table 1 below.

For the sake of exposition, one can conveniently arrange the data into three groups, notably: output data, factor price data and cost data.

3.1. Outputs

In the official statistics the output is represented either in tons or in ton kilometers or in number of trains or in train kilometers, and analogously for passenger transportation. We believe, that ton- and passengen-kilometer are more adequate output measures than the remaining ones. It is very probable that the choice of the output measure has an incidence on the estimates. In order to be able to judge this effect, the calculations were executed also with train kilometers as output measures.

3.2. Factor prices

It was completely impossible to obtain the prices of the capital goods of the industry considered, since the Railway Authorities preserved a complete secrecy about these matters. For that reason we approximated the real price of capital through the rate of return on long term state bonds. It admittedly is a poor substitute for the real thing.

(*) The main sources for our data were: Statistisch Jaarboek van de NMBS (Statistical Yearbook of the National Belgian Rail ay Company) and Jaarverslagen van de NMBS (Annual Report).

The prices of the other factors, notably labour and energy could be calculated as "average costs per unit". For energy the numerator is compound: it consists of KWH's and tons of gasoil which are converted into equivalent KWH's.

3.3. Costs

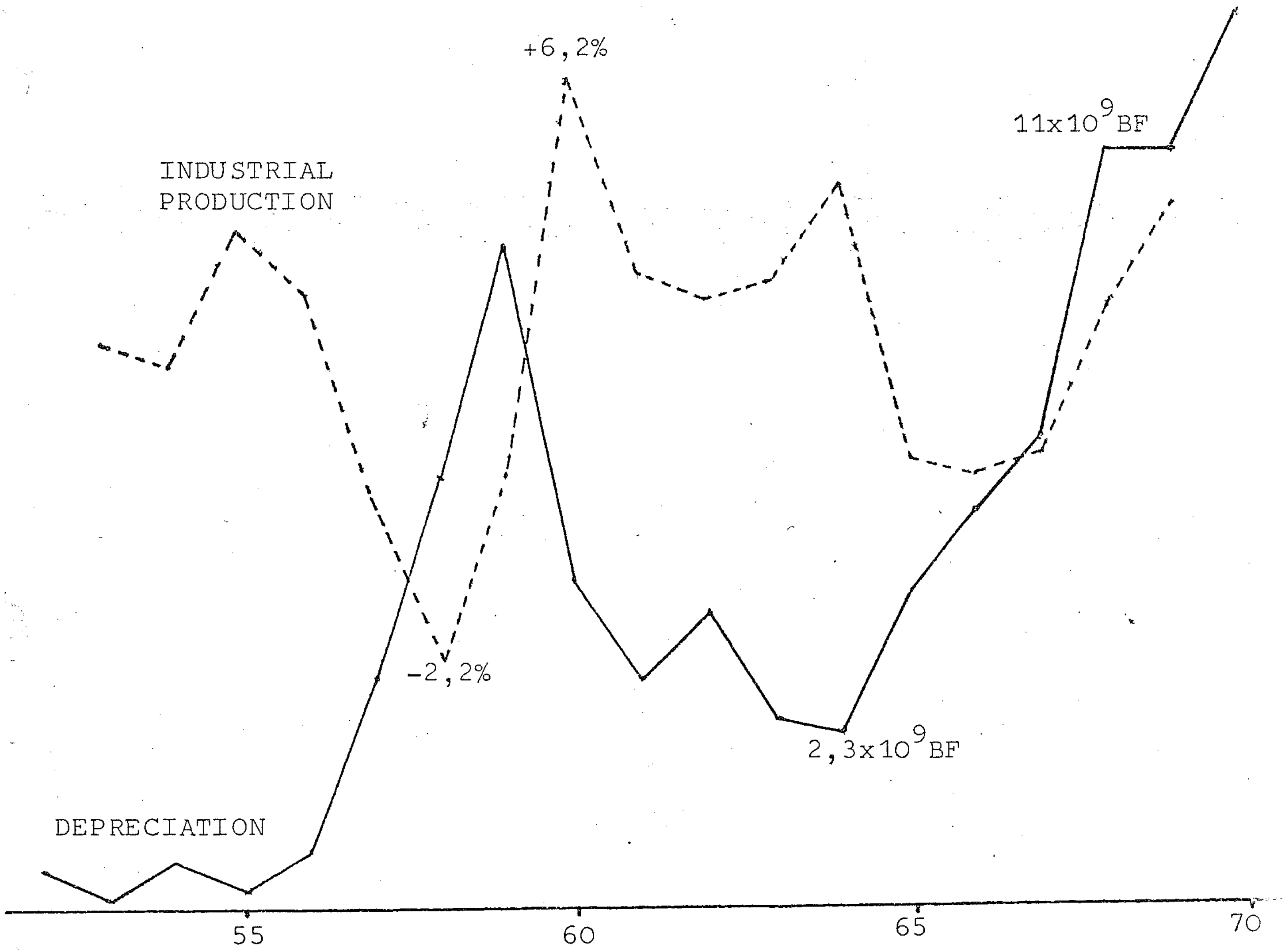
In the accompanying illustration one can see that the cost statement which is released to the public is very simple.

Cost statement of the NMBS, 1970 (in 10^6 Belgian francs and in % of total)

Personnel	: 13.850	60,1 %
Materials	: 1.601	7,0 %
Other costs	: 3.175	13,8 %
Contribution to the fund of renewal	: 4.400	19,1 %
	23.026	100,0 %

The heading "materials" consists mainly of fuels, while the heading "other costs" stands for electricity cost and maintenance of the rolling stock. As to the last head, the official interpretation is "economic replacement cost", while in fact it conceals the pure annual expenses on some investment. The accompanying figure shows what happens when this last head is included in total costs as a part of the capital cost. During the main part of the picture, "depreciation" in the sense of the officials, goes up when industrial production comes down. This is either a coincidence or it illustrates the anti-cyclical policy of the authorities towards the heavy basic industry of the country. Anyhow, ton kilometers fall fairly well in line with industrial production so that a regression of ton-kilometers and other explanatory variables on costs as a dependent variable leads to a predictable yet nonsensical negative sign for the marginal cost of freight transportation.

Figure 2. Depreciation, compared with cyclical variations of industrial production



We conclude from the previous exposition that the official's concept of the economic replacement cost cannot be used in our study. In the previous subsection (3.2) it was noticed, that the price of capital is unknown, so that an ex post calculation of the capital cost is ruled out too.

Suppose that the cost function were estimated on the basis of cost data exclusive of capital cost? That is, suppose that, on account of imperfect data the capital variable z, in the cost equation (2.2.4) becomes zero. The cost function K that we will estimate will not coincide with the true cost function C. But this is not relevant if we are interested only in a short run marginal cost function. The crucial question is about the shape of K as compared with C, and there may be serious trouble when the time capital cost has either oscillated or moved up appreciably during the period of observation, so that C-K is no constant.

Circumstances that are bound to moderate the discrepancy between (C-K) and some constant are: (1) when properly deflated, the true capital cost is most likely to be a very smooth curve, since the depreciation periods are very large in most cases. (2) In the present study, the original data are first transformed to procentual differences. (3) A scale factor will be inserted, accounting for the bias that might occur on account of the ommission of the original cost.

The cost series that was actually used for the calculation was:

$K = p_x . x + p_y . y$, where x = the personnel cost
y = fuel, electricity, and maintenance cost.

Table 1. Data used for computations as to the cost function (*)

Year	Output series				Price series			Cost series
	A Passenger-km ($x10^6$)	B Ton km ($x10^6$)	C Train km (passengers) ($x10^6$)	D Train km (freight) ($x10^6$)	A Labour BF per man-year ($x10^3$)	B Energy BF per equivalent KWH	C Capital %	Exclusive of capital cost ($x10^3$BF)
1952	7.546	6.114	43,67	19,28	74,9	1,57	4 53	6.644
1953	7.528	5.785	42,27	17,25	80,6	1,60	4,45	6.734
1954	7.562	5.697	43,52	16,97	82,8	1,59	4,31	6.646
1955	7.846	6.618	47,44	20,63	80,8	1,59	4,17	6.552
1956	8.333	6.922	54,14	21,12	83,1	1,57	4,18	6.997
1957	8.555	6.585	56,74	19,98	86,1	1,63	4;65	7.275
1958	9.057	5.870	59,01	15,79	98,4	1,56	4,52	8.176
1959	8.519	6.062	56,03	16,15	101,6	1,59	4,25	8.001
1960	8.578	6.306	54,97	16,95	105,0	1,43	4,31	7.787
1961	8.693	6.455	54,69	17,51	106,8	1,45	4,40	7.694
1962	8.958	6.467	57,57	18,32	113,8	1,23	4,40	7.977
1963	9.009	6.825	57,80	19,38	121,2	1,24	4,15	8.338
1964	9.041	6.925	58,69	20,32	120,1	1,23	4,21	8.120
1965	8.975	6.758	58,70	19,41	131,1	1,25	4,23	8.635
1966	8.708	6.234	56,76	16,94	154,0	1,31	4,23	9.806
1967	8.534	6.082	55,15	16,06	164,2	1,34	4,02	10.179
1968	8.177	6.675	53,12	17,91	174,0	1,39	4,08	10.616
1969	8.238	7.419	51,75	21,31	176,5	1,18	4,16	10.596
1970	8.260	7.816	52,08	23,29	180,4	1,16	4,19	11.046

(*) All cost and prices have been deflated with the index of wholesale prices; the price series was not deflated since it is a procentual rate.

4. THE RESULTS

After making the adjustment, mentioned in the last paragraph, the cost function is of the form:

$$C = B.p_x^{a'}.p_y^{b'}.p_z^{c'}.X^{\alpha'}.Y^{\beta'}.e^{\theta S}.e^{\varepsilon' t}.e^{\mu}$$

where S is the (new) scale variable, and where otherwise the same simplifying notation was used as before (see section 2).

After taking procentual differences of the original variables (which is approximately the same as using a log transformation), the cost function may be written as

$$\tilde{C} = a'.\tilde{p}_x + b'.\tilde{p}_y + c'.\tilde{p}_2 + \alpha'\tilde{X} + \beta'\tilde{Y} + \theta S + \varepsilon' t + \mu$$

where C = d log of C etc.

The estimation of this function was executed using ordinary least squares (OLS) with a restriction on the sum of the coefficients; the procedure yields estimates of the (constant) elasticities directly.

Two variants of this function were estimated: in the first version the outputs (X and Y) were represented in ton-km and passenger km respectively, whereas in the second, the outputs were represented in train km for freight and passenger transformation.

The results that were obtained from the OLS estimation procedure are presented in table 2.

Table 2 reveals the striking fact that the choice of the output measure is of crucial importance with respect to the cost elasticity of passenger transportation: the latter being about 0.60 in case (2) is well over unity in case (1). With respect to freight transportation

Table 2. Coefficients estimates of the cost function

DEPENDENT VARIABLE	X(pass.)	Y(freight)	pp_x(L)	p_y(E)	p_2(K)	S	t
(1) Cost	1.27**	0.12**	0.89**	0.06**	0.05*	-0.007**	30.00**
(2) Cost	0.58**	0.02	0.91**	0.04**	0.05*	-0.007**	28.05**

(1) = output measured in ton km & passenger km

(2) = output measured in train km

** = significant at 5% level

** = significant at 1% level

the choice of the measure still makes a lot of difference, but any-wayway the elasticity is very small.

According to what was explained in section three, our main interest for the analysis to come is in the first case (output measured in ton kilometers and passenger kilometers).

4.1. Returns to scale

The coefficient of the scale variable in the cost function would indicate the existence of very small economies of scale. But the typical way of judging economies of scale is to proceed from the coefficiënts of the production function. Suppose one keeps the level of ton kilometers as fixed, then the production function can be written

$$X = (Y^{\circ})^{-\beta} . x^{\frac{a}{\alpha}} . y^{\frac{b}{\alpha}} . z^{\frac{c}{\alpha}} . e^{\frac{\varepsilon t}{\alpha}}$$

If all the input levels were augmented with λ%, the output X could increase with $\gamma = \lambda \exp.(\frac{a+b+c}{\alpha})$%. If γ were larger than λ, there would be economies of scale in the production of passenger kilometers, but

$$\frac{a+b+c}{\alpha} = 0.79$$

This means that the production of passenger kilometers is characterised by decreasing returns to scale.

Analogously with the former case, one can imagine the level of passenger kilometers to be fixed. The production function would be

$$Y = (X^{\circ})^{-\alpha} . x^{\frac{a}{\beta}} . y^{\frac{b}{\beta}} . z^{\frac{c}{\beta}} . e^{\frac{\varepsilon}{\beta}t} .$$

An augmentation by λ%, of the levels of all the inputs would now lead to an increase of $\gamma\% = \exp.(\frac{a+b+c}{\beta})\%$. The last expression is far greater than λ, since $\frac{a+b+c}{\beta} = 8{,}33$. According to our estimates freight transportation is characterised by considerable economies of scale.

4.2. The marginal cost curve

Using the definition that marginal cost is the first derivative of the total cost function, yields for passenger transportation:

$$MC_X = \frac{\delta C}{\delta X} = 1.27X^{0.27} . Y^{0.12} . P_x^{0.89} . P_y^{0.06} . P_z^{0.05} . e^{-0.0075} . e^{30.0t}$$

and for freight transportation:

$$MC_Y = \frac{\delta C}{\delta Y} = 0.12Y^{-0.88} . X^{1.27} . P_x^{0.89} . P_y^{0.06} . P_z^{0.05} . e^{-0.0075} . e^{30.0t}$$

When the values of 1970 are filled out in the marginal cost equations, we derive the following relations:

$$MC_X = 0{,}163\ X^{0.27}$$

and

$$MC_Y = 374{,}3\ Y^{-0.88}$$

Table 3 shows the marginal cost schedules, computed for a realistic range of output data.

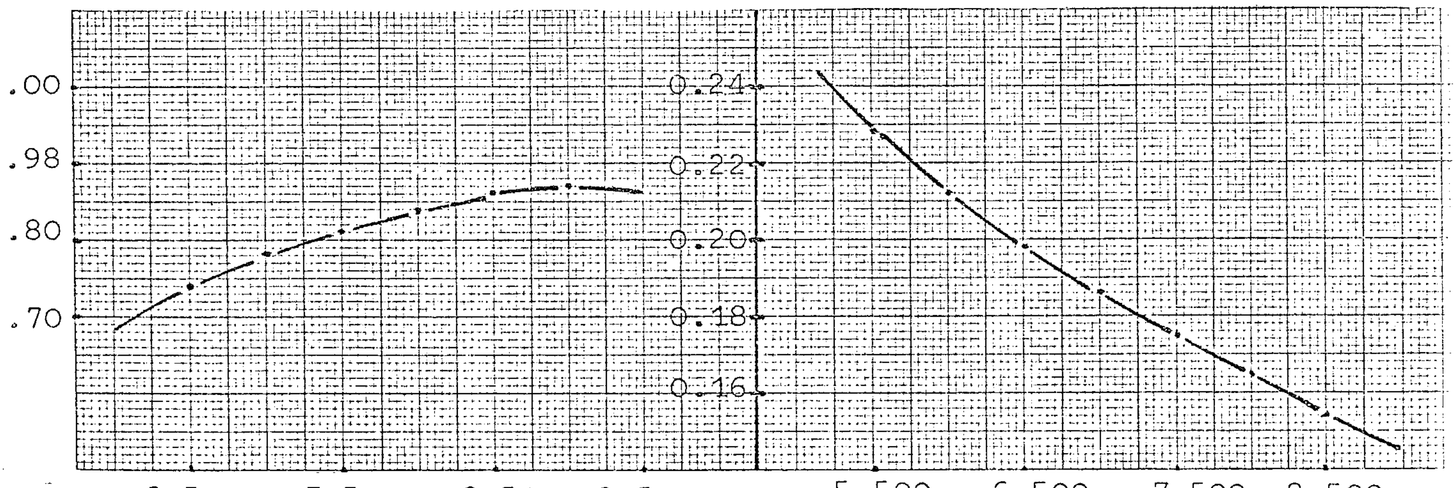
Table 3. Marginal cost schedules for data of passenger kilometers (X) and ton kilometers (Y) when the remaining cost variables are fixed on their 1970 level

Passengers		Freight	
Output levels in 10^6 pass.km	Marginal Cost in BF per pass.km	Output levels	Marginal Cost in BF per ton km
6.500	1,744	5.500	0,229
7.000	1,780	6.000	0,212
7.500	1,813	6.500	0,198
8.000	1,845	7.000	0,186
8.500	1,860	7.500	0,175
9.000	1,975	8.000	0,165
		8.500	0,157

These data are graphed in figure 3. An inspection of this figure shows that the rmarginal cost of passenger transportation is sloping upward, while the marginal cost of freight transportation is sloping downward.

Figure 3.a. Marginal Cost Passenger Transportation

Figure 3.b. Marginal Cost Freight Transporta



In the case of freight transportation there will be no short run profit, unless the price can be set so high that the average costs can be met. This would imply a policy of "charging what the traffic can bear".

In the case of passenger transportation, there are potential opportunities for making profits, which depends in fact on the slope and position of the demand curve. It is clear, that it is necessary to establish the properties of demand in order to be able to continue this discussion. This will be done in the next section.

5. FURTHER ANALYSIS

5.1. The demand curves

With respect to the estimation of the demand curve, abstraction is made of the interactions between supply and demand in the transport market. Previous SESO-research (1) revealed that crucial variables in the determination of demand are:

PASS. TRANSPORTATION	FREIGHT TRANSPORTATION
- Price (P_1) - time, weighted with opportunity cost of travelling (P_2) - disposable income (y_d) - price of intercity car transportation (P_c)	- Price (P_R) - price in inland waterways (P_B) - industrial production

So the demand equations can be written as:

$$x_p = \alpha . P_1 + \beta . P_2 + \gamma . P_c + \delta . y_d + \varepsilon + \mu$$

$$x_F = \alpha ' P_R + \beta ' P_B + \gamma ' D + \varepsilon ' + \mu '$$

where x_p = demand for passenger transportation, and
x_F = demand for freight transportation.

For the calculations, we could again dispose of time series with an overall length of 19 years (1952-1970). The data that were used are represented in the accompanying table.

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(1) See J. Busschaert en E. Van Broekhoven, The cost of time and the demand for passenger transportation, SESO paper nr. 7212/751, and J. Busschaert en E. Van Broekhoven, An Econometric model of Belgian Inland Waterway Transportation, SESO paper nr. 7313/851.

Table 5. Data used for computations with respect to railroad demand equations

Year	Passenger transportation					Freight transportation			
	Quantity demanded x (passenger km $\times 10^6$)	Material price p_1 (BF) (*)	Time cost p_2 (BF)(*)	Price of car transp. p_2 (BF)(*)	Disponable income Y_d (BF $\times 10^9$) (*)	Quantity demanded x (ton km) ($\times 10^6$)	Price Railways p_R (BF)	Price Inland Waterways p_B (index)	Industrial Production D (index)
1952	7.546	0.5136	0.6095	1,5588	368,8	6.114	1,1430	188,8	100,1
1953	7.528	0.5379	0.6184	1,6004	374,9	5.785	1,1560	187,5	100,0
1954	7.562	0.5426	0.6171	1,5776	390,7	5.697	1,1650	189,0	104,0
1955	7.846	0.5668	0.5668	1,5463	402,2	6.618	1,0710	198,3	112,2
1956	8.333	0.5360	0.5371	1,5062	410,2	6.922	1,0630	107,5	119,2
1957	8.555	0.5319	0.5286	1,4847	418,4	6.585	1,0900	225,0	119,4
1958	9.057	0.5444	0.5577	1,4784	426,7	5.870	1,0670	230,0	113,7
1959	8.519	0.6564	0.5798	1,5324	435,8	6.062	1,0060	230,0	118,6
1960	8.578	0.6142	0.6119	1,6015	459,3	6.306	0,9770	233,8	127,6
1961	8.693	0.5296	0.6222	1,5845	475,3	6.455	0.9540	237,5	134,8
1962	8.958	0.5352	0.6424	1,5543	502,0	6.467	0.9610	243,3	142,6
1963	9.009	0.5326	0.6690	1,5692	521,0	6.825	1,0050	251,9	153,4
1964	9.041	0.5518	0.6899	1,5046	547,1	6.925	0,9900	264,8	167,5
1965	8.975	0.6096	0.7245	1,4148	577,5	6.758	0,9690	287,5	169,5
1966	8.708	0.6507	0.7518	1,4842	591,3	6.234	1,0310	307,5	173,5
1967	8.534	0.7224	0.7603	1,4286	605,0	6.082	1,0310	312,5	177,3
1968	8.177	0.7380	0.7805	1,3906	630,4	6.675	1,0100	324,0	183,3
1969	8.238	0.7171	0.8165	1,4032	658,5	7.419	0,9990	350,5	199,9
1970	8.260	0.7233	0.8609	1,3367	699,1	7.816	1,1140	376,0	211,8
Mean value	8.427	0.5971	0.6603	1,5030	499,7	6.506	1,0422	255,0	143,6

(*) Deflated with BNP price index.

Two specifications were estimated for each of the demand equation: one was linear and the other linear in the logarithms.

Table 6 presents the regression results of the demand equations, while table 7 shows the price elasticities. As one knows, the price elasticities are directly supplied when a double log specification is used. In the linear case the elasticities were calculated as:

$\eta = \alpha \dfrac{\bar{P}}{\bar{X}}$, where α is the estimated price coefficient and $\bar{p}$ and $\bar{X}$ are the averages of the price and output series respectively.

Table 6. Regression results

Demand for passenger transportation						
DEPENDENT	P_1	P_2	P_c	y_d	constant	specification
x_p	-0.27 (0.08)	-0.28 (0.18)	-0.33 (0.20)	-0.72 (0.44)	3.45 (1.72)	double logarithm
x_p	-2751 (1220)	-5955 (2445)	-1804 (1243)	-3,03 (2,65)	6525 (1153)	linear

Demand for freight transportation					
DEPENDENT	P_R	P_B	D	constant	specification
x_F	-0.72 (0.35)	0.49 (0.27)	0.97 (0.24)	-2.77 (1.32)	double logarithm
x_F	-5370 (1437)	5.52 (3.99)	51.25 (9.24)	6979 (1590)	linear

The figures between brackets in table 6 are the standard deviations.

Table 7. Price elasticities in demand for

PASSENGER TRANSPORTATION		FREIGHT TRANSPORTATION	
double log	linear	double log	linear
-0.27	-0.19	-0.72	-0.86

The demand for both goods is inelastic, however, at a different rate: passenger transportation is very irresponsive to price, while the elasticity of freight transportation is in the range of -0.70 to _0.90.

Our calculations also show that the mean value of the price ($\bar{p}$) of passenger transportation is well beneath the marginal cost curve, which was graphed in the former section. Actually the mean amounts to only 0,66BF per passenger kilometer , while MC was in the range of 1,70 to 2,00BF per passenger kilometer. In fact, the price is even lower than average cost, ranging from 1,26 to 1,37BF. So, in order to reach a break-even situation, an amount of about 0,60 to 0,70BF per passenger kilometer must be added. For the mere sake of testing the reliability of these calculations, we state that total subsidies ranged between 2607 x 10^6BF in 1958 to 8.925 x 10^6BF in 1970.The per passenger kilometer subsidy varies between 0.30 and 1.00BF.

The case of freight transportation is slightly different. The average price per ton kilometer (1,04BF)is far above marginal cost (0,15 à 0,22) but not as high as average cost. The latter ranging between 1,32 and 1,94 per ton kilometer. The policy of "charging what the traffic can bear", which is apparently applied, is not pushed so far that costs are covered entirely. The question is whether there are opportunities for that with an aggregate demand elasticity of almost 0.90.

5.2. Equilibrium

In this subsection we shall apply some elementary static equilibrium theory to the case of the Belgian Railroad industry. Two equilibrium positions will be opposed to one another: the equilibrium under competition on the one hand and the monopolistic equilibrium on the other.

We have to make two important remarks first.

1. The previous section shows that demand for both passenger and freight transportation is inelastic in our case. An important corollary is, that where a constant elasticity has been estimated, marginal revenue is negative, for whatever output level. This rules out the use of the double logarithmic form and leaves us with the linear form.

2. Our cost function happens to be in double logarithmic form. Again constant elasticities are obtained, from a set of data. If an extrapolation is required on the basis of this cost function, it becomes less reliable the more the outcome of the extrapolation is outside the range of the data. One of the extrapolations that is not reliable is for instance: finding the intersection of marginaland average costs.

The competitive equilibrium is formed at the intersection of the demand curve and the marginal cost curve, while the monopolistic equilibrium is found at the intersection of the marginal revenue and the marginal cost curve.

The equilibrium can be found either algebraically through solving a set of equations for output, or they can be found by graphical analysis. The latter was used in this study.

The results can be inspected either in table 8 or in the figures 4 and 5.

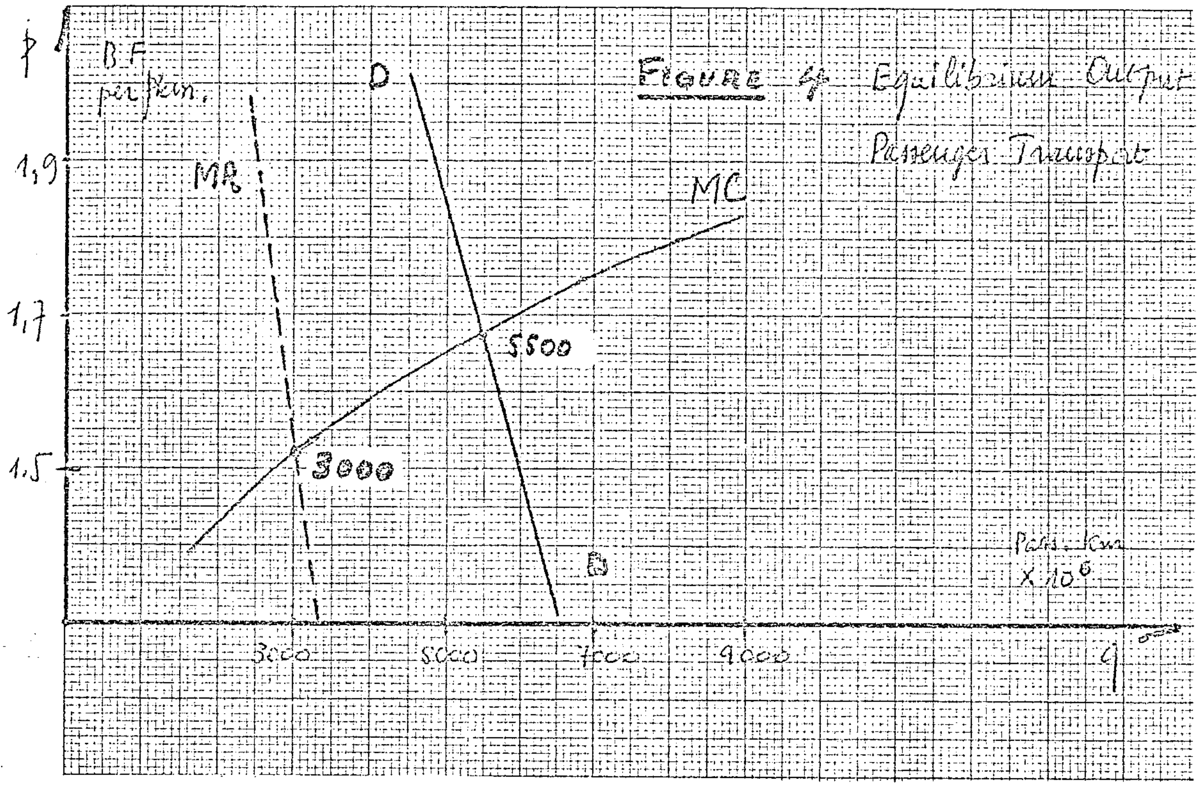


Figure 4 Equilibrium Output
Passenger Transport
p
BF per Pkm.
D
MR
MC
1,9
1,7
1,5
5500
3000
Pass. Km
x 10⁶
3000
5000
7000
9000
q

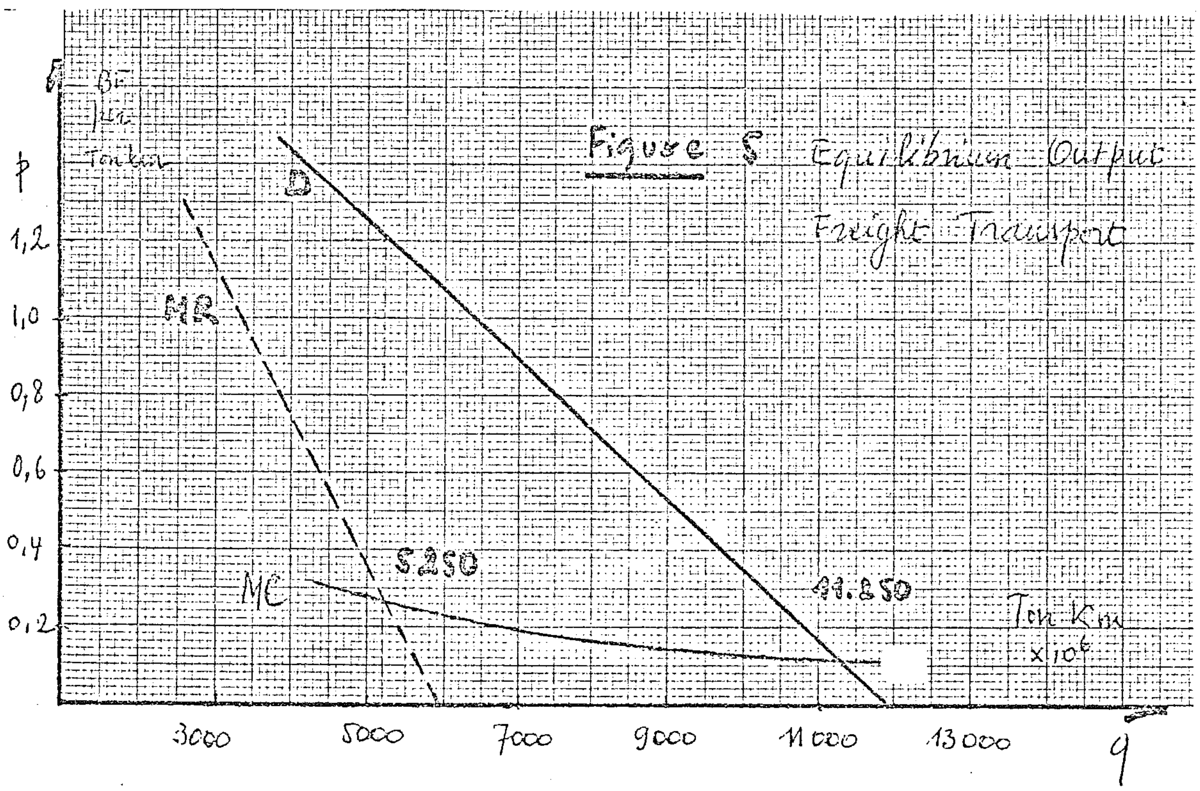
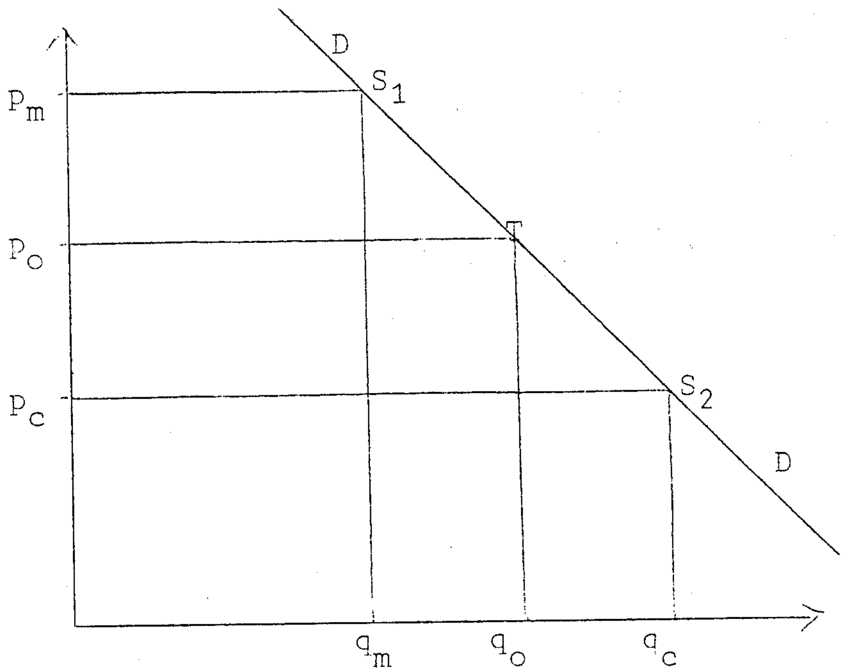


Figure 5 Equilibrium Output
Freight Transport
p
BF per Tonkm
D
MR
MC
1,2
1,0
0,8
0,6
0,4
0,2
5.250
11.250
Ton Km
x 10⁶
3000
5000
7000
9000
11 000
13 000
q

Table 8. Equilibrium Coordinates

	PASSENGERS		FREIGHT	
	competitive	monopolistic	competitive	monopolistic
Price (BF)	1,65	1,50	0,15	1,20
Output ($x10^6$)	5.500	3.000	11.250	5.250

The appraisal of the actual equilibrium positions with respect to either of the two equilibriums will be done in terms of linear algebra. Suppose that the DD-curve in the accompanying figure is our demand curve, and S_1 and S_2 are respectively the competitive and the monopolistic output, then the position of each point on the line can be determined in function of the coordinates of S_1 and S_2.



Indeed, the parameter representation of the straight line through two points $\{(p_m, q_m)$ and $(p_c, q_c)\}$ in any pair of axes is

$$\begin{bmatrix} p_o \\ q_o \end{bmatrix} = \frac{1}{1+\lambda} \begin{bmatrix} p_m \\ q_m \end{bmatrix} + \begin{bmatrix} \frac{\lambda}{1+\lambda} \end{bmatrix} \begin{bmatrix} p_c \\ q_c \end{bmatrix} \qquad (5.2.1.)$$

where p_o and q_o are the coordinates of any point on the straight line.

The point T, with unknown coordinates (p_o, q_o) can be any point of the DD-line, on the condition that p_o and q_o are solutions of system (5.2.1.). One can see that a solution of that system defines a value for λ, so that conversely, any point on the DD-line can be defined through one scalar λ. The solutions can be written either as

$$p_o = \frac{p_m + \lambda p_c}{1+\lambda}$$
$$q_o = \frac{q_m + \lambda q_c}{1+\lambda} \qquad (5.2.2.)$$

$$\text{or as} \quad \lambda = \frac{p_o - p_m}{p_c - p_o} = \frac{q_o - q_m}{q_c - q_o} \qquad (5.2.3.)$$

In expression (5.2.3.) one observes

(1) that either the p or the q coordinates suffice to determine λ

(2) that $\lambda > 0$ if $q_n < q_o < q_c$

$\lambda < 1$ if $q_o < q_m$ or $q_o > q_c$

In the case $\lambda > 0$ we can distinguish between $0 < \lambda < 1$ and $\lambda > 1$. In the latter case the equilibrium approaches the competitive equilibrium, while in the former case the equilibrium is closer to the monopolistic equilibrium.

A similar distinction can be made in the case of $\lambda < 0$. Here, the equilibrium will be to the <u>left</u> of the monopolistic situation if $-1 < \lambda < 0$. The closer λ approaches -1, the larger will be the deviation from the monopolistic equilibrium.

On the other hand, the equilibrium will be to the right of the competitive situation if $-\infty<\lambda<-1$. The closer λ approaches $-\infty$, the smaller the discrepancy between actual and competitive equilibrium.

In table 9 the values of λ are represented for various equilibrium positions. The figures were so arranged as to allow an inspection of the influence of government composition on the divergence with respect to the equilibriums.

Table 9.a. Passenger Transportation

<table>
<tr><th>Government composition</th><th>Value of λ</th><th>Situation as compared with either monopolistic or competitive equilibrium</th></tr>
<tr><td>1952-53 : CVP</td><td>-1,736</td><td rowspan="8">Larger than competitive output in each of the subperiods</td></tr>
<tr><td>1954-57 : CVP-LIB</td><td>-1,583</td></tr>
<tr><td>1958-61 : CVP-LIB</td><td>-1,469</td></tr>
<tr><td>1961-65 : BSP-CVP</td><td>-1,437</td></tr>
<tr><td>1965-66 : BSP-CVP</td><td>-1,448</td></tr>
<tr><td>1966-68 : PVV-CVP(*)</td><td>-1,505</td></tr>
<tr><td>1968-69 : CVP-BSP</td><td>-1,554</td></tr>
<tr><td>1969-71 : CVP-BSP</td><td>-1,546</td></tr>
<tr><td colspan="3">(*) The Liberal party (LIB) changed its name and became "Partij voor Vrijheid en Vooruitgang" (PVV).</td></tr>
</table>

In the case of passenger transportation, the real equilibria are to the right of the competitive equilibrium in each of the subperiods. Government composition has apparently no appreciable impact on this equilibrium.

Table 9.b. Freight transportation

Government composition	Value of λ	Situation as compared with either monopolistic or competitive equilibrium
1952-53 : CVP	0,132	Between Competitive and Monopolistic Equilibrium
1953-57 : BSP-LIB	0,252	
1958-61 : CVP-LIB	0,182	
1961-65 : BSP-CVP	0,315	
1965-66 : BSP-CVP	0,262	
1966-68 : CVP-PVV	0,220	
1968-69 : CVP-BSP	0,428	
1969-71 : CVP-BSP	0,652	

<u>Legend</u> : CVP = Christian Democratic Party
BSP = Social Democratic Party
LIB(PVV) = Liberal Party

The value of λ is positive and smaller than unity, which means that the equilibrium is between the competitive and the monopolistic situation, but closer to the latter than to the former.

If we would rank the subperiods according to the value of λ we would see that the lowest λ's are scored when the Christian Democratic Party governs alone or in combination with the Liberals. The highest scores for λ occur when the Social Democratic Party participates in government.

This result points to the conclusion, that the level of output of railroad freight transportation and government composition are not independent. The participation of the Socialist Democratic Party might well carry the output from the monopoly position towards the competitive position.

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