DEPARTEMENT BEDRIJFSECONOMIE

NETWORK PRICING IN ELECTRICITY

by

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INTRODUCTION

Network pricing is an important regulatory issue in the
European electricity industry, especially since the industry
is staged for gradual unbundling of production and
transmission. Consequently, power and transmission will have
to be priced separately. This paper discusses pricing rules
with special reference to electricity networks.

Section I reviews the basic model and some implementation
problems of regulatory pricing in a multi-product firm. In
section II access pricing is discussed. In section III a
specific model, including main features of electricity
production and transmission, is presented and analyzed.
Finally, practical implementation issues for an integrated as
well as unbundled industry are derived.

I. Regulatory pricing in a multi-product firm[1]

Electric utilities are multi-product firms as they generate power at different times of the day and for residential as well as industrial customers. There are two kinds of issues in regulating multi-product firms considered in modern regulatory economic theory i.e. pricing and incentives. Pricing is the subject of traditional regulatory theory, focusing on proper price discrimination and proper pricing in case of competitors or industry links. "New" regulatory economics also includes the analysis of asymmetry of information between regulator and the regulated industry, and consequently, focuses on incentive mechanisms to correct for this asymmetry and moral hazard. In this paper we consider primarily the problem of proper price discrimination.

1. Generalized Boiteux-Ramsey rule

In regulation four parties are involved: the regulator, taxpayers, consumers and the firm. The regulator is assumed to be "benevolent", maximizing total surplus in society. Consumers of the regulated firm's output derive value of use of the firm's output in return for part of their income. The firm - i.e. its shareholders or other residual claimants - retain profits. Finally, in case subsidies are required to sustain the financial viability of the firm, taxpayers carry the burden of difference between revenues and cost plus profit. In the case of taxed profits, taxpayers enjoy tax revenue.

If q is a vector of outputs of a regulated multi-product firm, total surplus is as follows.

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[1] Cfr Laffont & Tirole (1994) Chapter 3

Consumers of the firm's outputs enjoy a surplus equal to

$$S(q) - R(q)$$

with $S(q)$ gross consumer value of use of outputs and $R(q) = \Sigma p_i . q_i$ or revenue. Note that $p_i = \delta S(q)/\delta q_i = S_i(q)$ or the inverse demand curve for output i.

If the firm is left a rent t, then taxpayers have to foot the following bill

$$C(q) + t - R(q)$$

Money spent by the government is raised by distortionary taxes on labor, capital or consumption. A dollar raised costs the economy $1+\lambda$ with λ the shadow cost of public funds $(\lambda > 0)$[2]. Consequently the social value of the taxpayers' bill equals

$$(1+\lambda)[C(q) + t - R(q)]$$

Hence, the regulator maximizes total welfare W equal to

$$W = S(q) - R(q) + t - (1+\lambda)[C(q) + t - R(q)]$$

$$= S(q) - C(q) + \lambda[R(q) - C(q) - t]$$

Note that in the above formulation the shadow cost of public funds λ is exogenous and that rent of the firm is endogenous.

If transfers from the regulator to the regulatee are unacceptable, an alternative approach is to preset the rent of

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[2] The shadow cost of public funds is an economy-wide parameter and depends on tax rates, elasticities of demand and supply for consumption and factors of production. A typical estimate for the US is $\lambda = 0.3$ (Ballard, Shoven & Walley 1985; Hausman & Poterba 1987). The shadow cost of public funds is likely to be higher in EU countries since tax rates are higher.

the firm. In most cases revenue reconciliation is the norm and the rent is targeted to be zero. In this case λ is endogenous and equals the shadow cost of profit or total surplus forgone for an extra dollar of rent left to the firm. In this case the regulator maximizes social surplus or the difference between consumer value and resource costs, subject to the profit constraint. This yields the same objective function as the one formulated above, but with λ a variable rather than a preset value.

First order conditions for an optimum are

$$\frac{\delta W}{\delta q_i} = 0 \equiv S_{q_i} - C_{q_i} + \lambda \left(p_i + \sum_j q_j \cdot \frac{\delta p_j}{\delta q_i} - C_{q_i} \right)$$

from which follows the generalized Ramsey-Boiteux pricing rule viz.

$$\frac{p_i - C_{q_i}}{p_i} = -\frac{\lambda}{1+\lambda} \cdot \sum_j \frac{q_j}{p_i} \cdot \frac{\delta p_j}{\delta q_i} = -\frac{\lambda}{1+\lambda} \cdot \frac{1}{\eta_i}$$

The generalized pricing rule states that the Lerner index of good i (price minus marginal cost as a fraction of price) equals the Ramsey number $\lambda/(1+\lambda)$ times the (negative) inverse of the "super-elasticity" η of good i.

The "super-elasticity" of good i equals (Brown and Sibley 1985)

$$\eta_i = \cfrac{1}{\sum_j \cfrac{r_{ji}}{\epsilon_{ij}}}$$

with $r_{ji} = (p_j q_j)/(p_i q_i)$ and $\epsilon_{ij} = (\delta q_i/q_i)/(\delta p_j/p_j)$ or the cross-price elasticities of i. A "super-elasticity" does not only consider the own quantity reactions of consumers in "taxing" a specific commodity to generate revenue, but also includes the quantity reactions for substitutes or complements to determine the proper Lerner-index.

The above rule includes several well-known rules such as the standard "inverse elasticity" rule, or Boiteux-Steiner "peak-load" pricing.

The above generalized Ramsey-Boiteux pricing rule is equivalent to the familiar "inverse elasticity" rule if demand functions are independent. In this case "super-elasticity" reduces to the own price elasticity of demand (or $\eta_i = \epsilon_{ii}$).

Peak-load pricing or time dependent prices can equally be formulated in terms of the generalized rule. If demand functions for specific time periods are specified, and assuming time independent demand functions, the generalized Boiteux-Ramsey rule also covers peak-load pricing or spot pricing. In this case, the generalized rule reduces to

$$\frac{p_{it} - mc_{it}}{p_{it}} = -\frac{\lambda}{1+\lambda} \cdot \frac{1}{\epsilon_{iit}}$$

where prices, marginal costs and elasticities are time dependent.

Example. Two-product firm

Suppose a regulated firm produces two products (e.g. off-peak and on-peak electricity or commercial and household electricity demand) with linear inverse demand functions

$$p_1=30-q_1-0.25q_2 \qquad \text{and} \qquad p_2=20-0.25q_1-0.5q_2$$

Inverse demand functions express marginal willingness to pay by consumers for given levels of quantities available. Note that inverse demand function are symmetric in cross-quantity effects or $\delta q_1/\delta p_2 = \delta q_2/\delta p_1$.

Total consumer valuation in this case is

$$S(q_1,q_2)=30q_1+20q_2-0.5(q_1{}^2+0.5q_2{}^2+0.5q_1q_2)$$

since $\delta S/\delta q_1 = p_1$ and $\delta S/\delta q_2 = p_2$.

Product 1 has total cost $C_1=25+10q_1$ and product 2 costs $C_2=10+5q_2$.

Given that this firm has decreasing average costs, it would not cover fixed costs under first-best marginal cost pricing.

If the regulator fixes prices so that total surplus is maximized but subject to revenue reconciliation his appropriate objective function is

$$W=S(q_1,q_2)-C_1-C_2+\lambda(p_1q_1+p_2q_2-C_1-C_2)$$

Solving first-order conditions for outputs and λ yields optimal values
$$q_1=13.44$$
$$q_2=21.50$$
Corresponding prices are
$$p_1=11.18$$
$$p_2=5.89$$

Lerner indices i.e. the excess of price over marginal cost in terms of price are
$$L_1=0.1058$$
$$L_2=0.1508.$$
The price-cost margin on product 2 is higher than on 1 as the super-elasticity of 2 turns out to be -0.42 (own price elasticity -0.55) compared to -0.59 (own price elasticity -0.83) of product 1, and the most elastic product is "taxed" least to generate revenue.

In this case total surplus equals 278.18 and the firm's revenue equals total costs or 276.92.

2. Implementation issues

- Principal problems

Real world implementation of Ramsey-Boiteux pricing is far more complex than solving an example.

First, the model assumes a benevolent and independent regulator. In practice, independence and benevolence of regulators is not guaranteed. Regulators are rent-seekers, influenced by long-run direct rewards (Chicago view, Kahn (1971), Stigler (1971)) or indirect rewards (Virginian school, Buchanan (1972). It is not evident to design appropriate institutions to constrain rent-seeking discretion of regulators to realign their conduct in the public interest.

Second, optimal pricing also assumes that the regulator has a vast knowledge on costs and demand. In reality, asymmetry of information is typical of the regulator-regulatee setting. The regulator's main source of information is the regulatee, whose profit incentive will induce him to present information selectively. Regulatory capture, arising when control of information is complete so that the regulator acts in line with the interests of the regulatee, is an extreme case (Stigler 1971). In practice, institutional provisions are made to weaken the regulatee's informational grip by requiring independent cost audits, yardstick competition (Schleifer 1985), etc.

Third, Ramsey-Boiteux pricing assumes exogenous cost functions, implying that the managers and employees of the regulated firm have no effect on costs. This means that the regulated firm buys input prices at competitive prices and select the cost minimizing input combination. However, a practical system of rate setting is based on expected costs, consisting of operating and capital cost. Regulated firms will tend to inflate costs and the relative importance given to

operating and capital costs in rate making will not escape the regulatee's attention. The Averch-Johnson (1962) model shows how rate-of-return regulation induces the regulated firm to extend the rate base and to select excessive capital-labor ratios. Rate-of-return regulation in electricity e.g. would induce the industry to resist peak-load pricing, power pooling, and similar policies reducing the need for capacity. On the other hand high standards of reliability and other policies or norms boosting capital expenditure would be stimulated.

Finally, Ramsey-Boiteux pricing implies rigorous price discrimination based on marginal cost and taking into account willingness (ability) to pay. This might conflict with the principle of nondiscrimination among consumers on equity grounds even if consumers have widely different marginal costs (e.g. rural vs urban consumers).

- Implementation mechanisms

Apart from extensive modeling of aspects such as the information asymmetry and regulatee incentives, different mechanisms have been proposed in the literature to induce a regulated firm to approximate Ramsey-Boiteux pricing.

A mechanism proposed by Laffont and Tirole (Laffont & Tirole 1994) is the so-called price tax. In the Laffont and Tirole regulatory environment λ is an economy-wide exogenous cost of public funds. Consider the simple case of a single product firm. A welfare optimum requires maximization of

$$W = S(q)-C(q)+\lambda[pq-C(q)-t]$$

yielding an optimality condition (from $dW/dq=0$) that can be written as

$$(p-mc)(1+\lambda)+\lambda qp'=0$$

with $p'=dp/dq$.

In the Laffont-Tirole proposal the regulated firm is allowed to determine price but a price tax $q°/(1+\lambda)$ is imposed, where $q°$ is an estimate made by the regulator from survey or time series data of the quantity demanded at the optimal price.

The regulated firm determines price by maximizing its profits but subject to the price-tax or

$$\max \quad \pi=pq-C(q)-p[q°/(1+\lambda)]$$

From the first-order condition ($d\pi/dp=0$) it follows that

$$(p-mc)(1+\lambda)+\lambda qp'+(q-q°)p'=0$$

If the estimated quantity at optimal prices is approximately equal to the effective quantity, profit maximization under a price tax regime yields the welfare maximizing price as optimality conditions are identical in that case.

In the case no transfers between regulator and regulatee are allowed and λ is dependent upon a profit constraint and consequently endogenous, a dynamic mechanism, such as suggested by Vogelsang and Finsinger (1979), induces the regulated firm to move, over time, to Ramsey prices and outputs.

The mechanism is simple and is based on historical information readily available to the regulator viz. the price and quantity of each good and total cost in the previous period. Price determination is left to the regulated firm but subject to the following constraint:

$$\Sigma p_t q_{t-1} < C_{t-1}$$

with p_t the vector of current prices and q_{t-1} the vector of quantities in the previous period and C_{t-1} total cost in the previous period. Hence, the regulated firm is allowed to determine prices but the output produced in a previous period valued at current prices should not exceed costs in the previous period.

In determining current prices, the firm maximizes current period profit subject to the above constraint or maximizes the following Lagrangean

$$L = \Sigma p_t q_t - C(q_t) + \mu [C_{t-1} - \Sigma p_t q_{t-1}]$$

First-order conditions $(\delta L / \delta p_t = 0)$ and rewriting $\mu = 1/(1+\lambda)$, leads to the following equality

$$(p_t - mc_t)(1+\lambda) + \lambda q_t p_t' + (q_t - q_{t-1})p_t' = 0$$

with $p_t' = dp_t/dq_t$.

A welfare optimum (see above) requires that

$$(p-mc)(1+\lambda) + \lambda qp' = 0$$

which is reached by this mechanism in steady state ($p_t = p_{t-1} = p$ and $q_t = q_{t-1} = q$)

The common characteristic of the Vogelsang-Finsinger mechanism and the price-tax suggested by Laffont-Tirole is that both mechanisms are negative price incentive schemes. The firm is penalized for increasing its prices. In the Laffont-Tirole case an economy-wide shadow cost of public funds is used, whereas the Vogelsang-Finsinger mechanism uses a firm specific implicit shadow cost.

Strategic misrepresentation of costs is not eliminated by these mechanisms. Overstatement of costs slows the move towards optimum prices. Also, if costs depends on a cost-reducing activity (e.g. negotiating competitive input prices, introducing new technology, etc.) this will be undersupplied. Cost auditing with penalties for misreporting (see Train 1995) or yardstick competition is required to counter these perverse effects.

All regulatory systems must set or constrain prices and use specific institutional designs to accomplish this. In some systems one relies on rate-of-return regulation (e.g. US) whereby prices are fixed so that a "fair" rate-of-return on capital invested is guaranteed. Rate-of-return regulation does not mean (although it is sometimes presented that way) that costs are automatically passed on in prices. Costs are

thoroughly audited and excessive returns are corrected. In other settings (e.g. UK) price caps for broad commodity groups are fixed and pricing of commodities within this group is decentralized. UK practice of "RPI-X" is typical, indexing price caps to the retail price index (RPI) minus some growth factor X, fixed by the regulator, whereby X reflects productivity gains through technological progress or scale economies realized through demand growth. Whatever system is used, all regulators have to cope with the information asymmetry and the almost information monopoly of the regulatee and the incentive problems leading to regulatory failures.

II. Access pricing

Relaxing complete vertical integration in electricity and competition through entry of independent producers poses the problem of pricing access and use of the transmission network operated by the integrated producer.

1. A model of optimal access pricing

This section analyzes this problem in a more general framework viz. the appropriate pricing of an input (e.g. the access charge to use a common network) used and produced by an integrated regulated firm but also used by firms competing in the final product market of the regulated firm[3]. Incentive problems and complications of asymmetric information are left out and a "perfect" (i.e. a perfectly informed and benevolent) regulator is assumed.

Suppose the regulated firm and its competitors produce final commodities in quantities q_1 and q_2 with specific total costs (exclusive of the cost of the common resource) of $C_1(q_1)$ and $C_2(q_2)$. Final products are assumed to be close but not perfect substitutes. Further, assume that each unit of q_1 and each unit of q_2 requires one unit of the common resource produced by the regulated firm so that total production of the common input is $Q=q_1+q_2$ which costs $C_0(Q)$.

Suppose the competitors of the regulated firm are a competitive fringe with free entry and exit. These firms will act as price takers in maximizing profits and adjust output so that price p_2 equals overall marginal cost, being the sum of the marginal cost of production $c_2=\delta C_2/\delta q_2$ and the access (and user) charge p_a. Also, profits for a firm in the competitive

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[3] Crf. Laffont & Tirole (1994). Chapter 5

fringe will be zero so that revenue equals total access charges $(A=p_a \cdot q_2)$ plus production costs C_2.

. What is the regulator's appropriate objective function in this case?

Again, two approaches are possible yielding a formally equivalent objective function. First, transfers to the regulated firm are possible and the shadow cost of public funds is exogenously fixed. Alternatively, no transfers are possible but revenue reconciliation (or some other profit target) is required so that the shadow cost of funds is endogenous.

Suppose transfers are possible and the shadow cost of public funds is fixed.

Consumers enjoy a net surplus equivalent to

$$S(q_1, q_2) - (R_1 + R_2)$$

S being gross consumer value and R expenditures (revenue).

If the regulated firm gets a transfer t, and as the competitive fringe has zero profits, taxpayers also have to finance, in addition to profits, costs of the regulated firm minus its revenues. Tax payers face a bill equal to

$$C_0 + C_1 + t - R_1 - A$$

or

$$C_0 + C_1 + C_2 + t - R_1 - R_2$$

as $A = R_2 - C_2$.

This amount enters the regulator's objective function at

social cost i.e. each dollar augmented with the shadow cost of taxation.

Consequently, the regulator's objective function is:

$$W = S(q_1, q_2) - (R_1 + R_2) + t - (1 + \lambda)(C_0 + C_1 + C_2 + t - R_1 - R_2)$$

or after rearranging

$$W = S(q_1, q_2) - (C_0 + C_1 + C_2) + \lambda[(R_1 + R_2) - (C_0 + C_1 + C_2) - t]$$

If transfers are not allowed but profits of the regulated firm should equal a pre-fixed level t, the regulator's objective is to maximize

$$W = S(q_1, q_2) - (C_0 + C_1 + C_2)$$

subject however to the profit constraint of the regulated firm or

$$R_1 + A - C_0 - C_1 = t$$

but since $A = R_2 - C_2$, this is equivalent to

$$R_1 + R_2 - C_0 - C_1 - C_2 = t$$

The Lagrangean of this objective function is identical to the objective function formulated above.

First, note that the regulator's objective function in case of a common input is identical to the objective function obtained by considering the industry as a single multi-product firm with two outputs, taking into account an additional joint cost component. This leads to the first conclusion, viz. in case of a competitive fringe and a common input, the regulator should set prices for final outputs as if this industry is a multi-

product firm. The generalized Ramsey-Boiteux pricing rule derived in the previous paragraph shows how to determine these prices.

Second, the appropriate access price can be derived. The competitive fringe produces at prices equal to marginal cost inclusive of the access charge or

$$p_2 = c_2 + p_a$$

Also, the proper price for p_2 is such that the Lerner index (L_2) or $(p_2 - mc_2)/p_2$ equals the Ramsey number times the (negative) inverse of the super-elasticity η_2. Note that $mc_2 = c_0 + c_2$ or overall marginal cost of a fringe competitor equals the marginal cost of the common input plus the marginal cost of production. From these conditions, the appropriate value for the access charge p_a can be derived viz.

$$p_a = \frac{c_0 + L_2 c_2}{1 - L_2}$$

As the Lerner index L_2 2 is positive, it follows that the optimal access charge will exceed the marginal cost of access.

Example 2. Common costs and access to rivals

Reconsider example 1 where two products 1 and 2 with inverse demand functions

$$p_1 = 30 - q_1 - 0.25q_2 \qquad \text{and} \qquad p_2 = 20 - 0.25q_1 - 0.5q_2$$

were produced by a single multi-product firm. These products are imperfect substitutes. In example 1 costs were

$$C_1 = 25 + 10q_1 \qquad \text{and} \qquad C_2 = 10 + 5q_2$$

These costs are decomposed as follows. Suppose product 1 is produced by a regulated firm and product 2 by a competitive fringe and that both products require a common input, produced by firm 1. Say that the common input has a constant marginal cost of 2 and a fixed cost of 15. The industry has the following cost structure

$$\text{firm 1: } C_1 = 20 + 8q_1 \qquad \text{and} \qquad C_0 = 15 + 2(q_1 + q_2)$$
$$\text{or } C'_1 = 35 + 10q_1 + 2q_2$$

Firm 2 faces constant average production cost of 3 per unit produced. Also it has to buy a unit of the common input from firm 1 at price p_a (access price). Firm 2's cost are

$$C_2 + aq_2 = (5 + p_a)q_2.$$

Optimal regulation requires that the regulator considers the industry as a multi-product firm. He maximizes social value of output (gross consumer value minus total production cost inclusive the common input) subject to an overall profit constraint or

$$\max W = S(q_1, q_2) - (C_0 + C_1 + C_2) + \lambda(R_1 + R_2 - C_0 - C_1 - C_2)$$

This yields Ramsey-Boiteux prices identical with example 1 as demand and total cost are identical or

$$p_1 = 11.18 \quad \text{and } p_2 = 5.89$$

with Lerner-indices equal to

$$L_1 = 0.1058 \text{ and } L_2 = 0.1508$$

The optimal access prices equals $(c_0 + L_2 c_2)/(1 - L_2)$ or $p_a = 2.89$ which is larger than the marginal cost $c_0 = 2$ and than the average cost $C_0/(q_1 + q_2) = 2.43$ of producing the common input. Actually the Lerner index of the optimal access price is 0.3074.

2. Implementation problems

In principle, the information required to apply the optimal
rule derived above is not readily available. Therefore,
practical and more easily applicable rules are proposed,
maintaining however broad efficiency requirements. A rule
advocated in pricing telephone networks (Baumol & Sidak 1994)
is the so-called efficient component-pricing rule. The rule
states that the optimal price of the common input equals the
sum of marginal cost of production of the common input plus
the opportunity cost to the input supplier of the sale of a
unit of input. The latter component equals foregone profit or
forgone contribution to fixed costs due to enabling a rival to
sell in the final market.

Say a regulated firm and a competitor produce a perfect
substitute, so that inverse demand is $P=P(Q)=P(q_1+q_2)$.

The regulated firm produces a common input, each unit of the
final output q requiring one unit of the common input. Total
costs of the common output, production by the regulated firm
and its competitor are resp. C_0, C_1 and C_2. Total fixed cost of
the regulated firm are equal to F. Note that total fixed costs
include fixed costs of production and of producing the common
input.

Suppose now the regulator fixes - using one or another welfare
criterion - an optimal price of the final output, say P^o. Once
this price is fixed, total market quantity demanded is given
by $Q^o=P^{-1}(P^o)$, where P^{-1} is the inverse function of $P(Q)$.
According to the efficient component-pricing rule the optimal
price for the common input equals

$$p_a = mc_0 - (P^o - mc_0 - mc_1)\frac{dq_1}{dq_2}$$

or the marginal cost of producing the common input plus forgone marginal profit due to substitution in the final market. Note that $p_a = mc_0 + L.P^{\circ}$ or marginal cost of the common input plus the Lerner-index L in the final market times price of the final good. Also, in case of a perfect substitute $dq_1/dq_2 = -1$, so that the optimal input price is simply $p_a = P - mc_1$.

The efficient component-price rule is efficient as it ensures that total production Q° is produced at minimum cost to society at the regulated price P°. Profit functions are

$$\pi_1 = P^{\circ}q_1 + p_a q_2 - C_0(Q^{\circ}) - C_1(q_1) - F$$

and

$$\pi_2 = P^{\circ}q_2 - C_2(q_2) - p_a q_2$$

Profits are maximized if both firms produces an output so that

$$mc_1 = mc_2 = P - p_a$$

implying equality of marginal cost in both firms ensuring production at minimum societal cost.

Furthermore, under this rule, firm 1 is indifferent to providing access to a rival or producing a unit of final output itself. The effect of extra production of q_2 on profits of firm 1 is zero as the loss of profits due to loss of sales of final output is compensated by the price paid for increased sales of the common input.

Example 3. Efficient component pricing

Suppose inverse demand for a final product is P=30-Q. Say the product is produced by a monopoly. To produce a unit of the final output, one unit of a specific input must be produced. The marginal cost of this input is 2 per unit and the additional marginal cost to produce the final output is 8 per unit. Suppose the firm has fixed costs of 40. The monopoly is regulated and the regulator fixes price so that the monopoly earns zero profits. It is easily calculated by equating revenue (PQ) to total costs (10Q+40) and solving for P and Q that $P°=12.254$ and $Q°=17.746$. Total revenue is 217.46 and equals total costs.

Now a competitor enters the industry. He has a more favorable production technique for a small volume of output as his marginal costs are $2q_2$. From a societal point of view, a welfare gain is possible by having a volume up to 4 units produced by this competitor and the remainder by the (former) monopoly.

The regulator coerces the monopoly to yield the competitor access to the common input and fixes a price equal to marginal cost of production of the common input (2) plus the profit contribution of the monopoly's output or 2.254 (price minus marginal cost). Consequently, the price put on a unit of the common input is 4.254.

The new entrant, faced with a price of 12.254 for final output and a cost of 4.254 for the common input, will maximize his profits at a production level of 4. His marginal profit will be zero as marginal cost - including the cost of the common input - will be 8+4.254 or equal to the price of the final output. He will earn a profit of 16.

What is the profit position of the former monopoly? This will remain unchanged. Output will now be less (viz. 13.746 versus 17.746) but revenues from output and from sales of the common input will exactly cover production cost of output, of the common input and fixed cost. However, total revenue (and total cost) will be at a lower level (185.46 vs. 217.46).

In sum, pricing a common input at cost plus opportunity cost due to foregone contribution to fixed cost (or forgone profit) is a rule that is incentive compatible with allocating production at minimum cost. Also, informational requirements to impute this rule are reasonable.

III. Pricing of electricity network use

In previous sections, the models did not explicitly considered the specific characteristics of electricity production, transmission and distribution. Extensive theoretical work on peak-load pricing, pricing under stochastic demand, etc. (see e.g. Bös 1994, Berg & Tschirhart 1988) relevant to electricity pricing and capital budgeting usually ignore network costs and pricing. A particularly useful theory of electricity pricing that reflects the physical properties of electricity was developed by Bohn, Caramanis and Schweppe (1984). The next section is based on this model, omitting however stochastic demand but adding revenue reconciliation.

1. A model of optimal pricing in electrical networks

Models of electricity pricing usually focus on temporal differentiation. However, prices should also be spatially differentiated. Transportation of electricity is not costless and for every unit injected at one end of the network, less than one unit can be removed at another node, the difference being transmission losses. Also, maximum capacities must be observed in transmission lines, also adding to spatially differentiation of cost. Consequently, prices p_{it} should be defined for a specific location i and a specific time spot (spot pricing) or time period (peak-load pricing) t.

Say a utility operates J generating units. Production at time t of a unit located at j is denoted by y_{jt}. Each unit has marginal operating costs c_j (fuel and variable maintenance costs) and capacity Y_j. For simplicity, all units are indexed according to merit order i.e. $c_1 < c_2 < ... < c_J$.

The network serves I customers. Each customer has a demand at time t of x_{it}, depending upon price p_{it} and with a gross value to the consumer of $S_{it}(x_{it})$. Note that $\delta S / \delta x = p$ or the inverse

demand function. For simplicity, demand at i and t is assumed to depend solely on p_{it}.

The electricity network consists of K lines, each line having a maximum safety capacity of Z_k. The flow along a line k at time t are z_{kt} depends upon the net injections or withdrawals at each node, network configuration and line capacities so that

$$z_{kt}=z_{kt}(\mathbf{x},\mathbf{y},\mathbf{a})$$

where $\mathbf{a}$ is a vector of network characteristics. This relationship can be further specified using electrical laws and assuming approximate relationships (see e.g. Bohn et. al. Appendix).

The flows in the network determine total power losses L or $L_t=L(\mathbf{z})$.

Characteristic for an electric system is that an energy balance must be maintained at all times as excess or insufficient generation would almost instantaneously result in breakdown and power failure. Energy balance requires that

$$\Sigma_j y_{jt}=\Sigma_i x_{it}+L_t$$

The regulator's objective is the standard welfare criterion i.e. maximizing total surplus or consumer value minus, however subject to a profit constraint viz.

with F fixed costs of generation and transmission.

This yields the following Lagrangean ($\mathcal{L}$) to be maximized

total surplus $\qquad\qquad\qquad \Sigma_i S_{it}(x_{it})-\Sigma_j c_j y_{jt}$

energy balance constraint $\qquad +\Theta_t(\Sigma_j y_{jt}-\Sigma_i x_{it}-L_t)$

generating unit capacities $\qquad -\Sigma_j\mu_{jt}(y_{jt}-Y_j)$

network link capacities $\qquad -\Sigma_k\eta_k(z_k-Z_k)$

profit constraint $\qquad +\lambda_t(\Sigma_ip_{it}x_{it}-\Sigma_jc_jy_{jt}-F_t-\pi_t)$

The first-order condition for an optimum (and omitting the subscript t for convenience) yields

$$\frac{\delta\mathcal{L}}{\delta x_i} = 0 \equiv p_i-\Theta\frac{\delta L}{\delta x_i}-\Theta-\sum\eta_k\frac{\delta z_k}{\delta x_i}+\lambda[p_i+x_i\frac{\delta p_i}{\delta x_i}] = 0$$

and

$$\frac{\delta\mathcal{L}}{\delta y_j} = 0 \equiv -c_j+\Theta(1-\frac{\delta L}{\delta y_j})-\mu_j-\sum\eta_k\frac{\delta z_k}{\delta y_j}-\lambda c_j = 0$$

After rearranging the following price rule is obtained

$$p_i = \alpha_i[\Theta(1+\frac{\delta L}{\delta x_i})+\sum\eta_k\frac{\delta z_k}{\delta x_i}]$$

with

$$\alpha_i = \frac{1}{1+\lambda(1+\frac{1}{\epsilon_i})}$$

and

$$\Theta = \frac{(1+\lambda)\,c_j + \mu_j + \sum \eta_k \frac{\delta z_k}{\delta y_j}}{1 - \frac{\delta L}{\delta y_j}}$$

The Lagrangean multiplier Θ is a value which is equal to all customers and is interpreted as the shadow cost on demand. The numerator of this expression consists of the marginal cost of production (fuel) of the marginal unit j, valued at social cost (i.e. augmented with the shadow cost of profits). If capacity constrains production, μ is positive and equal to the premium required to curtail demand to capacity of the marginal unit. If network flows are constrained by link capacity, η is positive on these flows so that the summation term expresses the shadow cost of transmission constraints. The denominator augments the numerator since less than one unit produced is available to consumers due to losses generated by the marginal increase in production.

It is easy to see that if there is excess capacity in production ($\mu=0$) and transmission ($\eta=0$) and losses are trivial, the shadow cost on demand is simply the short term marginal cost of production valued at social cost.

This shadow cost on demand (Θ) is the basis of pricing a consumer at location i, as can be seen from the price rule. First, the shadow cost on demand, equal for all customers, is augmented with the cost of losses due the marginal increase in demand at i. Additionally, increased demand at i might cause capacity problems in the network. On those lines η is positive and consumer i has to pay the cost of these constraints. Finally, in order to maintain profits at a specific level, total shadow cost of demand on point i is augmented by a factor, taking into account the inverse of elasticity of demand i.

It is easily verified that in the absence of losses and network constraints, the price rule simply reduces to Ramsey-Boiteux prices, with Lerner indices proportional to the inverse elasticity of demand.

Another interesting conclusion is that, in the absence of line capacity constraints ($\eta=0$), optimal price at point i is approximately linear in line flows. Say a point i is connected with a line k. Using a DC network as an approximation, network losses are a quadratic function of flows. Hence, $\delta L/\delta x_i \approx 2R_k.z_k$ with R resistance on line k linking customer i with the network. Optimal price is then

$$p_i = \alpha_i \Theta (1+2R_k z_k)$$

so that price at point i is a linear function of the flow on the line connecting customer i with the line.

2. Implementation problems

- Integrated utility

The above model assumes a fully integrated regulated utility. Practical implementation of the above model in an integrated utility would require some simplifications to adapt it to available information and reduce its dimensions to make it computational. Also, the model assumes full spot pricing but in a practical setting periodical time-of-day pricing seems a more plausible arrangement. Prices could be recalculated periodically and announced for the next period. This would guarantee customers some price stability, avoiding unduly transaction costs and allowing customers to take into account price information in long range decision.

Apart from demand elasticities, most other information required to solve the model, such as network characteristics, short-run production costs, etc. should be available to a utility. Demand information could be introduced for broad categories of consumers.

- Unbundling and competition

The model also provides useful insights to price the use of transmission services in a partially integrated industry or in case of complete unbundling. Consider the following drastic simplification of the model, whereby capacity constraints are reflected in costs and losses. V_i is gross consumer value of demand x_i at location i and C_j is total cost of production y_j at location j. Losses L are a function of demand $\mathbf{x}$ and production $\mathbf{y}$. A (first best) social optimum and maintaining energy balance requires optimization of the following Lagrangean

$$\mathcal{L} = \sum S_i(x_i) - \sum C_j(y_j) + \Theta \left[\sum y_j - L - \sum x_i \right]$$

Apart from the energy balance constraints, first-order conditions require that

$$p_i = \Theta + \Theta \frac{\delta L}{\delta x_i}$$

and

$$c_j = \Theta - \Theta \frac{\delta L}{\delta y_j}$$

Again Θ may be construed as a system wide cost of electricity or the cost of electricity at a "swing bus". Consumer i should buy electricity up to the point where the price he is willing to pay equals the system wide cost augmented with the value of increased (or decreased) losses at the margin. Producer j should produce electricity up to the point where the marginal cost of production equals the system wide cost less the value of decreased (or increased) losses at the margin. Combining both equations yield

$$p_i = c_j \frac{1 + \dfrac{\delta L}{\delta x_i}}{1 - \dfrac{\delta L}{\delta y_j}} = c_j(1 + \tau_{ij})$$

with

$$\tau_{ij} \approx \frac{\delta L}{\delta x_i} - \frac{\delta L}{\delta y_j}$$

This latter term, τ, may be interpreted as a cost of transportation between producer j and consumer i.

Suppose the industry would consist of competitive producers and a network operator. A network operator can ensure a first best optimum by charging a price for use of his network equal to the term τ. He could establish this by fixing a swing bus or reference point and charging consumers and producers the value of the marginal contribution to losses.

It can be shown (for a DC network) that nature allocates flows by minimizing losses. Losses increase linearly with resistance (approximated with distance) and are quadratic in flows. Consequently, the values of the terms $\delta L/\delta x$ and $\delta L/\delta y$ can be found in a practical way by solving the following quadratic programming problem

minimize $\qquad L = \gamma \Sigma_i \Sigma_j d_{ij} z_{ij}^2$

subject to

$$\Sigma_i z_{ij} = y_j \qquad \forall j$$

$$\Sigma_j z_{ij} = x_i \qquad \forall i$$

where z_{ij} is the flow between nodes i and j, d_{ij} the distance, x_i demand in node i, y_j supply in node j and γ a scaling factor converting the value of the objective function to the total value of losses of the system. The dual values of the constraints are the resulting losses. This model could be extended with capacity constraints on links if applicable e.g.

$$z_{ij} \leq Z_{ij} \qquad \forall ij$$

If these constraints are binding, dual values could be added to the cost of transmission use.

This quadratic programming problem could be solved on a

regular basis to determine marginal cost of network use, informing consumers and producers of the location advantages or disadvantages of their particular position in the network.

In a second-best optimization, the network operator could achieve revenue reconciliation by using Ramsey pricing equating the product of the Lerner-index and price elasticity in all locations.

In a "partially" unbundled industry i.e. an industry with a regulated vertically integrated producer/network operator and competitive producers, the information on the marginal costs of network use should be used in combination with the efficient component pricing rule. The proper price of network use would be equal to the cost of use (component τ) plus the opportunity cost or forgone contribution to fixed (production) costs.

Example 4. Network pricing (unbundled environment)

Say a network of 5 nodes. Nodes 1 to 3 are net producers (Y). Nodes 4 and 5 have net demand (X). Production, demand and distances are given in the table below. E.g. production in node 2 is 1000. The distance from production node 3 to demand node 5 is 100.

node		4	5	
		X1	X2	output
1	Y1	50	100	250
2	Y2	25	80	1000
3	Y3	20	100	750
	demand	1200	800	2000

In this example capacity on all lines is assumed to be very large. Solving the following quadratic programming problem

$$\min \quad \Sigma\Sigma d_{ij} z_{ij}^2$$

subject to $\Sigma_j z_{ij} = Y_i$

$$\Sigma_i z_{ij} = X_j$$

yields a value for the objective function equal to $0.356(10_3)$ and the optimal flow values given in the following table.

	X1	X2	dual	mc_i
Y1	71	179	-24175	-1.358
Y2	625	375	0	0
Y3	505	245	-11036	-0.620
dual	31230	60065		
mc_j	+1.754	3.374		

E.g. Flows from production node Y2 to demand node X1 are 625 and to demand node X2 375, the sum being equal to total production at node Y2 viz. 1000.

The table also lists the dual values of the restrictions. E.g. an additional unit of production in node 3 yields a reduction in the objective function of 11036 units (distance*flow^2 which in a DC network is proportional to losses).

The units of the objective function can be converted into monetary values. Say that total losses in monetary units are valued at 2000. The unit of measure of the objective function $(d.z^2)$ is easily converted in monetary units by multiplying

with 2000/(0.356*10^8) or 5.618(10^{-5}). Hence, converting the dual values of the constraints yields the marginal cost (or gain) of an extra unit demand (or production).

Note that in this example production node 2 operates as a "swing bus" or reference point. Spatially differentiated prices are calculated on the basis of these marginal cost estimates.

IV. Conclusions

Although the real world is far more complex than the theoretical environment envisaged in regulatory theory, optimal pricing theory offers some guidelines in developing practical pricing principles in alternative industry settings.

In a vertically integrated electricity industry with, a monopoly covering distribution, production and transmission, the generalized Boiteux-Ramsey rule - i.e. price discrimination so that the Lerner index of each good times its "super-elasticity" is equal for all goods - discussed in section I, is a fundamental guideline for establishing efficient prices. Although incentive and information problems (such as non-benevolent regulators, asymmetry of information between regulator and regulatee, non-cost minimizing regulatee behavior, etc.) may hamper full implementation, institutional designs such as negative price incentive schemes and price capping, may over time yield sufficiently approximate solutions.

A more specific model discussed in section III, explicitly taking into account the specific characteristics of electricity production, transmission and distribution (e.g. capacity constraints, maintenance of energy balance at all time and losses due to transmission), yields even more precise pricing guidelines for a fully integrated regulated monopoly. Ramsey-Boiteux prices should not only be temporally differentiated, but also spatially, with prices depending upon the (system) cost of electricity but also upon the value of marginal losses and the shadow cost of net capacity constraints.

The models discussed in section II are relevant to an electricity industry with incomplete vertical integration. In this setting, apart from independent electricity producers, a regulated firm not only produces electricity but also acts as

grid operator for the industry and hence producing transmission services as a common input. First, the model discussed shows that the regulator should consider the industry as a multi-product firm and consequently apply Boiteux-Ramsey prices for all outputs and firms operating in the industry. Second, the model shows that independent producers should get access to the transmission system but at a price that exceeds the marginal cost of net use.

An operational rule used in telephone network pricing, the so-called efficient component rule, which is compatible with a regulatory strategy of price capping of final outputs, puts the difference between proper access price and marginal cost of net use at the opportunity cost of the foregone contribution to fixed costs. In case production capacity of the vertically integrated firm is optimal relative to demand, enabling new rivals access to the final market through the use of the common transmission net, leads to surplus production capacity, stranded investment and hence, non-recoverable fixed cost. This foregone contribution to fixed cost, due to missed sales, should be added to marginal cost of net use.

Finally, section III also offers some guidelines to analyze optimal network pricing in a fully unbundled industry. The basic idea of unbundling production and transmission, is that "workable competition" in production is feasible and, consequently can be deregulated, whereas transmission should be monopolized and subject to regulation in view of economies of scale and scope. In a competitive production market for electricity, transmission costs keep prices consumers are willing to pay (in the absence of consumer monopsony power) from equalizing marginal costs of production (in the absence of producer monopoly power). An optimal allocation of resources requires minimization of transmission costs. A quadratic programming problem - minimizing the sum of link distance times link flow squared - is suggested as a possible approach of finding the marginal cost of use of transmission

at a specific node in the network, taking into account losses as well as shadow costs of link capacity constraints. Such quadratic programming problem yields flows in a DC approximation of the network and dual values associated with constraints yield proper estimates of marginal costs of net use. It should be noted that such model studies marginal transformations and yields cost effects of increases or decreases of demand or production in specific nodes at the margin. Also, link capacities are assumed given and close to an optimal design. Large variations in node net demand or production (e.g. important new producers or consumers entering the market) should be considered as structural transformations, requiring perhaps re-optimation of net structure and incremental cost analysis rather than marginal cost estimates.

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Antwerp, October 1996

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