A sequential insertion heuristic for the Vehicle Routing Problem with Time Windows with relatively few customers per route

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Abstract

In this paper we study the performance of Solomon's (1987) sequential insertion heuristic I1 for Vehicle Routing Problems with Time Windows (VRPTWs) in which the number of customers per route is small with respect to the customers' time windows and the scheduling horizon. Solomon's (1987) time insertion $c_{12}(i,u,j)$ underestimates the additional time needed of inserting a new customer u between the depot, $i = i_0$ and the first customer j in the partially constructed route $(i_0 = i, i_1 = j, i_2, \ldots, i_m)$. This can cause the insertion criterion to select suboptimal insertion places for unrouted customers and the error in the corresponding insertion cost can cause the selection criterion to choose the wrong customer for insertion. Since the advance of the depot departure time is taken into account too cheaply, or even not at all, a route with a relatively small number of customers can have a larger schedule time than necessary. If, on the other hand, the number of customers per route is high, there is a high probability that the waiting time at the customers, that were at position 1 during the construction of the route, can be used to insert other customers. An easy way to solve this problem and hence improve the quality of the heuristic for short-routed VRPTWs is suggested.

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1 Introduction

The Vehicle Routing and Scheduling Problem with Time Windows (VRSPTW) refers to a situation in which homogeneous, capacitated vehicles, are required to service geographically scattered customers over a limited scheduling period (e.g. a day). Each customer i has a known demand q_i to be serviced (either for pick-up or delivery but not both) at time b_i chosen by the carrier. If time windows are hard, b_i is chosen within a time window, starting at the earliest

time e_i and ending at the latest time l_i that customer i permits the start of service. In the soft time window case, a vehicle is allowed to arrive too late at a customer but a penalty is incurred. In both cases, a vehicle arriving too early at customer j, has to wait until e_j. If t_{ij} represents the direct travel time from customer i to customer j, and s_i the service time at customer i, then the moment at which service begins at customer j, b_j, equals $\max\{e_j, b_i + s_i + t_{ij}\}$ and the waiting time w_j is equal to $\max\{0, e_j - (b_i + s_i + t_{ij})\}$. A time window can also be defined for the depot in order to define a 'scheduling horizon' in which each route must start and end (Potvin and Rousseau 1993).

In his seminal paper on sequential route-building heuristics for the VRSPTW, Solomon (1987) showed one of his insertion heuristics, I1, to be superior to two other insertion heuristics, two savings heuristics, the nearest neighbor and the sweep heuristic on a set of routing problems which were initially suggested by Christofides, Mingozzi and Toth (1979).

Since then, the excellent performance of the insertion heuristic under time-related side-constraints have been confirmed by Van Breedam (1994) on a different problem set. Also the heuristic has been used in numerous parallel and metaheuristics. We will show that the insertion heuristic proposed by Solomon (1987) uses a time insertion formulation that causes short routes to have a larger schedule time than necessary, often leading to worse solutions in general. Since this problem is related to the way in which the feasibility of the time schedule is checked, we introduce the necessary and sufficient conditions for time feasibility in section 2. In section 3 we briefly discuss the insertion heuristic and focus on the time insertion. Our modification of the insertion criterion is presented in section 4. Computational results are given in section 5 and conclusions are formulated in section 6.

2 Checking time feasibility

To check the time feasibility of the schedule after inserting an unrouted customer, Solomon (1987) developes necessary and sufficient conditions for time feasibility when time windows are hard.

If we denote by $b_{i_p}^{\text{new}}$ the new time at which service begins at customer i at position p after the insertion of customer u in the partially constructed route $(i_0, i_1, \ldots, i_m)$, and if the triangle equality holds for both distances and travel times, then the *push forward* in the schedule at customer i_p is defined as:

$$\text{PF}_{i_p} - b_{i_p}^{\text{new}} - b_{i_p} \geq 0 \tag{1}$$

and

$$\text{PF}_{i_{r+1}} = \max\left\{0, \text{PF}_{i_r} - w_{i_{r+1}}\right\}, p \leq r \leq m-1. \tag{2}$$

Solomon (1987) assumed that all vehicles leave the depot at e_0 to use the idea of the maximum possible push forward generated by inserting an unrouted customer u between two adjacent stops i_{p-1} and i_p. The necessary and sufficient conditions for time feasibility when inserting a customer u between i_{p-1} and $i_p, 1 \leq p \leq m$, on a partially constructed feasible route $(i_0, i_1, \ldots, i_m), i_0 = i_m$ − depot, are

$$b_u \leq l_u \text{ and } b_{i_r} + \mathrm{PF}_{i_r} \leq l_{i_r},\ p \leq r \leq m \tag{3}$$

Indeed, if $\mathrm{PF}_{i_p} > 0$, the schedule at customer i and some of its successors, i.e. customers i_r, $p \leq r \leq m$ could become infeasible. These customers have to be sequentially examined for time feasibility until we find a customer i_r whose waiting time and the one of its predecessors before the insertion of u, has nullified the push forward, i.e. $\mathrm{PF}_{i_r} = 0$, or which is serviced after l_{i_r}, making the schedule infeasible.

3 Solomon's (1987) sequential insertion heuristic I1

A sequential insertion heuristic considers the insertion of an unrouted customer u between two adjacent customers i_{p-1} and i_p in a partially finished route $(i_0, i_1, \ldots, i_m)$ in which i_0 represents the origin destination of a vehicle and i_m its destination location. Without loss of generality, Solomon (1987) assumes $i_0 = i_m =$ depot and that the start of service of all stops in the route, $b_{i_r}, 0 \leq r \leq m$ and the waiting time at i_r, w_{i_r} for $p \leq r \leq m$ are known. We focus on Solomon's (1987) sequential insertion heuristic I1, although the reasoning applies to all three insertion heuristics in Solomon (1987) as they contain the same time insertion component $c_{12}(i,u,j)$ in their insertion criteria.

After starting the current route with an initialization criterion, insertion criterion $c_1(i,u,j)$ is used to calculate for each unrouted stop u its best place and associated cost for insertion between two adjacent customers i and j in the current partial route $(i_0, i_1, ..., i_m)$. The cheapest insertion cost and the associated insertion place is determined for each unrouted customer u as

$$c_1(i(u), u, j(u)) = \min_p [c_1(i_{p-1}, u, i_p)], \quad p = 1, ..., m \tag{4}$$

in which

$$\begin{aligned} c_1(i,u,j) &= \alpha_1 c_{11}(i,u,j) + \alpha_2 c_{12}(i,u,j) \text{ with} \\ \alpha_1, \alpha_2 &\geq 0 \text{ and } \alpha_1 + \alpha_2 = 1 \\ c_{11}(i,u,j) &= d_{iu} + d_{uj} - \mu d_{ij}, \mu \geq 0 \\ c_{12}(i,u,j) &= b_j^{\text{new}} - b_j. \end{aligned} \tag{5}$$

In a second step, the customer that is best according to the selection criterion $c_2(i,u,j)$ is selected. The selected customer u^* is then inserted in the route between $i(u^*)$ and $j(u^*)$.

$$c_2(i(u^*), u^*, j(u^*)) = \max_u [c_2(i(u), u, j(u))] \quad \text{with } u \text{ unrouted and feasible} \tag{6}$$

in which

$$c_2(i,u,j) = \lambda d_{0u} - c_1(i,u,j), \lambda \geq 0 \tag{7}$$

If all remaining unrouted customers have no feasible insertion places, a new route is initialized and identified as the current route.

The selection criterion $c_2(i,u,j)$ of I1 is a generalization of the Clarke and Wright savings algorithm (Clarke and Wright 1964) since it tries to maximize the benefit derived from inserting a customer in the current partial route rather than on a new, direct route.

The insertion criterion $c_1(i,u,j)$, a generalization of the Mole and Jameson (1976) approach for the Vehicle Routing Problem (VRP), is time-space based since it is a weighted average of the distance insertion $c_{11}(i,u,j)$ and the time insertion $c_{12}(i,u,j)$. The distance insertion reflects the additional distance to be traveled to include customer u in the route. Solomon (1987) equals the time insertion, the time needed to include a customer in the current partial route, to the maximum push forward that is generated by inserting customer u between i and j. Nevertheless, it is well possible that the push forward generated at customer j can be weakened or even nullified by the waiting time at the successors of customer j. Although the calculation of the exact time insertion, i.e. the exact push forward over the entire schedule, is calculated while checking the time feasibility (see 3), using the maximum push forward can be considered to be an acceptable simplification (see section 4.2).

However, this approach is only valid if an unrouted customer u is inserted between two adjacent customers $i_{p-1} \neq i_0$ and i_p. Inserting customer u between i_0 and i_1 is a different case. Since all vehicles are assumed to leave the depot at e_0, a waiting time $\max\{0, e_{i_1}\}$ is generated at i_1, the first customer of the route. Unlike the waiting time at all other customers in the route i_r, $p < r \leq m$, it is fictitious, meaning that every reduction of this waiting time actually implies an advance of the depot departure time, and hence, an increase of the schedule time. By assuming that all vehicles leave the depot at e_0 and by equaling the time insertion to the maximum push forward, the time insertion of inserting a customer i before 1 is always underestimated. It may even be be wrongfully equaled to zero.

As an example, consider the partially finished route $(i_0 = i, i_1 = j, i_2, \ldots, i_m)$ in which $e_i = 0$ and all other time windows are represented by line segments bounded by dots (see Figure 1). The start of service at the depot i, b_i, cannot be equaled to $b_j - t_{ij}$ since that would erroneously hamper the heuristic from inserting customers before b_j. Therefore, a fictitious waiting time w_j is created at customer j. Given that $e_j = 100$ and $t_{ij} = 30$, the scheduling period starts at $t = 70$ and $w_j = 70$. Now consider the insertion of customer u_1 between i and j that leads to a shift in the start of service at customer j, i.e. $e_{u_1} + s_{u_1} + t_{u_1 j} > b_j$. After inserting customer u_1 at $t = 60$, and given that $t_{iu_1} = 30$, the scheduling period starts at $t = 30$, implying that the fictitious waiting time at the first customer in the route, u_1, equals 30. Using Solomon's (1987) definition, the maximum time insertion should equal $b_j^{\text{new}} - b_j = 110 - 100 = 10$. In reality, the beginning of the scheduling period advances from $t = (e_j - t_{ij}) = 70$ to $t = (e_{u_1} - t_{iu_1}) = 30$ and the time insertion is therefore at least equal to 40. In fact, it is equal to the advance of the beginning of the schedule (40) plus the push forward at the end of the schedule, whose maximum equals 10.

Figure 1: Insertion if $e_u + s_u + t_{uj} > b_j$

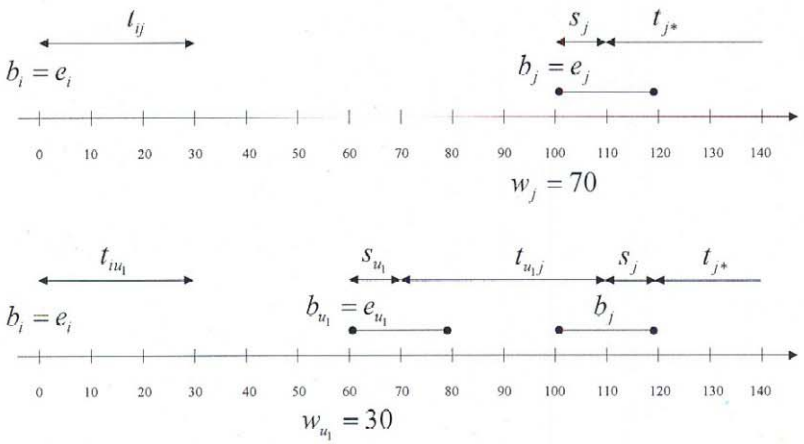
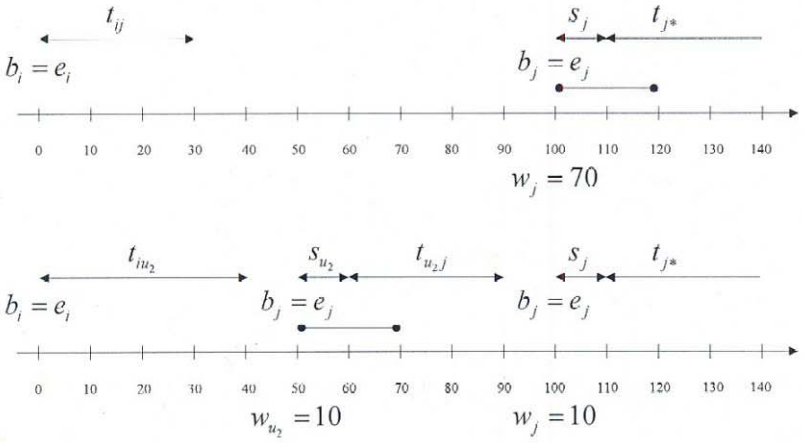


Figure 2: Insertion if $e_u + s_u + t_{uj} < b_j$



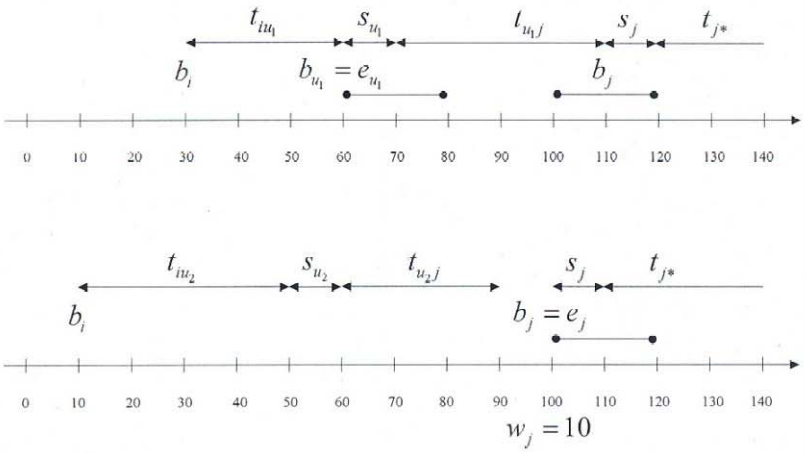
The error in calculating the time insertion gets worse if $e_u + s_u + l_{u_j} < b_j$. In this case (see Figure 2) the time insertion is put equal to zero although it equals $(b_j - t_{ij}) - (b_{u_2} - t_{iu_2})$. Yet, these errors do not prevent the correct calculation of the schedule time of the finished routes since

> "After the complete vehicle schedules have been created, we can adjust the depot departure time separately for each vehicle to eliminate any unnecessary waiting time" (Solomon 1987, p. 265, op. cit.).

Figure 3 clearly shows the increase of the schedule time after inserting u_1 and u_2 by advancing the depot departure time for the routes in Figures 1 and 2.

The main problem is that the miscalculation of the time insertion leads to an underestimated insertion cost for the insertion between the depot and the first customer. In some cases this causes the insertion criterion to select suboptimal insertion places for unrouted customers. The error in the corresponding insertion cost can cause the selection criterion to choose the wrong customer for insertion. Indeed, if customers u_1 and u_2 have the same distance insertion, the selection criterion will select u_2, since it seems to have the lowest insertion cost although in reality it has the largest one. Since the advance of the start of the schedule time is taken into account too cheaply, or even not at all, Solomon's (1987) insertion time encourages the heuristic to insert customers with a low or no push forward between i_0 and i_1 regardless of their push backward, creating large waiting times at customers that were scheduled first during the route construction. This can lead to routes with a larger schedule time than necessary if the number of customers in the route is small with respect to the scheduling horizon. If the number of customers in a route is relatively high, the high waiting times in the starting phase of the heuristic can be used to insert customers during the remainder of the route construction.

Figure 3: Routes after adjusting depot departure times



4 A modified sequential insertion heuristic I1

4.1 Introducing a push forward

Our sequential insertion heuristic uses the same distance insertion in the insertion criterion and the same selection criterion as the one described in Solomon (1987), but uses a more accurate definition of the time insertion. Vehicles are still assumed to leave the depot at e_0 and fictitious waiting time is generated at the first customer in the route, but it is now taken into account in the calculation of the time insertion by defining a *push backward* of the schedule. The push backward generated at the first customer of the route, j, by inserting an unrouted customer u between the depot $i = i_0$ and $j = i_1$ is equal to

$$\mathrm{PB} = (b_j - t_{ij}) - (b_u - t_{iu}) \tag{8}$$

From Figures 1 and 2, it can be seen that the time insertion of inserting customer u between $i = i_0$ and $j = i_1$, is equal to

$$c_{12}(i, u, j) = \mathrm{PB} + \mathrm{PF}_j. \tag{9}$$

Hence, we can reformulate the insertion criterion as follows:

$$c_{12}(i, u, j) = \begin{cases} [(b_j - t_{ij}) - (b_u - t_{iu})] + (b_j^{\mathrm{new}} - b_j) & \text{if } i = i_0 \\ b_j^{\mathrm{new}} - b_j & \text{if } i \neq i_0 \end{cases} \tag{10}$$

Following Solomon (1987), we adjust the depot departure time of each vehicle after finishing the routes to eliminate the remaining fictitious waiting time at the first customer in the route, i_1.

Note that in (10) the maximum push forward generated at customer j, $\mathrm{PF}_j = b_j^{\mathrm{new}} - b_j$, is used to approximate the impact of inserting a customer u before j on the depot arrival time. As stated before, this maximum push forward can be reduced or even nullified by the waiting time at j or its successors. The actual push forward at j and its successors is given by (2).

4.2 Using the actual push forward

Because checking the necessary and sufficient conditions for time feasibility (3), gives information on the latest calculated push forward, there is no need to use the maximum push forward as an approximation for the additional time that is needed to include a customer in a route. The latest calculated push forward represents the actual push forward over the entire schedule, PF. $\mathrm{PF} = 0$ if the push forward generated at j is equal to zero or if it is nullified by waiting times at customers r, $p \leq r \leq m$.

$$\mathrm{PF} = \mathrm{PF}_{i_r} = 0 \quad p \leq r \leq m - 1 \tag{11}$$

If the maximum push forward at j resulted in an actual push forward of the schedule,

$$PF = PF_{m-1} \neq 0 \tag{12}$$

This allows us to redefine the insertion time as

$$c_{12}(i,u,j) = \begin{cases} \text{PB+PF} & \text{if } i = i_0 \\ \text{PF} & \text{if } i \neq i_0. \end{cases} \tag{13}$$

However, using the actual push forward also has a disadvantage. Since the push forward is defined to be equal to or larger than zero, the actual push forward can be equal to zero for a number of different unrouted customers although they each can have a different maximum push forward. Thus if both push forwards are nullified by the waiting time at the successors, the selection criterion $c_2(i,u,j)$ is unable to distinguish between customers with a low maximum push forward that "consume" a small proportion of the waiting time at its successors and those with a high maximum push forward. Since the selection criterion can therefore choose the "wrong" customers for insertion, the solution quality can decrease.

5 Computational study

To evaluate Solomon's (1987) definition of the time insertion (maxPF), the push backward and maximum push forward (PBmaxPF) or the push backward and actual push forward (PBactualPF), we use the problem sets developed by Solomon (1987). The Solomon problem sets consist of VRPTW instances of 100 customers with randomly generated coordinates (set R), clustered coordinates (set C) or both (the so-called semi-clustered sets RC). Despite the fact that Solomon (1987) constructs sets with a small and a large average number of customers per route, they cannot be used directly to evaluate the different time insertion criteria. As will be explained below, they have almost the same relative route length.

The benefit of introducing a push backward in the time insertion criterion depends on the possibility of facing large waiting time at the customers that have been scheduled first at the end of the route construction phase. This possibility is negatively correlated with the number of customers per route and the customers' time window sizes relative to the scheduling horizon. Also, the use of the customer with the earliest deadline (l_i) as initialization criterion, limits the possible benefits from using PBmaxPF and PBactualPF as time insertion criteria. Indeed, if a route is initialized with the customer with the earliest deadline, there is a small probability that customers can be inserted between the depot $i = i_0$ and the first customer i_1 in the route, keeping the probability for high waiting times small. The short scheduling horizon and limited vehicle capacity of 200 units in sets R1, C1 and RC1 compared to longer scheduling horizon and higher vehicle capacities of 1000, 700 and 1000 units in R2, C2 and RC2, lead to routes having low and respectively high absolute route lengths. But taking the customers' time window sizes and the scheduling horizon into account, all problem sets have relatively long routes.

To control for the combined effect of customers' time window sizes and the scheduling horizon, the capacity constraints on the R2, C2 and RC2 sets were

replaced by a restriction on the maximum number of customers per route. Problem instances with routes ranging from 5 up to 40 customers were first solved for an extended set of Solomon's (1987) original parameter settings using the lexicographic preference ordering and second using real-life cost data to allow an easier representation of solution quality.

Solomon (1987) uses two initialization criteria: the farthest unrouted customer and the customer with the earliest deadline, and four $(\mu, \lambda, \alpha_1, \alpha_2)$ settings: $(1,1,1,0)$, $(1,2,1,0)$, $(1,1,0,1)$, and $(1,2,0,1)$. The best of these eight runs of each problem instance were used to calculate the average solution quality for each problem set. Solution quality is measured by a lexicographic preference ordering on the number of vehicles, minimum schedule time, minimum distance and sometimes also minimum waiting time. The calculations were coded in C++ using double floating-point precision.

Since α_2 is either 0 or 1 in Solomon's original parameter set, half the runs per instance ignore the time related component of inserting a customer in a route. Therefore $(\alpha_1, \alpha_2) = (0.5, 0.5)$ was added to the list of possible weighting factors, giving equal weighting to the distance and time related component of an insertion.

To isolate the effect of initialization criteria and parameter levels, results are reported for each of the initialization criteria with the full parameter set $\alpha_2 = (0, 0.5, 1)$ and the reduced set containing strictly positive weighting factors for the time insertion, $\alpha_2 = (0.5, 1)$. Finally, the averages over the best solutions generated by both initialization criteria and $\alpha_2 = (0, 0.5, 1)$ are used to evaluate each time insertion criterion's overall performance.

The highest impact of introducing a new time insertion criterion can be expected for insertion heuristics initializing routes with the farthest customer from the depot and using strictly positive α_2 parameters. For each solution in Table 1 the first signed deviation is printed in bold to help identify the lexicographically best solution. For the R and RC sets, PBmaxPF and PBactualPF produce solutions with highly reduced average schedule times for routes up to 15 customers. Solution improvements range from 70.02% to 5.77% and tamper off as route length increases. In general, PBmaxPF generates better solutions than PBactualPF because PBactualPF is unable to distinguish between customers with a low maximum push forward that "consume" a small or a large proportion of the waiting time at its successors. Despite the fact that maxPF is almost always the best time insertion criterion for the C set, it is striking that the lexicographic preference ordering prevents a small increase in route length to be compensated by a large reduction in schedule time to be (e.g. +3.61% versus -40.96% for routes with 10 customers).

Table 1: Farthest customer, stricty positive time insertion weights

n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
R SET						
5	routes	20.00	20.00	0.00	20.00	0.00
	time	8533.68	3192.62	**-62.59**	5019.33	**-70.02**
	distance	2189.16	1839.50	-15.97	5481.59	11.78
10	routes	10.00	10.00	0.00	10.00	0.00
	time	5358.24	2925.20	**-45.41**	4209.96	**-27.28**
	distance	1706.53	1581.64	-7.32	1750.33	2.50
15	routes	7.00	7.00	0.00	7.00	0.00
	time	3989.36	2877.10	**-27.88**	3771.82	**-5.77**
	distance	1535.42	1506.26	-1.90	1542.58	0.46
20	routes	5.00	5.09	**1.82**	5.36	**6.78**
	time	3286.18	2847.48	-13.35	3346.58	1.80
	distance	1546,96	1449.83	-6.28	1644.54	5.93
25	routes	4.09	4.55	**11.11**	4.55	**10.00**
	time	2825.05	2746.18	-2.79	3083.68	8.39
	distance	1490.44	1491.10	0.04	1580.28	5.69
30	routes	4.00	4.09	**2.27**	4.18	**4.35**
	time	2976.92	2618.97	-12.02	2850.70	-4.43
	distance	1426.91	1443.04	1.13	1544.02	7.58
35	routes	3.45	3.73	**7.89**	4.09	**15.56**
	time	2584.77	2552.00	-1.27	2883.55	10.36
	distance	1442.11	1409.29	-2.28	1571.54	8.24
40	routes	3.45	3.64	**5.26**	3.82	**9.52**
	time	2608.17	2615.69	0.29	2.88.42	7.59
	distance	1411.03	1440.17	2.06	1566.36	9.92
C SET						
5	routes	20.13	20.13	0.00	20.50	1.83
	time	35605.00	13878.75	**-61.02**	21782.24	**-62.79**
	distance	2658.19	2033.04	-23.52	2594.83	-2.44
10	routes	10.38	10.75	**3.61**	11.25	**7.78**
	time	21933.64	12950.71	-40.96	13084.57	-67.63
	distance	1935.57	1500.31	-22.49	2416.58	19.90
15	routes	7.38	7.50	**1.69**	8.00	**7.81**
	time	16476.31	13513.65	-17.98	12220.29	-34.83
	distance	1424.64	1399.83	-1.74	2210.55	35.55
20	routes	6.00	6.00	0.00	6.63	**9.43**
	time	14458.62	12271.84	**-15.12**	11568.39	-24.98
	distance	1543.84	1384.40	-10.33	1595.43	3.23
25	routes	5.00	5.50	**10.00**	5.88	**14.89**
	time	12698.29	11277.75	-11.19	11155.43	-13.83
	distance	1438.06	1257.64	-12.55	1728.89	16.82
30	routes	4.50	4.75	**5.56**	5.13	**12.20**
	time	11578.39	11499.02	-0.69	11195.22	-3.42
	distance	1118.61	1246.19	11.40	1471.51	23.98

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n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
35	routes	4.13	3.88	**-6.06**	4.25	**2.94**
	time	10173.99	10981.83	7.94	11366.82	10.49
	distance	1076.24	1143.96	6.29	1453.03	25.93
40	routes	4.13	3.88	**-6.06**	4.25	**2.94**
	time	10173.99	10981.83	7.94	11366.82	10.49
	distance	1076.24	1143.96	6.29	1453.03	25.93
RC SET						
5	routes	20.00	20.00	0.00	20.00	0.00
	time	8431.94	3726.12	**-55.81**	5530.10	**-52.47**
	distance	2729.33	2417.62	-11.42	2402.57	-13.60
10	routes	10.00	10.00	0.00	10.00	0.00
	time	5357.98	3273.99	**-38.90**	4514.19	**-18.69**
	distance	2203.18	1871.14	-15.07	1963.53	-12.21
15	routes	7.00	7.00	0.00	7.00	0.00
	time	7312.11	3165.93	**-26.58**	3866.34	**-11.53**
	distance	1942.53	1755.95	-9.61	1998.56	2.80
20	routes	5.00	5.50	**10.00**	5.50	**9.09**
	time	3462.20	3103.85	-10.35	3549.69	2.46
	distance	1793.46	1814.08	1.15	2015.80	11.03
25	routes	4.50	4.63	**2.78**	4.75	**5.26**
	time	3157.63	2999.30	-5.01	3398.06	7.08
	distance	1676.06	1717.55	2.48	2057.45	18.54
30	routes	4.25	4.25	0.00	4.25	0.00
	time	3075.01	2974.17	**-3.28**	3185.44	**3.47**
	distance	1764.56	1814.46	2.83	1924.00	8.29
35	routes	4.13	4.25	**3.03**	4.13	0.00
	time	2884.22	2949.11	2.25	3154.74	**8.58**
	distance	1701.92	1801.26	5.84	1926.48	11.66
40	routes	4.00	4.13	**3.13**	4.00	0.00
	time	2883.28	2930.76	1.65	3166.86	**8.95**
	distance	1701.65	1816.81	6.77	1905.07	10.68

Also for the full parameter set $\alpha_2 = (0, 0.5, 1)$, sets R and RC confirm the hypothesis that PBmaxPF and PBactualPF generate better solutions than maxPF for relatively routes (up to 15 to 20 customers) by taking the push forward into account when a customer is inserted between i_0 and i_1. Apart form $n = 10$, PBmaxPF is also superior to maxPF when the customers are clustered (C set).

Table 2: Farthest customer, all time insertion weights

n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
R SET						
5	routes	20.00	20.00	0.00	20.00	0.00
	time	8442.52	3192.62	**-62.18**	5019.33	**-68.20**
	distance	2134.84	1839.50	-13.83	2481.59	13.97
10	routes	10.00	10.00	0.00	10.00	0.00
	time	5325.24	2925.20	**-45.07**	4209.96	**-26.49**
	distance	1665.92	1581.64	-5.06	1750.33	4.82
15	routes	7.00	7.00	0.00	7.00	0.00
	time	3969.49	2877.10	**-27.52**	3771.82	**-5.24**
	distance	1521.82	1506.26	-1.02	1542.58	1.35
20	routes	5.00	5.00	0.00	5.18	**3.51**
	time	3276.67	2972.75	**-9.28**	3467.38	5.50
	distance	1494.92	1422.49	-4.84	1589.81	5.97
25	routes	4.09	4.27	**4.44**	4.27	**4.26**
	time	2825.05	2941.11	4.11	3154.67	10.45
	distance	1490.44	1456.90	-2.25	1483.39	-0.48
30	routes	4.00	4.09	**2.27**	4.18	**4.35**
	time	2964.56	2615.65	-11.77	2828.53	-4.81
	distance	1410.83	1415.17	0.31	1473.04	4.22
35	routes	3.45	3.73	**7.89**	3.82	**9.52**
	time	2567.43	2551.03	-0.64	2740.07	6.30
	distance	1417.31	1391.92	-1.79	1339.59	-5.80
40	routes	3.45	3.55	**2.63**	3.55	**2.56**
	time	2573.36	2594.63	0.83	2644.33	2.68
	distance	1376.37	1328.64	-3.47	1282.88	-7.29
C SET						
5	routes	20.13	20.13	0.00	20.25	**0.62**
	time	28362.86	13878.75	**-51.07**	22922.50	-23.73
	distance	1854.55	2033.04	9.62	2154.10	13.91
10	routes	10.25	10.38	**1.22**	10.38	**1.20**
	time	20113.58	14972.05	-25.56	18074.44	-11.28
	distance	1504.87	1307.38	-13.12	1418.40	-6.10
15	routes	7.38	7.38	0.00	7.50	**1.67**
	time	15688.88	13938.29	**-11.16**	14162.25	-10.78
	distance	1117.53	1311.95	17.40	1580.56	29.30
20	routes	5.88	5.88	0.00	5.88	0.00
	time	13946.64	12471.09	**-10.58**	13486.21	**-3.41**
	distance	1278.69	1196.08	-6.46	1166.39	-9.63
25	routes	5.00	5.13	**2.50**	5.25	**4.76**
	time	12698.29	11691.01	-7.93	12570.45	-1.02
	distance	1438.06	1103.12	-23.29	1309.52	-9.82
30	routes	4.38	4.38	0.00	4.38	0.00
	time	11792.95	12091.04	**2.53**	12047.69	**2.11**
	distance	1116.50	1007.46	-9.77	1098.80	-1.61

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n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
35	routes	3.38	3.50	**3.70**	3.50	**3.57**
	time	10299.54	20261.52	-0.37	10301.47	0.02
	distance	787.84	807.99	2.56	783.99	-0.49
40	routes	3.38	3.50	**3.70**	3.50	**3.57**
	time	10299.54	10261.52	-0.37	10301.47	0.02
	distance	787.84	807.99	2.56	783.99	-0.49
RC SET						
5	routes	20.00	20.00	0.00	20.00	0.00
	time	8173.68	3726.12	**-54.41**	5530.10	**-47.80**
	distance	2363.38	2417.62	2.30	2402.57	1.63
10	routes	10.00	10.00	0.00	10.00	0.00
	time	5211.71	3273.99	**-37.18**	4514.19	**-15.45**
	distance	2125.03	1871.14	-11.95	1963.53	-8.23
15	routes	7.00	7.00	0.00	7.00	0.00
	time	4307.37	3165.93	**-26.50**	3866.34	**-11.41**
	distance	1897.35	1755.95	-7.45	1998.56	5.06
20	routes	5.00	5.38	**7.50**	5.38	**6.98**
	time	3396.22	3213.31	-5.39	3511.84	3.29
	distance	1694.86	1756.67	3.65	1661.45	-2.01
25	routes	4.38	4.38	0.00	4.50	**2.78**
	time	3120.87	3072.44	-1.55	3214.75	2.92
	distance	1624.14	1657.03	2.02	1582.54	-2.63
30	routes	4.25	4.25	0.00	4.13	**-3.03**
	time	3047.47	2933.35	**-3.74**	3126.57	2.53
	distance	1719.89	1731.46	0.67	1782.50	3.51
35	routes	4.13	4.13	0.00	4.00	**-3.13**
	time	2828.90	2878.32	**1.75**	2912.81	2.88
	distance	1637.29	1713.34	4.65	1630.84	-0.40
40	routes	4.00	4.00	0.00	3.88	**-3.23**
	time	2836.01	2865.75	**1.05**	2912.88	2.64
	distance	1643.02	1696.15	3.23	1702.46	3.49

By comparing the performance of the three time insertion criteria for routes initialized with the customer with the earliest deadline and using $\alpha_2 = (0.5, 1)$ to guide the heuristic (see Table 3), we obtain an indication whether customers are regularly inserted before the initialization customer. Despite the fact that the reductions of total schedule time for each problem instance are a lot smaller than the corresponding ones in Table 1, PBmaxPF and PBactualPF are still highly successful for respectively the R and C sets. The slight increase in route length for $n = 5$ makes that PBmaxPF does not dominate the entire range up to 20 customers per route for the RC set.

Table 3: Earliest deadline, strictly positive time insertion weights

n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
R SET						
5	routes	20.00	20.00	0.00	20.00	0.00
	time	4220.98	3053.37	**-27.66**	5842.34	**27.75**
	distance	2503.28	1999.35	-20.13	2138.22	-17.07
10	routes	10.00	10.00	0.00	10.00	0.00
	time	3067.74	2857.77	**-6.84**	4822.29	**36.38**
	distance	1862.62	1629.21	-12.53	1976.50	5.76
15	routes	7.00	7.00	0.00	7.00	0.00
	time	2901.09	2830.64	**-2.43**	4003.59	**27.54**
	distance	1606.36	1594.88	-0.71	1647.05	2.47
20	routes	5.45	5.27	**-3.33**	5.18	**-5.26**
	time	2897.66	2784.11	-3.92	3353.91	13.60
	distance	1579.06	1514.13	-4.11	1507.96	-4.71
25	routes	4.18	4.18	0.00	4.55	**8.00**
	time	2833.86	2671.05	**-5.64**	3015.68	6.03
	distance	1540.48	1492.66	-3.10	1482.93	-3.88
30	routes	4.09	4.09	0.00	4.00	**-2.27**
	time	2841.43	2681.47	**-5.63**	2945.93	3.55
	distance	1455.37	1433.81	-1.48	1508.60	3.53
35	routes	3.64	3.82	**5.00**	4.09	**11.11**
	time	2574.96	2518.11	-2.21	2699.30	4.61
	distance	1410.62	1406.16	-0.32	1470.69	4.08
40	routes	3.45	3.73	**7.89**	3.73	**7.32**
	time	2614.13	2533.37	-3.09	2746.10	4.81
	distance	1389.85	1413.90	1.73	1492.20	6.86
C SET						
5	routes	20.00	20.00	0.00	20.00	0.00
	time	13638.92	14344.80	**5.18**	29265.17	**53.40**
	distance	2503.73	2330.83	-6.91	2143.29	-16.82
10	routes	10.63	10.63	0.00	10.00	**-6.25**
	time	11137.40	11799.87	**5.95**	22654.80	50.84
	distance	1716.88	1674.63	-2.46	1389.08	-23.60
15	routes	7.38	7.38	0.00	7.00	**-5.36**
	time	11285.93	11293.97	**0.07**	18764.46	39.85
	distance	1411.53	1379.59	-2.26	1810.75	22.05
20	routes	6.00	6.00	0.00	5.88	**-2.13**
	time	11712.10	11590.73	**-1.04**	14575.39	19.64
	distance	1288.95	1269.80	-1.49	1714.88	24.84
25	routes	5.13	5.00	**-2.44**	5.00	**-2.50**
	time	11597.79	11821.28	1.93	12664.84	8.43
	distance	1248.01	1226.43	-1.73	1414.65	11.78
30	routes	4.25	4.25	0.00	4.13	**-3.03**
	time	11773.10	11926.08	**1.30**	11901.43	1.08
	distance	1101.24	1091.21	-0.91	1527.54	27.91

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n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
35	routes	3.75	3.75	0.00	3.88	**3.23**
	time	10153.98	10164.73	**0.11**	10340.16	1.80
	distance	919.29	917.82	-0.16	979.20	6.12
40	routes	3.75	3.75	0.00	3.88	**3.23**
	time	10153.98	10164.73	**0.11**	10340.16	1.80
	distance	919.29	917.82	-0.16	979.20	6.12
RC SET						
5	routes	20.00	20.13	**0.63**	20.00	0.00
	time	4820.71	3723.31	-22.76	6839.07	**29.51**
	distance	2781.44	2521.03	-9.36	2447.41	-13.65
10	routes	10.00	10.00	0.00	10.00	0.00
	time	3955.08	3442.03	**-12.97**	5156.87	**23.30**
	distance	2223.36	2064.15	-7.16	1888.44	-17.74
15	routes	7.00	7.00	0.00	7.00	0.00
	time	3465.01	3162.09	**-8.74**	4219.35	**17.88**
	distance	2050.68	1891.27	-7.77	1727.04	-18.74
20	routes	5.38	5.25	**-2.33**	5.63	**4.44**
	time	3291.17	3057.50	-7.10	3575.34	7.95
	distance	1971.09	1861.04	-5.58	1806.63	-9.10
25	routes	4.38	4.50	**2.86**	4.88	**10.26**
	time	3086.41	2995.08	-2.96	3294.79	6.32
	distance	1857.29	1810.52	-2.52	1909.10	2.71
30	routes	4.25	4.25	0.00	4.38	**2.86**
	time	3003.56	2997.32	**-0.21**	3173.12	5.34
	distance	1735.85	1817.22	4.69	1897.26	8.51
35	routes	4.00	4.13	**3.13**	4.38	**8.57**
	time	2831.88	2822.88	-0.32	3073.98	7.88
	distance	1727.38	1722.88	-0.26	1887.86	8.50
40	routes	4.00	4.13	**3.13**	4.25	**5.88**
	time	2843.02	2859.01	0.56	3082.02	7.75
	distance	1727.72	1770.04	2.45	1887.46	8.46

Table 3 shows that problem instances with relatively short routes whose customers' locations possess a certain randomness (R and RC) are better solved using PBmaxPF. For purely clustered problems, the picture is less clear. Only in four cases the best solution is found using PBmaxPF. For all three problem sets, PBactualPF generated the worst results.

Table 4: Earliest deadline, all time insertion weights

n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
R SET						
5	routes	6596.90	5072.72	**-23.10**	8000.56	**17.54**
	time	20.00	20.00	0.00	20.00	0.00
	distance	4212.71	3053.37	-27.52	5842.34	27.89
10	routes	10.00	10.00	0.00	10.00	0.00
	time	3067.74	2857.77	**-6.84**	4822.29	**36.38**
	distance	1862.62	1629.21	-12.53	1976.50	5.76
15	routes	7.00	7.00	0.00	7.00	0.00
	time	2901.09	2830.64	**-2.43**	4003.59	**27.54**
	distance	1606.36	1594.88	-0.71	1647.05	2.47
20	routes	5.18	5.18	0.00	5.09	**-1.79**
	time	3229.60	2897.94	**-10.27**	3388.02	4.68
	distance	1514.47	1501.09	-0.88	1474.20	-2.73
25	routes	4.18	4.18	0.00	4.27	**2.13**
	time	2833.86	2674.05	**-5.64**	3065.41	7.55
	distance	1540.48	1492.66	-3.10	1371.52	-12.32
30	routes	4.00	4.00	0.00	4.00	0.00
	time	2833.21	2706.76	**-4.46**	2927.16	**3.21**
	distance	1415.05	1421.92	0.49	1446.27	2.16
35	routes	3.64	3.64	0.00	3.64	0.00
	time	2573.82	2554.62	**-0.75**	2639.65	**2.49**
	distance	1392.75	1378.44	-1.03	1343.47	-3.67
40	routes	3.36	3.45	**2.70**	3.45	**2.63**
	time	2608.88	2578.03	-1.18	2625.77	0.64
	distance	1355.78	1344.48	-0.83	1359.06	0.24
C SET						
5	routes	20.00	20.00	0.00	20.00	0.00
	time	13638.92	14344.80	**5.18**	22659.31	**39.81**
	distance	2503.73	2330.83	-6.91	1955.59	-28.03
10	routes	10.00	10.00	0.00	10.00	0.00
	time	18653.13	19380.11	**3.90**	21539.57	**13.40**
	distance	1373.52	1454.36	5.89	1353.88	-1.45
15	routes	7.00	7.00	0.00	7.00	0.00
	time	14360.34	14076.36	**-1.98**	17775.75	**19.21**
	distance	1333.42	1299.93	-2.51	1426.16	6.50
20	routes	5.88	5.88	0.00	5.88	0.00
	time	12081.90	11960.53	**-1.00**	13782.79	**12.34**
	distance	1225.30	1206.15	-1.56	1107.93	-10.59
25	routes	4.75	4.75	0.00	4.88	**2.56**
	time	12463.36	12294.77	**-1.35**	12521.94	0.47
	distance	1190.54	1228.18	3.16	1322.43	9.97
30	routes	4.13	4.13	0.00	4.00	**-3.13**
	time	11718.93	11699.57	**-0.17**	11866.90	1.25
	distance	1053.42	1037.32	-1.53	1467.19	28.20

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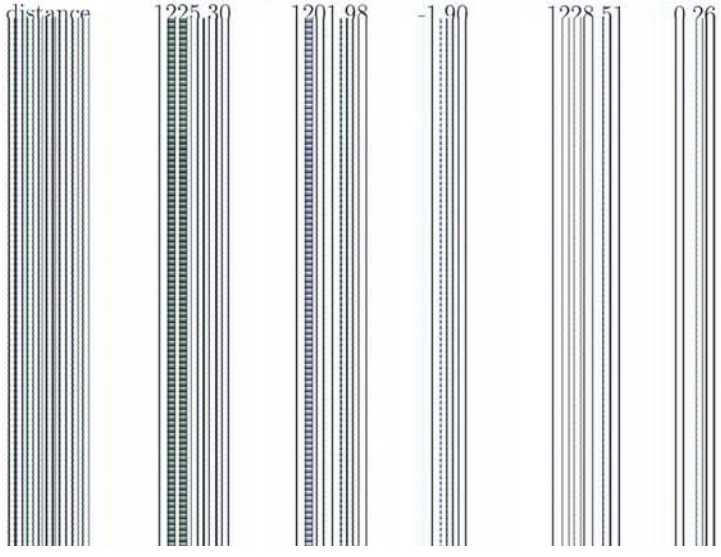
n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
35	routes	3.50	3.50	0.00	3.63	**3.45**
	time	9910.59	9929.21	**0.19**	9954.56	0.44
	distance	793.71	799.92	0.78	798.71	0.63
40	routes	3.50	3.50	0.00	3.63	**3.45**
	time	9910.59	9929.21	**0.19**	9954.56	0.44
	distance	793.71	799.92	0.78	798.71	0.63
RC SET						
5	routes	20.00	20.00	0.00	20.00	0.00
	time	4820.71	4539.29	**-5.84**	6839.07	**29.51**
	distance	2781.44	2474.89	-11.02	2447.41	-13.65
10	routes	10.00	10.00	0.00	10.00	0.00
	time	3955.08	3442.03	**-12.97**	5156.87	**23.30**
	distance	2223.36	2064.15	-7.16	1888.44	-17.74
15	routes	7.00	7.00	0.00	7.00	0.00
	time	3465.01	3162.09	**-8.74**	4219.35	**17.88**
	distance	2050.68	1891.27	-7.77	1727.04	-18.74
20	routes	5.25	5.25	0.00	5.38	**2.33**
	time	3360.37	3057.50	**-9.01**	3658.90	8.16
	distance	1859.80	1861.04	0.07	1621.32	-14.71
25	routes	4.38	4.38	0.00	4.50	**2.78**
	time	3086.41	3052.08	**-1.11**	3270.68	5.63
	distance	1857.29	1815.08	-2.27	1688.49	-10.00
30	routes	4.25	4.25	0.00	4.25	0.00
	time	3001.15	2997.32	**-0.13**	3133.80	**4.23**
	distance	1744.41	1817.22	4.17	1739.67	-0.27
35	routes	4.00	4.13	**3.13**	4.13	**3.03**
	time	2827.34	2806.72	-0.73	2920.68	3.20
	distance	1704.57	1695.73	-0.52	1707.91	0.20
40	routes	4.00	4.00	0.00	4.00	0.00
	time	2843.02	2860.08	**0.60**	2931.92	**3.03**
	distance	1727.72	1680.78	-2.72	1707.90	-1.16

Table 5 reports on the overall performance of the three time insertion criteria. Since PBmaxPF is a modification of maxPF to better calculate the time needed to insert a customer in route, its main effect on solution quality consists of reducing schedule time for routes up to 30 customers, besides for $n = 20$ for RC. Although PBactualPF obtained a few impressive improvements over maxPF in the previous tables, its overall quality is rather poor, making it inferior to maxPF and PBmaxPF.

Table 5: best solutions of farthest customer, earliest deadline and the full parameter set

n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
R SET						
5	routes	20.00	20.00	0.00	20.00	0.00
	time	4212.71	3041.86	**-27.79**	5008.07	**15.88**
	distance	2364.19	1978.15	-16.33	2469.00	4.25
10	routes	10.00	10.00	0.00	10.00	0.00
	time	3067.74	2802.04	**-8.66**	4196.03	**26.89**
	distance	1862.62	1598.17	-14.20	1730.89	-7.61
15	routes	7.00	7.00	0.00	7.00	0.00
	time	2901.09	2767.05	**-4.62**	3274.98	**22.12**
	distance	1606.36	1532.52	-4.60	1539.24	-4.36
20	routes	5.00	5.00	0.00	5.09	**1.79**
	time	3129.81	2801.85	**-10.48**	3334.82	6.15
	distance	1519.75	1489.94	-1.96	1476.75	-2.91
25	routes	4.09	4.18	**2.22**	4.27	**4.26**
	time	2779.47	2654.57	-4.49	3035.99	8.45
	distance	1462.26	1455.53	-0.46	1407.75	-3.87
30	routes	4.00	4.00	0.00	4.00	0.00
	time	2813.75	2587.15	**-8.05**	2788.49	**-0.91**
	distance	1429.44	1404.70	-1.73	1455.90	1.82
35	routes	3.45	3.64	**5.26**	3.64	**5.00**
	time	2545.38	2511.89	-1.321	2613.03	2.59
	distance	1405.66	1333.97	-5.10	1308.31	-7.44
40	routes	3.36	3.36	0.00	3.36	0.00
	time	2549.62	2549.76	**0.01**	2583.40	**1.31**
	distance	1364.90	1301.45	-4.65	1291.07	-5.72
C SET						
5	routes	20.00	20.00	0.00	20.00	0.00
	time	13587.80	11940.45	**-12.12**	21856.09	**37.83**
	distance	2349.92	2126.80	-9.49	2060.55	-14.04
10	routes	10.00	10.00	0.00	10.00	0.00
	time	17686.73	17213.42	**-2.68**	19658.64	**10.03**
	distance	1597.55	1366.56	-14.46	1300.86	-22.81
15	routes	7.00	7.00	0.00	7.00	0.00
	time	13509.95	12836.61	**-4.98**	16548.40	**18.36**
	distance	1421.23	1365.68	-3.91	1503.82	5.49
20	routes	5.88	5.88	0.00	5.88	0.00
	time	12081.90	11485.81	**-4.93**	13067.63	**7.54**
	distance	1225.30	1201.98	-1.90	1228.51	0.26



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n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
	distance	1051.46	1027.44	**-2.28**	1497.07	29.77
35	routes	3.38	3.38	0.00	3.50	**3.57**
	time	9876.66	9885.57	**0.09**	9901.12	0.25
	distance	761.27	762.72	0.19	769.69	1.09
40	routes	3.38	3.38	0.00	3.50	**3.57**
	time	9876.66	9885.57	**0.09**	9901.12	0.25
	distance	761.27	762.72	0.19	769.69	1.09
RC SET						
5	routes	20.00	20.00	0.00	20.00	0.00
	time	4807.54	3684.05	**-23.37**	5530.10	**13.07**
	distance	2703.97	2424.31	-10.34	2402.57	-12.55
10	routes	10.00	10.00	0.00	10.00	0.00
	time	3955.08	3232.72	**-18.26**	4505.36	**12.21**
	distance	2223.36	1945.26	-12.51	1978.49	-12.38
15	routes	7.00	7.00	0.00	7.00	0.00
	time	3465.01	3062.35	**-11.62**	3866.34	**10.38**
	distance	2050.68	1811.34	-11.67	1998.56	-2.61
20	routes	5.00	5.25	**5.00**	5.38	**6.98**
	time	3332.88	2994.45	-10.15	3430.09	2.83
	distance	1759.30	1818.52	3.37	1698.36	-3.59
25	routes	4.38	4.38	0.00	4.50	**2.78**
	time	3060.50	2991.96	**-2.24**	3197.25	4.28
	distance	1781.20	1702.33	-4.43	1608.52	-10.74
30	routes	4.25	4.25	0.00	4.13	**3.03**
	time	2991.22	2902.96	**-2.95**	3065.35	2.42
	distance	1691.52	1728.56	2.19	1759.99	3.89
35	routes	4.00	4.13	**3.13**	4.00	0.00
	time	2802.28	2794.99	-0.26	2862.37	**2.10**
	distance	1648.93	1674.15	1.53	1664.65	0.94
40	routes	4.00	4.00	0.00	3.88	**-3.23**
	time	2800.95	2811.03	**0.36**	2881.78	2.80
	distance	1647.43	1653.63	0.38	1668.83	1.28

The multidimensionality of the lexicographic solution hampers the evaluation of the time insertion criterion. Also, the use of a lexicographic preference ordering on vehicles, schedule time and distance may not always be appropriate. In the short run, capacity is fixed and a focus on schedule time and travel distance may be more appropriate. By using cost data, solution quality can be expressed as a simple cost measurement. Depending the definition of costs, different cost components have to be taken into account

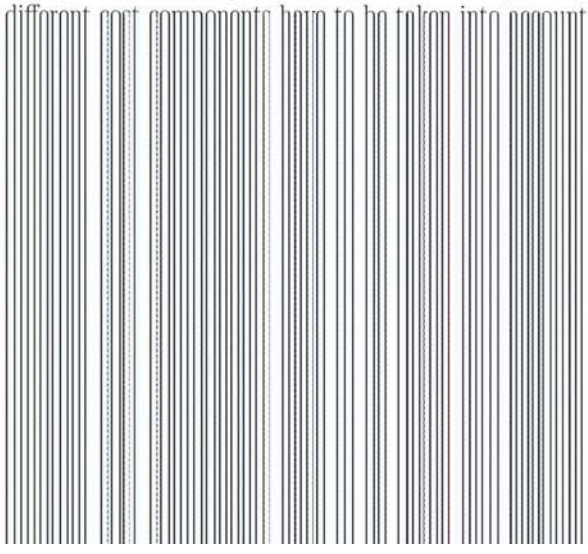


Table 6: Hour and time coefficients in Euros for 1999 (Blauwens et al. 2000)

Carrying capacity	hour coefficient	kilometer coefficient
delivery van 0.5 t	16.03	0.10
lorry 5 t	17.14	0.15
lorry 8 t	18.06	0.17
lorry 20 t	20.88	0.21
truck and trailer 28 t	21.75	0.24

however, are only incurred when distance is travelled. Time costs consist of the labor cost of the lorry driver, the time-related component of the depreciation cost[1], the annual insurance premium, and all costs that automatically result from holding a certain capacity, such as rent of garages and administration costs. Kilometer costs consist of fuel consumption, kilometer-related depreciation and maintenance, mileage allowances of staff, fines and damages. Costs that cannot be divided into one of these categories, such as commissions, accommodation expenses, tollage, and port dues can be taken separately into account. The figures in Table 6 are averages of sample data from different companies of vehicles with different engine powers. They are calculated for firms respecting all statutory regulations with wage-earning truck drivers.

Since a pick-up or delivery problem is modelled, average sample data on the cost structure of a lorry with a loading capacity of 20 tons is used. Because the lorry's fixed costs are expressed per hour (20.88 EUR), they are multiplied with the maximum statutory driving time (9 hours) to obtain the daily cost of owning the lorry (187.92 EUR). The hour coefficient from Blauwens et al. (2000) is divided by 60 to approximate the time coefficient δ_t, expressing the opportunity cost of time. Indeed, in the long run the average opportunity cost of time equals the average cost of owning a vehicle. In the short run, the opportunity cost will depend on the carrier's potential customers of that moment, making it higher during peak periods than during off-peak periods. The distance coefficient is equaled to the kilometer coefficient.

One could easily transform the solutions of each run, generated by the previous parameter set, into costs and then select the cheapest solution per problem set. It seems more interesting though, especially to VRPTW practitioners, to use the actual cost data to minimize total costs instead of arbitrarily chosen values from $[0, 1]$. For this purpose, the insertion heuristic has to be slightly modified.

The insertion distance $c_{11}(i, u, j)$ is multiplied by the kilometer cost δ_d. The insertion time $c_{12}(i, u, j)$ is valued at the opportunity cost of time δ_t. If we would steer the heuristic only by the kilometer cost δ_d and the opportunity cost of time δ_t, we would obtain only one solution based on the actual cost

data. Since it is well possible to obtain a better solution based on factors that differ from the original δ_d and δ_t, they are multiplied by two weighting factors $\alpha_1, \alpha_2 \geq 0$, which no longer have to sum up to 1.

$$c_1^* (i(u), u, j(u)) = \max_p [c_1 (i_{p-1}, u, i_p)], \quad p = 1, ..., m \tag{14}$$

$$\begin{aligned} c_1 (i,u,j) &= \alpha_1 \delta_d c_{11} (i,u,j) + \alpha_2 \delta_t c_{12} (i,u,j) \text{ in which} \\ c_{11} (i,u,j) &= d_{iu} + d_{uj} - \mu d_{ij}, \mu \geq 0 \\ c_{12} (i,u,j) &= b_j^{\text{new}} - b_j. \end{aligned} \tag{15}$$

If $\alpha_1 = \alpha_2 = 1$, the original cost levels are used to guide the insertion heuristic. The selection criterion $c_2 (i,u,j)$ is rewritten in the same way

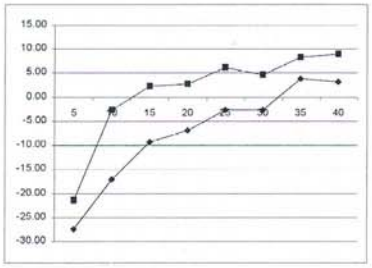
$$c_2^* (i(u^*), u^*, j(u^*)) = \max_u [c_2 (i(u), u, j(u))] \quad u \text{ unrouted and feasible} \tag{16}$$

$$c_2 (i,u,j) = \lambda (\alpha_1 \delta_d d_{ou} + \alpha_2 \delta_t t_{ou}) + c_f - c_1 (i,u,j), \lambda \geq 0. \tag{17}$$

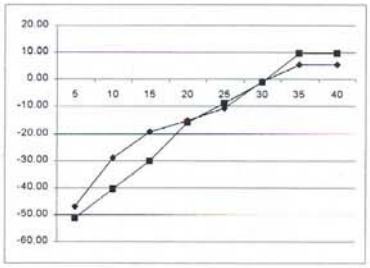
By using cost data we can replace the traditional lexicographic preference ordering by short or long run costs minimization. Figures 4 to 8 depict the time insertion criterion's impact on the long run cost level. The PBmaxPF and PBactualPF curves are deviations in terms of percentages from the maxPF cost level. Details on the impact on costs, route length, schedule time and distance can be found in the Appendix.

Figure 4 shows a similar picture for the R and RC sets. Both PBmaxPF as PBactualPF outclass maxPF by offering cost reductions starting from more than 50 % tampering off to more than 10% for routes with 15 customers. Over the range of route lengths considered, the use of the actual push forward could never improve on the combination of the push backward and maximum push forward. Not only do cost deviations generated by PBmaxPF always outstrip PBactualPF, they also tend to stay negative for longer routes (up to $n = 30$). Cost savings tend to decrease as the number of customers per route increases because it increases the probability that the high waiting time, stored by maxPF at customers that were at the first position during route construction, can be used to insert other customers. For the C set, PBactualPF leads to very significant improvements (more than 50%). The deviations from the maxPF cost level are more moderate for PBmaxPF, making both the negative and positive deviations from maxPF smaller.

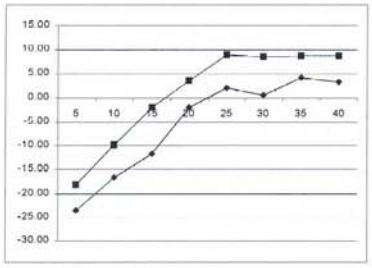
Figure 4: Farthest customer, strictly positive time insertion weights



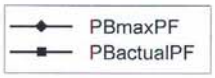
(a) R SET



(b) C SET

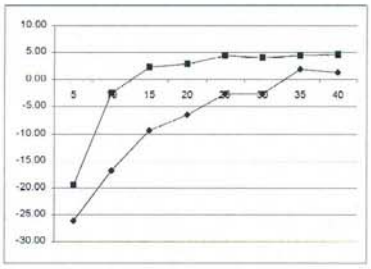


(c) RC SET

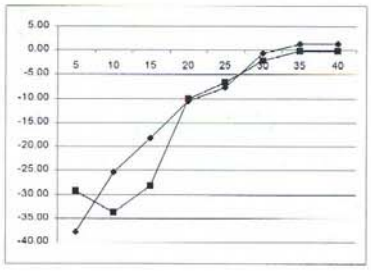


(d) LEGEND

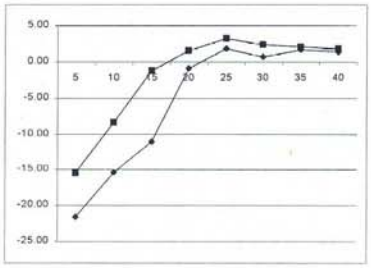
Figure 5: Farthest customer, all time insertion weights



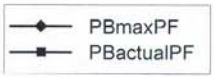
(a) R SET



(b) C SET



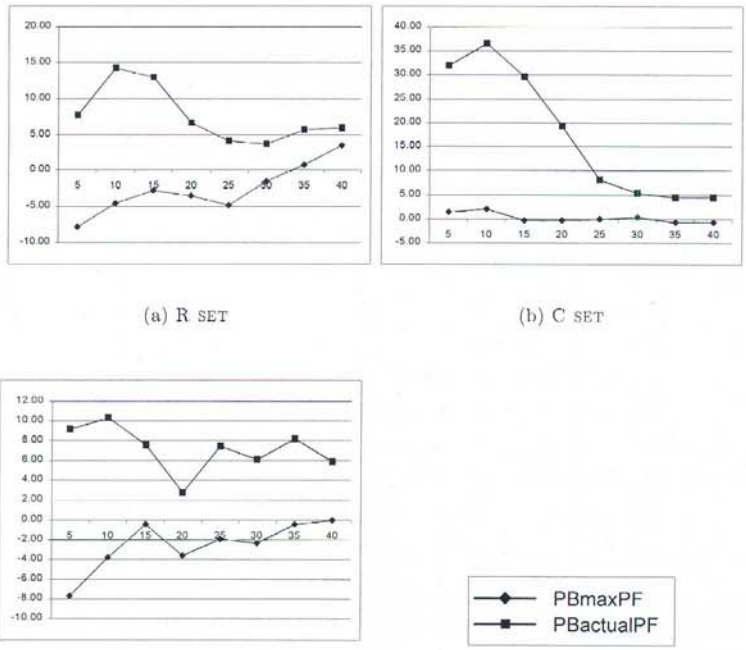
(c) RC SET



(d) LEGEND

Additional costs savings from adding $\alpha_2 = 0$ to the parameter list are bound to be small since it makes maxPF and PBmaxPF solutions identical. For the C set it reduces the cost gap between maxPF and PBmaxPF and PBactualPF. In general, the pattern of the PBmaxPF and PBactualPF curves in Figure 5 remain largely the same.

Figure 6: Earliest deadline, strictly positive time insertion weights

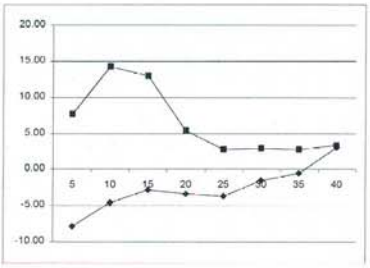


(a) R SET (b) C SET

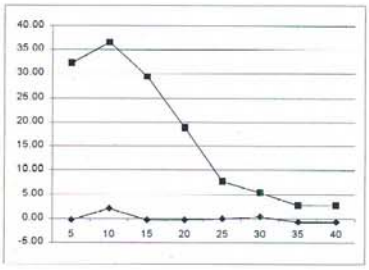
(c) RC SET (d) LEGEND

Under the conditions of Figure 6 an insertion heuristic can only benefit from using PBmaxPF instead of maxPF, if customers' locations possess a certain randomness. Cost reductions eroding from almost 8% up to 30 customers (R set) or 35 (RC set) remain important. PBactualPF produces extremely bad results especially for short routes in the C set.

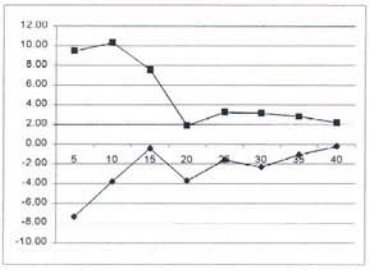
Figure 7: Earliest deadline, all time insertion weights



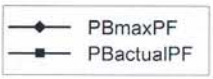
(a) R SET



(b) C SET



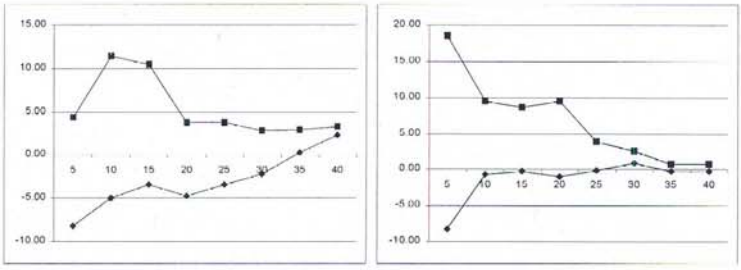
(c) RC SET



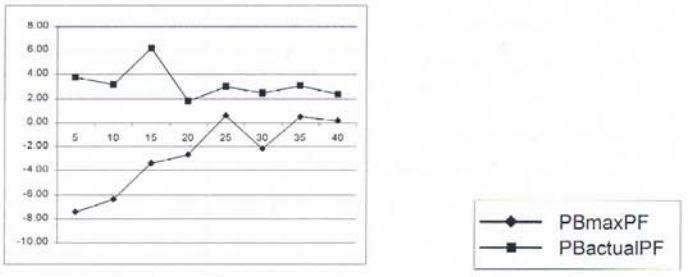
(d) LEGEND

Using the full parameter set to guide the heuristic makes hardly any difference to the time insertion criterion's relative performance as can be seen in Figure 7.

Figure 8: Best solution of farthest customer, earliest deadline and all time insertion weights



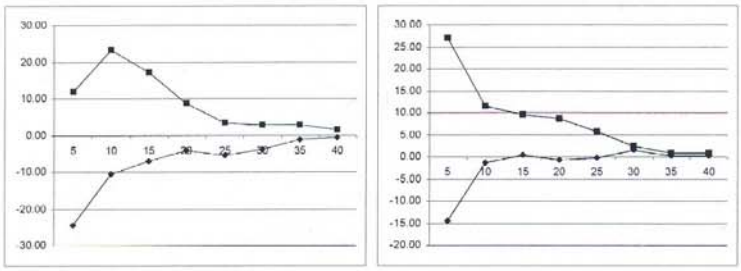
(a) R SET (b) C SET



(c) RC SET (d) LEGEND

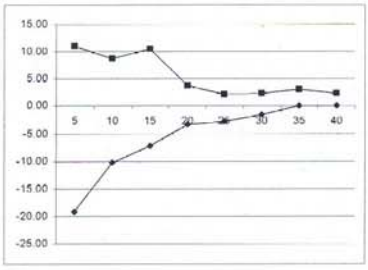
The impressive cost reductions depicted in Figures 4 and 5 are weakened by selecting the best solution from both initialization criteria. Since initializing a route with the customer with the earliest deadline keeps the possibility of inserting a customer between i_0 and i_1 low, its maxPF solutions are of a higher relative quality than the ones based on the farthest customer from the depot. Because the use of PBmaxPF does not change the heuristic's complexity, its cost savings over Solomon's (1987) time insertion criterion, which we called maxPF, can be realized without any additional computation time. PBmaxPF is the best time insertion criterion when the routes relative length is relatively small, especially when routes are only initialized with the farthest customer from the depot.

Figure 9: Best solution of farthest customer, earliest deadline and all time insertion weights (short run costs)

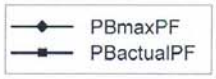


(a) R SET

(b) C SET



(c) RC SET



(d) LEGEND

6 Conclusion

In this paper attention was given to the behavior of Solomon's (1987) time insertion criterion (maxPF), for problem instances with varying relative route lengths. Relative route length is defined as the combination of the absolute number of customers per route, and the customers' time window sizes relative to the scheduling horizon.

Because Solomon's (1987) time insertion $c_{12}(i,u,j)$ underestimates the additional time needed to insert a customer between the depot, $i = i_0$ and the first customer $j = i_1$, the insertion heuristic tends to select customers with an underestimated high waiting time for insertion between i_0 and i_1. Since the advance of the depot departure time is taken into account too cheaply, or even not at all, a route with a relatively small number of customers can have a larger schedule time than necessary. If, on the other hand, the number of customers per route is high, there is a high probability that the waiting time at the customers, that were at position i_1 during the construction of the route, can be used to insert other customers.

By introducing a push forward, taking the advance of the depot departure time into account, and using the actual push forward to measure the impact on the depot arrival time, two new time insertion criteria were created. Their relative effectiveness to maxPF was evaluated by the classical lexicographic preference ordering on vehicles, schedule time and distance, and by using real-life cost data. Solution improvements generated by PBmaxPF are the largest when relatively short routes are initialized by the farthest customer from the depot (up to 47%). The impact of using PBmaxPF on the best solution from both initialization criteria and all insertion weights considered, is larger for short run cost than for long run cost minimization. Because capacity costs do not have to be taken into account in the short run, the possible increase in the number of routes by focussing on schedule time minimization, does not deteriorate solution quality. This makes PBmaxPF an even more interesting time insertion criterion for daily routing and scheduling problems. The improvements over the original Solomon (1987) criterion, maxPF, tend to decrease as route length increases. For larger routes, the waiting time, stored by maxPF at the customers that were inserted before the first customer during the route construction, can be used to insert other customers, allowing maxPF to surpass the other time insertion criteria in solution quality.

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Appendix

Table 7: Farthest customer, strictly positive time insertion weights

n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
R SET						
5	costs	7215.55	5237.79	**-27.41**	5946.75	**-21.34**
	routes	20.00	20.00	0.00	20.00	0.00
	time	8576.39	3012.62	-64.87	5011.43	-71.14
	distance	1969.03	1795.81	-8.80	1851.56	-6.34
10	costs	3905.12	3239.48	**-17.05**	3801.68	**-2.72**
	routes	10.00	10.00	0.00	10.00	0.00
	time	4692.80	2849.21	-39.29	4470.54	-4.97
	distance	1636.79	1536.49	-6.13	1528.04	-7.12
15	costs	2906.70	2636.49	**-9.30**	2975.37	**2.31**
	routes	7.09	7.00	-1.28	7.00	-1.30
	time	3465.92	2775.40	-19.92	3738.30	7.29
	distance	1533.47	1480.03	-3.48	1495.85	-2.51
20	costs	2422.84	2251.66	**-7.07**	2491.16	**2.74**
	routes	5.36	5.09	-5.08	5.55	3.28
	time	3046.80	2722.06	-10.66	3151.59	3.33
	distance	1477.59	1448.75	-1.95	1467.93	-0.66
25	costs	2147.20	2087.87	**-2.76**	2286.55	**6.09**
	routes	4.18	4.36	4.35	4.73	11.54
	time	2855.61	2659.11	-6.88	2949.00	3.17
	distance	1531.66	1427.03	-6.83	1549.80	1.17
30	costs	2077.56	2020.53	**-2.75**	2181.33	**4.76**
	routes	4.00	4.09	2.27	4.36	8.33
	time	2809.64	2619.82	-6.76	2837.11	0.97
	distance	1450.52	1416.94	-2.31	1558.33	6.92
35	costs	1878.61	1951.76	**3.89**	2048.32	**8.29**
	routes	3.45	3.73	7.89	3.73	7.32
	time	2546.54	2614.16	2.66	2795.08	8.89
	distance	1430.14	1423.35	-0.47	1563.36	8.52
40	costs	1858.57	1919.09	**3.26**	2043.46	**9.05**
	routes	3.27	3.45	5.56	3.64	10.00
	time	2585.93	2640.87	2.12	2829.75	8.62
	distance	1431.92	1462.06	2.10	1564.02	8.45
C SET						
5	costs	16691.82	8842.80	**-47.02**	11030.41	**-51.33**
	routes	20.13	20.38	1.24	20.75	3.01
	time	35357.24	13023.40	-63.17	18709.29	-88.98

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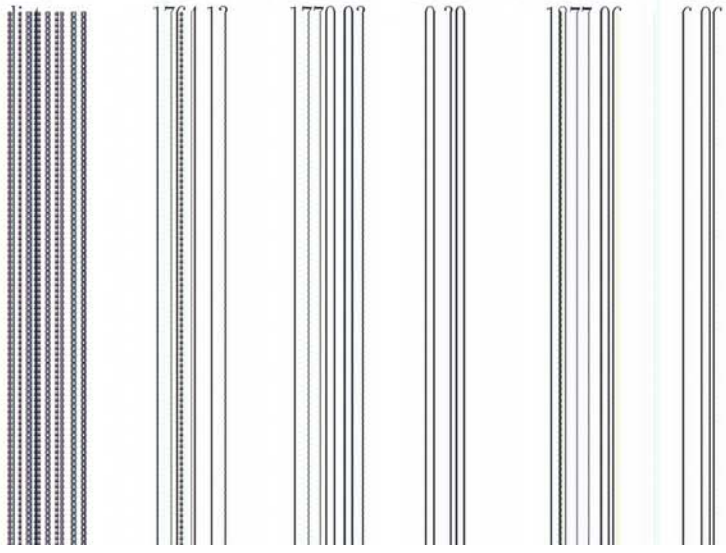
n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
15	costs	7787.57	6265.87	**-19.54**	5981.74	**-30.19**
	routes	7.50	7.75	3.33	8.38	10.45
	time	17211.14	12881.23	-25.16	11439.98	-50.45
	distance	1619.56	1361.76	-15.92	1778.32	8.93
20	costs	6522.17	5542.13	**-15.03**	5618.21	**-16.09**
	routes	6.13	6.63	8.16	6.88	10.91
	time	14421.34	11411.97	-20.87	11187.85	-28.90
	distance	1468.88	1357.46	-7.59	1803.70	18.56
25	costs	5797.37	5187.55	**-10.52**	5340.36	**-8.56**
	routes	5.13	5.63	9.76	5.88	12.77
	time	12927.48	10964.11	-15.19	11076.64	-16.71
	distance	1398.00	1312.45	-6.12	1590.24	12.09
30	costs	5225.11	5193.04	**-0.61**	5175.39	**-0.96**
	routes	4.50	5.00	11.11	5.38	16.28
	time	11861.06	11394.51	-3.93	10995.04	-7.88
	distance	1049.26	1200.65	14.43	1412.70	25.73
35	costs	4517.84	4769.58	**5.57**	4991.49	**9.49**
	routes	3.75	4.00	6.67	4.50	16.67
	time	10323.18	10774.96	4.38	10968.45	5.88
	distance	919.47	1117.56	21.54	1370.10	32.89
40	costs	4517.84	4769.58	**5.57**	4991.49	**9.49**
	routes	3.75	4.00	6.67	4.50	16.67
	time	10323.18	10774.96	4.38	10968.45	5.88
	distance	919.47	1117.56	21.54	1370.10	32.89
RC SET						
5	costs	7292.25	5576.79	**-23.52**	6166.67	**-18.25**
	routes	20.00	20.00	0.00	20.00	0.00
	time	8525.90	3628.42	-57.44	5275.98	-61.60
	distance	2361.82	2315.40	-1.97	2384.28	0.94
10	costs	4119.86	3436.24	**-16.59**	3750.17	**-9.86**
	routes	10.00	10.00	0.00	10.13	1.23
	time	5116.32	3197.43	-37.51	3927.16	-30.28
	distance	1917.41	1851.41	-3.44	2003.47	4.30
15	costs	3178.75	2804.81	**-11.76**	3119.08	**-1.91**
	routes	7.00	7.00	0.00	7.25	3.45
	time	4107.49	3083.55	-24.93	3722.20	-10.35
	distance	1807.91	1734.58	-4.06	1922.24	5.95
20	costs	2586.81	2534.01	**-2.04**	2682.70	**3.57**
	routes	5.25	5.63	7.14	5.75	8.70
	time	3381.72	3022.82	-10.61	3308.75	-2.21



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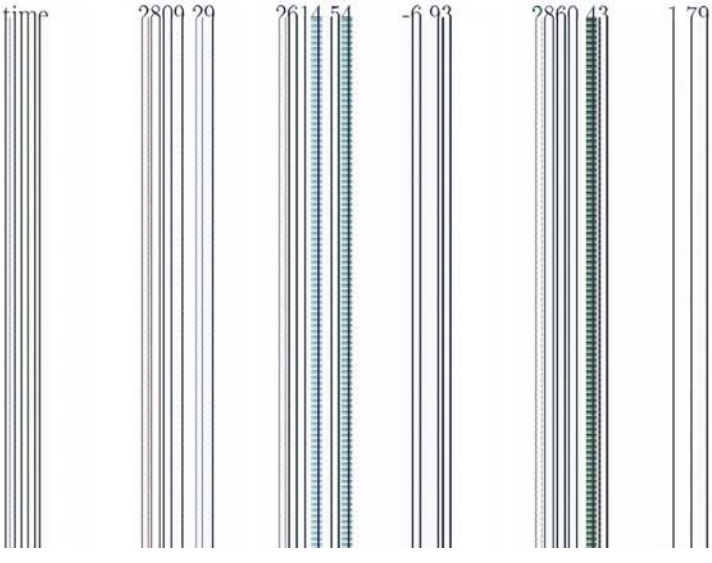
n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
	distance	1717.93	1747.88	1.74	1931.00	11.03
30	costs	2255.71	2270.74	**0.67**	2466.78	**8.56**
	routes	4.25	4.25	0.00	4.63	8.11
	time	3026.18	2988.56	-1.24	3233.27	6.41
	distance	1683.07	1800.23	6.96	1968.65	14.51
35	costs	2159.05	2248.54	**4.14**	2368.95	**8.86**
	routes	4.00	4.25	6.25	4.25	5.88
	time	2882.37	2954.90	2.52	3169.91	9.07
	distance	1684.63	1756.55	4.27	1946.50	13.45
40	costs	2149.19	2223.46	**3.46**	2352.80	**8.65**
	routes	4.00	4.00	0.00	4.13	3.03
	time	2846.96	2975.21	4.50	3185.10	10.62
	distance	1694.88	1818.38	7.29	1955.04	13.31

Table 8: Farthest customer, all time insertion weights

n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
R SET						
5	costs	7100.60	5237.79	**-26.23**	5946.75	**-19.40**
	routes	20.00	20.00	0.00	20.00	0.00
	time	8351.22	3012.62	-63.93	5011.43	-66.64
	distance	1816.58	1795.81	-1.14	1851.56	1.89
10	costs	3897.52	3239.48	**-16.88**	3801.68	**-2.52**
	routes	10.00	10.00	0.00	10.00	0.00
	time	4699.32	2849.21	-39.37	4470.54	-5.12
	distance	1595.65	1536.49	-3.71	1528.04	-4.42
15	costs	2906.70	2636.49	**-9.30**	2975.37	**2.31**
	routes	7.09	7.00	-1.28	7.00	-1.30
	time	3465.92	2775.40	-19.92	3738.30	7.29
	distance	1533.47	1480.03	-3.48	1495.85	-2.51
20	costs	2408.80	2251.66	**-6.52**	2481.63	**2.93**
	routes	5.36	5.09	-5.08	5.45	1.67
	time	3045.75	2722.06	-10.63	3195.31	4.68
	distance	1420.61	1448.75	1.98	1436.01	1.07
25	costs	2137.31	2082.24	**-2.58**	2236.83	**4.45**
	routes	4.18	4.27	2.17	4.36	4.17
	time	2861.57	2695.78	-5.79	3109.29	7.97
	distance	1481.84	1421.55	-4.07	1394.91	-6.23
30	costs	2072.54	2017.88	**-2.64**	2159.27	**4.02**
	routes	4.00	4.09	2.27	4.27	6.38
	time	2809.29	2614.54	-6.93	2860.43	1.79



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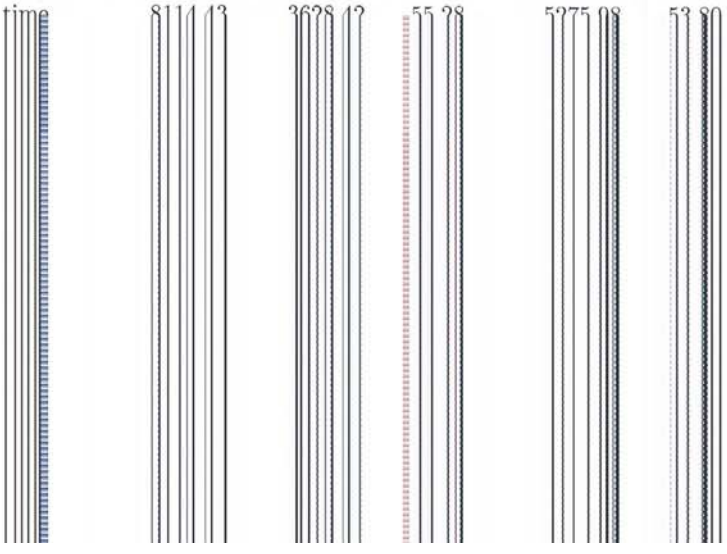
n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
40	costs	1850.22	1877.16	**1.46**	1939.44	**4.60**
	routes	3.27	3.36	2.78	3.45	5.26
	time	2572.84	2616.27	1.69	2728.84	5.72
	distance	1416.09	1394.16	-1.55	1419.25	0.22
C SET						
5	costs	14249.50	8842.80	**-37.94**	11030.41	**-29.18**
	routes	20.38	20.38	0.00	20.75	1.81
	time	28723.13	13023.40	-54.66	18709.29	-53.52
	distance	1770.77	2007.45	13.37	2584.30	31.48
10	costs	9069.37	6766.84	**-25.39**	6785.66	**-33.65**
	routes	10.38	10.88	4.82	11.00	5.68
	time	19530.52	12579.20	-35.59	12157.33	-60.65
	distance	1346.15	1440.20	6.99	2032.46	33.77
15	costs	7660.95	6265.87	**-18.21**	5981.74	**-28.07**
	routes	7.38	7.75	5.08	8.38	11.94
	time	17205.32	12881.23	-25.13	11439.98	-50.40
	distance	1198.27	1361.76	13.64	1778.32	32.62
20	costs	6187.89	5542.13	**-10.44**	5618.21	**-10.14**
	routes	6.00	6.63	10.42	6.88	12.73
	time	13783.62	11411.97	-17.21	11187.85	-23.20
	distance	1098.65	1357.46	23.56	1803.70	39.09
25	costs	5623.41	5187.55	**-7.75**	5271.32	**-6.68**
	routes	5.25	5.63	7.14	5.63	6.67
	time	12496.73	10964.11	-12.26	11208.43	-11.49
	distance	1199.85	1312.45	9.38	1307.24	8.21
30	costs	5206.99	5174.67	**-0.62**	5103.94	**-2.02**
	routes	4.50	5.00	11.11	5.00	10.00
	time	11793.98	11385.82	-3.46	11171.68	-5.57
	distance	1071.01	1136.67	6.13	1152.46	7.07
35	costs	4371.58	4425.65	**1.24**	4367.54	**-0.09**
	routes	3.50	3.50	0.00	3.50	0.00
	time	10065.41	10264.41	1.98	10026.66	-0.39
	distance	879.59	816.31	-7.20	918.93	4.28
40	costs	4371.58	4425.65	**1.24**	4367.54	**-0.09**
	routes	3.50	3.50	0.00	3.50	0.00
	time	10065.41	10264.41	1.98	10026.66	-0.39
	distance	879.59	816.31	-7.20	918.93	4.28
RC SET						
5	costs	7111.79	5576.79	**-21.58**	6166.67	**-15.33**
	routes	20.00	20.00	0.00	20.00	0.00
	time	8114.43	3628.42	-55.28	5975.08	-26.36



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n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
	time	5059.89	3197.43	-36.81	3927.16	-28.84
	distance	1764.69	1851.41	4.91	2003.47	11.92
15	costs	3152.49	2804.81	**-11.03**	3116.26	**-1.16**
	routes	7.00	7.00	0.00	7.13	1.75
	time	4129.95	3083.55	-25.34	3837.47	-7.62
	distance	1665.94	1734.58	4.12	1841.19	9.52
20	costs	2545.80	2522.57	**-0.91**	2586.90	**1.59**
	routes	5.25	5.25	0.00	5.25	0.00
	time	3381.37	3264.39	-3.46	3534.00	4.32
	distance	1593.77	1666.60	4.57	1543.71	-3.24
25	costs	2282.35	2325.64	**1.90**	2361.11	**3.34**
	routes	4.38	4.63	5.71	4.50	2.78
	time	3040.36	3108.39	2.24	3303.15	7.96
	distance	1675.64	1561.63	-6.80	1524.89	-9.89
30	costs	2217.11	2231.95	**0.67**	2272.88	**2.45**
	routes	4.25	4.25	0.00	4.38	2.86
	time	2953.46	2981.52	0.95	3096.66	4.62
	distance	1627.68	1648.82	1.30	1554.53	-4.71
35	costs	2142.70	2180.27	**1.75**	2187.95	**2.07**
	routes	4.00	4.13	3.13	4.00	0.00
	time	2867.33	2885.09	0.62	2977.61	3.70
	distance	1638.29	1671.20	2.01	1666.92	1.72
40	costs	2120.26	2148.94	**1.35**	2159.96	**1.84**
	routes	4.00	4.00	0.00	4.00	0.00
	time	2826.61	2879.30	1.86	2905.41	2.71
	distance	1603.83	1646.93	2.69	1654.98	3.09

Table 9: Earliest deadline, strictly positive time insertion weights

n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
R SET						
5	costs	5661.93	5217.42	**-7.85**	6128.47	**7.61**
	routes	20.00	20.00	0.00	20.00	0.00
	time	4007.99	2931.03	-26.87	5522.37	27.42
	distance	2119.79	1829.28	-13.70	1867.86	-13.49
10	costs	3345.05	3193.67	**-4.53**	3898.26	**14.19**
	routes	10.00	10.00	0.00	10.00	0.00
	time	3017.42	2701.82	-10.46	4804.75	37.20
	distance	1732.45	1559.32	-9.99	1445.85	-19.82
15	costs	2659.52	2584.95	**-2.80**	3053.55	**12.90**
	routes	7.00	7.00	0.00	7.09	1.28

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n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
25	costs	2168.20	2062.32	**-4.88**	2260.94	**4.10**
	routes	4.36	4.36	0.00	4.82	9.43
	time	2809.78	2587.54	-7.91	2953.64	4.87
	distance	1543.25	1424.33	-7.71	1365.16	-13.05
30	costs	2026.55	1995.50	**-1.53**	2101.94	**3.59**
	routes	4.00	4.00	0.00	4.18	4.35
	time	2670.71	2609.98	-2.27	2808.05	4.89
	distance	1439.42	1398.10	-2.87	1412.07	-1.94
35	costs	1921.94	1935.79	**0.72**	2035.72	**5.59**
	routes	3.64	3.73	2.50	3.91	6.98
	time	2577.91	2557.35	-0.80	2735.49	5.76
	distance	1422.85	1439.17	1.15	1454.88	2.20
40	costs	1868.28	1933.53	**3.49**	1987.45	**6.00**
	routes	3.36	3.73	10.81	3.64	7.50
	time	2588.27	2560.53	-1.07	2724.16	4.99
	distance	1397.79	1425.16	1.96	1483.75	5.79
C SET						
5	costs	9029.57	9159.06	**1.43**	13290.63	**32.06**
	routes	20.13	20.13	0.00	20.00	-0.63
	time	13438.05	13991.76	4.12	25976.40	48.27
	distance	2380.17	2116.84	-11.06	2051.83	-16.00
10	costs	6142.01	6268.87	**2.07**	9669.15	**36.48**
	routes	10.50	10.63	1.19	10.13	-3.70
	time	10931.23	11219.66	2.64	21437.45	49.01
	distance	1519.92	1532.41	0.82	1275.95	-19.12
15	costs	5461.74	5448.26	**-0.25**	7754.08	**29.56**
	routes	7.75	7.63	-1.61	7.00	-10.71
	time	10600.38	10645.31	0.42	17697.42	40.10
	distance	1318.45	1295.01	-1.78	1166.41	-13.03
20	costs	5085.85	5067.48	**-0.36**	6300.41	**19.28**
	routes	6.13	6.13	0.00	5.63	-8.89
	time	10494.44	10461.50	-0.31	14291.62	26.57
	distance	1178.20	1149.42	-2.44	1124.45	-4.78
25	costs	5015.54	5018.02	**0.05**	5462.32	**8.18**
	routes	5.75	5.75	0.00	4.88	-17.95
	time	10558.01	10542.40	-0.15	12372.06	14.66
	distance	1086.73	1119.69	3.03	1003.04	-8.34
30	costs	4803.66	4819.55	**0.33**	5076.45	**5.37**
	routes	4.38	4.50	2.86	4.25	-2.94
	time	10696.02	10675.59	-0.19	11521.23	7.16
	distance	1080.41	1078.36	-0.19	1118.25	3.20

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n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
	distance	815.37	810.09	-0.65	903.57	9.76
40	costs	4298.81	4270.53	**-0.66**	4499.24	**4.45**
	routes	3.38	3.25	-3.70	3.88	12.90
	time	9968.09	9957.95	-0.10	10213.19	2.40
	distance	815.37	810.09	-0.65	903.57	9.76
RC SET						
5	costs	6000.02	5538.58	**-7.69**	6605.25	**9.16**
	routes	20.13	20.13	0.00	20.00	-0.63
	time	4549.62	3455.39	-24.05	6563.12	30.68
	distance	2645.25	2309.23	-12.70	2345.33	-12.79
10	costs	3630.71	3492.66	**-3.80**	4046.57	**10.28**
	routes	10.25	10.25	0.00	10.00	-2.50
	time	3457.73	3119.59	-9.78	5040.96	31.41
	distance	2088.50	2003.62	-4.06	1721.30	-21.33
15	costs	2885.70	2874.83	**-0.38**	3123.46	**7.61**
	routes	7.00	7.38	5.36	7.00	0.00
	time	3179.67	2997.59	-5.73	4043.07	21.35
	distance	1932.23	1857.34	-3.88	1670.98	-15.63
20	costs	2615.93	2521.33	**-3.62**	2688.64	**2.70**
	routes	5.75	5.25	-8.70	5.63	-2.22
	time	3138.20	3102.27	-1.15	3491.71	10.12
	distance	1847.08	1896.50	2.68	1735.29	-6.44
25	costs	2341.13	2297.78	**-1.85**	2531.00	**7.50**
	routes	4.50	4.63	2.78	4.88	7.69
	time	3051.86	2939.26	-3.69	3369.69	9.43
	distance	1806.00	1690.78	-6.38	1842.67	1.99
30	costs	2232.43	2180.94	**-2.31**	2378.05	**6.12**
	routes	4.13	4.00	-3.03	4.38	5.71
	time	2990.61	2931.88	-1.96	3207.22	6.75
	distance	1735.53	1704.00	-1.82	1832.46	5.29
35	costs	2157.23	2148.42	**-0.41**	2350.51	**8.22**
	routes	4.00	4.00	0.00	4.38	8.57
	time	2859.56	2828.93	-1.07	3111.21	8.09
	distance	1710.12	1717.80	0.45	1856.91	7.91
40	costs	2160.00	2160.51	**0.02**	2294.98	**5.88**
	routes	4.00	4.00	0.00	4.38	8.57
	time	2860.65	2846.39	-0.50	2992.13	4.39
	distance	1720.07	1742.85	1.32	1798.18	4.34

Table 10: Earliest deadline, all time insertion weights

n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
R SET						
5	costs	5661.93	5217.42	**-7.85**	6128.47	**7.61**
	routes	20.00	20.00	0.00	20.00	0.00
	time	4007.99	2931.03	-26.87	5522.37	27.42
	distance	2119.79	1829.28	-13.70	1867.86	-13.49
10	costs	3345.05	3193.67	**-4.53**	3898.26	**14.19**
	routes	10.00	10.00	0.00	10.00	0.00
	time	3017.42	2701.82	-10.46	4804.75	37.20
	distance	1732.45	1559.32	-9.99	1445.85	-19.82
15	costs	2659.52	2584.95	**-2.80**	3053.55	**12.90**
	routes	7.00	7.00	0.00	7.09	1.28
	time	2784.10	2644.73	-5.01	3977.09	30.00
	distance	1563.38	1454.80	-6.95	1404.17	-11.34
20	costs	2354.54	2272.88	**-3.47**	2488.74	**5.39**
	routes	5.36	5.36	0.00	5.00	-7.27
	time	2812.43	2629.99	-6.49	3521.86	20.14
	distance	1532.82	1457.09	-4.94	1348.04	-13.71
25	costs	2141.90	2062.32	**-3.72**	2204.14	**2.82**
	routes	4.27	4.36	2.13	4.45	4.08
	time	2839.79	2587.54	-8.88	3013.38	5.76
	distance	1461.35	1424.33	-2.53	1326.61	-10.16
30	costs	2026.55	1995.50	**-1.53**	2088.40	**2.96**
	routes	4.00	4.00	0.00	4.09	2.22
	time	2670.71	2609.98	-2.27	2850.85	6.32
	distance	1439.42	1398.10	-2.87	1364.76	-5.47
35	costs	1902.73	1892.69	**-0.53**	1956.40	**2.74**
	routes	3.55	3.55	0.00	3.64	2.50
	time	2592.98	2567.55	-0.98	2748.31	5.65
	distance	1392.14	1387.16	-0.36	1319.36	-5.52
40	costs	1844.94	1900.83	**3.03**	1907.50	**3.28**
	routes	3.27	3.64	11.11	3.55	7.69
	time	2603.32	2572.51	-1.18	2659.19	2.10
	distance	1349.92	1342.72	-0.53	1315.98	-2.58
C SET						
5	costs	9005.50	8980.15	**-0.28**	13290.63	**32.24**
	routes	20.13	20.13	0.00	20.00	-0.63
	time	13520.39	13496.16	-0.18	25976.40	47.95
	distance	2160.47	2090.00	-3.26	2051.83	-5.29
10	costs	6142.01	6268.87	**2.07**	9669.15	**36.48**
	routes	10.50	10.63	1.19	10.13	3.70

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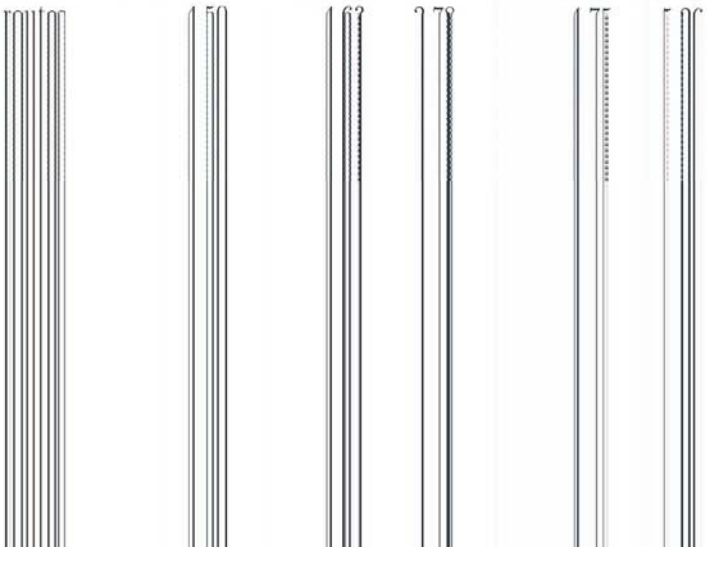
n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
	time	10600.38	10645.31	0.42	17692.01	40.08
	distance	1318.45	1295.01	-1.78	1147.43	-14.90
20	costs	5085.85	5067.48	**-0.36**	6277.61	**18.98**
	routes	6.13	6.13	0.00	5.50	-11.36
	time	10494.44	10461.50	-0.31	14282.72	26.52
	distance	1178.20	1149.42	-2.44	1140.26	-3.33
25	costs	5015.20	5018.02	**0.06**	5438.77	**7.79**
	routes	5.63	5.75	2.22	4.75	-18.42
	time	10630.98	10542.40	-0.83	12373.92	14.09
	distance	1077.38	1119.69	3.93	1000.11	-7.73
30	costs	4803.66	4819.55	**0.33**	5076.45	**5.37**
	routes	4.38	4.50	2.86	4.25	-2.94
	time	10696.02	10675.59	-0.19	11521.23	7.16
	distance	1080.41	1078.36	-0.19	1118.35	3.39
35	costs	4268.90	4243.73	**-0.59**	4393.77	**2.84**
	routes	3.38	3.25	-3.70	3.63	6.90
	time	9884.13	9885.41	0.01	10112.29	2.26
	distance	812.47	803.60	-1.09	806.19	-0.78
40	costs	4268.90	4243.73	**-0.59**	4393.77	**2.84**
	routes	3.38	3.25	-3.70	3.63	6.90
	time	9884.13	9885.41	0.01	10112.29	2.26
	distance	812.47	803.60	-1.09	806.19	-0.78
RC SET						
5	costs	5977.89	5538.58	**-7.35**	6605.25	**9.50**
	routes	20.13	20.13	0.00	20.00	-0.63
	time	4506.14	3455.39	-23.32	6563.12	31.34
	distance	2616.09	2309.23	-11.73	2345.33	-11.54
10	costs	3630.71	3492.66	**-3.80**	4046.57	**10.28**
	routes	10.25	10.25	0.00	10.00	-2.50
	time	3457.73	3119.59	-9.78	5040.96	31.41
	distance	2088.50	2003.62	-4.06	1721.30	-21.33
15	costs	2885.70	2874.83	**-0.38**	3122.66	**7.59**
	routes	7.00	7.38	5.36	7.00	0.00
	time	3179.67	2997.59	-5.73	4053.52	21.56
	distance	1932.23	1857.34	-3.88	1652.48	-16.93
20	costs	2601.42	2504.45	**-3.73**	2651.67	**1.90**
	routes	5.50	5.13	-6.82	5.38	-2.33
	time	3264.46	3147.03	-3.60	3567.26	8.49
	distance	1799.27	1859.13	3.33	1667.48	-7.90
25	costs	2334.55	2297.78	**-1.58**	2412.98	**3.25**



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n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
	routes	4.13	4.00	-3.03	4.25	2.94
	time	2990.61	2931.88	-1.96	3128.56	4.41
	distance	1735.53	1704.00	-1.82	1736.37	0.05
35	costs	2147.88	2125.03	**-1.06**	2211.65	**2.88**
	routes	4.00	4.00	0.00	4.13	3.03
	time	2857.69	2804.37	-1.87	2967.01	3.68
	distance	1673.86	1655.96	-1.07	1683.17	0.55
40	costs	2141.70	2137.49	**-0.20**	2189.83	**2.20**
	routes	3.88	3.88	0.00	4.00	3.13
	time	2883.10	2881.69	-0.05	2969.81	2.92
	distance	1709.11	1693.62	-0.91	1686.09	-1.37

Table 11: Best solutions of farthest customer, earliest deadline and all time insertion weights

n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
R SET						
5	costs	5661.76	5196.32	**-8.22**	5918.94	**4.35**
	routes	20.00	20.00	0.00	20.00	0.00
	time	4037.52	2891.19	-28.39	4943.71	18.33
	distance	2076.26	1799.11	-13.35	1833.87	-13.22
10	costs	3345.05	3178.25	**-4.99**	3775.12	**11.39**
	routes	10.00	10.00	0.00	10.00	0.00
	time	3017.42	2675.68	-11.33	4401.90	31.45
	distance	1732.45	1532.99	-11.51	1516.89	-14.21
15	costs	2659.52	2568.46	**-3.42**	2970.02	**10.45**
	routes	7.00	7.00	0.00	7.00	0.00
	time	2784.10	2617.85	-5.97	3725.13	25.26
	distance	1563.38	1425.04	-8.85	1492.64	-4.74
20	costs	2345.22	2234.91	**-4.70**	2438.03	**3.81**
	routes	5.27	5.09	-3.45	5.18	-1.75
	time	2869.94	2670.56	-6.95	3241.23	11.46
	distance	1481.78	1453.62	-1.90	1401.29	-5.74
25	costs	2095.24	2024.14	**-3.39**	2178.05	**3.80**
	routes	4.09	4.18	2.22	4.36	6.25
	time	2824.06	2586.14	-8.42	2972.67	5.00
	distance	1432.12	1409.65	-1.57	1348.11	-6.23
30	costs	2019.28	1973.23	**-2.28**	2076.77	**2.77**
	routes	4.00	4.00	0.00	4.09	2.22
	time	2666.41	2546.25	-4.51	2800.94	4.80
	distance	1415.38	1397.71	-1.25	1388.67	-1.92

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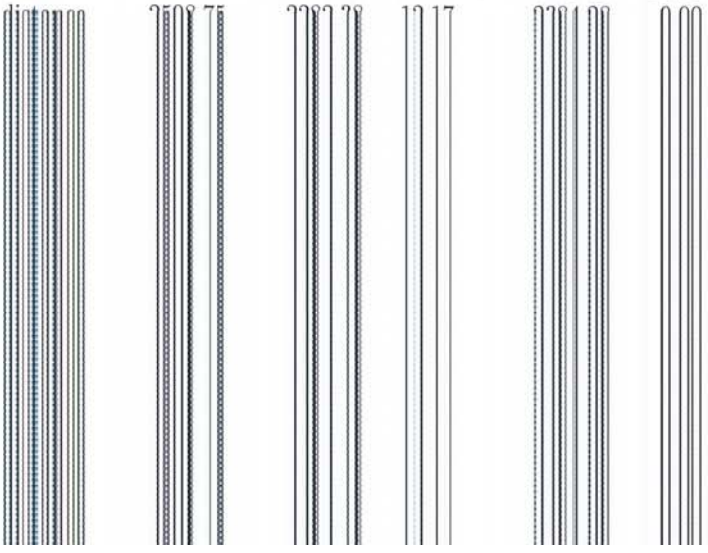
n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
	routes	3.18	3.36	5.71	3.36	5.41
	time	2548.64	2554.76	0.24	2663.76	4.32
	distance	1370.33	1397.33	1.97	1315.48	-4.17
C SET						
5	costs	8977.49	8236.75	**-8.25**	11030.41	**18.61**
	routes	20.13	20.13	0.00	20.75	3.01
	time	13393.15	11483.11	-14.26	18709.29	28.41
	distance	2228.26	1911.41	-14.22	2584.30	13.78
10	costs	6142.01	6098.65	**-0.71**	6785.66	**9.49**
	routes	10.50	10.63	1.19	11.00	4.55
	time	10931.23	10790.97	-1.28	12157.33	10.09
	distance	1519.92	1444.76	-4.94	2032.46	25.22
15	costs	5461.74	5448.26	**-0.25**	5981.74	**8.69**
	routes	7.75	7.63	-1.61	8.38	7.46
	time	10600.38	10645.31	0.42	11439.98	7.34
	distance	1318.45	1295.01	-1.78	1778.32	25.86
20	costs	5085.85	5039.46	**-0.91**	5618.21	**9.48**
	routes	6.13	6.00	-2.04	6.88	10.91
	time	10494.44	10457.97	-0.35	11187.85	6.20
	distance	1178.20	1135.69	-3.61	1803.70	34.68
25	costs	5003.85	4995.57	**-0.17**	5205.13	**3.87**
	routes	5.50	5.63	2.27	5.50	0.00
	time	10655.03	10543.83	-1.04	11122.14	4.20
	distance	1093.08	1121.95	2.64	1254.44	12.86
30	costs	4773.22	4819.55	**0.97**	4902.51	**2.64**
	routes	4.63	4.50	-2.70	4.75	2.63
	time	10394.19	10675.59	2.71	10735.94	3.18
	distance	1195.46	1078.36	-9.80	1140.76	-4.80
35	costs	4232.16	4219.46	**-0.30**	4265.65	**0.78**
	routes	3.38	3.25	-3.70	3.38	0.00
	time	9824.58	9850.66	0.27	9894.81	0.71
	distance	745.74	752.87	0.96	783.43	4.81
40	costs	4232.16	4219.46	**-0.30**	4265.65	**0.78**
	routes	3.38	3.25	-3.70	3.38	0.00
	time	9824.58	9850.66	0.27	9894.81	0.71
	distance	745.74	752.87	0.96	783.43	4.81
RC SET						
5	costs	5935.07	5495.05	**-7.41**	6166.67	**3.76**
	routes	20.13	20.00	-0.62	20.00	-0.63
	time	4395.04	3416.31	-22.27	5275.98	16.70



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n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
	distance	2088.50	1915.43	-8.29	2003.47	-4.24
15	costs	2885.70	2789.50	**-3.33**	3076.98	**6.22**
	routes	7.00	7.00	0.00	7.00	0.00
	time	3179.67	3017.50	-5.10	3861.49	17.66
	distance	1932.23	1766.55	-8.57	1740.57	-11.01
20	costs	2532.56	2463.68	**-2.72**	2578.21	**1.77**
	routes	5.38	5.13	-4.65	5.25	-2.38
	time	3226.59	3086.38	-4.35	3546.52	9.02
	distance	1665.15	1777.20	6.73	1489.33	-11.81
25	costs	2277.93	2291.35	**0.59**	2349.49	**3.05**
	routes	4.38	4.63	5.71	4.50	2.78
	time	3023.17	2921.01	-3.38	3280.78	7.85
	distance	1682.14	1690.43	0.49	1508.92	-11.48
30	costs	2205.60	2158.17	**-2.15**	2262.04	**2.50**
	routes	4.13	4.00	-3.03	4.25	2.94
	time	2956.87	2889.14	-2.29	3080.41	4.01
	distance	1672.66	1671.12	-0.09	1630.80	-2.57
35	costs	2108.20	2119.56	**0.54**	2176.29	**3.13**
	routes	3.88	4.00	3.23	4.00	3.13
	time	2826.56	2809.57	-0.60	2935.88	3.72
	distance	1651.54	1625.61	-1.57	1678.87	1.63
40	costs	2097.85	2101.45	**0.17**	2148.75	**2.37**
	routes	3.88	3.88	0.00	4.00	3.13
	time	2809.82	2827.66	0.64	2900.50	3.13
	distance	1632.68	1621.79	-0.67	1615.40	-1.07

Table 12: Best solutions of farthest customer, earliest deadline and all time insertion weights (short run costs)

n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
R SET						
5	costs	1903.36	1437.92	-24.45	2160.54	11.90
	routes	20.00	20.00	0.00	20.00	0.00
	time	4037.52	2891.19	-28.39	4943.71	18.33
	distance	2076.26	1799.11	-13.35	1833.87	-13.22
10	costs	1451.96	1299.05	-10.53	1895.92	23.42
	routes	10.09	10.00	-0.90	10.00	-0.91
	time	2983.42	2675.68	-10.31	4401.90	32.22
	distance	1723.88	1532.99	-11.07	1516.89	-13.65
15	costs	1344.08	1249.15	-7.06	1622.13	17.14
	routes	7.00	7.09	1.30	7.36	4.94

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n	$c_{12}(i,u,j)$	maxPF	PBmaxPF	%Δ	PBactualPF	%Δ
25	costs	1285.99	1216.18	-5.43	1328.71	3.22
	routes	4.73	4.64	-1.92	4.82	1.89
	time	2704.30	2533.51	-6.32	2890.47	6.44
	distance	1437.07	1393.82	-3.01	1345.12	-6.84
30	costs	1261.77	1212.21	-3.93	1296.53	2.68
	routes	4.18	4.18	0.00	4.36	4.17
	time	2667.08	2533.04	-5.03	2738.37	2.60
	distance	1390.10	1377.96	-0.87	1431.55	2.90
35	costs	1196.93	1182.61	-1.20	1231.91	2.84
	routes	3.82	3.91	2.38	4.00	4.55
	time	2528.64	2483.66	-1.78	2643.81	4.36
	distance	1320.70	1326.25	0.42	1299.42	-1.64
40	costs	1203.55	1194.15	-0.78	1220.65	1.40
	routes	3.64	3.73	2.50	3.82	4.76
	time	2536.79	2494.63	-1.66	2613.50	2.94
	distance	1336.46	1358.41	1.64	1296.46	-3.08
C SET						
5	costs	5195.60	4448.71	-14.38	7131.07	27.14
	routes	20.13	20.25	0.62	20.75	3.01
	time	13393.15	11464.02	-14.40	18709.29	28.41
	distance	2228.26	1913.46	-14.13	2584.30	13.78
10	costs	4159.47	4101.44	-1.40	4707.33	11.64
	routes	10.63	10.75	1.18	11.25	5.56
	time	10897.65	10785.60	-1.03	12122.43	10.10
	distance	1529.52	1450.21	-5.19	2036.34	24.89
15	costs	3984.20	4000.01	0.40	4407.91	9.61
	routes	7.88	7.75	-1.59	8.38	5.97
	time	10542.14	10595.55	0.51	11439.98	7.85
	distance	1314.73	1303.14	-0.88	1778.32	26.07
20	costs	3934.84	3911.94	-0.58	4310.18	8.71
	routes	6.13	6.00	-2.04	7.00	12.50
	time	10494.44	10457.97	-0.35	11168.67	6.04
	distance	1178.20	1135.69	-3.61	1764.50	33.23
25	costs	3935.00	3928.15	-0.17	4171.57	5.67
	routes	5.75	5.75	0.00	5.50	-4.55
	time	10558.01	10512.73	-0.43	11122.14	5.07
	distance	1086.73	1123.83	3.41	1254.44	13.37
30	costs	3904.09	3961.98	1.48	4006.25	2.55
	routes	4.63	4.88	5.41	4.88	5.13
	time	10394.19	10711.12	3.05	10714.68	2.99
	distance	1195.46	977.14	-18.26	1156.40	-3.38