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STUDIECENTRUM VOOR ECONOMISCH EN SOCIAAL ONDERZOEK

PORT COST FUNCTIONS

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This paper is part of a research project on port economics which adresses itself to the study of the pricing and investment decision in ports, as an issue in public economics. The development of a systematic body of knowledge in this field would not only prove valuable in a Belgian context, but it may also be assumed that in the development of a European port policy the need will arise for a systematic study on optimal government decisions regarding ports. Such a study must start from a model for an individual port as the basic operating unit. At a later stage the interactions between ports can be worked out.

This study will mainly concentrate on the basic issues of portmanagement viz. organisation, and pricing and investment policy. These topics will be dealt with as problems of economic theory i.e. problems of principle. It is evident that in order that such a theory should not be developed in a vacuum, one should have very concrete examples and potential applications in one's mind.

At first the individual port will be studied. To do so, we fall back upon the ordinary models of the firm, which means that we distinguish elements of demand and supply.

As regards demand a number of basic determinants of arriving and departing tonnage can be distinguished. Among the basic factors we mention: the level of international trade and the regional development of the hinterland. In the present paper these will not be discussed.

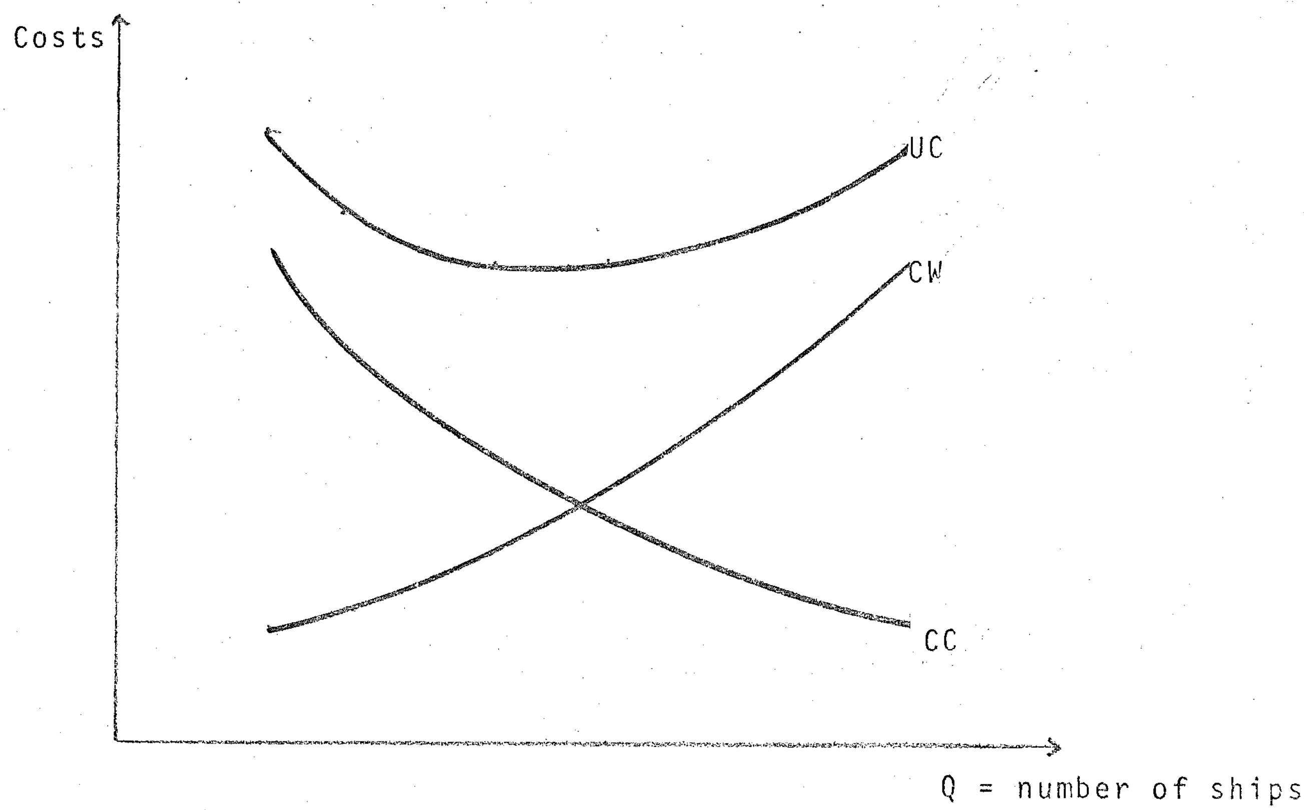
As to supply the port cost function must be considered. In the total of port operations one can distinguish between cargo-handling services and the provision of waterinfrastructure services. This distinction is made for two reasons. First, it occurs in most of the major ports that cargo-handling and the management of waterinfrastructure are separated with regards to organisation.

Cargo-handling is for the major part carried out by private firms on which the port authority has practically no control; the waterinfrastructure on the other hand is managed by a public authority (municipality, state, ...). Second, the inputs for producing 'cargo-handling' and 'waterinfrastructure' can be separated, although both products are complementary in their use. A public port policy obviously wants to consider both elements. Yet, in the present paper we will concentrate on the characteristics of supply of waterinfrastructure i.e. qua walls, locks, navigable water, and so on, and especially on the cost function for waterinfrastructure.

1. PRODUCTION COST AND USER'S COST

Before the model is described the distinction between the cost of producing waterinfrastructure facilities and services and the cost of using it will be made. It is obvious that the largest part of the production cost of waterinfrastructure consists of capital cost, more specifically the construction cost of the quay walls, or, what comes down to the same, of berths. The other production costs, such as maintenance of the quay walls, can be expressed as a percentage of the annual capital charges for quay walls. Therefore, the properly accounted annual capital charges for quay walls, increased with a percentage for maintenance and other costs may be regarded as the total annual cost of production. The average cost of production therefore is shaped as curve CC in graph 1, for given numbers of berths M.

The cost of production is not equal to the social cost or the cost of using the facility. For a given number of berths and a certain cargo-handling capacity - measured as a rate of output, such as a certain number of ships that can be loaded and/or unloaded during a period of time - it is clear that congestion



Graph 1.

appears as demand increases. Beyond a certain level of demand ships can not be served as they call at the port. Therefore, the users' cost will be equal to the production cost increased with the waiting cost of ships, which increases with the level of demand. The users' cost will have a shape as curve UC in graph 1, viz. the sum of the production cost CC and the cost of waiting CW.

Since both the production and the user's cost function should play a prominent rôle in all decisions of public pricing and investment, it will be investigated, how the parameters of this function can be computed. This is essential for any later realistic application.

2. THE MODEL

2.1. The variables

In order to derive the user's cost curve, we now develop a procedure based on queueing theory (1). In this model the demand pattern for a commodity flow X is taken for granted. This means that the quantity of commodity X, the number of ships required to transport this cargo and the time patterns of arrivals is known. When these data are available one can easily derive a mean arrival rate and its distribution (or interarrival time and its distribution). That part of supply, which refers to cargo-handling facilities and services, is taken also as a datum. This means that cargo-handling rates of service times and their frequency of occurance are given.

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(1) Other research along the line of waiting time theory has been done in the context of ports. See e.g. references (1), (6) (10) and (11).

The foregoing data variables can quantitatively take different forms. The demand variable can come into the analysis as the tonnage of cargo, so that the relationship between this quantity and the number of ships required for transport must be fixed with a certain state of shipping technology (1). Assuming a certain state of shipping technology means that ship size does not come in this analysis as a variable. Another possibility is to introduce the demand variable as the number of ships calling at the port. Cargo-handling can be measured by means of cargo-handling rates of through service times.

The variables regulated by the port authority is the number of berths of specific dimensions and with suitable short-equipment for a certain commodity flow. The port authority can either by construction or reallocation provide additional berthage for a certain traffic. This means that, from a suited queueing process the total waiting time that all ships incur by using the port, is calculated for different numbers of berths. Given the unit cost of waiting and the unit cost of berthage, total costs can be found (2).

2.2. The queueing process

After the variables are discussed we now turn to the subject of the queueing process to be used in order to derive total waiting time. Two methods to build a computable queueing model are available viz. analytic solutions and simulation (3). At the present stage of our research only an analytic solution is worked out. However, with simulated queueing processes one could proceed in the same way as it is the case with the analytic one.

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(1) Constant technology in cargo-handling as well as in shipping should be assumed in order to derive the cost functions. See e.g. Joel Dean (2), pp. 306-307 about measuring costs.
(2) Although the estimation of these data variables is a difficult problem as well theoretically as practically, this will not be treated in this research note. Some work in this field has been done already with regards to ship costs by Goss (3) and Heaver (4); the cost of port structures was studied by Bertlin (9) and Nonneman (8).
(3) A through analysis of solutions for different queueing processes is found in Morse (7) or in Kaufman & Cruon (5).

Consider a time pattern of arrivals at a certain port system. The number of arrivals, say n, during a period of time Δt is a stochastic variable N. The probability that N equals n is given by $p_n(\Delta t)$. The conditions, that the process described by the distribution of N is of the Poisson type, are:

1° $p_n(\Delta t)$ should only depend on Δt and not on the initial time t_o at the beginning of the period;

2° the number of arrivals in a certain interval should be independent of the number of arrivals in a previous interval;

3° the probability that there is more than one arrival in a small period of time should be very small.

These conditions are not always fullfilled in a port system. Certain commodity flows are seasonally fluctuating so that the number of ships undergoes seasonal influences also. This means that the probability of a certain number of arrivals depends upon a point in time and not upon the interval. Also the first and second condition are not valid for liners. They have regular shedules so that at a certain point in time the probability of an arrival in the next period could be zero and at another period one. This remark limits out analysis to tramp shipping. The user's cost in the case of liners will be close to the port production cost. Because of the regularity in this part of shipping, through sheduling or other techniques congestion can be reduced to a minimum.

If the probability of n arrivals during a period of time t follows a Poisson distribution, it can be proved that the probability of an interval of duration T in which no arrivals take place follows an exponential distribution. This means that if arrival rates, i.e. the number of arrivals per unit of time is Poisson distributed with expected value say λ, arrival times i.e. the interval between two arrivals, are exponential distributed with expectation λ^{-1}.

For various ports the hypothesis of random arrivals was tested. (See Agerschou & Korsgaard (1)). In a major number of cases it turned out to be a valid assumption.

The service time is defined as the period of time necessary for handling a ship at the berth. According to the foregoing rule, service rates, i.e. the number of ships that could be handled at full utilization of cargo-handling capacity during a period of time, is assumed to be Poisson distributed. This is the case when the previously stated conditions hold.

Only berths of the same specification and ships of the same size can be dealth with in this analysis, The rule "any ship can be sent to any berth" limits the size of the subsystems in port which could be treated with this method. Such an hypothesis can be relaxed by use of simulation processes. By means of this simplification, several parts of a port can be studied by means of the present method.

Among others, Kaufmann and Cruon derived a solution for a system with poisson arrivals and service rates and several identical stations. It is beyond the scope of this paper to give the whole mathematical calculus. Only the essential formulas and derivations will be stated.

The mean arrival rate is equal to λ so during a period of time the number of ships calling at the port system equals λ. The mean service rate equals μ. This value is the total number of ships a port system can handle during that period of time at full utilization of cargo-handling capacity. When the mean time a ship stays at the quay is equal to t then the mean service rate of one berth, say μ', is t^{-1}. So the mean service rate of the total port system equals $M.t^{-1}$ where M stands for the number of berths.

Berth utilization is defined as the ratio of mean arrival rate to mean service rate. So $\phi = \lambda/\mu$ which is always less than unity.

Given these identities, and assuming homogeneous berths, poisson arrival and service rates, the resulting mean waiting time of a ship in the system is equal to

$$\bar{w} = \frac{p_o}{\mu} \cdot \frac{M^M \phi^M}{M!(1-\phi)^2}$$

where M stands for the number of berths and p_o for the probability that there are no ships in port or

$$p_o = \frac{1}{\frac{M^M \phi^M}{M!(1-\phi)} + \sum_{n=o}^{M-1} \frac{M^M \phi^M}{n!}}$$

Hence, total waiting time W, i.e. the waiting time of all λ ships arrived during the period is

$$W = p_o \frac{M^M \phi^{M+1}}{M!(1-\phi)^2}$$

2.3. The cost function

After we have established a procedure to find the total waiting time of ships, the cost function can be derived. If the cost of waiting per ship and per unit of time, say α, and the capital charge per unit of time for a berth, say β, is known then the total cost function equals

$$TC = \alpha M + \beta W$$

where $W = f(M,Q)$

Berth utilization equals arrival rate or demand Q devided by the service rate so

$$\phi = \gamma . Q$$

where $\gamma = \mu^{-1}$

and Q = demand

Hence, substituting this in the cost equation results in

$$TC = \alpha M + \beta . g(M,Q)$$

This cost function is a long term cost equation. For M equal to a constant number of berths the equation describes a short run cost function. Average cost can easily be derived. These short run average cost functions have a minimum value as they consist in an increasing part, viz. congestion costs and a decreasing part, viz. capital costs. The minimum value however can not be found analytically for larger values of M so that it is more convenient to find it numerically for given parameters by generating the average cost function for different levels of demand and numbers of berths.

2.4. An example

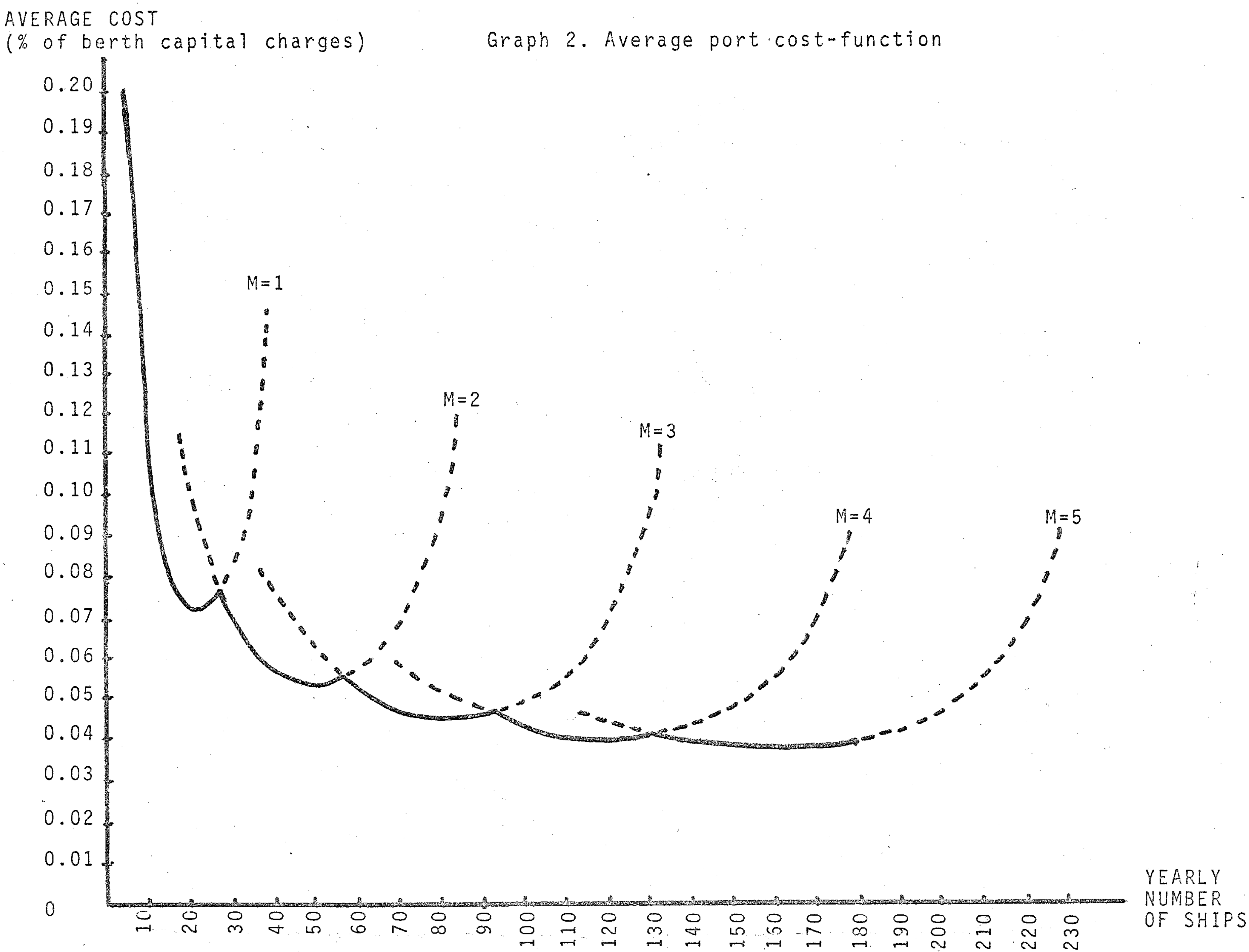
A computor program UCØST was written in which a subroutine QUPØR handles the queueing process. The following parameters are needed:

- annual capital charges for a berth;
- annual cost of a laid-up ship;
- mean time in days a ship spends at the quays for cargo-handling operations;
- the number of berths for which a cost function is needed.

This program results in average cost data for each number of berths in the range 1 to the required number and for demands in the range of berth utilization from 0.1 to 0.9.

A graphed version of the results is found in graph 2. In this example the capital charges where initialised at one, and the yearly cost of a waiting ship was estimated at five times the capital charges for a berth. The mean times required for cargo-handling was 5 days. The resultant average user's cost is expressed as a percentage of annual capital charges for a berth.

Graph 2. Average port cost-function



As one can see from graph 2 short run cost curves have the textbook U-shape and minimum cost demand levels can easily be read from the graph. The long-run cost curve, if smooted, results in L-shape, which illustrates the point that ports are decreasing cost industries.

3. CONCLUSIONS AND FURTHER PROBLEMS

It is clear that this graph could give a decision maker some important indications with regards to optimal utilization of berthage. For those subsystems in the port which more or less meet the assumptions made in this analysis, the functions could be calculated with their appropriate data variables. This would lead to a practical criterium to select those subsystems in which over-or undercapacity exists, for a more thorough analysis. Such non-optimal systems could be analysed by a suited cost-benefit approach in order to reallocate or invest in berthage.

With regards to pricing social marginal costs could also be identified for several subsystems of the port, so that marginal cost pricing as a rule would not be in a vacuum.

However, a number of tricky problems in gathering the data and determining the parameters especially measuring capital costs remain unsolved. It is our intention to study these problems in the future. Also, the problem of aggregation will be studied in order to derive a computable cost function for a whole port. This would solve the problem of allocating investment to different ports.

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